Interval Valued Neutrosophic Graphs

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Abstract: The notion of interval valued neutrosophic sets is a generalization of fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy sets, interval valued intuitionstic fuzzy sets and single valued neutrosophic sets. We apply for the first time the concept of interval valued neutrosophic sets, an instance of neutrosophic sets, to graph theory. We introduce certain types of interval valued neutrosophc graphs (IVNG) and investigate some of their properties with proofs and examples.

Keywords: Interval valued neutrosophic set, interval valued neutrosophic graph, strong interval valued neutrosophic graph, constant interval valued neutrosophic graph, complete interval valued neutrosophic graph, degrre of interval valued neutrosophic graph .

1. Introduction:

Neutrosophic sets (NSs) proposed by Smarandache [13, 14] is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. they are a generalization of the theory of fuzzy sets [31], intuitionistic fuzzy sets [28, 30], interval valued fuzzy set [23] and interval-valued intuitionistic fuzzy sets [29]. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity-membership function (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. In order to practice NS in real life applications conveniently, Wang et al. [17] introduced the concept of a single-valued neutrosophic sets (SVNS), a subclass of the neutrosophic sets. The same authors [16,18] introduced the concept of interval valued neutrosophic sets (IVNS), which is more precise and flexible than single valued neutrosophic sets. The IVNS is a generalization of single valued neutrosophic sets, in which three membership functions are independent and their value belong to the unit interval [0, 1]. Some more work on single valued neutrosophic sets, interval valued neutrosophic sets and their applications may be found on [3, 4, 5,6, 19, 20, 21, 22, 24, 25, 26, 27, 39, 41, 42, 43, 44, 45, 49].

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving a combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. Most important thing which is to be noted is that, when we have uncertainty regarding either the set of vertices or edges or both, the model becomes a fuzzy graph. The extension of fuzzy graph [7, 9, 38] theory have been developed by several researchers including intuitionistic fuzzy graphs [8, 32, 40] considered the vertex sets and edge sets as intuitionistic fuzzy sets. Interval valued fuzzy graphs [33, 34] considered the vertex sets and edge sets as interval valued fuzzy sets. Interval valued intuitionstic fuzzy graphs [2, 48] considered the vertex sets and edge sets as interval valued intuitionstic fuzzy sets. Bipolar fuzzy graphs [35, 36] considered the vertex sets and edge sets as bipolar fuzzy sets. M-polar fuzzy graphs [37] considered the vertex sets and edge sets as m-polar fuzzy sets. But, when the relations between nodes(or vertices) in problems are indeterminate, the fuzzy graphs and their extensions are failed. For this purpose, Samarandache [10, 11, 12, 51] have defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), which called them; I-edge neutrosophic graph and I-vertex neutrosophic graph, these concepts are studied deeply and has gained popularity among the researchers due to its applications via real world problems [1, 15, 50, 52]. The two others graphs are based on (t, i, f) components and called them; The (t, i, f)-Edge neutrosophic graph and the (t, i, f)-vertex neutrosophic graph, these concepts are not developed at all. Later on, Broumi et al.[47] introduced a third neutrosophic graph model. This model allows the attachment of truth-membership (t), indeterminacy – membership (i) and falsity- membership degrees (f) both to vertices and edges, and investigated some of their properties. The third neutrosophic graph model is called single valued neutrosophic graph (SVNG for short). The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionstic fuzzy graph. Also the same authors [46] introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. In the literature the study of interval valued neutrosophic graphs (IVN-graph) is still blank, we shall focus on the study of interval valued neutrosophic graphs in this paper.

In this paper, some certain types of interval valued neutrosophic graphs are developed and some interesting properties are explored.

2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, fuzzy graph, intuitionistic fuzzy graph, single valued neutrosophic graphs, relevant to the present work. See especially [2, 7, 8,13,18, 47] for further details and background.

Definition 2.1 [13]. Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X→]⁻0,1⁺[define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈ X to the set A with the condition:

$$^{-}0 \le T_A(x)+ I_A(x)+ F_A(x)\le 3^{+}. \qquad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^{-}0,1^{+}[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [16] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [17]. Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\} \qquad (2)$$

Definition 2.3[7]. A fuzzy graph is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. i.e $\sigma : V \to [0,1]$ and $\mu: VxV\to[0,1]$ such that $\mu(uv) \le \sigma(u) \wedge \sigma(v)$ for all u, v ∈ V where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

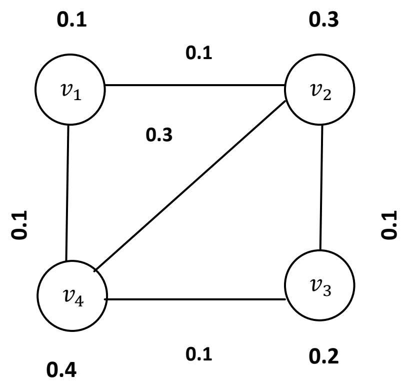

Figure 1: Fuzzy Graph

Definition 2.4 [7]**.** The fuzzy subgraph H = (τ, ρ) is called a fuzzy subgraph of G = (σ, μ)

If τ(u) ≤ σ(u) for all u ∈ V and ρ(u, v) ≤ μ(u, v) for all u, v ∈ V

Definition 2.5 [8]**.** An Intuitionistic fuzzy graph is of the form G =(V, E) where

i. V={v_1, v_2,...., v_n} such that μ_1: V→ [0,1] and γ_1: V → [0,1] denote the degree of membership and nonmembership of the element v_i ∈ V, respectively, and 0 ≤ $\mu_1(v_i)$ + $\gamma_1(v_i)$) ≤ 1 for every v_i ∈ V, (i = 1, 2, ……. n),

ii. E ⊆ V x V where μ_2: VxV→[0,1] and γ_2: VxV→ [0,1] are such that $\mu_2(v_i, v_j)$ ≤ min [$\mu_1(v_i)$, $\mu_1(v_j)$] and $\gamma_2(v_i, v_j)$ ≥ max [$\gamma_1(v_i)$, $\gamma_1(v_j)$] and 0 ≤ $\mu_2(v_i, v_j)$ + $\gamma_2(v_i, v_j)$ ≤ 1 for every (v_i, v_j) ∈ E, (i, j = 1,2, ……. n)

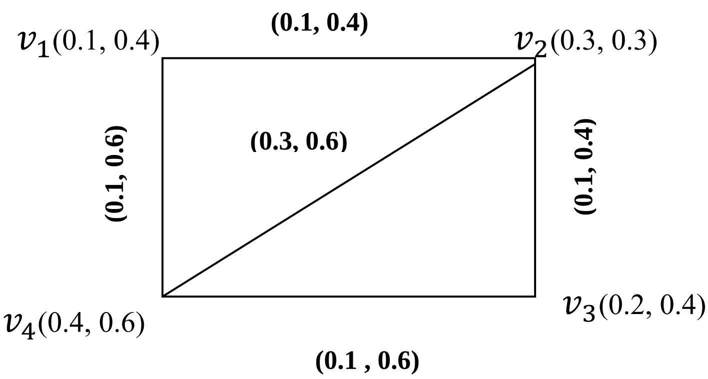

Figure 2: Intuitionistic Fuzzy Graph

Definition 2.6 [2] An interval valued intuitionistic fuzzy graph with underlying set V is defined to be a pair G= (A, B) where

1) The functions M_A : V→ D [0, 1] and N_A : V→ D [0, 1] denote the degree of membership and non membership of the element x ∈ V, respectively, such that 0 such that 0≤M_A(x)+ N_A(x) ≤ 1 for all x ∈ V.

