Conjectured Primality Criteria for Specific Classes of $k \cdot b^n - 1$

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Abstract: Conjectured polynomial time primality tests for specific classes of numbers of the form $k \cdot b^n - 1$ are introduced.

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1 Introduction

In 1969 Hans Riesel provided polynomial time primality test for numbers of the form $3 \cdot 2^n - 1$ with $n > 2$, see Theorem 5 in [1]. In this note I present polynomial time primality tests for specific classes of numbers of the form $k \cdot b^n - 1$ that may be considered as generalization of the Riesel primality test for $3 \cdot 2^n - 1$.

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left( (x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where $m$ and $x$ are nonnegative integers.

Conjecture 2.1. Let $N = k \cdot b^n - 1$ such that $n > 2$ and

\[
\begin{align*}
&k \equiv 3 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4} \\
&k \equiv 3 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\
&k \equiv 3 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\
&k \equiv 3 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4}
\end{align*}
\]

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(5778))$, then

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.2. Let $N = k \cdot b^n - 1$ such that $n > 2$ and
\[
\begin{aligned}
  &k \equiv 9 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 1 \pmod{4} \\
  &k \equiv 9 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod{4} \\
  &k \equiv 9 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\
  &k \equiv 9 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 3 \pmod{4}
\end{aligned}
\]

Let \( S_i = P_b(S_{i-1}) \) with \( S_0 = P_{b/2}(P_{b/2}(5778)) \), then

\( N \) is prime iff \( S_{n-2} \equiv 0 \pmod{N} \)

References