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A Model of Quantum Black Holes and Dark Matter Production

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Abstract

I propose a model of black holes and dark matter to extend the standard model. I assume that at the center of any black hole there is a Kerr (Schwarzschild) black hole core object. The core is proposed to replace the singularity of the hole. During early inflation gravitons condensate gradually around the cores to form a primordial quantum black holes which evolve into dark matter in the universe together with the standard model particles.

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1 Introduction and Summary

The motivation behind the model described here is to find an economic way to go beyond the Standard Model (BSM), including inflation and a model of renormalization group based quantum gravity. This short note is hoped to be a step forward in exploring the role of Planck scale gravity in particle physics and big bang universe while any complete theory of quantum gravity remains beyond the scope of this note.

In particular I pay attention to the nature of mini quantum black holes at zero temperature. I made earlier a gedanken experiment of what might happen when exploring a mini black hole deep inside with a probe. In [1, 2] I made two assumptions

(**i**) inside any black hole there is a 3D integral part core, a Kerr (Scwarzschild) mini black hole of spin $\frac{1}{2}(0)$, or higher. The core of the hole has a high mass, of the order of the Planck mass. The core is called here gravon,

(**ii**) the core is a point of condensation of gravitons. The black hole singularity is replaced by the core field. Einstein equations hold outside the hole, but in the inner region quantum effects need to be included, using renormalization group (RG) equation methods, see Sec. 5.

What is not discussed here is the horizon, which has been extensively treated in the literature after the AMPS paper [3]. [2] Dark energy is left for future considerations as well.

The core is introduced to illustrate the case of singularity free black hole. A core is formed under high curvature regions in spacetime. It is at the same time a candidate for dark matter by being a condensation point for bosons, gravitons in this case. The core couples to gravitons and to the Higgs. Couplings to other standard model particles are weak. On the other hand, it may be easier to see the core to be the T=0 remnant of a thermally radiated black hole (possibly without a horizon) [4].

The point of this note is that the core under inflation [5, 6, 7, 8, 9] is a key missing element of the standard model. The cores with a relatively long life time are formed during early inflation when the curvature is high. Inflation thus produces cores developing into thermal black holes as well as standard model particles.

The material discussed in this note makes use of abundant results calculated in the references. In fact, since much of the material is widely spread out in the literature, I try to reproduce some sources in detail for the possible benefit of the younger readers. I put the different pieces together into a model with my interpretation of the role of quantum black holes, as best as understood today.

With the Planck scale having its the conventional value 10^{19} GeV finding a gravon is hard. Gamma-ray signals from the sky may be a promising way. A gamma-ray, or particle, with energy half the Planck mass would be a clear signal of the models of

[2]Their paper introduced the field to this author.

this type.

The physical picture of gravity we are looking for in our scheme is as follows. It is between quantum chromodynamics (QCD) and quantum electrodynamics (QED). For color and electric charge the relevant objects, hadrons and atoms, respectively, are neutral, of course, as seen from a proper distance while mass cannot be similarly "hidden". Even for a single massless core or hole in vacuum, very close to the core virtual particle pairs are created and destroyed, making a cloud of mass/energy around the core, all objects interacting gravitationally with the core and each other. At shortest scales neighboring gravons and gravitons interact. When the scale is increased gradually larger blocks of gravons and virtual particles interact. With the Wilsonian renormalization group method it will be seen that an asymptotically safe (AS) gravity [10] is created in the UV. In the classical limit general relativity (GR) will be obtained from the action. All this happens within the gravitational field of everything else in the universe. It means that quantum cosmology needs simultaneously be developed.

This model is a simplified attempt to define a gravity-gauge theory connection. It all intakes place in four dimensions directly in a few stages, unlike the more ambitious string theory based AdS/CFT dualities [11]. Extra strength and beauty may be available from higher dimensions.

2 The Black Hole Core

I propose that this phenomenological model, based on particle approach rather than geometrical, of the black hole core gives reasonable quantitative answers to a few but important long term problems in astro-particle physics. The model is as concrete as possible and under control of present day technology. In particular, I propose

(**i**) the core is "virtually" there in the form a M_{Planck} black hole object, which is of the order of the from nature constants obtained final mass of a classical black hole before it disappears totally with information loss. Alternatively, or even more probably, the M_{Planck}-object may be a remnant, either stable or with long lifetime. Remnants have no singularity or information loss problems, see eg. the recent review [12],

(**ii**) the core works well together with the inflation model of Sec. 5 with the core, gravitons and the Higgs creating a universe with substantial amount of dark matter together with the standard model matter,

(**iii**) there should not be problems with unitarity of the present model, and

(**iv**) the model gives a physically appealing interpretation to the nature of thermal Hawking radiation: bosons emitted from the hole.

Furthermore, the model most obviously can be extended or generalized to more sophisticated mathematical disciplines.

Other theories have been studied extensively like the various versions of the inflationary model (IM) [13, 14, 15] and the minimally supersymmetric model (MSSM), see eg. [16]. The IM has an inflaton, the Higgs is a good candidate, with no new extra degrees of freedom but with (some six) fine tuning problems [17]. The MSSM doubles particle spectrum with 120 new parameters. The MSSM has candidates for dark matter. But neither IM or MSSM can solve the singularity or information loss problem, as far as the author is aware. It is, of course, ultimately an experimental verification what is needed to accept one model or some other.

3 Inflation

3.1 Introduction

Inflation [13, 14, 15] is perhaps one of the most natural way to stretch the initial quantum vacuum fluctuations to the size of the current Hubble patch, seeding the initial perturbations for the cosmic microwave background (CMB) radiation and large scale structure in the universe [19] (for a theoretical treatment, see [21]). Since inflation dilutes all matter it is pertinent that after the end of inflation the universe is filled with the right thermal degrees of freedom, i.e. the Standard Model (SM) degrees of freedom together with dark matter (for a review on pre- and post-inflationary dynamics, see [18]). The most economical, with no new degrees of freedom, way to achieve this would be via the vacuum energy density stored within the SM Higgs, whose properties are now being measured at the Large Hadron Collider (LHC) [23, 24]. Naturally, the decay of the Higgs would create all the SM quarks and leptons observed within the visible sector of the universe. Albeit, with just alone SM Higgs and minimal coupling to gravity, it is hard to explain the temperature anisotropy observed in the CMB radiation without invoking physics beyond the SM [3].