2) The functions M_B : E ⊆ $V \times V$ → D [0, 1] and N_B : : E ⊆ $V \times V$ → D [0, 1] are defined by $M_{BL}(x,y)$)≤min ($M_{AL}(x)$, $M_{AL}(y)$) and $N_{BL}(x,y)$) ≥max ($N_{AL}(x)$, $N_{AL}(y)$)

$M_{BU}(x,y)$)≤min ($M_{AU}(x)$, $M_{AU}(y)$) and $N_{BU}(x,y)$) ≥max ($N_{AU}(x)$, $N_{AU}(y)$) such that

0≤$M_{BU}(x,y)$)+ $N_{BU}(x,y)$) ≤ 1 for all (x,y) ∈ E.

Definition 2.7 [47]. Let A = (T_A, I_A, F_A) and B = (T_B, I_B, F_B) be single

valued neutrosophic sets on a set X. If A = (T_A, I_A, F_A) is a single valued neutrosophic relation on a set X, then A =(T_A, I_A, F_A) is called a single valued neutrosophic relation on B = (T_B, I_B, F_B) if

$T_B(x, y) \leq \min(T_A(x), T_A(y))$

$I_B(x, y) \geq \max(I_A(x), I_A(y))$ and

$F_B(x, y) \geq \max(F_A x), F_A(y))$ for all x, y ∈ X.

A single valued neutrosophic relation A on X is called symmetric if $T_A(x, y)$ = $T_A(y, x)$, $I_A(x, y)$ = $I_A(y, x)$, $F_A(x, y)$ = $F_A(y, x)$ and $T_B(x, y)$ = $T_B(y, x)$, $I_B(x, y)$ = $I_B(y, x)$ and $F_B(x, y)$ = $F_B(y, x)$, for all $x, y \in X$.

Definition 2.8 [47]. A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G= (A, B) where

1.The functions T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element ∈ V, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \text{ for all } \in V \ (i=1, 2, \ldots,n)$$

2. The functions T_B: E ⊆ V x V →[0, 1], I_B:E ⊆ V x V →[0, 1] and : E ⊆ V x V →[0, 1] are defined by

$T_B(\{v_i, v_j\}) \leq \min [T_A(v_i), T_A(v_j)]$,

$I_B(\{v_i, v_j\}) \geq \max [I_A(v_i), I_A(v_j)]$ and

$F_B(\{v_i, v_j\}) \geq \max [F_A(v_i), F_A(v_j)]$

Denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3$ for all $\{v_i, v_j\} \in E$ (i, j = 1, 2,…, n)

We call A the single valued neutrosophic vertex set of V, B the single valued neutrosophic edge set of E, respectively, Note that B is a symmetric single valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E Thus, G = (A, B) is a single valued neutrosophic graph of G*= (V, E) if

$T_B(v_i, v_j) \leq \min [T_A(v_i), T_A(v_j)]$,

$I_B(v_i, v_j) \geq \max [I_A(v_i), I_A(v_j)]$ and

$F_B(v_i, v_j) \geq \max [F_A(v_i), F_A(v_j)]$ for all $(v_i, v_j) \in E$.

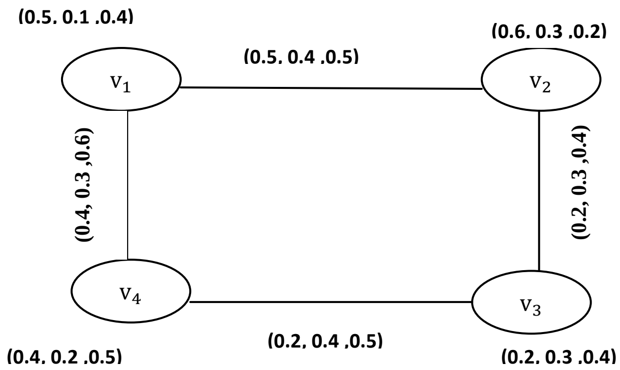

Figure 3: Single valued neutrosophic graph

Definition 2.9 [47]. A partial SVN-subgraph of SVN-graph G= (A, B) is a

SVN-graph H = (V', E') such that

(i) $\boldsymbol{V' \subseteq V}$, where $\boldsymbol{T'_A(v_i) \leq T_A(v_i)}$, $\boldsymbol{I'_A(v_i) \geq I_A(v_i)}$, $\boldsymbol{F'_A(v_i) \geq F_A(v_i)}$, for all $\boldsymbol{v_i \in V}$.

(ii) $\boldsymbol{E' \subseteq E}$, where $\boldsymbol{T'_B(v_i, v_j) \leq T_B(v_i, v_j)}$, $\boldsymbol{I'_{Bij} \geq I_B(v_i, v_j)}$, $\boldsymbol{F'_B(v_i, v_j) \geq F_B(v_i, v_j)}$, for all $\boldsymbol{(v_i\ v_j) \in E}$.

Definition 2.10 [47]. A SVN-subgraph of SVN-graph G= (V, E) is a SVN-graph H = ($\boldsymbol{V'}$, $\boldsymbol{E'}$) such that

(i) $\boldsymbol{V'} = \boldsymbol{V}$, where $\boldsymbol{T'_A(v_i)} = \boldsymbol{T_A(v_i)}$, $\boldsymbol{I'_A(v_i)} = \boldsymbol{I_A(v_i)}$, $\boldsymbol{F'_A(v_i)} = \boldsymbol{F_A(v_i)}$for all $\boldsymbol{v_i}$ in the vertex set of $\boldsymbol{V'}$.

(ii) $\boldsymbol{E'} = \boldsymbol{E}$, where $\boldsymbol{T'_B(v_i, v_j)} = \boldsymbol{T_B(v_i, v_j)}$, $\boldsymbol{I'_B(v_i, v_j)} = \boldsymbol{I_B(v_i, v_j)}$, $\boldsymbol{F'_B(v_i, v_j)} = \boldsymbol{F_B(v_i, v_j)}$ for every $(\boldsymbol{v_i}\ \boldsymbol{v_j}) \in \boldsymbol{E}$ in the edge set of $\boldsymbol{E'}$.

Definition 2.11 [47]. Let G= (**A**, **B**) be a single valued neutrosophic graph. Then the degree of any vertex **v** is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex **v** denoted by d(v)= $(d_T(v), d_I(v), d_F(v))$ where

$d_T(v)$=$\sum_{u \neq v} T_B(u, v)$ denotes degree of truth-membership vertex.

$d_I(v)$=$\sum_{u \neq v} I_B(u, v)$ denotes degree of indeterminacy-membership vertex.

$d_F(v)$=$\sum_{u \neq v} F_B(u, v)$ denotes degree of falsity-membership vertex.

