Electro-Magnetic Field Equation and Lorentz gauge, Wave Function, Wave Equation in Rindler spacetime

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ABSTRACT

In the general relativity theory, we find the electro-magnetic field transformation and the electro-magnetic field equation (Maxwell equation) in Rindler spacetime. We treat Lorentz gauge transformation in Rindler spacetime. We find the electro-magnetic wave equation and the electro-magnetic wave function in Rindler space-time. Specially, this article say the uniqueness of the accelerated frame because the accelerated frame can treat electro-magnetic field equation.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic field equation in Rindler space-time.

The Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2 , z = \xi^3 \qquad (1)$$

In this time, the tetrad $e^a{}_\mu$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2]$$

$$= -\frac{1}{c^2}\eta_{ab}\frac{\partial x^a}{\partial \xi^\mu}\frac{\partial x^b}{\partial \xi^\upsilon}d\xi^\mu d\xi^\upsilon$$

$$= -\frac{1}{c^2}\eta_{ab}e^a{}_\mu e^b{}_\upsilon d\xi^\mu d\xi^\upsilon = -\frac{1}{c^2}g_{\mu\upsilon}d\xi^\mu d\xi^\upsilon \quad , \quad e^a{}_\mu = \frac{\partial x^a}{\partial \xi^\mu} \qquad (2)$$

$$e^\alpha{}_0(\xi^0) = \frac{\partial x^\alpha}{c\partial \xi^0} = ((1 + \frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c}), (1 + \frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c}), 0, 0) \qquad (3)$$

About y -axis's and z -axis's orientation

$$e^\alpha{}_2(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^2} = (0,0,1,0) \quad , \quad e^\alpha{}_3(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^3} = (0,0,0,1) \qquad (4)$$

The other unit vector $e^\alpha{}_1(\xi^0)$ is

$$e^\alpha{}_1(\xi^0) = \frac{\partial x^\alpha}{\partial \xi^1} = (\sinh(\frac{a_0\xi^0}{c}), \cosh(\frac{a_0\xi^0}{c}), 0, 0) \qquad (5)$$

Therefore,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$= c\cosh(\frac{a_0\xi^0}{c})d\hat{\xi}^0 + \sinh(\frac{a_0\xi^0}{c})d\hat{\xi}^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1$$

$$= c\sinh(\frac{a_0\xi^0}{c})d\hat{\xi}^0 + \cosh(\frac{a_0\xi^0}{c})d\hat{\xi}^1, dy = d\xi^2 = d\hat{\xi}^2,\ dz = d\xi^3 = d\hat{\xi}^3 \quad (6)$$

The vector transformation is

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}, \qquad U'_{\mu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} U_{\alpha} \quad (7)$$

Therefore, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A}) = A^{\alpha}$ is

$$A^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} A'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} A_{\xi}^{\ \mu} = e^{\alpha}{}_{\mu} A_{\xi}^{\ \mu} \ ,\ e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}}$$

$$dx^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} dx'^{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} d\xi^{\mu} = e^{\alpha}{}_{\mu} d\xi^{\mu} \ , e^{\alpha}{}_{\mu} = \frac{\partial x^{\alpha}}{\partial \xi^{\mu}} \quad (8)$$

Hence, the transformation of the electro-magnetic 4-vector potential $(\phi, \vec{A})$ in inertial frame and the eletro-magnetic 4-vector potential $(\phi_{\xi}, \vec{A}_{\xi})$ in uniformly accelerated frame is

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\phi = 4\pi\rho$$

$$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\vec{A} = \frac{4\pi}{c}\vec{j}$$

4-vector $(c\rho, \vec{j}) = \rho_0 \dfrac{dx^{\alpha}}{d\tau}$ (9)

Lorentz gauge transformation is in Rindler spacetime,

$A^{\mu} \to A^{\mu} + \partial^{\mu}\Lambda = A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda$, Λ is a scalar function.

$$g^{00} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1$$

Hence,

$\phi_{\xi} \to \phi_{\xi} - \dfrac{1}{c}\dfrac{\partial\Lambda}{\partial\xi^0}\dfrac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_{\xi} \to \vec{A}_{\xi} + \vec{\nabla}_{\xi}\Lambda$, Λ is a scalar function. (10)

Lorentz gauge fixing condition is in Rindler spacetime,

$$A^{\mu}{}_{;\mu} = \frac{\partial A^{\mu}}{\partial \xi^{\mu}} + \Gamma^{\mu}{}_{\mu\rho} A^{\rho},$$

$$\Gamma^{\mu}{}_{\mu\rho} = \Gamma^{0}{}_{01} = \frac{1}{2} g^{00} (\frac{\partial g_{00}}{\partial \xi^{1}}) = \frac{a_0}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^1)}$$

$$g^{00} = -\frac{1}{(1+\frac{a_0 \xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1 \tag{11}$$

$$0 = \frac{\partial \phi}{c \partial t} + \vec{\nabla} \cdot \vec{A} = \frac{\partial \phi_{\xi}}{c \partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} + \frac{A_{\xi^1} a_0}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})}$$

$$A^{\mu} \to A^{\mu} + \partial^{\mu} \Lambda = A^{\mu} + g^{\mu\upsilon} \partial_{\upsilon} \Lambda$$

$$A^{\mu}{}_{;\mu} = \frac{\partial A^{\mu}}{\partial \xi^{\mu}} + \Gamma^{\mu}{}_{\mu\rho} A^{\rho} \to \partial_{\mu} (A^{\mu} + g^{\mu\upsilon} \partial_{\upsilon} \Lambda) + \Gamma^{0}{}_{01} (A^1 + \frac{\partial \Lambda}{\partial \xi^1})$$

$$= \partial_{\mu} A^{\mu} + (g^{\mu\upsilon} \partial_{\mu} \partial_{\upsilon}) \Lambda + \Gamma^{0}{}_{01} (A^1 + \frac{\partial \Lambda}{\partial \xi^1})$$

$$0 = \frac{\partial \phi_{\xi}}{c \partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} + \frac{A_{\xi^1} a_0}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})}$$

$$\to \frac{1}{c} \frac{\partial \phi_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} - [\frac{1}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}{}^2] \Lambda$$

$$+ \frac{A_{\xi^1} a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2} \xi^1)} + \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})} = 0$$

$$[\frac{1}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}{}^2] \Lambda + \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})} = 0 \tag{12}$$

$$\phi = \cosh(\frac{a_0 \xi^0}{c})(1+\frac{a_0}{c^2} \xi^1) \phi_{\xi} + \sinh(\frac{a_0 \xi^0}{c}) A_{\xi^1}$$

$$= \cosh(\frac{a_0 \xi^0}{c}) \hat{\phi}_{\xi} + \sinh(\frac{a_0 \xi^0}{c}) \hat{A}_{\xi^1}$$

$$A_x = \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}$$

$$= \sinh(\frac{a_0\xi^0}{c})\hat{\phi}_\xi + \cosh(\frac{a_0\xi^0}{c})\hat{A}_{\xi^1}$$

$$A_y = A_{\xi^2} = \hat{A}_{\xi^2}, A_z = A_{\xi^3} = \hat{A}_{\xi^3} \qquad (13)$$

$$g = \begin{pmatrix} -(1+\frac{a_0\xi^1}{c^2})^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e^a{}_\mu e_b{}^\mu = \delta^a{}_b \ , \ e^a{}_\mu e_a{}^\upsilon = \delta_\mu{}^\upsilon$$

$$e^a{}_\mu e^b{}_\upsilon \eta_{ab} = g_{\mu\upsilon} \ \rightarrow \ A^T \eta A = g$$