However, a very interesting possibility may arise within the SM if the Higgs were to couple to gravity non-minimally - such as in the context of extended inflation [27], which has recently received particular attention after the Higgs discovery at the LHC in the context of Higgs inflation [28]. By tuning this non-minimal coupling constant, ξ, between the Ricci scalar of the Einstein-Hilbert term and the SM Higgs, it is possible to explain sufficient amount of e-folds of inflation, and also fit other observables such as the amplitude of temperature anisotropy and the spectral tilt in the CMB data. Indeed, this is very nice an satisfactory, except that the non-minimal coupling, ξ, turns out to be very large (at the classical level $\xi \sim 10^4$) in order to explain the CMB observables. This effectively redefines the Planck's constant during inflation,

[3] Within supersymmetry it is indeed possible to invoke the flat direction composed of the Higgses to realize inflation with minimal gravitational interaction, see [26], which can explain the current CMB observations.

and invites new challenges for this model, whose consequences have been debated vigorously in many papers, such as [25].

One particular consequence of such large non-minimal coupling is that there is a new scale in the theory, $\bar{M}_{\rm Planck}/\sqrt{\xi}$, lower than the standard reduced Planck mass, $\bar{M}_{\rm Pl} \approx 2.4 \times 10^{18}$ GeV. Typically inflation occurs above this scale, the Higgs field takes a vacuum expectation value (VEV) above $\bar{M}_{\rm Pl}/\sqrt{\xi}$ in order to sustain inflation sufficiently. In fact, the inflaton potential, in the Einstein frame, approaches a constant plateau for sufficiently large field values. Effectively, the inflaton becomes a flat direction, where it does not cost any energy for the field to take any VEV beyond this cut-off.

Given this constraint on the initial VEV of the inflaton and the new scale, the authors wish to address two particularly relevant issues concerning the Higgs inflation model [28], one on the classical front and the other on the quantum front.

(**i**) Classically, a large VEV of the inflaton does not pose much of a problem as long as the initial energy density stored in the inflaton system, in the Einstein frame, is below the cut-off of the theory. Since, the potential energy remains bounded below this cut-off, the question remains - what should be the classical initial condition for the kinetic energy of the inflaton?

Apriori there is no reason for the inflaton to move slowly on the plateau, therefore the question the authors wish to settle in this paper is what should be the range of phase space allowed for a sustainable inflation to occur with almost a flat potential? The aim of this paper is to address this classical initial condition problem [4]. Here the authors strictly assume homogeneity of the universe from the very beginning; they do not raise the issue of initial homogeneity condition required for a successful inflation; this issue has been discussed earlier in a generic inflationary context in many classic papers (see [34, 35]). In this paper, instead the authors look into the possibility of initial phase space for a spatially flat universe, and study under what pre-inflationary conditions Higgs inflation could prevail.

(**ii**). At quantum level, the original Higgs model poses a completely different challenge. A large ξ will inevitably modify the initial action. One may argue that there will be quantum corrections to the Ricci scalar, R, such as a Higgs-loop correction - leading to a quadratic in curvature action, i.e. $R+\alpha R^2$ type correction, where α is a constant, whose magnitude the author shall discuss in this paper. The analysis is based on the renormalization group equations (RGEs) of the SM parameters and the gravitational interactions. The RGE analysis will yield a gravitational action which will become very similar to the Starobinsky type inflationary model [5] [5].

[4]Some single monomial potentials and exponential potentials exhibit a classic example of late time attractor where the inflaton field approaches a slow roll phase from large initial kinetic energy, see [32, 33].

[5]In principle, large ξ may also yield higher derivative corrections up to quadratic in order, see [36], and also higher curvature corrections, but in this paper, the author will consider for simplicity the lowest order corrections. We will argue that the αR^2 is necessarily generated unless one is at the critical point of Ref. [38] or invokes a fine-tuning on the initial values of the running parameters.

With the R^2 nonlinear modification of the EinsteinHilbert action, one and the same gravity takes care of both inflation and subsequent reheating of the Universe. Only one new degree of freedom a scalar in the gravity sector emerges and only one new parameter (in front of the R^2-term) is present. The Starobinsky action is

$$S = \int d^4x\sqrt{-g}\big(\frac{1}{16\pi G}R + \frac{1}{b}R^2\big) \tag{1}$$

with the dimensionless coupling $b = 6M^2/M^2_{\text{Planck}}$, where M is a constant of mass dimension one, $M_{\text{Planck}} = G^{-1/2}$, G is the Newton's constant with scale dependence and g is the determinant of the metric.

Another important property of the Starobinsky action is that making a non-perturbative renormalization group (RG) analysis it leads to asymptotically safe (AS) gravity. There exists a non-trivial, or non-Gaussian, UV fixed point, where G is asymptotically safe and the R^2 coupling b vanishes. This vanishing of the coupling b in the UV turns out to be of great importance for a succesful inflationary behavior.

Asymptotic safety was first introduced by Weinberg [10] and it states that a UV complete theory for gravity is obtained by assuming that gravity is non-perturbatively renormalizable through the existence of a non-trivial interacting fixed point under the RG. The starting point for RG calculations is an exact renormalization group equation (ERGE) in Wilsonian context [6].

The aim of [9] will be to address both the classical and quantum issues. The latter issue is more of a challenge, but the authors will perform both of them carefully.

They, [9], briefly begin our discussion with essential ingredients of Higgs inflation in section 3.2, then they discuss the classical pre-inflationary initial conditions for Higgs inflation in section 4. In this section, they discuss both analytical 4.1, and numerical results 4.2. In section 5, they discuss the quantum correction to the original Higgs inflation model, i.e. they discuss the RGEs of the Planck mass in subsection 5.1, SM parameters in 5.2, and the gravitational correction arising due to large ξ in subsection 5.3, respectively.

3.2 Higgs Inflaton

The Higgs inflation model is defined by the action

$$\mathcal{S} = \int d^4x\sqrt{-g}\big[\mathcal{L}_{\mathcal{SM}} - \big(\frac{\bar{M}^2_{\text{Planck}}}{2} + \xi|\mathcal{H}|^2\big)R\big] \tag{2}$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian minimally coupled to gravity, ξ determines the non-minimal coupling between the the Higgs and the Ricci scalar R and $\mathcal{H}$ is the Higgs doublet. The part of the action that depends on the metric and the Higgs field only (the scalar-tensor part) is

$$\mathcal{S}_{st} = \int d^4x\sqrt{-g}\big[|\partial\mathcal{H}|^2 - V - \big(\frac{\bar{M}^2_{\text{Planck}}}{2} + \xi|\mathcal{H}|^2\big)R\big] \tag{3}$$

where $V = \lambda(|H|^2 - \nu^2/2)^2$ is the Higgs potential ans ν is the electroweak Higgs VEV. Inflation requires a large non-minimal coupling $\xi > 1$.