Definition 2.12[47]. A single valued neutrosophic graph G=(A, B) of G^*= (V, E) is called strong single valued neutrosophic graph if

$T_B(v_i, v_j) = \min\ [T_A(v_i),\ T_A(v_j)]$

$I_B(v_i, v_j) = \max\ [I_A(v_i),\ I_A(v_j)]$

$F_B(v_i, v_j) = \max\ [F_A(v_i), F_A(v_j)]$

For all $(v_i, v_j) \in$ E.

Definition 2.13 [47]. A single valued neutrosophic graph G= (A, B) is called complete if

$T_B(v_i, v_j) = \min\ [T_A(v_i),\ T_A(v_j)]$

$I_B(v_i, v_j) = \max\ [I_A(v_i),\ I_A(v_j)]$

$F_B(v_i, v_j) = \max\ [F_A(v_i), F_A(v_j)]$

for all $v_i, v_j \in$ V.

Definition 2.14[47]. The complement of a single valued neutrosophic graph G (A, B) on G^* is a single valued neutrosophic graph $\bar{G}$ on G^* where:

1. $\bar{A}$ =A

2. $\overline{T_A}(v_i)$= $T_A(v_i)$, $\overline{I_A}(v_i)$= $I_A(v_i)$, $\overline{F_A}(v_i)$ = $F_A(v_i)$, for all $v_j \in$ V.

3. $\overline{T_B}(v_i, v_j)$= min $[T_A(v_i), T_A(v_j)] - T_B(v_i, v_j)$

$\overline{I_B}(v_i, v_j)$= max $[I_A(v_i), I_A(v_j)] - I_B(v_i, v_j)$ and

$\overline{F_B}(v_i, v_j)$= max $[F_A(v_i), F_A(v_j)] - F_B(v_i, v_j)$, For all $(v_i, v_j) \in$ E.

Definition 2.15 [18]. Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS A) A in X is characterized by truth-membership function T_A(x), indeteminacy-membership function I_A(x) and falsity-membership function F_A(x). For each point x in X, we have that T_A(x)= $[\boldsymbol{T_{AL}}(\mathbf{x}), \boldsymbol{T_{AU}}(\mathbf{x})]$, I_A(x) = $[\boldsymbol{I_{AL}}(\boldsymbol{x}), \boldsymbol{I_{AU}}(\boldsymbol{x})]$, F_A(x) = $[\boldsymbol{F_{AL}}(\boldsymbol{x}), \boldsymbol{F_{AU}}(\boldsymbol{x})] \subseteq [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.16 [18]. An IVNS A is contained in the IVNS B, A ⊆ B, if and only if T_{AL}(x) ≤ T_{BL}(x), T_{AU}(x) ≤ T_{BU}(x), I_{AL}(x) ≥ I_{BL}(x), I_{AU}(x) ≥ I_{BU}(x), F_{AL}(x) ≥ F_{BL}(x) and F_{AU}(x) ≥ F_{BU}(x) for any x in X.

Definition 2.17 [18]. The union of two interval valued neutrosophic sets A and B is an interval neutrosophic set C, written as C = A ∪ B, whose truth-membership, indeterminacy-membership, and false membership are related to those A and B by

$$T_{CL}(x) = \max(T_{AL}(x), T_{BL}(x))$$
$$T_{CU}(x) = \max(T_{AU}(x), T_{BU}(x))$$
$$I_{CL}(x) = \min(I_{AL}(x), I_{BL}(x))$$
$$I_{CU}(x) = \min(I_{AU}(x), I_{BU}(x))$$
$$F_{CL}(x) = \min(F_{AL}(x), F_{BL}(x))$$

$F_{CU}(x) =$ min ($F_{AU}(x)$, $F_{BU}(x)$) for all x in X.

Definition 2.18 [18]. Let X and Y be two non-empty crisp sets. An interval valued neutrosophic relation R(X, Y) is a subset of product space X × Y, and is characterized by the truth membership function $T_R(x, y)$, the indeterminacy membership function $I_R(x, y)$, and the falsity membership function $F_R(x, y)$, where x ∈ X and y ∈ Y and $T_R(x, y)$, $I_R(x, y)$, $F_R(x, y) \subseteq [0, 1]$.

3. Interval Valued Neutrosophic Graphs

Throught this paper, we denote G^* = (V, E) a crisp graph, and G =(A, B) an interval valued neutrosophic graph

Definition 3.1. By an interval-valued neutrosophic graph of a graph G^* = (V, E) we mean a pair G = (A, B), where A =< [T_{AL}, T_{AU}], [I_{AL}, I_{AU}], [F_{AL}, F_{AU}]> is an interval-valued neutrosophic set on V and B =< [T_{BL}, T_{BU}], [I_{BL}, I_{BU}], [F_{BL}, F_{BU}]> is an interval-valued neutrosophic relation on E satisfies the following condition:

1.V= $\{v_1, v_2, \ldots, v_n\}$ such that T_{AL}:V→[0, 1], T_{AU}:V→[0, 1], I_{AL}:V→[0, 1], I_{AU}:V→[0, 1] and F_{AL}:V→[0, 1], F_{AU}:V→[0, 1] denote the degree of truth-membership, the degree of indeterminacy- membership and falsity-membership of the element $y \in$ V, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \text{ for all } v_i \in \text{V (i=1, 2, ...,n)}$$

2. The functions T_{BL}:V x V →[0, 1], T_{BU}:V x V →[0, 1], I_{BL}:V x V →[0, 1], I_{BU}:V x V →[0, 1] and F_{BL}:V x V →[0,1], F_{BU}:V x V →[0, 1] are such that

$T_{BL}(\{v_i, v_j\}) \leq \min\ [T_{AL}(v_i), T_{AL}(v_j)]$

$T_{BU}(\{v_i, v_j\}) \leq \min\ [T_{AU}(v_i), T_{AU}(v_j)]$

$I_{BL}(\{v_i, v_j\}) \geq \max[I_{BL}(v_i), I_{BL}(v_j)]$

$I_{BU}(\{v_i, v_j\}) \geq \max[I_{BU}(v_i), I_{BU}(v_j)]$ And

$F_{BL}(\{v_i, v_j\}) \geq \max[F_{BL}(v_i), F_{BL}(v_j)]$

$F_{BU}(\{v_i, v_j\}) \geq \max[F_{BU}(v_i), F_{BU}(v_j)]$

Denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in$ E respectively, where

$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3$ for all $\{v_i, v_j\} \in$ E (i, j = 1, 2,…, n)

We call A the interval valued neutrosophic vertex set of V, B the interval valued neutrosophic edge set of E, respectively, Note that B is a symmetric interval valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E Thus, G = (A, B) is an interval valued neutrosophic graph of G*= (V, E) if

$T_{BL}(v_i, v_j) \leq \min\ [T_{AL}(v_i), T_{AL}(v_j)]$

$T_{BU}(v_i, v_j) \leq \min\ [T_{AU}(v_i), T_{AU}(v_j)]$

$I_{BL}(v_i, v_j) \geq \max[I_{BL}(v_i), I_{BL}(v_j)]$

$I_{BU}(v_i, v_j) \geq \max[I_{BU}(v_i), I_{BU}(v_j)]$ And

$F_{BL}(v_i, v_j) \geq \max[F_{BL}(v_i), F_{BL}(v_j)]$

$F_{BU}(v_i, v_j) \geq \max[F_{BU}(v_i), F_{BU}(v_j)]$ for all $(v_i, v_j) \in$ E.