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon} = \eta_{ab} \rightarrow \ (A^T)^{-1} g A^{-1} = (A^T)^{-1} A^T \eta A A^{-1} = \eta$$

$$e^a{}_\mu = \eta^{ab} g_{\mu\upsilon} e_b{}^\upsilon \rightarrow \eta^{-1}(A^T)^{-1} A^T \eta A = A = \eta^{-1}(A^T)^{-1} g \qquad (14)$$

$$\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= A \begin{pmatrix} cd\xi^0 \\ d\xi^1 \\ d\xi^2 \\ d\xi^3 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cd\hat{\xi}^0 \\ d\hat{\xi}^1 \\ d\hat{\xi}^2 \\ d\hat{\xi}^3 \end{pmatrix}$$

(15)

$$e_{\mu}{}^{\alpha} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} = A^{-1} = \begin{pmatrix} \frac{c\partial \xi^0}{c\partial t} & \frac{c\partial \xi^0}{\partial x} & \frac{c\partial \xi^0}{\partial y} & \frac{c\partial \xi^0}{\partial z} \\ \frac{\partial \xi^1}{c\partial t} & \frac{\partial \xi^1}{\partial x} & \frac{\partial \xi^1}{\partial y} & \frac{\partial \xi^1}{\partial z} \\ \frac{\partial \xi^2}{c\partial t} & \frac{\partial \xi^2}{\partial x} & \frac{\partial \xi^2}{\partial y} & \frac{\partial \xi^2}{\partial z} \\ \frac{\partial \xi^3}{c\partial t} & \frac{\partial \xi^3}{\partial x} & \frac{\partial \xi^3}{\partial y} & \frac{\partial \xi^3}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & 0 & 0 \\ -\sinh(\frac{a_0 \xi^0}{c}) & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = (A^{-1})^T \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix} = (A^T)^{-1} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\cosh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & -\sinh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ -\frac{\sinh(\frac{a_0 \xi^0}{c})}{(1+\frac{a_0 \xi^1}{c^2})} & \cosh(\frac{a_0 \xi^0}{c}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial \xi^0} \\ \frac{\partial}{\partial \xi^1} \\ \frac{\partial}{\partial \xi^2} \\ \frac{\partial}{\partial \xi^3} \end{pmatrix}$$

$$=\begin{pmatrix}\cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0\\ -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{c}\frac{\partial}{\partial\hat{\xi}^0}\\ \frac{\partial}{\partial\hat{\xi}^1}\\ \frac{\partial}{\partial\hat{\xi}^2}\\ \frac{\partial}{\partial\hat{\xi}^3}\end{pmatrix} \quad (17)$$

$$\frac{1}{c}\frac{\partial}{\partial t}=\frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0}+\frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$=\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$=\cosh(\frac{a_0}{c}\xi^0)\frac{\partial}{c\partial\hat{\xi}^0}-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\hat{\xi}^1}$$

$$\frac{\partial}{\partial x}=\frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0}+\frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}$$

$$=-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$=-\sinh(\frac{a_0\xi^0}{c})\frac{\partial}{c\partial\hat{\xi}^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\hat{\xi}^1}$$

$$\frac{\partial}{\partial y}=\frac{\partial}{\partial\xi^2}=\frac{\partial}{\partial\hat{\xi}^2},\frac{\partial}{\partial z}=\frac{\partial}{\partial\xi^3}=\frac{\partial}{\partial\hat{\xi}^3}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2=\frac{1}{c^2(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2=\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2$$

$$\vec{\nabla}=(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}),\quad \vec{\nabla}_{\xi}=(\frac{\partial}{\partial\xi^1},\frac{\partial}{\partial\xi^2},\frac{\partial}{\partial\xi^3}),\ \vec{\nabla}_{\hat{\xi}}=(\frac{\partial}{\partial\hat{\xi}^1},\frac{\partial}{\partial\hat{\xi}^2},\frac{\partial}{\partial\hat{\xi}^3}) \quad (18)$$

2. Electro-magnetic Field in the Rindler space-time

The electro-magnetic field $(\vec{E},\vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \vec{B} = \vec{\nabla}\times\vec{A} \tag{19}$$

$$E_x = -\frac{\partial\phi}{\partial x} - \frac{\partial A_x}{c\partial t}$$

$$= -[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0\xi^1}{c^2})\phi_\xi + \cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$= -\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial A_{\xi^1}}{c\partial\xi^0} - (1+\frac{a_0\xi^1}{c^2})\frac{\partial\phi_\xi}{\partial\xi^1} - 2\phi_\xi\frac{a_0}{c^2}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}[(1+\frac{a_0}{c^2}\xi^1)^2\phi_\xi] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0}$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\hat{\xi}^1}[(1+\frac{a_0}{c^2}\xi^1)\hat{\phi}_\xi] - \frac{\partial\hat{A}_{\xi^1}}{c\partial\hat{\xi}^0} \tag{20}$$

$$E_y = -\frac{\partial\phi}{\partial y} - \frac{\partial A_y}{c\partial t} = -\frac{\partial}{\partial\xi^2}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^2}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial\phi_\xi}{\partial\xi^2} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^2}}{c\partial\xi^0}$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial\xi^1} - \frac{\partial A_{\xi^1}}{\partial\xi^2}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^2}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial \xi^0}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^2}[\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})] - \frac{\partial \hat{A}_{\xi^2}}{c\partial \hat{\xi}^0}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^2}] \qquad (21)$$

$$E_z = -\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{c\partial t} = -\frac{\partial}{\partial \xi^3}[\cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \sinh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial \xi^1}]A_{\xi^3}$$

$$= -(1+\frac{a_0\xi^1}{c^2})\cosh(\frac{a_0\xi^0}{c})\frac{\partial \phi_\xi}{\partial \xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\cosh(\frac{a_0\xi^0}{c})\frac{\partial A_{\xi^3}}{c\partial \xi^0}$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial \xi^0}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^3}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^3}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^3}[\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})] - \frac{\partial \hat{A}_{\xi^3}}{c\partial \hat{\xi}^0}]$$

$$+\sinh(\frac{a_0}{c}\xi^0)[\frac{\partial\hat{A}_{\xi^3}}{\partial\hat{\xi}^1}-\frac{\partial\hat{A}_{\xi^1}}{\partial\hat{\xi}^3}] \quad (22)$$