The non-minimal coupling $-\xi|\mathcal{H}|^2 R$ can be eliminated through a conformal transformation

$$g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi|\mathcal{H}|^2}{\bar{M}^2_{\text{Planck}}}. \tag{4}$$

The original frame, where the Lagrangian has the form 2, is called Jordan frame, while the one where gravity is canonically normalized is called the Einstein frame. In the unitary gauge, where the only scalar field is the radial mode $\phi = \sqrt{2|\mathcal{H}|^2}$, one has

$$S_{\text{st}} = \int d^4x\sqrt{-g}\left[K\frac{(\partial\phi)^2}{2} - \frac{V}{\Omega^4} - \frac{\bar{M}^2_{\text{Planck}}}{2}R\right], \tag{5}$$

where $K = \left(\Omega^2 + 6\xi^2\phi^2/\bar{M}^2_{\text{Planck}}\right)/\Omega^4$. The non-canonical Higgs kinetic term can be made canonical through the fiels redefinition $\phi = \phi(\chi)$ defined by

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/\bar{M}^2_{\text{Planck}}}{\Omega^4}}, \tag{6}$$

with the conventional condition $\phi(\chi = 0) = 0$. One can find a closed expression of χ as a function of ϕ:

$$\chi(\phi) = \bar{M}_{\text{Planck}}\sqrt{\frac{1+6\xi}{\xi}}\sinh^{-1}x_1 - \sqrt{6}\bar{M}_{\text{Planck}}\tanh^{-1}x_2.$$

where $x_1 = \left[\frac{\sqrt{\xi(1+6\xi)}\phi}{\bar{M}_{\text{Planck}}}\right]$ and $x_2 = \left[\frac{\sqrt{6}\xi\phi}{\sqrt{\bar{M}^2_{\text{Planck}} + \xi(1+6\xi)\phi^2}}\right]$

Thus, χ feels a potential

$$U = \frac{V}{\Omega^4} = \frac{\lambda(\phi(\chi)^2 - v^2)^2}{4(1 + \xi\phi(\chi)^2/\bar{M}^2_{\text{Planck}})^2} \tag{7}$$

Let us now recall how slow-roll inflation emerges. From (6) and (7) it follows [28] that U is exponentially flat when $\chi \gg \bar{M}_{\text{Planck}}$, which is the key property to have inflation. Indeed, for such high field values the slow-roll parameters

$$\epsilon = \frac{\bar{M}^2_{\text{Planck}}}{2}\left(\frac{1}{U}\frac{dU}{d\chi}\right)^2, \quad \eta = \frac{\bar{M}^2_{\text{Planck}}}{U}\frac{d^2U}{d\chi^2} \tag{8}$$

are guaranteed to be small. Therefore, the region in field configurations where $\chi > \bar{M}_{\text{Planck}}$ (or equivalently [28] $\phi > \bar{M}_{\text{Planck}}/\sqrt{\xi}$) corresponds to inflation. We will investigate whether successful sow-roll inflation emerges also for large initial field kinetic energy in the next section. Here it is simply assumed that the time derivatives are small.

All the parameters of the model can be fixed through experiments and observations, including ξ [28, 29]. ξ can be obtained by requiring that the measured power spectrum [22],

$$P_R = \frac{U/\epsilon}{24\pi^2 \bar{M}^4_{\rm Planck}} = (2.14 \pm 0.05) \times 10^{-9}, \tag{9}$$

is reproduced for a field value $\phi = \phi_{\rm b}$ corresponding to an appropriate number of e-folds of inflation [29]:

$$N = \int_{\phi_{\rm end}}^{\phi_{\rm b}} \frac{U}{\bar{M}^2_{\rm Planck}} \left(\frac{dU}{d\phi}\right)^{-1} \left(\frac{d\chi}{d\phi}\right)^2 d\phi \approx 59, \tag{10}$$

where $\phi_{\rm end}$ is the field value at the end of inflation, that is

$$\epsilon(\phi_{\rm end}) \approx 1. \tag{11}$$

For $N = 59$, by using the classical potential the authors obtain

$$\xi = (5.02 \pm 0.06) \times 10^4 \sqrt{\lambda}, \qquad (N = 59) \tag{12}$$

where the uncertainty corresponds to the experimental uncertainty in Eq. (9). Note that ξ depends on N:

$$\xi = (4.61 \pm 0.06) \times 10^4 \sqrt{\lambda} (N = 54), \quad \xi = (5.43 \pm 0.06) \times 10^4 \sqrt{\lambda} (N = 64). \tag{13}$$

This result indicates that ξ has to be much larger than one because $\lambda \sim 0.1$ (for precise determinations of this coupling in the SM see Refs. [50, 46]).

4 Pre-inflationary dynamics: classical analysis

Let us now analyze the dynamics of this classical system in the homogeneous case without making any assumption on the initial value of the time derivative $\dot{\chi}$. We will assume that the universe is sufficiently homogeneous to begin inflation.

In the Einstein frame $S_{\rm st}$ is given by:

$$S_{\rm st} = \int d^4x \sqrt{-g} \left[\frac{(\partial\chi)^2}{2} - U - \frac{\bar{M}^2_{\rm Planck}}{2} R\right], \tag{14}$$

where U is the Einstein frame potential given in Eq. (7).

Let us assume a universe with three dimensional translational and rotational symmetry, that is a Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{15}$$

with $k = 0, \pm 1$.

Then the Einstein equations and the scalar equations imply the following equations for $a(t)$ and the spatially homogeneous field $\chi(t)$

$$\ddot{\chi} + 3H\dot{\chi} + U' = 0, \tag{16}$$

$$\frac{\dot{a}^2 + k}{a^2} - \frac{\dot{\chi}^2 + 2U}{6\bar{M}_{\rm Pl}^2} = 0, \tag{17}$$

$$\frac{k}{a^2} - \dot{H} - \frac{\dot{\chi}^2}{2\bar{M}_{\rm Pl}^2} = 0. \tag{18}$$

where $H = \dot{a}/a$, a dot denotes a derivative with respect to t and a prime is a derivative with respect to χ.

Notice that Eq. (16) tells us that χ cannot be constant before inflation unless U is flat. From Eqs. (16) and (17) one can derive (18), which is therefore dependent.

