The notion of mar constants

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I present a notion based on the digital root of a number, namely "mar constant", that highlights the periodicity of some infinite sequences of non-null positive integers (sequences of squares, cubes, triangular numbers, polygonal numbers etc).

Definition:

We understand by "mar constants" the numbers with n digits obtained by concatenation from the values of the digital root of the first n terms of an infinite sequence of non-null positive integers, if the mar values of the terms of such a sequence form themselves a periodic sequence, with a periodicity equal to n. We consider that it is interesting to see, from some well known sequences of positive integers, which one is characterized by a mar constant and which one it isn't.

Example:

The values of the digital root of the terms of the cubic numbers sequence (1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, ...) are 1, 8, 9, 1, 8, 9 (...) so these values form a sequence with a periodicity equal to three, the terms 1, 8, 9 repeating infinitely. Concatenating these three values is obtained a mar constant, i.e. the number 189.

Let's take the following sequences:

(1) The cubic numbers sequence

S_n is the sequence of the cubes of positive integers and, as it can be seen in the example above, is characterized by a mar constant with three digits, the number 189.

(2) The square numbers sequence

S_n is the sequence of the square of positive integers (A000290 in OEIS): 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441 (...) and is characterised by a mar constant with nine digits, the number 149779419.

(3) The triangular numbers sequence

S_n is the sequence of the numbers of the form (n*(n + 1))/2 = 1 + 2 + 3 + ... + n (A000217 in OEIS): 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300 (...) and is characterised by a mar constant with nine digits, the number 136163199.

(4) The centered square numbers sequence

S_n is the sequence of the numbers of the form m = 2*n*(n + 1) + 1 (A001844 in OEIS): 1, 5, 13, 25, 41, 61, 85, 113, 145, 181, 221, 265, 313, 365, 421, 481, 545, 613 (...) and is characterised by a mar constant with nine digits, the number 154757451.

(5) The centered triangular numbers sequence

S_n is the sequence of the numbers of the form m = 3*n*(n + 1)/ 2 + 1 (A005448 in OEIS): 1, 4, 10, 19, 31, 46, 64, 85, 109, 136, 166, 199, 235, 274, 316, 361, 409, 460 (...) and is characterised by a mar constant with three digits, the number 141.

(6) The Devlali numbers sequence

S_n is the sequence of the Devlali numbers (defined by the Indian mathematician D.R. Kaprekar, born in Devlali), which are the numbers that can not be expressed like n + S(n), where n is integer and S(n) is the sum of the digits of n. The sequence of these numbers is (A003052 in OEIS): 1, 3, 5, 7, 9, 20, 31, 42, 53, 64, 75, 86, 97, 108, 110, 121, 132, 143, 154, 165, 176, 187, 198 (...).

This sequence is characterized by a mar constant with 9 digits, the number 135792468.

(7) The Demlo numbers sequence

S_n is the sequence of the Demlo numbers (defined by the Indian mathematician D.R. Kaprekar and named by him after a train station near Bombay), which are the numbers of the form (10^n - 1)/9)^2. The sequence of these numbers is (A002477 in OEIS): 1, 121, 12321, 1234321, 123454321, 12345654321, 1234567654321, 123456787654321, 12345678987654321, 1234567900987654321 (...).

This sequence is characterized by a mar constant with 9 digits, the number 149779419.

Comment:

I conjecture that any sequence of polygonal numbers, *i.e.* numbers with generic formula ((k^2*(n – 2) – k*(n – 4))/2, is characterized by a mar constant:

: The sequence of pentagonal numbers, numbers of the form n*(3*n – 1)/2, *i.e.* 1, 5, 12, 22, 35, 51, 70, 92, 117, 145, 176, 210, 247, 287, 330, 376, 425, 477, 532, 590, ...(A000326) is characterized by the mar constant 153486729;
: The sequence of hexagonal numbers, numbers of the form n*(2*n – 1), *i.e.* 1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, 325, 378, 435, 496, 561, 630, 703, 780, ...(A000326) is characterized by the mar constant 166193139 etc.

Conclusion:

We found so far eight mar constants, six with nine digits, *i.e.* the numbers 149779419, 136163199, 154757451, 135792468, 153486729, 166193139 and two with three digits, *i.e.* the numbers 189 and 141.