$$B_x=\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}=\frac{\partial A_{\xi^3}}{\partial\xi^2}-\frac{\partial A_{\xi^2}}{\partial\xi^3}=\frac{\partial\hat{A}_{\xi^3}}{\partial\hat{\xi}^2}-\frac{\partial\hat{A}_{\xi^2}}{\partial\hat{\xi}^3} \quad (23)$$

$$B_y=\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}=\frac{\partial A_x}{\partial\xi^3}-\frac{\partial A_{\xi^3}}{\partial x}$$

$$=\frac{\partial}{\partial\xi^3}[\sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi+\cosh(\frac{a_0\xi^0}{c})A_{\xi^1}]$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^1}}{\partial\xi^3}-\frac{\partial A_{\xi^3}}{\partial\xi^1}]$$

$$-\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}[\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^3}}{c\partial\xi^0}]$$

$$=\cosh(\frac{a_0}{c}\xi^0)[\frac{\partial\hat{A}_{\xi^1}}{\partial\hat{\xi}^3}-\frac{\partial\hat{A}_{\xi^3}}{\partial\hat{\xi}^1}]$$

$$-\sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^3}[\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})]-\frac{\partial\hat{A}_{\xi^3}}{c\partial\hat{\xi}^0}] \quad (24)$$

$$B_z=\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}=\frac{\partial A_{\xi^2}}{\partial x}-\frac{\partial A_x}{\partial\xi^2}$$

$$=[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}+\cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]A_{\xi^3}$$

$$-\frac{\partial}{\partial \xi^2}[\sinh(\frac{a_0 \xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\phi_\xi + \cosh(\frac{a_0 \xi^0}{c})A_{\xi^1}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial A_{\xi^2}}{\partial \xi^1} - \frac{\partial A_{\xi^1}}{\partial \xi^2}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \xi^2}[\phi_\xi(1+\frac{a_0 \xi^1}{c^2})^2] - \frac{1}{(1+\frac{a_0 \xi^1}{c^2})}\frac{\partial A_{\xi^2}}{c\partial \xi^0}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial \hat{A}_{\xi^2}}{\partial \hat{\xi}^1} - \frac{\partial \hat{A}_{\xi^1}}{\partial \hat{\xi}^2}]$$

$$+ \sinh(\frac{a_0}{c}\xi^0)[-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial \hat{\xi}^2}[\hat{\phi}_\xi(1+\frac{a_0 \xi^1}{c^2})] - \frac{\partial \hat{A}_{\xi^2}}{c\partial \hat{\xi}^0}] \qquad (25)$$

Hence, we can define the electro-magnetic field $(\vec{E}_\xi, \vec{B}_\xi)$ in Rindler spacetime.

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0 \xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0 \xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0 \xi^1}{c^2})}\frac{\partial \vec{A}_\xi}{c\partial \xi^0}$$

$$= -\frac{1}{(1+\frac{a_0 \xi^1}{c^2})}\vec{\nabla}_{\hat{\xi}}\{\hat{\phi}_\xi(1+\frac{a_0 \xi^1}{c^2})\} - \frac{\partial \vec{\hat{A}}_\xi}{c\partial \hat{\xi}^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi \times \vec{A}_\xi = \vec{\nabla}_{\hat{\xi}} \times \vec{\hat{A}}_\xi$$

In this time, $\vec{\nabla}_\xi = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}), \vec{A}_\xi = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$

$$\vec{\nabla}_{\hat{\xi}} = (\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3}), \vec{\hat{A}}_\xi = (\hat{A}_{\xi^1}, \hat{A}_{\xi^2}, \hat{A}_{\xi^3}) \qquad (26)$$

Lorentz gauge transformation is in Rindler spacetime,

$$\phi_\xi \to \phi_\xi - \frac{1}{c}\frac{\partial \Lambda}{\partial \xi^0}\frac{1}{(1+\frac{a_0 \xi^1}{c^2})^2}, \quad \vec{A}_\xi \to \vec{A}_\xi + \vec{\nabla}_\xi \Lambda, \quad \Lambda \text{ is a scalar function.} \qquad (27)$$

$$\vec{E}_\xi = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\frac{\partial\Lambda}{c\partial\xi^0}$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\Lambda$$

$$= -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{A}_\xi}{c\partial\xi^0}$$

$$\vec{B}_\xi = \vec{\nabla}_\xi\times\vec{A}_\xi + \vec{\nabla}_\xi\times\vec{\nabla}_\xi\Lambda = \vec{\nabla}_\xi\times\vec{A}_\xi \qquad (28)$$

Lorentz gauge fixing condition is in Rindler spacetime,

$$0 = \frac{\partial\phi_\xi}{c\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi + \frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\to \frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0} + \vec{\nabla}_\xi\cdot\vec{A}_\xi - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda$$

$$+\frac{A_{\xi^1}a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)} + \frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2]\Lambda + \frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0$$

$$\phi - \frac{1}{c}\frac{\partial\Lambda}{\partial t} = \cosh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\{\phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\}$$

$$+\sinh(\frac{a_0\xi^0}{c})(A_{\xi^1} + \frac{\partial\Lambda}{\partial\xi^1})$$

$$A_x + \frac{\partial\Lambda}{\partial x} = \sinh(\frac{a_0\xi^0}{c})(1+\frac{a_0}{c^2}\xi^1)\{\phi_\xi - \frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$$

$$+\cosh(\frac{a_0\xi^0}{c})(A_{\xi^1}+\frac{\partial\Lambda}{\partial\xi^1})$$

$$A_y+\frac{\partial\Lambda}{\partial y}=A_{\xi^2}+\frac{\partial\Lambda}{\partial\xi^2},A_z+\frac{\partial\Lambda}{\partial z}=A_{\xi^3}+\frac{\partial\Lambda}{\partial\xi^3} \quad (29)$$

We obtain the transformation of the electro-magnetic field.

$$E_x=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\xi^1}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial A_{\xi^1}}{c\partial\xi^0}$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{\partial\hat{\xi}^1}\{\hat{\phi}_\xi(1+\frac{a_0\xi^1}{c^2})\}-\frac{\partial\hat{A}_{\xi^1}}{c\partial\hat{\xi}^0}=E_{\xi^1} \quad ,$$

$$E_y=E_{\xi^2}\cosh(\frac{a_0\xi^0}{c})+B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z=E_{\xi^3}\cosh(\frac{a_0\xi^0}{c})-B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$B_x=B_{\xi^1},$$

$$B_y=B_{\xi^2}\cosh(\frac{a_0\xi^0}{c})-E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$$

$$B_z=B_{\xi^3}\cosh(\frac{a_0\xi^0}{c})+E_{\xi^2}\sinh(\frac{a_0\xi^0}{c}) \quad (30)$$

Hence,

$$E_x=E_{\xi^1},B_x=B_{\xi^1},$$

$$\begin{pmatrix}E_y\\B_y\\E_z\\B_z\end{pmatrix}=H\begin{pmatrix}E_{\xi^2}\\B_{\xi^2}\\E_{\xi^3}\\B_{\xi^3}\end{pmatrix}$$

$$H = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & \sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & -\sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & -\sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ \sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \quad (31)$$