Thus, one has to solve the following system with initial conditions

$$\begin{cases} \dot{\Pi} + 3H\Pi + U' = 0, & \Pi(\bar{t}) = \overline{\Pi}, \\ \dot{\chi} = \Pi, & \chi(\bar{t}) = \overline{\chi}, \\ \dot{a}^2 + k = \frac{a^2}{6\bar{M}_{\rm Pl}^2}(\Pi^2 + 2U), & a(\bar{t}) = \bar{a}, \end{cases} \tag{19}$$

where $\bar{t}$ is some initial time before inflation and $\overline{\chi}$, $\bar{\Pi}$ and $\bar{a}$ are the initial conditions for the three dynamical variables. In the case $k = 0$ the previous system can be reduced to a single second order equation. Indeed, by setting $k = 0$ in Eq. (17) and inserting it in Eq. (16), one obtains

$$\ddot{\chi} + \sqrt{\frac{3\dot{\chi}^2 + 6U}{2\bar{M}_{\rm Planck}^2}}\,\dot{\chi} + U' = 0, \qquad (k = 0). \tag{20}$$

This equation has to be solved with two initial conditions (for χ and $\dot{\chi}$). The initial condition for a is not needed in this case as its overall normalization does not have a physical meaning for $k = 0$.

We confine our attention to the regime where quantum Einstein gravity corrections are small:

$$U \ll \bar{M}_{\rm Planck}^4, \quad \dot{\chi}^2 \ll \bar{M}_{\rm Planck}^4, \quad \frac{|k|}{a^2} \ll \bar{M}_{\rm Planck}^2 \tag{21}$$

such that one can ignore the details of the ultraviolet (UV) completion of Einstein gravity. However, the authors do not always require to be initially in a slow-roll regime. The first and second conditions in (21) come from the requirement that the energy-momentum tensor is small (in units of the Planck scale) so that it does not source a large curvature; the third condition ensures that the three-dimensional curvature is also small. The first condition is automatically fulfilled by the Higgs inflation potential, Eq. (7): the quartic coupling λ is small [49, 50, 46] and the non-minimal coupling ξ is large (see Eq. (12)). The second and third conditions in (21) are implied by the requirement of starting from an (approximately) de Sitter space, which is maximally symmetric; therefore the authors do not consider them as a fine-tuning in the initial conditions. In de Sitter one has to set $k = 0$ and $\dot{H} = 0$, which then implies $\dot{\chi} = 0$ from Eq. (18). Notice also that one cannot start from an exact de Sitter, given Eq. (16): the potential U is almost, but not exactly flat in the large field case (see Eq. (7)).

In order for the Higgs to trigger inflation sooner or later one should have a slow-roll regime, where the kinetic energy is small compared to the potential energy, $\dot{\chi}^2/2 \ll U$, and the field equations are approximately

$$\frac{\dot{a}^2 + k}{a^2} \approx \frac{U}{3\bar{M}_{\rm Pl}^2}, \quad \dot{\chi} \approx -\frac{1}{3H}U', \quad \text{(slow-roll equations)}. \tag{22}$$

The conditions for this to be true are

$$\dot{\chi}^2 \ll 2U, \quad |\ddot{\chi}| \ll 3|H\dot{\chi}| \qquad \text{(slow-roll regime)}. \tag{23}$$

We will use these conditions rather than the standard $\epsilon \ll 1$ and $\eta \ll 1$ as one does not assume a priori a small kinetic energy.

4.1 Analytic approximations in simple cases

Let us assume, for simplicity, that the parameter k in the FRW metric vanishes, i.e. a spatially flat metric, and consider the case $\dot{\chi}^2 \gg U$, such that the potential energy can be neglected compared to the kinetic energy. In this case, combining Eqs. (17) and (18) gives

$$\dot{H} + 3H^2 + \frac{2k}{a^2} = 0, \qquad (\dot{\chi}^2 \gg U), \tag{24}$$

which for spatially flat curvature, $k = 0$, leads to

$$H(t) = \frac{\bar{H}}{1 + 3\bar{H}(t - \bar{t})}, \qquad (\dot{\chi}^2 \gg U,\ k = 0), \tag{25}$$

where $\bar{H} = H(\bar{t})$. By inserting this result into Eq. (18), one finds

$$\dot{\chi}^2 = \frac{6\bar{M}_{\rm Planck}^2 \bar{H}^2}{\left[1 + 3\bar{H}(t - \bar{t})\right]^2}, \qquad (\dot{\chi}^2 \gg U,\ k = 0). \tag{26}$$

that is the kinetic energy density scales as $1/t^2$ by taking into account the time dependence of H. This result [34] tells us that an initial condition with large kinetic energy is attracted towards one with smaller kinetic energy, but it also shows that dropping the potential energy cannot be a good approximation for arbitrarily large times. Moreover, notice that Eqs. (25) and (26) imply

$$\ddot{\chi} = -3H\dot{\chi} \tag{27}$$

so the dynamics is *not* approaching the second condition in (23). Therefore, the argument above is not conclusive and one needs to solve the equations with U included in order to see if the slow-roll regime is an attractor.

4.2 Numerical studies

We studied numerically the system in (19) assuming $k = 0$; this case is realistic and is the simplest one: it does not require an initial condition for a. We found that even for an initial kinetic energy density $\overline{\Pi}^2$ of order $10^{-3}\bar{M}_{\rm Planck}^4$ (which they regard as the maximal order of magnitude to have negligibly small quantum gravity), one should start from an initial field value $\overline{\chi}$ of order $10\bar{M}_{\rm Planck}$ to inflate the universe for an appropriate number of e-folds, i.e. $N = 59$. This value of $\overline{\chi}$ is only one order of magnitude bigger than the one needed in the ordinary case, $\overline{\Pi}^2 \ll U(\overline{\chi}) \sim 10^{-10}\bar{M}_{\rm Planck}$, where the initial kinetic energy is much smaller than the potential energy.

Fig. 1 presents these results more quantitatively. There the initial conditions for $\overline{\Pi}$ have been chosen to be negative because positive values favor slow-roll even with respect to the case where the initial kinetic energy is much smaller than the potential energy: this is because the potential in Eq. (7) is an increasing function of χ for $\chi \gg v$.

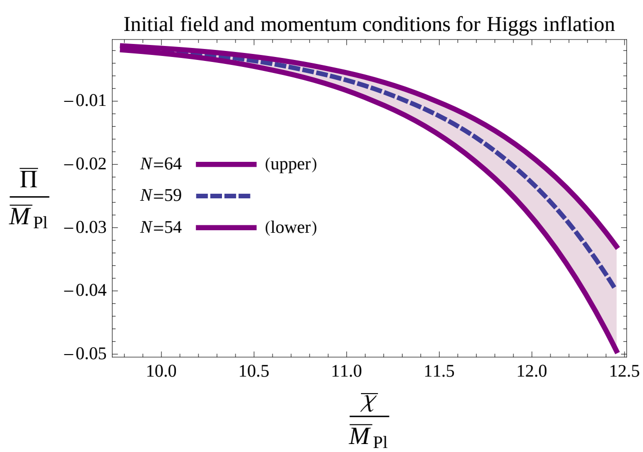

Figure 1: Initial conditions $\overline{\chi}$ and $\overline{\Pi}$ for the Higgs field and its momentum $\Pi = \dot{\chi}$ respectively. The thickness of the lines corresponds to 2σ uncertainty in the value of the power spectrum, Eq. (9).