Example 3.2. Consider a graph G^* such that V= $\{v_1, v_2, v_3, v_4\}$, E=$\{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. Let A be a interval valued neutrosophic subset of V and let B a interval valued neutrosophic subset of E denoted by

	v_1	v_2	v_3
T_{AL}	0.3	0.2	0.1
T_{AU}	0.5	0.3	0.3
I_{AL}	0.2	0.2	0.2
I_{AU}	0.3	0.3	0.4

	v_1v_2	v_2v_3	v_3v_1
T_{BL}	0.1	0.1	0.1
T_{BU}	0.2	0.3	0.2
I_{BL}	0.3	0.4	0.3
I_{BU}	0.4	0.5	0.5

F_{AL}	0.3	0.1	0.3
F_{AU}	0.4	0.4	0.5

F_{BL}	0.4	0.4	0.4
F_{BU}	0.5	0.5	0.6

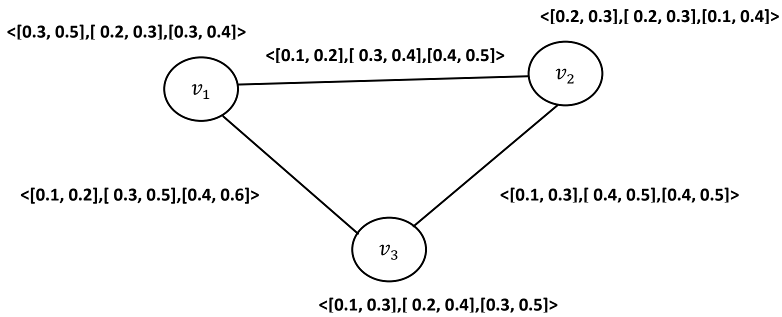

Figure 4: G: Interval valued neutrosophic graph

In figure 4, (i) (v_1, <[0.3, 0.5],[0.2, 0.3],[0.3, 0.4]>) is an interval valued neutrosophic vertex or IVN-vertex.

(ii) (v_1v_2, <[0.1, 0.2], [0.3, 0.4], [0.4, 0.5]>) is an interval valued neutrosophic edge or IVN-edge.

(iii) (v_1, <[0.3, 0.5], [0.2, 0.3], [0.3, 0.4]>) and (v_2, <[0.2, 0.3],[0.2, 0.3],[0.1, 0.4]>) are interval valued neutrosophic adjacent vertices.

(iv) (v_1v_2, <[0.1, 0.2], [0.3, 0.4], [0.4, 0.5]>) and (v_1v_3, <[0.1, 0.2],[0.3, 0.5], [0.4, 0.6]>) are an interval valued neutrosophic adjacent edge.

Note 1. (i) When $T_{BL}(v_i, v_j)$ = $T_{BU}(v_i, v_j)$ =$I_{BL}(v_i, v_j)$ = $I_{BU}(v_i, v_j)$= $F_{BL}(v_i, v_j)$ = $F_{BU}(v_i, v_j)$ for some i and j, then there is no edge between v_i and v_j . Otherwise there exists an edge between v_i and v_j .

(ii) If one of the inequalities is not satisfied in (1) and (2), then G is not an IVNG

The interval valued neutrosophic graph G depicted in figure 3 is represented by the following adjacency matrix $\boldsymbol{M_G}$

$\boldsymbol{M_G}$ =

$$\begin{bmatrix} <[0.3,0.5],[0.2,0.3],[0.3,0.4]> & <[0.1,0.2],[0.3,0.4],[0.4,0.5]> & <[0.1,0.2],[0.3,0.5],[0.4,0.6]> \\ <[0.1,0.2],[0.3,0.4],[0.4,0.5]> & <[0.2,0.3],[0.2,0.3],[0.1,0.4]> & <[0.1,0.3],[0.4,0.5],[0.4,0.5]> \\ <[0.1,0.2],[0.3,0.5],[0.4,0.6]> & <[0.1,0.3],[0.4,0.5],[0.4,0.5]> & <[0.1,0.3],[0.2,0.4],[0.3,0.5]> \end{bmatrix}$$

Definition 3.3. A partial IVN-subgraph of IVN-graph G= (A, B) is an IVN-graph H = ($\boldsymbol{V'}$, $\boldsymbol{E'}$) such that

(i) $\boldsymbol{V'} \subseteq \boldsymbol{V}$, where $\boldsymbol{T'_{AL}(v_i) \le T_{AL}(v_i), T'_{AU}(v_i) \le T_{AU}(v_i), I'_{AL}(v_i) \ge I_{AL}(v_i), I'_{AU}(v_i) \ge I_{AU}(v_i), F'_{AL}(v_i) \ge F_{AL}(v_i), F'_{AU}(v_i) \ge F_{AU}(v_i)}$, for all $\boldsymbol{v_i \in V}$.

(ii) $\boldsymbol{E'} \subseteq \boldsymbol{E}$, where $\boldsymbol{T'_{BL}(v_i, v_j) \le T_{BL}(v_i, v_j), T'_{BU}(v_i, v_j) \le T_{BU}(v_i, v_j), I'_{BL}(v_i, v_j) \ge I_{BL}(v_i, v_j), I'_{BU}(v_i, v_j) \ge I_{BU}(v_i, v_j), F'_{BL}(v_i, v_j) \ge F_{BL}(v_i, v_j), F'_{BU}(v_i, v_j) \ge F_{BU}(v_i, v_j)}$, for all $\boldsymbol{(v_i\, v_j) \in E}$.

Definition 3.4. An IVN-subgraph of IVN-graph G= (V, E) is an IVN-graph H = ($\boldsymbol{V'}$, $\boldsymbol{E'}$) such that

(i) $\boldsymbol{T'_{AL}(v_i) = T_{AL}(v_i), T'_{AU}(v_i) = T_{AU}(v_i), I'_{AL}(v_i) = I_{AL}(v_i), I'_{AU}(v_i) = I_{AU}(v_i), F'_{AL}(v_i) = F_{AL}(v_i), F'_{AU}(v_i) = F_{AU}(v_i)}$, for all $\boldsymbol{v_i}$ in the vertex set of $\boldsymbol{V'}$.

(ii) $\boldsymbol{E'} = \boldsymbol{E}$, where $\boldsymbol{T'_{BL}(v_i, v_j) = T_{BL}(v_i, v_j), T'_{BU}(v_i, v_j) = T_{BU}(v_i, v_j), I'_{BL}(v_i, v_j) = I_{BL}(v_i, v_j), I'_{BU}(v_i, v_j) = I_{BU}(v_i, v_j), F'_{BL}(v_i, v_j) = F_{BL}(v_i, v_j), F'_{BU}(v_i, v_j) = F_{BU}(v_i, v_j)}$, for every $\boldsymbol{(v_i\, v_j) \in E}$ in the edge set of $\boldsymbol{E'}$.