The inverse-transformation of the electro-magnetic field is

$$E_{\xi^1} = E_x , B_{\xi^1} = B_x$$

$$\begin{pmatrix} E_{\xi^2} \\ B_{\xi^2} \\ E_{\xi^3} \\ B_{\xi^3} \end{pmatrix} = H^{-1} \begin{pmatrix} E_y \\ B_y \\ E_z \\ B_z \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} \cosh(\frac{a_0\xi^0}{c}) & 0 & 0 & -\sinh(\frac{a_0\xi^0}{c}) \\ 0 & \cosh(\frac{a_0\xi^0}{c}) & \sinh(\frac{a_0\xi^0}{c}) & 0 \\ 0 & \sinh(\frac{a_0\xi^0}{c}) & \cosh(\frac{a_0\xi^0}{c}) & 0 \\ -\sinh(\frac{a_0\xi^0}{c}) & 0 & 0 & \cosh(\frac{a_0\xi^0}{c}) \end{pmatrix} \quad (32)$$

$$E_{\xi^1} = E_x , B_{\xi^1} = B_x$$

$$E_{\xi^2} = E_y \cosh(\frac{a_0\xi^0}{c}) - B_z \sinh(\frac{a_0\xi^0}{c}),$$

$$B_{\xi^2} = B_y \cosh(\frac{a_0\xi^0}{c}) + E_z \sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0\xi^0}{c}) + B_y \sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c}) \quad (33)$$

3. Electro-magnetic Field Equation(Maxwell Equation) in the Rindler space-time

Maxwell equation is

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \qquad \text{(34-i)}$$

$$\vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j} \qquad \text{(34-ii)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \text{(34-iii)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{c\partial t} \qquad \text{(34-iv)}$$

1. $\vec{\nabla}\cdot\vec{E} = 4\pi\rho$

$E_x = E_{\xi^1}$,

$$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c}),$$

$$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$$

$$4\pi\rho = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]E_{\xi^1}$$

$$+\frac{\partial}{\partial\xi^2}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+\frac{\partial}{\partial\xi^3}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_\xi\cdot\vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial\xi^2} - \frac{\partial B_{\xi^2}}{\partial\xi^3} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial E_{\xi^1}}{c\partial\xi^0}]$$

$$= \cosh(\frac{a_0}{c}\xi^0)(\vec{\nabla}_{\hat{\xi}}\cdot\vec{E}_\xi) + \sinh(\frac{a_0}{c}\xi^0)[\frac{\partial B_{\xi^3}}{\partial\hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial\hat{\xi}^3} - \frac{\partial E_{\xi^1}}{c\partial\hat{\xi}^0}] \qquad (35)$$

2. $\vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{c\partial t} + \frac{4\pi}{c}\vec{j}$

$$B_x = B_{\xi^1}$$

$$B_y = B_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})$$

$$B_z = B_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})$$

X-component) $\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$

$$= \frac{\partial}{\partial \xi^2} [B_{\xi^3} \cosh(\frac{a_0 \xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0 \xi^0}{c})]$$

$$- \frac{\partial}{\partial \xi^3} [B_{\xi^2} \cosh(\frac{a_0 \xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0 \xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c} \xi^0) [\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3}] + \sinh(\frac{a_0 \xi^0}{c}) [\frac{\partial E_{\xi^2}}{\partial \xi^2} + \frac{\partial E_{\xi^3}}{\partial \xi^3}]$$

$$= \frac{\partial E_x}{c \partial t} + \frac{4\pi}{c} j_x$$

$$= [\frac{\cosh(\frac{a_0 \xi^0}{c})}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial}{c \partial \xi^0} - \sinh(\frac{a_0 \xi^0}{c}) \frac{\partial}{\partial \xi^1}] E_{\xi^1} + \frac{4\pi}{c} j_x$$

Hence,

$$\frac{4\pi}{c} j_x$$

$$= \sinh(\frac{a_0 \xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{E}_\xi) + \cosh(\frac{a_0 \xi^0}{c}) [\frac{\partial B_{\xi^3}}{\partial \xi^2} - \frac{\partial B_{\xi^2}}{\partial \xi^3} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \frac{\partial E_{\xi^1}}{c \partial \xi^0}]$$

$$= \sinh(\frac{a_0 \xi^0}{c})(\vec{\nabla}_{\hat{\xi}} \cdot \vec{E}_\xi) + \cosh(\frac{a_0 \xi^0}{c}) [\frac{\partial B_{\xi^3}}{\partial \hat{\xi}^2} - \frac{\partial B_{\xi^2}}{\partial \hat{\xi}^3} - \frac{\partial E_{\xi^1}}{c \partial \hat{\xi}^0}] \quad (36)$$

Y-component) $\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}$

$$= \frac{\partial B_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \frac{\partial E_y}{c\partial t} + \frac{4\pi}{c} j_y$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{4\pi}{c} j_y$$

$$\frac{4\pi}{c} j_y = \frac{\partial B_{\xi^1}}{\partial\xi^3} - \frac{\partial B_{\xi^3}}{\partial\xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}B_{\xi^3} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial E_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^3}\{B_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^1}\{B_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} - \frac{\partial E_{\xi^2}}{c\partial\hat{\xi}^0}$$

(37)

Z-component) $\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$- \frac{\partial B_{\xi^1}}{\partial\xi^2}$$

$$= \frac{\partial E_z}{c\partial t} + \frac{4\pi}{c} j_z$$

$$= [\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{c\partial \xi^0} - \sinh(\frac{a_0\xi^0}{c}) \frac{\partial}{\partial \xi^1}] \cdot [E_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2} \sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{4\pi}{c} j_z$$

$$\frac{4\pi}{c} j_z = \frac{\partial B_{\xi^2}}{\partial \xi^1} - \frac{\partial B_{\xi^1}}{\partial \xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{a_0}{c^2} B_{\xi^2} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial E_{\xi^3}}{c\partial \xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial \xi^1} \{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial \xi^2} \{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial E_{\xi^3}}{c\partial \xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial \hat{\xi}^1} \{B_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial}{\partial \hat{\xi}^2} \{B_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} - \frac{\partial E_{\xi^3}}{c\partial \hat{\xi}^0}$$

(38)

3. $\vec{\nabla} \cdot \vec{B} = 0$

$$\vec{\nabla} \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial}{c\partial \xi^0} + \cosh(\frac{a_0\xi^0}{c}) \frac{\partial}{\partial \xi^1}] B_{\xi^1}$$

$$+ \frac{\partial}{\partial \xi^2} [B_{\xi^2} \cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3} \sinh(\frac{a_0\xi^0}{c})]$$

$$+ \frac{\partial}{\partial \xi^3} [B_{\xi^3} \cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2} \sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_\xi \cdot \vec{B}_\xi) + \sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial \xi^3} + \frac{\partial E_{\xi^3}}{\partial \xi^2}) - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{\partial B_{\xi^1}}{c\partial \xi^0}] = 0$$

$$= \cosh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\hat{\xi}} \cdot \vec{B}_{\xi}) + \sinh(\frac{a_0\xi^0}{c})[-(-\frac{\partial E_{\xi^2}}{\partial\hat{\xi}^3} + \frac{\partial E_{\xi^3}}{\partial\hat{\xi}^2}) - \frac{\partial B_{\xi^1}}{c\partial\hat{\xi}^0}] = 0$$