We conclude that at the classical level Higgs inflation does not suffer from a worrisome fine-tuning problem for the initial conditions.

5 Quantum Corrections

The theory of 2 is not renormalizable. This means that quantum corrections $\Delta\Gamma$ at a given order in perturbation theory can generate terms that are not combinations of those in the classical action S. In formulae the (quantum) effective action is given by:

$$\Gamma = S + \Delta\Gamma \tag{28}$$

where $S + \Delta\Gamma$ cannot generically be reproduced by substituting the parameters in S with some renormalized quantities.

A UV completion requires the existence of additional degrees of freedom that render the theory renormalizable or even finite. Much below the scale of this new physics, the effective action can be approximated by an expansion of the form

$$\Delta\Gamma = \int d^4x\sqrt{-g}\,(\delta\mathcal{L}_2 + \delta\mathcal{L}_4 + \dots) \tag{29}$$

where $\delta\mathcal{L}_n$ represents a combination of dimension n operators.

We consider the one-loop corrections generated by all fields of the theory, both the matter fields and gravity. Our purpose is to apply it to inflationary and pre-inflationary dynamics. At this order all divergences of the theory can be reabsorbed by operators of dimension 4 or lower. We therefore use the approximation

$$\Delta\Gamma \approx \int d^4x\sqrt{-g}\,(\delta\mathcal{L}_2 + \delta\mathcal{L}_4)\,. \tag{30}$$

We have

$$\delta\mathcal{L}_2 = -\frac{\delta\bar{M}^2_{\text{Planck}}}{2}R \tag{31}$$

$$\delta\mathcal{L}_4 = \alpha R^2 + \beta\left(\frac{1}{3}R^2 - R_{\mu\nu}R^{\mu\nu}\right) + \delta Z_{\mathcal{H}}|\partial\mathcal{H}|^2 - \delta\lambda|\mathcal{H}|^4 - \delta\xi|\mathcal{H}|^2 R + \dots$$

where for each parameter p_c in the classical action the authors have introduced a corresponding quantum correction δp and the dots represent the additional terms due to the fermions and gauge fields of the SM. Notice that they have added general [6] quantum corrections that are quadratic in the curvature tensors as they are also possible dimension 4 operators. These are parameterized by two dimensionless couplings α and β. We have neglected v as it is very small compared to inflationary energies.

Our purpose is now to determine the RGEs for the renormalized couplings

$$p = p_c + \delta p$$

[6] $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is a linear combination of R^2, $R_{\mu\nu}R^{\mu\nu}$ and a total derivative.

as well as for the new couplings α and β generated by quantum corrections. Indeed the RGEs encode the leading quantum corrections. We will use the dimensional regularization (DR) scheme to regularize the loop integrals and the modified minimal subtraction ($\overline{\text{MS}}$) scheme to renormalize away the divergences. This as usual leads to a renormalization scale that one denotes with $\bar{\mu}$.

5.1 RGE of the Planck mass

In the absence of the dimensionful parameter v, the only possible contributions to the RGE of $\bar{M}_{\text{Planck}}$ are the rainbow and the seagull diagram contributions to the graviton propagator due to gravity itself: the rainbow topology is the one of Fig. 2, while the seagull one is obtained by making the two vertices of Fig. 2 coincide without deforming the loop.

The seagull diagram vanishes as it is given by combinations of loop integrals of the form

$$\int d^d k \frac{k_\mu k_\nu}{k^2 + i\epsilon}, \qquad \int d^d k \frac{1}{k^2 + i\epsilon}, \tag{32}$$

where d is the space-time dimension in DR. These types of loop integrals vanish in DR. The rainbow diagram does not contribute to the RGE of $\bar{M}_{\text{Planck}}$ either. The reason is that each graviton propagator carries a factor of $1/\bar{M}^2_{\text{Planck}}$ and each graviton vertex carries a factor of $\bar{M}^2_{\text{Planck}}$ (because the graviton kinetic term $-\bar{M}^2_{\text{Planck}}R/2$ is proportional to $\bar{M}^2_{\text{Planck}}$): the rainbow diagram has two graviton propagators and two vertices, therefore this contribution is dimensionless and cannot contribute to the RGE of a dimensionful quantity. We conclude that $\bar{M}_{\text{Planck}}$ does not run in this case. This argument assumes that the graviton wave function renormalization is trivial, which the authors have checked to be the case at the one-loop level at hand.

5.2 RGEs of SM parameters

Having neglected v all SM parameters are dimensionless and thus cannot receive contributions from loops involving graviton propagators (that carry a factor of $1/\bar{M}^2_{\text{Planck}}$). Therefore, the SM RGEs apply and can be found (up to the three-loop level) in a convenient form in the appendix of Ref. [46].

5.3 RGEs of gravitational couplings

Finally, the authors consider the RGEs for ξ, α and β. The one of ξ does not receive contribution from loops involving graviton propagators as they carry a factor of $1/\bar{M}^2_{\text{Planck}}$ and ξ is dimensionless. So the RGE of ξ receives contribution from the SM couplings and ξ itself only [47, 48]:

$$(4\pi)^2 \frac{d\xi}{d\ln\bar{\mu}} = (1+6\xi)\left(y_t^2 - \frac{3}{4}g_2^2 - \frac{3}{20}g_1^2 + 2\lambda\right), \tag{33}$$

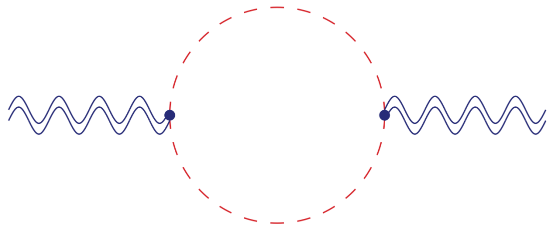

Figure 2: The leading loop diagram that generates the R^2 term in the effective action. The dashed lines correspond to the Higgs field, while the external double lines represent gravitons.

where y_t is the top Yukawa coupling and g_3, g_2 and $g_Y = \sqrt{3/5}g_1$ are the gauge couplings of $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ respectively.