Example 3.5. $\mathbf{G_1}$ in Figure 5 is an IVN-graph. $\mathbf{H_1}$ in Figure 6 is a partial IVN-subgraph and $\mathbf{H_2}$ in Figure 7 is a IVN-subgraph of $\mathbf{G_1}$

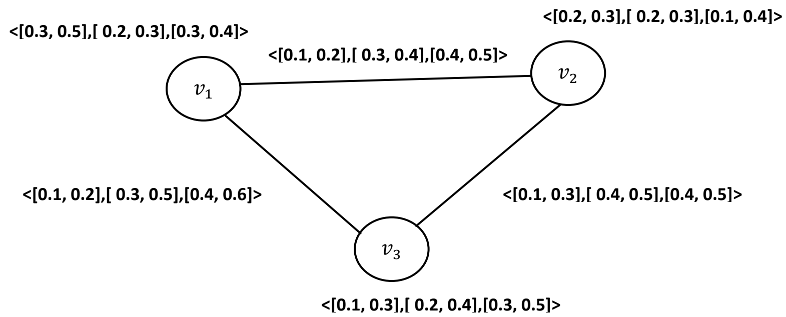

Figure 5: G_1, an interval valued neutrosophic graph

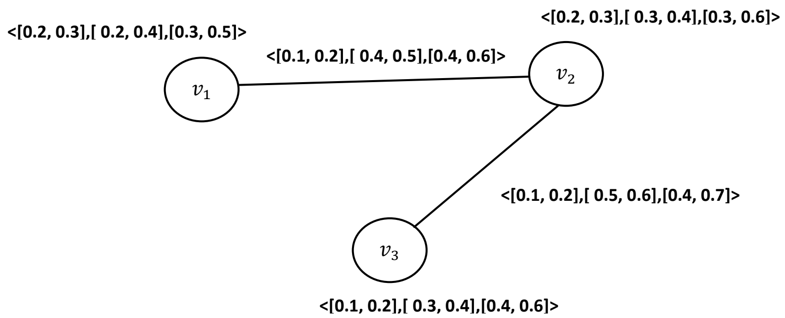

Figure 6: H_1, a partial IVN-subgraph of G_1

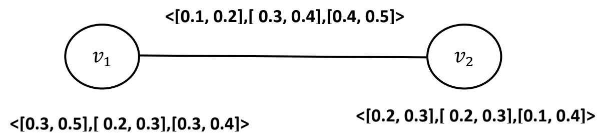

Figure 7: H_2, an IVN-subgraph of G_1.

Definition 3.6. The two vertices are said to be adjacent in an interval valued neutrosophic graph G= (**A**, **B**) if

$T_{BL}(v_i, v_j) = \min\,[T_{AL}(v_i), T_{AL}(v_j)]$,

$T_{BU}(v_i, v_j) = \min\,[T_{AU}(v_i), T_{AU}(v_j)]$,

$I_{BL}(v_i, v_j) = \max\,[I_{AL}(v_i), I_{AL}(v_j)]$

$I_{BU}(v_i, v_j) = \max\,[I_{AU}(v_i), I_{AU}(v_j)]$ and

$F_{BL}(v_i, v_j) = \max\,[F_{AL}(v_i), F_{AL}(v_j)]$

$F_{BU}(v_i, v_j) = \max\,[F_{AU}(v_i), F_{AU}(v_j)]$

In this case, v_i and v_j are said to be neighbours and (v_i, v_j) is incident at v_i and v_j also.

Definition 3.7. A path P in an interval valued neutrosophic graph G= (**A**, **B**) is a sequence of distinct vertices $v_0, v_1, v_3, \ldots v_n$ such that $T_{BL}(v_{i-1}, v_i) > 0$, $T_{BU}(v_{i-1}, v_i) > 0$, $I_{BL}(v_{i-1}, v_i) > 0$, $I_{BU}(v_{i-1}, v_i) > 0$ and $F_{BL}(v_{i-1}, v_i) > 0$, $F_{BU}(v_{i-1}, v_i) > 0$ for $0 \leq i \leq 1$. Here $n \geq 1$ is called the length of the path P. A single node or vertex v_i may also be considered as a path. In this case the path is of the length ([0, 0], [0, 0], [0, 0]). The consecutive pairs (v_{i-1}, v_i) are called edges of the path.We call P a cycle if $v_0 = v_n$ and n≥3.

Definition 3.8. An interval valued neutrosophic graph G= (A, B) is said to be connected if every pair of vertices has at least one interval valued neutrosophic path between them, otherwise it is disconnected.

Definition 3.9. A vertex $v_j \in V$ of interval valued neutrosophic graph G= (**A**, **B**) is said to be an isolated vertex if there is no effective edge incident at v_j.

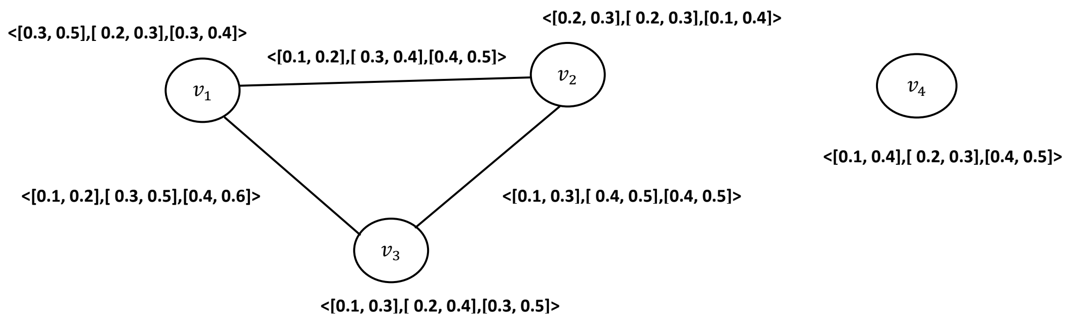

Figure 8:Example of interval valued neutrosophic graph

In figure 8, the interval valued neutrosophic vertex v_4 is an isolated vertex.

Definition 3.10. A vertex in an interval valued neutrosophic G= (**A**, **B**) having exactly one neighbor is called ***a pendent vertex***. Otherwise, it is called ***non-pendent vertex***. An edge in an interval valued neutrosophic graph incident with a pendent vertex is called a ***pendent edge***. Otherwise it is called ***non-pendent edge***. A vertex in an interval valued neutrosophic graph adjacent to the pendent vertex is called **a support** of the pendent edge

Definition 3.11. An interval valued neutrosophic graph G= (A, B) that has neither self loops nor parallel edge is called simple interval valued neutrosophic graph.

Definition 3.12. When a vertex $\mathbf{v_i}$ is end vertex of some edges $(\mathbf{v_i}, \mathbf{v_j})$ of any IVN-graph G= (A, B). Then $\mathbf{v_i}$ and $(\mathbf{v_i}, \mathbf{v_j})$ are said to be **incident** to each other.