(39)

4. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{c\partial t}$

$E_x = E_{\xi^1}$,

$E_y = E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})$,

$E_z = E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})$

X-component) $\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$

$$= \frac{\partial}{\partial\xi^2}[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial}{\partial\xi^3}[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$= \cosh(\frac{a_0}{c}\xi^0)[\frac{\partial E_{\xi^3}}{\partial\xi^2} - \frac{\partial E_{\xi^2}}{\partial\xi^3}] - \sinh(\frac{a_0\xi^0}{c})[\frac{\partial B_{\xi^2}}{\partial\xi^2} + \frac{\partial B_{\xi^3}}{\partial\xi^3}]$$

$$= -\frac{\partial B_x}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]B_{\xi^1}$$

Hence, $-\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\xi} \cdot \vec{B}_{\xi}) + \cosh(\frac{a_0\xi^0}{c})[(\frac{\partial E_{\xi^3}}{\partial\xi^2} - \frac{\partial E_{\xi^2}}{\partial\xi^3}) + \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial B_{\xi^1}}{c\partial\xi^0}]$

$$= -\sinh(\frac{a_0\xi^0}{c})(\vec{\nabla}_{\hat{\xi}} \cdot \vec{B}_{\xi}) + \cosh(\frac{a_0\xi^0}{c})[(\frac{\partial E_{\xi^3}}{\partial\hat{\xi}^2} - \frac{\partial E_{\xi^2}}{\partial\hat{\xi}^3}) + \frac{\partial B_{\xi^1}}{c\partial\hat{\xi}^0}] = 0 \quad (40)$$

Y-component) $\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$

$$= \frac{\partial E_{\xi^1}}{\partial \xi^3}$$

$$-[-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) - B_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$= -\frac{\partial B_y}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) - E_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^1}}{\partial \xi^3} - \frac{\partial E_{\xi^3}}{\partial \xi^1} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^3} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^2}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^3}\{E_{\xi^1}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^1}\{E_{\xi^3}(1+\frac{a_0\xi^1}{c^2})\} + \frac{\partial B_{\xi^2}}{c\partial\hat{\xi}^0} = 0$$

(41)

Z-component) $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$

$$= [-\frac{\sinh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[E_{\xi^2}\cosh(\frac{a_0\xi^0}{c}) + B_{\xi^3}\sinh(\frac{a_0\xi^0}{c})]$$

$$-\frac{\partial E_{\xi^1}}{\partial \xi^2}$$

$$= -\frac{\partial B_z}{c\partial t}$$

$$= -[\frac{\cosh(\frac{a_0\xi^0}{c})}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}]\cdot[B_{\xi^3}\cosh(\frac{a_0\xi^0}{c}) + E_{\xi^2}\sinh(\frac{a_0\xi^0}{c})]$$

$$\frac{\partial E_{\xi^2}}{\partial\xi^1} - \frac{\partial E_{\xi^1}}{\partial\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}E_{\xi^2} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial B_{\xi^3}}{c\partial\xi^0}$$

$$= \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^1}\{E_{\xi^2}(1+\frac{a_0}{c^2}\xi^1)\} - \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\hat{\xi}^2}\{E_{\xi^1}(1+\frac{a_0\xi^1}{c^2})\} + \frac{\partial B_{\xi^3}}{c\partial\hat{\xi}^0} = 0$$

(42)

Therefore, we obtain the electro-magnetic field equation by Eq (35)-Eq(42) in Rindler spacetime .

$$\vec{\nabla}_\xi\cdot\vec{E}_\xi = \vec{\nabla}_{\hat{\xi}}\cdot\vec{E}_\xi = 4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \quad (43\text{-i})$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0} + \frac{4\pi}{c}\vec{j}_\xi$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\hat{\xi}}\times\{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\} = \frac{\partial\vec{E}_\xi}{c\partial\hat{\xi}^0} + \frac{4\pi}{c}\vec{j}_\xi \quad (43\text{-ii})$$

$$\vec{\nabla}_\xi\cdot\vec{B}_\xi = \vec{\nabla}_{\hat{\xi}}\cdot\vec{B}_\xi = 0 \quad (43\text{-iii})$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0}$$

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_{\hat{\xi}}\times\{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\} = -\frac{\partial\vec{B}_\xi}{c\partial\hat{\xi}^0} \quad (43\text{-iv})$$

$$\vec{E}_{\xi} = (E_{\xi^1}, E_{\xi^2}, E_{\xi^3}), \vec{B}_{\xi} = (B_{\xi^1}, B_{\xi^2}, B_{\xi^3}),$$

$$\vec{\nabla}_{\xi} = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}), \vec{\nabla}_{\hat{\xi}} = (\frac{\partial}{\partial \hat{\xi}^1}, \frac{\partial}{\partial \hat{\xi}^2}, \frac{\partial}{\partial \hat{\xi}^3})$$

Hence, the transformation of 4-vector $(c\rho, \vec{j}) = \rho_0 \frac{dx^{\alpha}}{d\tau}$ is

$$\rho = \rho_{\xi}(1 + \frac{a_0 \xi^1}{c^2})\cosh(\frac{a_0 \xi^0}{c}) + \frac{j_{\xi^1}}{c}\sinh(\frac{a_0 \xi^0}{c})$$

$$j_x = j_{\xi^1}\cosh(\frac{a_0 \xi^0}{c}) + c\rho_{\xi}(1 + \frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0 \xi^0}{c}), \quad j_y = j_{\xi^2}, j_z = j_{\xi^3}$$

In this time, 4-vector $(c\rho_{\xi}, \vec{j}_{\xi}) = \rho_0 \frac{d\xi^{\alpha}}{d\tau}$ (44)

Generally, the continuity equation is in Rindler spacetime,

$$0 = j^{\mu}{}_{;\mu} = \frac{\partial j^{\mu}}{\partial \xi^{\mu}} + \Gamma^{\mu}{}_{\mu\rho} j^{\rho},$$

$$\Gamma^{\mu}{}_{\mu\rho} = \Gamma^{0}{}_{01} = \frac{1}{2} g^{00}(\frac{\partial g_{00}}{\partial \xi^1}) = \frac{a_0}{c^2}\frac{1}{(1 + \frac{a_0}{c^2}\xi^1)}$$

$$g^{00} = -\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})^2}, g^{11} = g^{22} = g^{33} = 1$$

$$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = \frac{\partial \rho_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{j}_{\xi} + \frac{j_{\xi^1} a_0}{c^2}\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} \quad (45)$$