The RGEs of α and β receive two contributions: one from pure gravity loops (a rainbow and a seagull diagram), which they denote with β^g, and one from matter loops, β^m:

$$(4\pi)^2 \frac{d\alpha}{d\ln\bar{\mu}} = \beta_\alpha^g + \beta_\alpha^m, (4\pi)^2 \frac{d\beta}{d\ln\bar{\mu}} = \beta_\beta^g + \beta_\beta^m. \tag{34}$$

One finds [44]

$$\beta_\alpha^g = -\frac{1}{4}, \qquad \beta_\beta^g = \frac{7}{10}, \tag{35}$$

and in the SM [48]

$$\beta_\alpha^m = -\frac{(1+6\xi)^2}{18}, \qquad \beta_\beta^m = \frac{283}{60}. \tag{36}$$

5.4 Quantum corrections: Higgs-to-Starobinsky inflation

Let us start this section by commenting on fine-tunings in the couplings, a relevant issue as inflation is motivated by cosmological fine-tuning problems. The first equation in (36) has an important implication; the Feynman diagram that leads to this contribution is given in Fig. 2. Generically Higgs inflation requires a rather large value of ξ, which implies a strong naturalness bound

$$|\alpha| \gtrsim \frac{\xi^2}{8\pi^2}. \tag{37}$$

A large value of ξ is necessary at the classical level (see Eq. (12) and the corresponding discussion). At quantum level one can obtain smaller values, but still $\xi \gg 1$ [45, 40].

A possible exception is Higgs inflation at the critical point [38]; however, $\xi \gtrsim 10$ to fulfill the most recent observational bounds, $r \lesssim 0.1$ [43]. Moreover, in previous analysis of Higgs inflation at the critical point the wave function renormalization of the Higgs field has been neglected, an approximation that is under control when ξ is large [45].

Since $\xi \gg 1$ generically, (37) indicates that the quantum mechanically generated R^2 term may participate in inflation. Therefore, the authors approximate the effective action as follows:

$$\Gamma = \int d^4x\sqrt{-g}\left[\mathcal{L}_{\rm SM}^{\rm eff} - \left(\frac{\bar{M}_{\rm Planck}^2}{2} + \xi|\mathcal{H}|^2\right) R + \alpha R^2\right], \tag{38}$$

where the $\mathcal{L}_{\rm SM}^{\rm eff}$ part corresponds to the effective SM action. The scalar-tensor effective action is

$$\Gamma_{\rm st} = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2 - V_{\rm eff} - \frac{1}{2}\left(\bar{M}_{\rm Planck}^2 + \xi\phi^2\right) R + \alpha R^2\right].$$

Here the author has neglected the wave function renormalization of the Higgs because ξ is large and one have fixed the unitary gauge. Moreover, $V_{\rm eff}$ is the SM effective potential.

As well-known, the R^2 term corresponds to an additional scalar. In order to see this one can add to the action the term

$$-\int d^4x\sqrt{-g}\;\alpha\left(R + \frac{\omega}{4\alpha}\right)^2,$$

where ω is an auxiliary field: indeed by using the ω field equation one obtains immediately that this term vanishes. On the other hand, after adding that term

$$\Gamma_{\rm st} = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2 - V - \frac{f}{2}R - \frac{\omega^2}{16\alpha}\right], \tag{39}$$

where $f = \bar{M}_{\rm Planck}^2 + \omega + \xi\phi^2$.

Note that there is the non-canonical gravitational term $-fR/2$. Like was done in section 3.2, one can go to the Einstein frame (where one has instead the canonical Einstein term $-\bar{M}_{\rm Planck}^2 R_E/2$) by performing a conformal transformation,

$$g_{\mu\nu} \to \frac{\bar{M}_{\rm Planck}^2}{f} g_{\mu\nu}. \tag{40}$$

One obtains [39]

$$\Gamma_{\rm st} = \int d^4x\sqrt{-g}\left[\mathcal{L}_{\phi z} - U_{\rm eff} - \frac{\bar{M}_{\rm Planck}^2}{2}R\right], \tag{41}$$

where

$$\mathcal{L}_{\phi z} = \frac{6\bar{M}_{\rm Planck}^2}{z^2}\frac{(\partial\phi)^2 + (\partial z)^2}{2},$$

$$U_{\rm eff}(\phi,z) = \frac{36\bar{M}_{\rm Planck}^4}{z^4}\left[V_{\rm eff}(\phi) + \frac{1}{16\alpha}\left(\frac{z^2}{6} - \bar{M}_{\rm Planck}^2 - \xi\phi^2\right)^2\right]$$

and the new scalar $z = \sqrt{6f}$ has introduced.

Notice that when $\alpha \to 0$, the potential $U_{\rm eff}$ forces $z^2 = 6(\bar{M}_{\rm Planck}^2 + \xi\phi^2)$ and one recovers the Higgs inflation action. For large α (as dictated by a large ξ), this conclusion cannot be reached. The absence of runaway directions in $U_{\rm eff}$ requires $\alpha > 0$ and $\lambda > 0$, which is possible within the SM, although in tension[7] with the measured values of some electroweak observables [46, 40].

Ref. [39] studied a system that includes (41) as a particular case[8]. It was found that inflation is never dominated by the Higgs, because its quartic self-coupling λ (which one assumes to be positive for the argument above) is unavoidably larger than the other scalar couplings, taking into account its RG flow. Even assuming that the Higgs has a dominant initial value, in our two-field context inflation starts only after the field evolution has reached an attractor where ϕ is subdominant. We have checked that this happens also when ξ is large.

Therefore, the predictions are closer to those of Starobinsky inflation, which are distinct from the Higgs inflation ones [37].

5.5 Conclusions on Quantum Corrections

In conclusion, the authors have studied two different aspects of standard Higgs inflation - to seek how fine-tuned the initial conditions should be to fall into a slow-roll attractor solution in an approximate exponentially flat Higgs potential in the Einstein frame. We started with a large kinetic energy, and they found that for an initial kinetic energy density of order $10^{-3}\bar{M}_{\rm Planck}^4$ (this is the maximum allowed order of magnitude to avoid quantum gravity corrections) the inflaton VEV should be $\sim 10\bar{M}_{\rm Planck}$ to sustain inflation long enough to give rise to enough e-folds.

In the second half of the paper, the authors focused on the question of viability of Higgs inflation in presence of large ξ, typically required for explaining the observed CMB power spectrum and the right tilt. We found that one would incur quantum corrections (at the lowest order) to the Ricci scalar, i.e. quadratic in Ricci scalar, αR^2, with a universality bound on α given by Eq. (37), unless the initial value of α is fine-tuned. Therefore, a natural outcome of SM Higgs inflation is effectively a Starobinsky-type inflation model where both the Higgs and a new scalar degree of freedom play key contributions to the curvature perturbations. For large $\xi \sim 10^2 - 10^4$, the potential would be effectively determined by the Starobinsky scalar component z, and the CMB predictions be different from that of Higgs inflation.

