A new conjecture in number theory

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Abstract

I propose a new conjecture of generalization of the Fermat last theorem and the Lander, Parkin, and Selfridge conjecture.

Conjecture 1. *Let n, m be two positive integers and $m \neq n$, let a polynomial $f(x) = a_k x^k + a_{k-1} x^{k-1} + ... + a_0$, where $a_0, a_1, a_2, a_3, ..., a_k$ be fixed integers and $a_k \neq 0$, exist k_0 be the positive integer such that when $k \geq k_0$ then no $(n + m)$ positive integers x_1, x_2, x_n, y_1, y_2,..., y_m can satisfy an equation as follows:*

$$f(x_1) + f(x_2) + ... + f(x_n) = f(y_1) + f(y_2) + ... + f(y_m) \tag{1}$$

Example 1: Let $f(x) = x^k$, exist k_0 such that when $k \geq k_0$ then no three positive integers x, y, z can satisfy an equations:

$$x^k + y^k = z^k$$

By the Fermat Last Theorem, in **example 1** we get $k_0 = 3$

In **example 1**, $a_k = 1$, $a_i = 0$ for $i \neq k$.

Example 2: Let n, m be two positive integers and $m \neq n$, let $f(x) = x^k$, exist k_0 such that when $k \geq k_0$ then no $(n + m)$ positive integers x_1, x_2,...,x_n, y_1, y_2,..., y_m can satisfy an equation:

$$x_1^k + x_2^k + ... + x_n^k = y_1^k + y_2^k + ... + y_m^k$$

By the Lander, Parkin, and Selfridge conjecture in **example 2**, $k_0 = n + m + 1$

In **example 2**, $a_k = 1$, $a_i = 0$ for $i \neq k$.

Example 3: Let $f(x) = 3x^k + 4x^2 + 5$, exist k_0 such that when $k \geq k_0$ then no three integers x, y, z can satisfy an equation:

$$3x^k + 4x^2 + 5 + 3y^k + 4y^2 + 5 = 3z^k + 4z^2 + 5$$

In **example 3**, $a_k = 3$, $a_2 = 4$, $a_0 = 5$, $a_i = 0$ for $i \neq k, 2, 0$.

Note that: a_k be the factor of x_k, so a_k and k are independent.

References

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