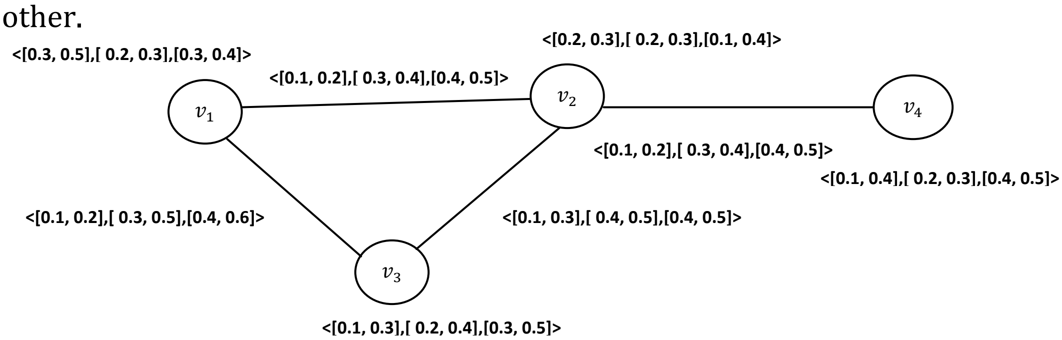

Figure 9 : Incident IVN-graph.

In this graph v_2v_1, v_2v_3 and v_2v_4 are incident on v_2.

Definition 3.13. Let G= $(\mathbf{A}, \mathbf{B})$ be an interval valued valued neutrosophic graph. Then the degree of any vertex **v** is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex **v** denoted by

d(v)= $([d_{TL}(v), d_{TU}(v)], [d_{IL}(v), d_{IU}(v)], [d_{FL}(v), d_{FU}(v)])$ where

$d_{TL}(v)=\sum_{u\neq v} T_{BL}(u, v)$ denotes degree of lower truth-membership vertex.

$d_{TU}(v)=\sum_{u\neq v} T_{BU}(u, v)$ denotes degree of upper truth-membership vertex,

$d_{IL}(v)=\sum_{u\neq v} I_{BL}(u, v)$ denotes degree of lower indeterminacy-membership vertex.

$d_{IU}(v)=\sum_{u\neq v} I_{BU}(u, v)$ denotes degree of upper indeterminacy-membership vertex.

$d_{FL}(v)=\sum_{u\neq v} F_{BL}(u, v)$ denotes degree of lower falsity-membership vertex.

$d_{FU}(v)=\sum_{u\neq v} F_{BU}(u, v)$ denotes degree of upper falsity-membership vertex.

Example 3.14. Let us consider an intreval valued neutrosophic graph G= (A, B) of G^* = (V, E) where V ={v_1, v_2, v_3, v_4} and E={v_1v_2, v_2v_3, v_3v_4 , v_4v_1}.

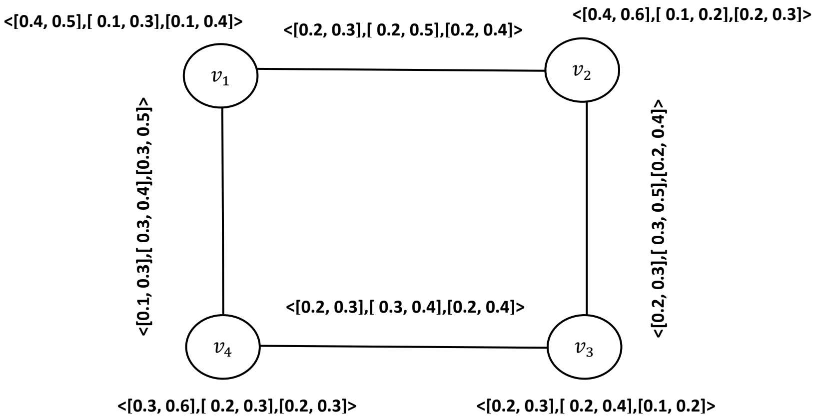

Figure 10: Degree of vertex of interval valued neutrosophic graph

We have, the degree of each vertex as follows:

$d(\mathrm{v}_1)$= ([0.3, 0.6], [0.5, 0.9], [0.5, 0.9]), $d(\mathrm{v}_2)$= ([0.4, 0.6], [0.5, 1.0], [0.4, 0.8]), $d(\mathrm{v}_3)$= ([0.4, 0.6], [0.6, 0.9], [0.4, 0.8]), $d(\mathrm{v}_4)$= ([0.3, 0.6], [0.6, 0.8], [0.5, 0.9]),

Definition 3.15 . An interval valued neutrosophic graph G= (A, B) is called constant if degree of each vertex is k =($[k_{1L}, k_{1U}]$, $[k_{2L}, k_{2U}]$, $[k_{3L}, k_{3U}]$),That is, d(v) =($[k_{1L}, k_{1U}]$, $[k_{2L}, k_{2U}]$, $[k_{3L}, k_{3U}]$), for all $v \in$ V.

Example 3.16. Consider An interval valued neutrosophic graph G such that V ={v_1, v_2, v_3, v_4} and E={$\mathrm{v}_1\mathrm{v}_2$, $\mathrm{v}_2\mathrm{v}_3$, $\mathrm{v}_3\mathrm{v}_4$, $\mathrm{v}_4\mathrm{v}_1$}.

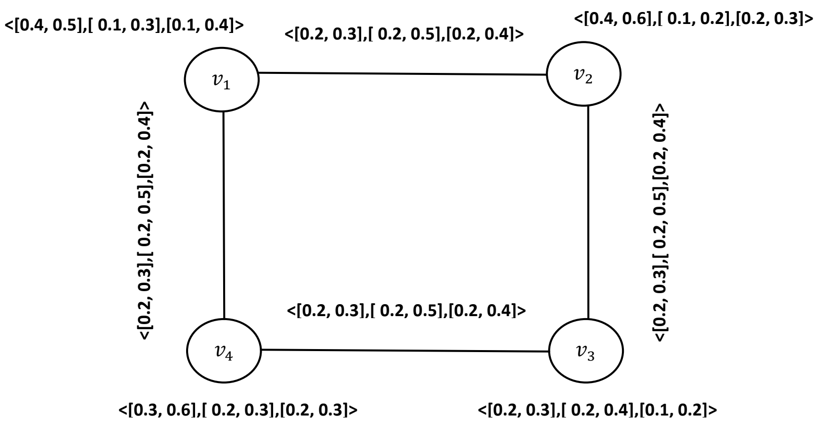

Figure 11: Constant IVN-graph.

Clearly, G is constant IVN-graph since the degree of $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$ and $\mathbf{v_4}$ is ([0.4, 0.6], [0.4, 1], [0.4, 0.8])

Definition 3.17. An interval valued neutrosophic graph G=(A, B) of G^*= (V, E) is called strong interval valued neutrosophic graph if

$T_{BL}(v_i, v_j) = \min\,[T_{AL}(v_i),\ T_{AL}(v_j)], T_{BU}(v_i, v_j) = \min\,[T_{AU}(v_i),\ T_{AU}(v_j)]$

$I_{BL}(v_i, v_j) = \max\,[I_{AL}(v_i),\ I_{AL}(v_j)],\ I_{BU}(v_i, v_j) = \max\,[I_{AU}(v_i),\ I_{AU}(v_j)]$

$F_{BL}(v_i, v_j) = \max\,[F_{AL}(v_i), F_{AL}(v_j)],\ F_{BU}(v_i, v_j) = \max\,[F_{AU}(v_i), F_{AU}(v_j)]$, for all $(v_i, v_j) \in$ E.

