Gravity is not geometry

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Abstract: In this paper was presented description of gravity without the geometry of the space-time.

1. Introduction

General Relativity (GR) is a theory which a since about 100 years describes the gravitational phenomena as a geometric properties of the space-time. Although GR is widely accepted as a fundamental theory of gravitation for the many physicists still this is not a perfect theory.

In GR the space-time plays a very important role. The space-time continuum is a mathematical model that joins three-dimensional space and one dimension time into a single idea, the four-dimensional space-time. Under influence outer gravitational field the space-time is curved.

2. Alternative description of the gravity

Suppose that there is an alternative description of gravitational phenomena. The arena, where gravitational phenomena take place is the material medium [1]. It is infinite collection of bodies filling the whole space-time about a certain mass density $\rho$, which has the capacity to propagate the gravitational interactions.

Let us assume that in the absence of the outer gravitational field the medium becomes the bare medium. This medium with the bare mass density $\rho^{bare}$ is homogeneous, isotropic and independent of the time. The bare medium is defined by the bare mass density tensor $\rho^{\mu\nu}_{bare}$

$$\rho^{\mu\nu}_{bare} = \rho^{bare} \cdot \eta_{\mu\nu} = \text{diag}(-\rho^{bare}, \rho^{bare}, \rho^{bare}, \rho^{bare})$$

where: $\eta_{\mu\nu}$ is the Minkowski tensor, $\mu, \nu = 0, 1, 2, 3$. The bare medium is equivalent, in a some sense, with the Minkowski space-time, where $\rho^{bare} = 0$.

Under influence outer gravitational field the bare medium is changes and deformed (is curved) and becomes the effective medium with the effective mass density tensor $\rho^{\mu\nu}$.

The effective medium is equivalent, in a some sense, with the Riemann four-dimensional space-time, where $\rho^{bare} = 0$.

In GR the Riemann four-dimensional space-time is active and curved. In the $m(GR)$ theory the Minkowski four-dimensional space-time is a passive background (is only a scaffolding) but the material medium is active and curved. All clocks with the effective mass runs slower than the clocks with the bare mass while a rods with the effective mass will show a length greater than the rods with the bare mass [2]. That is the main difference between the two theories.

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1 Our theory we will call the $m(GR)$ theory – the modified theory of GR.
3. The metric

To correctly describe gravity in the effective medium we use line element in the form

\[ ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}(x)}{\rho_{\text{bare}}} dx^\mu dx^\nu \]  

(2)

where: \( \rho_{\mu\nu}(x) \) is a symmetric, position dependent, the effective mass density tensor. This tensor describes the relationship between the effective medium and bare medium. For the simplicity, in further discussion we assume, that \( \rho_{\text{bare}} = 1 \).

The effective mass density tensor \( \rho_{\mu\nu}(x) \) is equivalent, in a some sense, to the metric tensor \( g_{\mu\nu}(x) \) in the four-dimensional space-time. In absence of the outer gravitational field, i.e. \( \rho_{\mu\nu}(x) = \rho_{\text{bare}}^{\mu\nu} \) the metric (2) has form of the Minkowski metric \( ds^2(\eta_{\mu\nu}) = \eta_{\mu\nu} dx^\mu dx^\nu \). It is equivalent to the metric in the flat space-time.

For example, the bare mass density tensor in the polar coordinates is

\[ \rho_{\mu\nu}(r) = \rho_{\text{bare}}^{\mu\nu} \cdot \text{diag}(-1, 1, r^2, r^2 \cdot \sin^2 \Theta) \]

4. The equations of motion

The Lagrangian function for the particle with the effective mass density tensor \( \rho_{\mu\nu}(x) \) has form

\[ L = \frac{1}{2} \rho_{\mu\nu}(x) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \]  

(3)

The Euler–Lagrange equations gives the equation of motion

\[ \frac{dp_\gamma(x)}{d\tau} - \frac{1}{2} \frac{\partial \rho_{\mu\nu}(x)}{\partial x^\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \]  

(4)

It is the geodesic equation, where: \( p_\gamma(x) = \rho_{\gamma\nu}(x) \frac{dx^\nu}{d\tau} \) is the density of the four-momentum, \( \tau \) is the proper time. In the particular case, if the mass density tensor \( \rho_{\mu\nu}(x) \) does not depend on a coordinate \( x^\epsilon \) (i.e. in the bare medium \( \frac{\partial \rho_{\mu\nu}(x)}{\partial x^\epsilon} = 0 \)), then \( \frac{dp_\gamma(x)}{d\tau} = 0 \) and finally \( p_\gamma \) is constant along the world line.