We treat Lorentz gauge transformation about the electro-magnetic field equation in Rindler spacetime. Eq(43-i) is

$$\vec{\nabla}_{\xi} \cdot \vec{E}_{\xi} = \vec{\nabla}_{\xi} \cdot \{-\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}\vec{\nabla}_{\xi}\{\phi_{\xi}(1 + \frac{a_0 \xi^1}{c^2})^2\} - \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}\frac{\partial \vec{A}_{\xi}}{c\partial \xi^0}\}$$

$$= -\vec{\nabla}_{\xi}\{\frac{1}{(1 + \frac{a_0 \xi^1}{c^2})}\} \cdot [\vec{\nabla}_{\xi}\{\phi_{\xi}(1 + \frac{a_0 \xi^1}{c^2})^2\} + \frac{\partial \vec{A}_{\xi}}{c\partial \xi^0}]$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}+\frac{\partial}{c\partial\xi^0}(\vec{\nabla}_\xi\cdot\vec{A}_\xi)]$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}[\frac{\partial}{\partial\xi^1}\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}+\frac{\partial A_{\xi^1}}{c\partial\xi^0}]$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{A_{\xi^1}a_0}{c^2}]$$,

$$\frac{1}{c}\frac{\partial\phi_\xi}{\partial\xi^0}+\vec{\nabla}_\xi\cdot\vec{A}_\xi=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}$$

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2}$$

$$=4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \qquad (46)$$

If we apply Lorentz gauge transformation to Eq (46),

$\phi_\xi\to\phi_\xi-\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_\xi\to\vec{A}_\xi+\vec{\nabla}_\xi\Lambda$, Λ is a scalar function.

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2}+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0}{c^2}\frac{\partial}{c\partial\xi^0}\frac{\partial\Lambda}{\partial\xi^1}$$

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0}\{[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\Lambda+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2} \qquad (47)$$

In this time,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]\Lambda+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}=0 \qquad (48)$$

Hence, Eq(43-i) is

$$\vec{\nabla}_\xi\cdot\vec{E}_\xi$$

$$=-\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}E_{\xi^1}-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}[\nabla_\xi{}^2-\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial A_{\xi^1}}{c\partial\xi^0}\frac{a_0}{c^2}$$

$$=4\pi\rho_\xi(1+\frac{a_0\xi^1}{c^2}) \qquad (49)$$

Eq(43-i) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-ii) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{B}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$=\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{\nabla}_\xi\times\vec{A}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$=\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\{\vec{\nabla}_\xi\times\vec{A}_\xi\}+\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\vec{A}_\xi$$

$$=\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(1,0,0)\times\vec{B}_\xi+\{-\nabla_\xi{}^2\vec{A}_\xi+\vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{A}_\xi)\}$$

$$=\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3},B_{\xi^2})+\{-\nabla_\xi{}^2\vec{A}_\xi+\vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{A}_\xi)\}$$

$$=\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{E}_\xi}{c\partial\xi^0}+\frac{4\pi}{c}\vec{j}_\xi$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{\partial}{c\partial\xi^0}[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}]-\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi+\frac{4\pi\vec{j}_\xi}{c}$$

$$=-\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)-\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi+\frac{4\pi\vec{j}_\xi}{c}$$

(50)

Therefore,

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$=\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}(0,-B_{\xi^3},B_{\xi^2})+\{-\nabla_\xi{}^2\vec{A}_\xi+\vec{\nabla}_\xi(\vec{\nabla}_\xi\cdot\vec{A}_\xi)\}$$

$$+\frac{\partial}{c\partial\xi^0}\vec{\nabla}_\xi\phi_\xi+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{A}_\xi$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_{\xi}{}^{2}+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_{\xi}+\vec{\nabla}_{\xi}[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}]$$

$$\frac{1}{c}\frac{\partial\phi_{\xi}}{\partial\xi^0}+\vec{\nabla}_{\xi}\cdot\vec{A}_{\xi}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1} \tag{51}$$

$$\frac{4\pi}{c}\vec{j}_{\xi}$$
$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_{\xi}}{c\partial\xi^0}(1,0,0)$$
$$+[-\nabla_{\xi}{}^{2}+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_{\xi}$$
$$+\vec{\nabla}_{\xi}[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}] \tag{52}$$

If we apply Lorentz gauge transformation to Eq (52),

$\phi_{\xi}\to\phi_{\xi}-\frac{1}{c}\frac{\partial\Lambda}{\partial\xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}$, $\vec{A}_{\xi}\to\vec{A}_{\xi}+\vec{\nabla}_{\xi}\Lambda$, Λ is a scalar function.

$$\frac{4\pi}{c}\vec{j}_{\xi}$$
$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_{\xi}}{c\partial\xi^0}(1,0,0)$$
$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$
$$+[-\nabla_{\xi}{}^{2}+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_{\xi}+[-\nabla_{\xi}{}^{2}+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{\nabla}_{\xi}\Lambda$$
$$+\vec{\nabla}_{\xi}[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}A_{\xi^1}]+\vec{\nabla}_{\xi}[-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial\Lambda}{\partial\xi^1}]$$

$$= \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})} (0, -B_{\xi^3}, B_{\xi^2}) + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{2a_0}{c^2} \frac{\partial \phi_\xi}{c\partial \xi^0} (1,0,0)$$

$$- \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3} \frac{2a_0}{c^2} \frac{1}{c^2} (\frac{\partial}{\partial \xi^0})^2 \Lambda (1,0,0)$$

$$+ [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{A}_\xi + [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{\nabla}_\xi \Lambda$$

$$+ \frac{1}{(1+\frac{a_0 \xi^1}{c^2})^2} \frac{a_0{}^2}{c^4} A_{\xi^1} (1,0,0) - \frac{1}{(1+\frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \vec{\nabla}_\xi A_{\xi^1}$$

$$+ \frac{1}{(1+\frac{a_0 \xi^1}{c^2})^2} \frac{a_0{}^2}{c^4} \frac{\partial \Lambda}{\partial \xi^1} (1,0,0) - \frac{1}{(1+\frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda \tag{53}$$

In this time,

$$[\frac{1}{c^2} \frac{1}{(1+\frac{a_0 \xi^1}{c^2})^2} (\frac{\partial}{\partial \xi^0})^2 - \nabla_\xi{}^2] \Lambda + \frac{1}{(1+\frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial \Lambda}{\partial \xi^1} = 0$$

$$0 = \vec{\nabla}_\xi [\{-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2 + \frac{1}{(1+\frac{a_0 \xi^1}{c^2})} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1}\} \Lambda]$$

$$= \vec{\nabla}_\xi \{\frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\} (\frac{\partial}{\partial \xi^0})^2 \Lambda + [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{\nabla}_\xi \Lambda$$

$$+ \vec{\nabla}_\xi \{\frac{a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\} \frac{\partial \Lambda}{\partial \xi^1} + \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \frac{a_0}{c^2} \frac{\partial}{\partial \xi^1} \vec{\nabla}_\xi \Lambda$$

$$= -\frac{2}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3} \frac{a_0}{c^2} (\frac{\partial}{\partial \xi^0})^2 \Lambda (1,0,0) + [-\nabla_\xi{}^2 + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} (\frac{\partial}{\partial \xi^0})^2] \vec{\nabla}_\xi \Lambda$$

$$-\frac{a_0{}^2}{c^4}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0)+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_\xi\Lambda \tag{54}$$