Example 3.18. Consider a graph G^* such that V= $\{v_1, v_2, v_3, v_4\}$, E=$\{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. Let A be an interval valued neutrosophic subset of V and let B an interval valued neutrosophic subset of E denoted by

	v_1	v_2	v_3
T_{AL}	0.3	0.2	0.1
T_{AU}	0.5	0.3	0.3
I_{AL}	0.2	0.2	0.2
I_{AU}	0.3	0.3	0.4
F_{AL}	0.3	0.1	0.3
F_{AU}	0.4	0.4	0.5

	v_1v_2	v_2v_3	v_3v_1
T_{BL}	0.2	0.1	0.1
T_{BU}	0.3	0.3	0.3
I_{BL}	0.2	0.2	0.2
I_{BU}	0.3	0.4	0.4
F_{BL}	0.3	0.3	0.3
F_{BU}	0.4	0.4	0.5

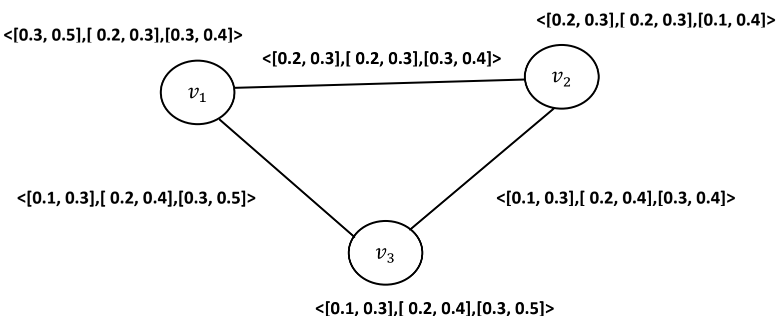

Figure12:Strong IVN-graph.

By routing computations, it is easy to see that G is a strong interval valued neutrosophic of G^*.

Proposition 3.19: An interval valued neutrosophic graph is the generalization of interval valued fuzzy graph

Proof: Suppose G=(V, E) be an interval valued neutrosophic graph. Then by setting the indeterminacy- membership and falsity- membership values of vertex set and edge set equals to zero reduces the interval valued neutrosophic graph to interval valued fuzzy graph.

Proposition 3.20: An interval valued neutrosophic graph is the generalization of interval valued intuitionistic fuzzy graph

Proof: Suppose G=(V, E) be an interval valued neutrosophic graph. Then by setting the indeterminacy- membership values of vertex set and edge set equals to zero reduces the interval valued neutrosophic graph to interval valued intuitionistic fuzzy graph.

Proposition 3.21: An interval valued neutrosophic graph is the generalization of intuitionistic fuzzy graph

Proof: Suppose G=(V, E) be an interval valued neutrosophic graph. Then by setting the indeterminacy- membership, upper truth-membership and upper falsity-membership values of vertex set and edge set equals to zero reduces the interval valued neutrosophic graph to intuitionistic fuzzy graph.

Proposition 3.22: An interval valued neutrosophic graph is the generalization of single valued neutrosophic graph.

Proof: Suppose G=(V, E) be an interval valued neutrosophic graph. Then by setting the upper truth-membership equals lower truh-membership, upper indeterminacy- membership equals lower indeterminacy-membership and upper falsity-membership equals lower falsity- membership values of vertex set and edge set reduces the interval valued neutrosophic graph to single valued neutrosophic graph.

Definition 3.23. The complement of an interval valued neutrosophic graph

G (A, B) on G^* is an interval valued neutrosophic graph $\bar{G}$ on G^* where:

1. $\bar{A}$ =A

2. $\overline{T_{AL}}(v_i)= T_{AL}(v_i)$, $\overline{T_{AU}}(v_i)= T_{AU}(v_i)$, $\overline{I_{AL}}(v_i)= I_{AL}(v_i)$, $\overline{I_{AU}}(v_i)= I_{AU}(v_i)$, $\overline{F_{AL}}(v_i) = F_{AL}(v_i)$, $\overline{F_{AU}}(v_i) = F_{AU}(v_i)$, for all $v_j \in$ V.

3. $\overline{T_{BL}}(v_i, v_j)=\min\left[T_{AL}(v_i), T_{AL}(v_j)\right] - T_{BL}(v_i, v_j)$,
$\overline{T_{BU}}(v_i, v_j)=\min\left[T_{AU}(v_i), T_{AU}(v_j)\right] - T_{BU}(v_i, v_j)$,

$\overline{I_{BL}}(v_i, v_j)=\max\left[I_{AL}(v_i), I_{AL}(v_j)\right] -$
$I_{BL}(v_i, v_j)$, $\overline{I_{BU}}(v_i, v_j)=\max\left[I_{AU}(v_i), I_{AU}(v_j)\right] - I_{BU}(v_i, v_j)$, and

$\overline{F_{BL}}(v_i, v_j)=\max\left[F_{AL}(v_i), F_{AL}(v_j)\right] - F_{BL}(v_i, v_j)$,
$\overline{F_{BU}}(v_i, v_j)=\max\left[F_{AU}(v_i), F_{AU}(v_j)\right] - F_{BU}(v_i, v_j)$, For all $(v_i, v_j) \in$ E

Remark 3.24. if G= (V, E) is an interval valued neutrosophic graph on G^*.

Then from above definition, it follow that $\bar{\bar{G}}$ is given by the interval valued neutrosophic graph $\bar{\bar{G}} = (\bar{\bar{V}}, \bar{\bar{E}})$ on G^* where $\bar{\bar{V}}$ =V and
$\overline{\overline{T_{BL}}}(v_i, v_j)=\min\left[T_{AL}(v_i), T_A(v_j)\right]-T_{BL}(v_i, v_j)$,
$\overline{\overline{T_{BU}}}(v_i, v_j)=\min\left[T_{AU}(v_i), T_A(v_j)\right]-T_{BU}(v_i, v_j)$,

$\overline{\overline{I_{BL}}}(v_i, v_j)=\max\left[I_{AL}(v_i), I_{AL}(v_j)\right]-I_{BL}(v_i, v_j)$,
$\overline{\overline{I_{BU}}}(v_i, v_j)=\max\left[I_{AU}(v_i), I_{AU}(v_j)\right]-I_{BU}(v_i, v_j)$, and

$\overline{\overline{F_{BL}}}(v_i, v_j) = \max\left[F_{AL}(v_i), F_{AL}(v_j)\right]-F_{BL}(v_i, v_j)$, $\overline{\overline{F_{BU}}}(v_i, v_j)$
$= \max\left[F_{AU}(v_i), F_{AU}(v_j)\right]-F_{BU}(v_i, v_j)$, For all $(v_i, v_j) \in$ E.