The equation (4) has the also different equivalent form (if \( \rho_{\gamma\nu}(x) \) does not depend explicitly on \( \tau \))

\[ \rho_{\gamma\nu}(x) \frac{d^2 x^\nu}{d\tau^2} + \Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}(x)) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \]  

(5)

where:
\[ \Gamma_{\mu \nu} (\rho_{\mu \nu} (x)) = \frac{1}{2} \left( \frac{\partial \rho_{\mu \nu} (x)}{\partial \chi^\rho} + \frac{\partial \rho_{\gamma \nu} (x)}{\partial \chi^\mu} - \frac{\partial \rho_{\mu \nu} (x)}{\partial \chi^\gamma} \right) \]

is the Christoffel symbols of the first kind, now is expressed by the effective mass density tensor \( \rho_{\mu \nu} (x) \). Dimension of the Christoffel symbols of the first kind is \([\text{mass density/meter}]\) or gradient of the mass density.

Analyzing the equation of motion (5), we can see that the distribution and motion of the surrounding masses, expressed by the term

\[ \Gamma_{\mu \nu} (\rho_{\mu \nu} (x)) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \]

is the source of the inertial forces, expressed by the term

\[ \rho_{\gamma \nu} (x) \frac{d^2 x^\nu}{d\tau^2} \]

The motion of the body does not depend on the properties of the space-time but the presence and motion of the surrounding masses and their distribution. These surrounding masses and their distribution are the source of the inertial forces.

The new quality of the understanding, kept in Mach’s spirit, has been reached\(^2\).

5. The bare medium is the reference frame

In particular case, when the surrounding masses are the bare masses, i.e.

\[ \Gamma_{\mu \nu} (\rho_{\mu \nu}^{bare}) = 0 \]

then the inertial forces disappear

\[ \rho_{\gamma \nu}^{bare} \frac{d^2 x^\nu}{d\tau^2} = 0 \], \hspace{1cm} (6)

note that is always \( \rho_{\gamma \nu}^{bare} \neq 0 \).

According to the equation (6) we can say that the body with the bare mass density \( \rho_{\gamma \nu}^{bare} \) is in the rest or moves in a straight line with the constant speed in respect to the surrounding masses and their distribution (not respect to the space-time).

The equation (6) locally determines the new reference frame – the bare medium reference frame.

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\(^2\) This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in not accelerated motion relatively to space, but relatively to the centre of all the other masses in the Universe; in this way the series of causes of mechanical phenomena was closed, in contrast to the mechanics of Newton and Galileo [3].
6. Mach’s equation

The surrounding masses and their distribution generates the weak and stationary gravitational field. The motion of these masses is $<\!<\!<$ speed of the light $c$. We can decompose $\rho_{\mu\nu}$ to following simple form

$$\rho_{\mu\nu}(x) = \rho_{\mu\nu}^{\text{bare}} + \rho_{\mu\nu}^*(x) \quad (7)$$

where $\rho_{\mu\nu}^*(x) << 1$ is a small perturbation in the effective mass density tensor.

The equation of motion (5) takes now the form $(i = 1, 2, 3)$

$$\left(\rho_{\mu\nu}^{\text{bare}} + \rho_{00}^*(x)\right) \frac{d^2x^i}{dt^2} = -\frac{1}{2} c^2 \frac{\partial \rho_{00}^*}{\partial x^i} \quad (8)$$

The motion of the body does not depend on the properties of space-time but from the gradient of the small perturbation in the effective mass density $\rho_{00}^*(x)$ of the surrounding masses, causing the mass density change: $\rho_{\mu\nu}^{\text{bare}} + \rho_{00}^*(x)^3$.

The equation (8), which we will call Mach’s equation of motion for the gravitation, is different than the classical Newton's equation.

By introducing the material medium for the describe the gravity we received physics in spirit of the Mach. The Principle of Equivalence, underlying the GR, lost raison d’etre.

References


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3 A similar result we get for the surrounding rotating masses.