Therefore,

$$\frac{4\pi}{c}\vec{j}_\xi$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}A_{\xi^1}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\vec{\nabla}_\xi A_{\xi^1}+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0)-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_\xi\Lambda$$

$$+\vec{\nabla}_\xi[\{-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2+\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\}\Lambda]$$

$$-\vec{\nabla}_\xi\{\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\}(\frac{\partial}{\partial\xi^0})^2\Lambda-\frac{a_0{}^2}{c^4}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial\Lambda}{\partial\xi^1}(1,0,0)$$

$$+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{a_0}{c^2}\frac{\partial}{\partial\xi^1}\vec{\nabla}_\xi\Lambda$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$-\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{2a_0}{c^2}\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}A_{\xi^1}(1,0,0)-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\vec{\nabla}_\xi A_{\xi^1}$$

$$+\frac{1}{c^2}\frac{2}{(1+\frac{a_0}{c^2}\xi^1)^3}\frac{a_0}{c^2}(\frac{\partial}{\partial\xi^0})^2\Lambda(1,0,0)$$

$$=\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}(0,-B_{\xi^3},B_{\xi^2})+\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{2a_0}{c^2}\frac{\partial\phi_\xi}{c\partial\xi^0}(1,0,0)$$

$$+[-\nabla_\xi{}^2+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2]\vec{A}_\xi$$

$$+\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}\frac{a_0{}^2}{c^4}A_{\xi^1}(1,0,0)-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\vec{\nabla}_\xi A_{\xi^1} \qquad (55)$$

Hence, Eq(43-ii) is invariant about Lorentz gauge transformation in Rindler spacetime.

Eq (43-iii) is

$$\vec{\nabla}_\xi\cdot\vec{B}_\xi=\vec{\nabla}_\xi\cdot(\vec{\nabla}_\xi\times\vec{A}_\xi+\vec{\nabla}_\xi\times\vec{\nabla}_\xi\Lambda)=\vec{\nabla}_\xi\times\vec{\nabla}_\xi\cdot\vec{A}_\xi=0 \qquad (56)$$

Eq (43-iv) is

$$\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0\xi^1}{c^2})\}$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times[\vec{\nabla}_\xi\{\phi_\xi(1+\frac{a_0\xi^1}{c^2})^2\}-\vec{\nabla}_\xi(\frac{\partial\Lambda}{c\partial\xi^0})+\frac{\partial\vec{A}_\xi}{c\partial\xi^0}+\frac{\partial}{c\partial\xi^0}(\vec{\nabla}_\xi\Lambda)]$$

$$=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\vec{\nabla}_\xi\times\frac{\partial\vec{A}_\xi}{c\partial\xi^0}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial(\vec{\nabla}_\xi\times\vec{A}_\xi)}{c\partial\xi^0}=-\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{\partial\vec{B}_\xi}{c\partial\xi^0} \qquad (57)$$

Hence, Eq (43-iii), Eq (43-iv) are invariant about Lorentz gauge transformation in Rindler spacetime.

Hence, the electro-magnetic field equations(Maxwell Equations) in Rindler spacetime are invariant about Lorentz gauge transformation.

4. Electro-magnetic wave equation in Rindler space-time

The electro-magnetic wave function is

$$E_x = E_{x0}\sin\Phi, E_y = E_{y0}\sin\Phi, E_z = E_{z0}\sin\Phi$$

$$B_x = B_{x0}\sin\Phi, B_y = B_{y0}\sin\Phi, B_z = B_{z0}\sin\Phi$$

$$E_{\xi^1} = E_x, B_{\xi^1} = B_x$$

$$E_{\xi^1} = E_{X0}\sin\Phi', B_{\xi^1} = B_{X0}\sin\Phi'$$

$$E_{\xi^2} = E_y\cosh(\frac{a_0\xi^0}{c}) - B_z\sinh(\frac{a_0\xi^0}{c}),$$

$$= (E_{y0}\sin\Phi')\cosh(\frac{a_0\xi^0}{c}) - (B_{z0}\sin\Phi')\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^2} = B_y\cosh(\frac{a_0\xi^0}{c}) + E_z\sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{y0}\sin\Phi')\cosh(\frac{a_0\xi^0}{c}) + (E_{z0}\sin\Phi')\sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z\cosh(\frac{a_0\xi^0}{c}) + B_y\sinh(\frac{a_0\xi^0}{c})$$

$$= (E_{z0}\sin\Phi')\cosh(\frac{a_0\xi^0}{c}) + (B_{y0}\sin\Phi')\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z\cosh(\frac{a_0\xi^0}{c}) - E_y\sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{z0}\sin\Phi')\cosh(\frac{a_0\xi^0}{c}) - (E_{y0}\sin\Phi')\sinh(\frac{a_0\xi^0}{c}) \qquad (58)$$

$$\Phi = \omega(t - l\frac{x}{c} - m\frac{y}{c} - n\frac{z}{c}),$$

$$\Phi' = \omega'(\hat{\xi}^0 - l'\frac{\hat{\xi}^1}{c} - m'\frac{\hat{\xi}^2}{c} - n'\frac{\hat{\xi}^3}{c})$$

In this time,

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c}) \quad , \quad x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3$$

$$\xi^0 = \frac{c}{a_0}\tanh^{-1}\left(\frac{ct}{x+\frac{c^2}{a_0}}\right), \xi^1 = \sqrt{\left(x+\frac{c^2}{a_0}\right)^2 - c^2t^2} - \frac{c^2}{a_0}$$

$$\lim_{a_0\to 0}\xi^0 = \lim_{a_0\to 0} c\tanh^{-1}\left(\frac{cta_0}{a_0x+c^2}\right)/a_0 = \lim_{a_0\to 0} c\tanh^{-1}\left(\frac{cta_0}{c^2}\right)/a_0 = \lim_{a_0\to 0} c\frac{1}{1-\left(\frac{a_0t}{c}\right)^2}\frac{t}{c} = t$$

$$\lim_{a_0\to 0}\xi^1 = \lim_{a_0\to 0}c^2\left(\sqrt{\left(1+\frac{a_0}{c^2}x\right)^2 - \frac{a_0{}^2t^2}{c^2}} - 1\right)/a_0 = \lim_{a_0\to 0}c^2\left(\sqrt{\left(1+\frac{a_0}{c^2}x\right)^2} - 1\right)/a_0$$

$$= \lim_{a_0\to 0}c^2\left(\frac{a_0}{c^2}x\right)/a_0 = x$$

Hence,

$$\lim_{a_0\to 0}\hat{\xi}^0 = \lim_{a_0\to 0}\int d\hat{\xi}^0 = \lim_{a_0\to 0}\int\left(1+\frac{a_0}{c^2}\xi^1\right)d\xi^0 = \lim_{a_0\to 0}\int d\xi^0 = \lim_{a_0\to 0}\xi^0 = t$$

$$\lim_{a_0\to 0}\hat{\xi}^1 = \lim_{a_0\to 0}\xi^1 = x,\ \ y = \xi^2 = \hat{\xi}^2, z = \xi^3 = \hat{\xi}^3$$