[7] Such tension, however, can be be eliminated by adding to the SM well-motivated new physics, which solve its observational problems [41].

[8] Ref. [39] has an additional scalar which, however, can be consistently decoupled by taking its mass large enough. For another treatment of the dynamical system in (41) see Ref. [42].

6 Condensed Boson Model

6.1 Bose-Einstein Condensate

It is assumed that one, or more gravon(s) behave as a nucleus around which gravitons condense. Gravons form the seed for black holes and dark matter in our scheme. Dark matter exist abundantly all over the galaxies. We must look for a process for the formation of core surrounded by a graviton condensate. This process takes place in the early stage of inflation when the Starobinsky R^2 dominates in the action.

A Bose-Einstein condensation model has been studied by Dvali and Gomez [51], see also [52]. Freely citing them, we give below a brief summary of their model to see the mechanism of condensation.

Graviton interaction is the starting point. The graviton-graviton interaction dimensionless coupling constant is

$$\alpha = L_{\mathrm{Planck}}^2 / L^2 \tag{42}$$

where L is a characteristic wave length of the gravitons participating in the interaction and L_{Planck} is the Planck length. Newton's constant G_{N} is related to the Planck length by $L_{\mathrm{Planck}}^2 = \hbar G_{\mathrm{N}}$. The physical meaning of the coupling α is understood as the relativistic generalization of Newtonian attraction between two gravitons. The attraction between two non-relativistic massive particles of mass m can be written in terms of α as

$$V(r)_{\mathrm{Newton}} = -\hbar \frac{\alpha}{r} \tag{43}$$

with the only difference that for a massive particle m is its Compton wave length, $L = \hbar/m$. The difference for gravitons is that the role of the Compton wave length is replaced by the actual wave length.

Gravitons can self-condense into black holes. To see this let us localize as many soft gravitons as possible around a core within a region of space of size L. We try to form a condensate of gravitons of characteristic wave length L by gradually increasing the occupation number N. For small N the gravitons behave like photons, and the condensate requires external binding forces. As one increasess N the effects of the graviton interaction become large. Individual gravitons feel strong collective binding potential and for the critical occupation number

$$N = N_c = \frac{1}{\alpha} \tag{44}$$

the graviton condensate becomes self-sustained. The condition for this can be obtained by equating the kinetic energies of individual gravitons, $E_k = \hbar/L$, with the collective binding potential $V = -\alpha N \hbar/L$

$$E_k + V = (1 - \alpha N)\frac{\hbar}{L} = 0 \tag{45}$$

The concept of maximal packing is that the system is so densely packed that its defining characteristics becomes simply N. In particular

$$L = \sqrt{N} L_{\text{Planck}}, \quad \alpha = \frac{1}{N}. \tag{46}$$

For gravitons being in an overpacked point means that further increase of N without increasing L becomes impossible. Any further increase of N results in the increase of the wave length in such a way that the system stays at the maximal packing point (46). Equation (45) indicates that the critical point (46) can be achieved for arbitrary N, but decrease of L beyond $L < \sqrt{N} L_{\text{Planck}}$ would result into an even stronger bound system. This collapse of L can happen but it cannot take the system out of the critical point (46). The reason is that the decrease of L is balanced by the decrease of N due to quantum depletion and leakage of the condensate. The condensate slowly collapses and it looses gravitons at the same rate. So the systems always stays at the critical point (46).

The reason for the leakage is that due to the interaction with the other gravitons some of the gravitons get excited above the ground state. The ground state energy is within $1/N$ from the escape level and the gravitons gaining energies above this tiny gap leave the condensate for the continuum. The condensate starts to leak with a depletion rate essentially given by

$$\Gamma_{leakage} = \frac{1}{\sqrt{N} L_{\text{Planck}}} + L_{\text{Planck}}^{-1} \mathcal{O}(N^{-3/2}) \tag{47}$$

This can be understood from the following. Since the graviton-graviton coupling in the condensate is $1/N$ the probability for any pair of gravitons to scatter is suppressed by the factor $1/N^2$, but this suppression is compensated by a combinatoric factor $\sim N^2$ counting the number of available graviton pairs.

The quantum depletion rate translates into the following leakage law

$$\dot{N} = -\frac{1}{\sqrt{N} L_{\text{Planck}}} + L_{\text{Planck}}^{-1} \mathcal{O}(N^{-3/2}) \tag{48}$$

where the dot means time derivative. This quantum leakage of the graviton condensate becomes Hawking radiation in the semi-classical limit, which is defined as the following double scaling limit

$$N \to \infty, \quad L_{\text{Planck}} \to 0, \quad L = \sqrt{N} L_{\text{Planck}} = \text{finite}, \quad \hbar = \text{finite} \tag{49}$$

Thus the semi-classical limit is the limit in which all the quantum physics of the condensate decouples as $1/N \to 0$ and becomes impossible to resolve. The condensate becomes now a collection of infinite number of infinitely soft non-interacting bosons.

The thermality of Hawking radiation follows from the leakage law. Rewriting N in terms of the black hole mass one gets the Stefan-Boltzmann law for a black hole with Hawking temperature $T = \hbar/L$

$$\dot{M} = -\frac{\hbar}{L^2}. \tag{50}$$

The exponential suppression of higher frequencies, usually attributed to the thermality of the source, follows from the combinatorics of the quantum depletion. The underlying quantum physics of this thermal-like spectrum has nothing to do with the thermality of the source, since condensate is in fact cold, but with the underlying quantum physics of BEC being at the overpacked critical point.

6.2 Microscopic Model

In [51] a simple prototype model, based on standard theory of Bogoliubov-de Gennes equation, is considered, which captures the key features of the phenomenon. Let $\Psi(x)$ be a field operator that describes the order parameter of the Bose gas. The particle number density is given by the correlator $n(x) = \langle \Psi(x)\Psi(x) \rangle$. A simple Hamiltonian that takes into account the self-interaction of the order parameter is

$$H = -\hbar L_0 \int d^3x \Psi(x) \nabla^2 \Psi(x) - g \int d^3x \Psi(x)^+ \Psi(x)^+ \Psi(x) \Psi(x) \tag{51}$$

where L_0 is a parameter of length dimensionality and g is an attractive interaction coupling constant of dimensionality $[\text{length}]^3[\text{mass}]$. We set the system in a finite box of size R with periodic boundary conditions $\Psi(0) = \Psi(2\pi R)$. The normalization condition is

$$\int d^3x \Psi^+ \Psi = N. \tag{52}$$

Performing the plane-wave expansion

$$\Psi = \sum_k \frac{a_k}{\sqrt{V}} \exp^{\mathrm{ikx/R}} \tag{53}$$

where $V = (2\pi R)^3$ is the volume and $a_k, {a_k}^+$ are particle creation and annihilation operators, The rewritten Hamiltonian is

$$\mathcal{H} = \sum_k k^2 {a_k}^+ a_k - \frac{1}{4} \alpha \sum_k a^+_{k+p} a^+_{k'-p} a_k a_{k'} \tag{54}$$

where $\alpha = \frac{4gR^2}{\hbar V L_0}$ and $\mathcal{H} = \frac{R^2}{\hbar L_0} H$.