Therefore, electro-magnetic wave function is

$$\lim_{a_0\to 0}\Phi' = \lim_{a_0\to 0}\omega'\left(\hat{\xi}^0 - l'\frac{\hat{\xi}^1}{c} - m'\frac{\hat{\xi}^2}{c} - n'\frac{\hat{\xi}^3}{c}\right) = \omega'\left(t - l'\frac{x}{c} - m'\frac{y}{c} - n'\frac{z}{c}\right)$$

$$= \omega\left(t - l\frac{x}{c} - m\frac{y}{c} - n\frac{z}{c}\right) = \Phi$$

$$\omega = \omega', l' = l, m' = m, n' = n$$
$$l^2 + m^2 + n^2 = l'^2 + m'^2 + n'^2 = 1$$

(59)

Hence,

$$\left[\frac{1}{c^2}\frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)^2}\left(\frac{\partial}{\partial\xi^0}\right)^2 - \nabla_{\xi}{}^2\right]E_{\xi^1}$$

$$= \left[\frac{1}{c^2}\left(\frac{\partial}{\partial\hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}}{}^2\right]E_{\xi^1} = 0$$

$$\left[\frac{1}{c^2}\frac{1}{\left(1+\frac{a_0}{c^2}\xi^1\right)^2}\left(\frac{\partial}{\partial\xi^0}\right)^2 - \nabla_{\xi}{}^2\right]B_{\xi^1}$$

$$= \left[\frac{1}{c^2}\left(\frac{\partial}{\partial\hat{\xi}^0}\right)^2 - \nabla_{\hat{\xi}}{}^2\right]B_{\xi^1} = 0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_y$$

$$=[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]E_y=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_y$$

$$=[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]B_y=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]E_z$$

$$=[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]E_z=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_\xi{}^2]B_z$$

$$=[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2-\nabla_{\hat{\xi}}{}^2]B_z=0 \qquad (60)$$

The electro-magnetic wave equation is in vacuum

$$\vec{\nabla}_\xi\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}$$

$$=\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}+(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\{\vec{E}_\xi(1+\frac{a_0}{c^2}\xi^1)\}$$

$$=\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_\xi\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$=\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{\nabla}_\xi(1+\frac{a_0}{c^2}\xi^1)\times\vec{E}_\xi+(1+\frac{a_0}{c^2}\xi^1)^2\vec{\nabla}_\xi\times\vec{\nabla}_\xi\times\vec{E}_\xi$$

$$= [\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{E}_{\xi}]\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1) - [\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)]\vec{E}_{\xi}$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_{\xi}(\vec{\nabla}_{\xi}\cdot\vec{E}_{\xi}) - \nabla_{\xi}{}^2\vec{E}_{\xi}]$$

$$= -\frac{1}{c}\frac{\partial}{\partial\xi^0}[\vec{\nabla}_{\xi}\times\{\vec{B}_{\xi}(1+\frac{a_0\xi^1}{c^2})\}] = -\frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_{\xi},$$

In this time, $\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1) = (\frac{a_0}{c^2},0,0)$ (61)

Hence,

$$\vec{\nabla}_{\xi}\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_{\xi}\times\{\vec{E}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\} + \frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_{\xi}$$

$$= [\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{E}_{\xi}]\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1) - [\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)]\vec{E}_{\xi}$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_{\xi}(\vec{\nabla}_{\xi}\cdot\vec{E}_{\xi}) - \nabla_{\xi}{}^2\vec{E}_{\xi}] + \frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{E}_{\xi}$$

$$= \frac{a_0{}^2}{c^4}(E_{\xi^1},0,0) - \frac{a_0{}^2}{c^4}(E_{\xi^1},E_{\xi^2},E_{\xi^3}) + (1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_{\xi}{}^2]\vec{E}_{\xi}$$

$$= \frac{a_0{}^2}{c^4}(0,-E_{\xi^2},-E_{\xi^3}) + (1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_{\xi}{}^2]\vec{E}_{\xi}$$

$$= \frac{a_0{}^2}{c^4}(0,-E_{\xi^2},-E_{\xi^3}) + (1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2 - \nabla_{\hat{\xi}}{}^2]\vec{E}_{\xi}$$

$$= \vec{0}$$ (62)

Hence, the magnetic wave equation is in vacuum

$$\vec{\nabla}_{\xi}\times(1+\frac{a_0}{c^2}\xi^1)\vec{\nabla}_{\xi}\times\{\vec{B}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\} + \frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{B}_{\xi}$$

$$= [\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{B}_{\xi}]\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1) - [\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)\cdot\vec{\nabla}_{\xi}(1+\frac{a_0}{c^2}\xi^1)]\vec{B}_{\xi}$$

$$+(1+\frac{a_0}{c^2}\xi^1)^2[\vec{\nabla}_{\xi}(\vec{\nabla}_{\xi}\cdot\vec{B}_{\xi}) - \nabla_{\xi}{}^2\vec{B}_{\xi}] + \frac{1}{c^2}(\frac{\partial}{\partial\xi^0})^2\vec{B}_{\xi}$$

$$= \frac{a_0{}^2}{c^4}(0,-B_{\xi^2},-B_{\xi^3}) + (1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_{\xi}{}^2]\vec{B}_{\xi}$$

$$= \frac{a_0{}^2}{c^4}(0,-B_{\xi^2},-B_{\xi^3}) + (1+\frac{a_0}{c^2}\xi^1)^2[\frac{1}{c^2}(\frac{\partial}{\partial\hat{\xi}^0})^2 - \nabla_{\hat{\xi}}{}^2]\vec{B}_\xi$$

$$= \vec{0} \tag{63}$$

The electromagnetic wave function, Eq(58),Eq(59) satisfy the electromagnetic wave equation, Eq(62),Eq(63).

5. Conclusion

We find the electro-magnetic field transformation and the electro-magnetic equation in uniformly accelerated frame.

Generally, the coordinate transformation of accelerated frame is

(I) $$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \tag{64}$$

(II) $$ct = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = \frac{c^2}{a_0}\exp(\frac{a_0}{c^2}\xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}, y = \xi^2, z = \xi^3 \tag{65}$$

Hence, this article say the accelerated frame is Rindler coordinate (I) that can treat electro-magnetic field equation.

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]W.Rindler, Am.J.Phys.**34**.1174(1966)

[3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[7]A.Miller, Albert Einstein's Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[8]W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)

[9]Massimo Pauri, Michele Vallisner, "Marzke-Wheeler coordinates for accelerated observers in special relativity":Arxiv:gr-qc/0006095(2000)

[10]A. Einstein, " Zur Elektrodynamik bewegter K¨orper", Annalen der Physik. 17:891(1905)

[11]J.W.Maluf and F.F.Faria,”The electromagnetic field in accelerated frames”:Arxiv:gr-qc/1110.5367v1(2011)