We will now study the spectrum of low lying excitatons about a uniform BEC. We assume that most particles occupy the $K = 0$ level and study the small quantum fluctuations about this state. The spectrum of fluctuations is determined by the Bogoliubov-De Gennes equation. In a first approximation one can use the Bogoliubov replacement

$${a_0}^+ = a_0 = \sqrt{N_0} \sim \sqrt{N} \tag{55}$$

of the ground state creation and annihilation operators into classical c-numbers. This approximation relies on taking $N \gg 1$ and $\hbar \neq 0$. Keeping only terms up to quadratic order in a_k^+, a_k for $k \neq 0$, and taking into account the normalization condition (52)

$$a_0 a_0 + \sum_{k \neq 0} a_k^+ a_k = N \tag{56}$$

leads to the following Hamiltonian describing the small quantum fluctuations

$$\mathcal{H} = \sum_{k \neq 0} (k^2 + \alpha N/2) a_k^+ a_k - \frac{1}{4} \alpha N \sum_{k \neq 0} (a_k^+ a_{-k} + a_k a_{-k}) \tag{57}$$

In order to diagonalize the Hamiltonian one performs a Bogoliubov transformation

$$a_k = u_k b_k + v_k^* b_k^+ \tag{58}$$

The Bogoliubov coefficients are given by

$$u, v = \pm \frac{1}{2} \Big(\frac{k^2 - \alpha N/2}{\epsilon(k)} \pm 1 \Big) \tag{59}$$

leading to the following spectrum of the Bogoliubov modes

$$\epsilon(k) = \sqrt{k^2 (k^2 - \alpha N)} \tag{60}$$

The Hamiltonian in terms of b-particles is diagonal and has the following form

$$\mathcal{H} = \sum_k \epsilon(k) b_k^+ b_k + \text{constant} \tag{61}$$

As is seen in (60) the first Bogoliubov energy vanishes for

$$N = N_c = \frac{1}{\alpha} \tag{62}$$

and the system undergoes a quantum phase transition. The essence of this phase transition is that fot $N > N_c$ the first Bogoliubov level becomes tachyonic and the uniform BEC is no longer a ground state. Taking into account $\frac{1}{N}$-corrections it is clear that the gap between the uniform ground state and the Bogoliubov modes collapses to $\frac{1}{N}$ and becomes extremely cheap to excite these modes. So by quantum fluctuations the system starts to be populated by Bogoliubov modes easily. This means that the condensate starts to undergo a very efficient quantum depletion. The number density of the depleted a-particles to each k-levels are given by

$$n_k = |v_k|^2 \tag{63}$$

Since n_k decreases as $1/|k|^4$ for large $|k|$, the total number of depleted particles is well approximated by the first -level depletion

$$\Delta N \sim n_1 = \Big(\frac{1-\alpha n/2}{\sqrt{1-\alpha N}} - 1\Big) \sim \sqrt{N} \tag{64}$$

The striking similarity of the above BEC physics with the black hole quantum picture suggests that in both cases we are dealing with one and the same physics of a quantum phase transition. Indeed the physics of the graviton condensate is reproduced for the particular case of $L_0 = R = L$ and $g = \hbar L_{\rm Planck}^2$.

The criticality condition (62) is nothing but the self-sustainability condition (44) which implies that the graviton condensate is maximaly packed (46). The energy gap too the first Bogoliubov level is given by

$$\epsilon_1 = \frac{\hbar}{L\sqrt{N}} = \frac{\hbar}{N L_{\rm Planck}} \tag{65}$$

This expression summarizes the remarkable property of maximally packed systems: The energy cost of a collective excitation can be made arbitrarily low by increasing the occupation number of bosons in the condensate.

Thus by increasing N one can encode essentially unlimited amount of information in these modes. In the semi-classical limit (49) the energy gap collapses to zero and the BEC, the black hole, becomes an infinite capacitor of information storage.

This is a very general property of overpacked BEC's which are at the critical point of quantum phase of quantum phase transition. In both cases the cold atomic system [53, 54] versus the graviton condensate the key point is the maximal packing. The overpacking of the system results in the collapse of the mass gap and the Bogliubov modes become degenerate within an $1/N$ window. These almost degenerate Bogoliubov modes are the quantum holographic degrees of freedom that are responsible both for the entropy as well as for the efficient depletion of the system.

6.3 Effect of Core in BEC

We first consider the effect of a highly localized δ-impurity on the BEC in one dimension [55]. This approach indicates that the density of the BEC is substantially increased in the vicinity of an attractive impurity, which enhances inelastic collisions and may result in the loss of the impurity atom. In addition, a scaling argument has been given to show that attractive impurity-BEC interactions can lead to a point-like ground state of the impurity in 2D and 3D.

It is reasonable to assume that at the Planck scale no point-like ground state is formed (that's what we want to avoid) because of uncertainty relations and possible repulsive action of gravity at the shortest didtances. Rather it is expected that the gravons help to provide seeds for graviton condensation.

7 Discussion and Conclusions

The present note contains a definite model, and references elsewhere, how to go a short but important step beyond the standard model towards a theory of Planck scale phenomena, assuming the standard model is valid up to that scale. At the Planck scale black holes are the key objects of quantum gravity to study. Unfortunately not all existing calculation results concerning Planck mass region black holes are in consensus. And a key idea is still missing. On the other hand, ERGE based calculations provide rather solid results for $f(R)$ gravity.

The next task is to find a real quantum action for the model of this note as a field theory, first for pure gravity later one and more standard model particles included. Pure gravity should be taken in this model as gravon and graviton terms in a quantum condensed state that will correspond the Einstein equation (5). A realistic model of quantum gravity should start from the microscopic entities operating at the quantum scale, the Planck scale. Then the methods of the new model theory, be it quantum field theory or something else, will be introduced to calculate the properties of the model like the UV behavior of the interaction.

The scheme I propose here can be summarized as having the gravon and the graviton the fundamental elementary particles of quantum gravity, to be considered in the standard model. The gravon, going through Starobinsky inflation and Bose-Einstein graviton condensation, is a natural candidate for non-singular black holes and dark matter in the universe.

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