Thus $\overline{\overline{T_{BL}}} = T_{BL}$, $\overline{\overline{T_{BU}}} = T_{BUL}$, $\overline{\overline{I_{BL}}} = I_{BL}$, $\overline{\overline{I_{BU}}} = I_{BU}$, and $\overline{\overline{F_{BL}}} = F_{BL}$, $\overline{\overline{F_{BU}}} = F_{BU}$ on V, where E =($[T_{BL}, T_{BU}]$, $[I_{BL}, I_{BU}]$, $[F_{BL}, F_{BU}]$) is the interval valued neutrosophic relation on V. For any interval valued neutrosophic graph G, $\bar{G}$ is strong interval valued neutrosophic graph and $G \subseteq \bar{G}$.

Proposition 3.25. G= $\bar{\bar{G}}$ if and only if G is a strong interval valued neutrosophic graph.

Proof. it is obvious.

Definition 3.26. A strong interval valued neutrosophic graph G is called self complementary if G$\cong \bar{G}$. Where $\bar{G}$ is the complement of interval valued neutrosophic graph G.

Example 3.27. Consider a graph G^* = (V, E) such that V =$\{v_1, v_2, v_3, v_4\}$, E=$\{v_1v_2, v_2v_3, v_3v_4, v_1v_4\}$. Consider an interval valued neutrosophic graph G.

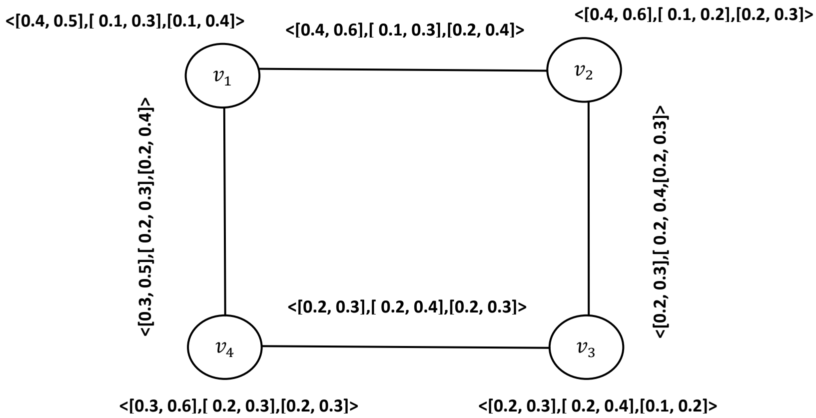

Figure13: G: Strong IVN- graph

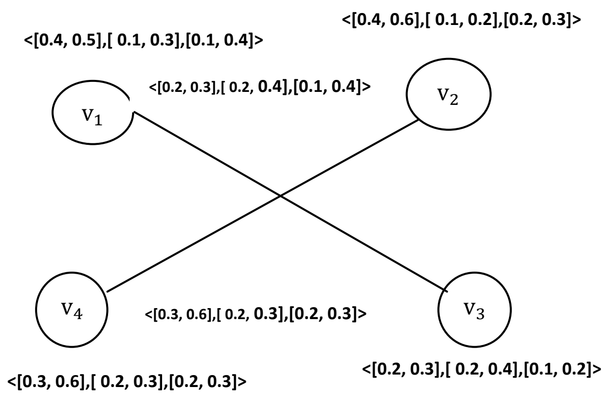

Figure14: $\bar{G}$ Strong IVN- graph

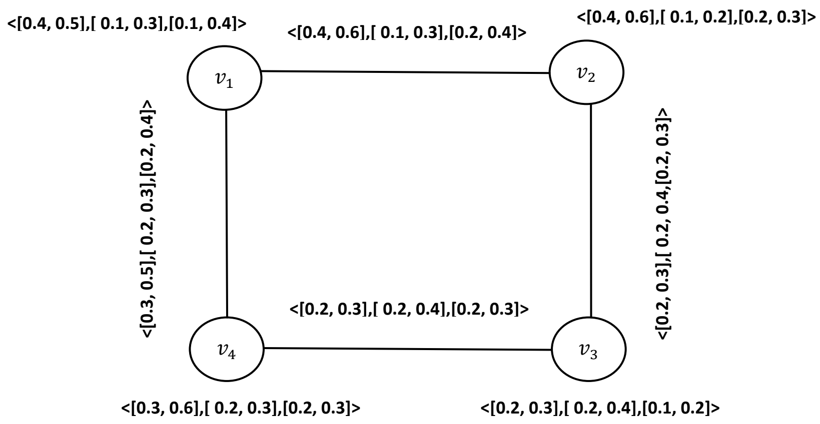

Figure15: $\bar{\bar{G}}$ Strong IVN- graph

Clearly, G≅ $\bar{\bar{G}}$. Hence G is self complementary.

Proposition 3.26. Let G=(A, B) be a ***strong*** interval valued neutrosophic graph. If

$T_{BL}(v_i, v_j) = \min\,[T_{AL}(v_i),\ T_{AL}(v_j)]$

$T_{BU}(v_i, v_j) = \min\,[T_{AU}(v_i),\ T_{AU}(v_j)]$

$I_{BL}(v_i, v_j) = \max\,[I_{AL}(v_i),\ I_{AL}(v_j)]$

$I_{BU}(v_i, v_j) = \max\,[I_{AU}(v_i),\ I_{AU}(v_j)]$

$F_{BL}(v_i, v_j) = \max\,[F_{AL}(v_i), F_{AL}(v_j)]$

$F_{BU}(v_i, v_j) = \max\,[F_{AU}(v_i), F_{AU}(v_j)]$

for all v_i, v_j ∈ V. Then G is self complementary.

Proof. Let G= (A, B) be a strong interval valued neutrosophic graph such that

$T_{BL}(v_i, v_j) = \min\,[T_{AL}(v_i),\ T_{AL}(v_j)]$

$T_{BU}(v_i, v_j) = \min\,[T_{AU}(v_i),\ T_{AU}(v_j)]$

$I_{BL}(v_i, v_j) = \max\,[I_{AL}(v_i),\ I_{AL}(v_j)]$

$I_{BU}(v_i, v_j) = \max\,[I_{AU}(v_i),\ I_{AU}(v_j)]$

$F_{BL}(v_i, v_j) = \max\,[F_{AL}(v_i), F_{AL}(v_j)]$

$F_{BU}(v_i, v_j) = \max\,[F_{AU}(v_i), F_{AU}(v_j)]$

For all $v_i, v_j \in$ V. Then G≈ $\overline{\overline{G}}$ under the identity map I: V →V. Hence G is self complementary .

Proposition 3.27. Let G be a self complementary interval valued neutrosophic graph. Then

$\sum_{v_i \neq v_j} T_{BL}(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \min\,[T_{AL}(v_i), T_{AL}(v_j)]$

$\sum_{v_i \neq v_j} T_{BU}(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \min\,[T_{AU}(v_i), T_{AU}(v_j)]$

$\sum_{v_i \neq v_j} I_{BL}(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \max\,[I_{AL}(v_i), I_{AL}(v_j)]$

$\sum_{v_i \neq v_j} I_{BU}(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \max\,[I_{AU}(v_i), I_{AU}(v_j)]$

$\sum_{v_i \neq v_j} F_{BL}(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \max\,[F_{AL}(v_i), F_{AL}(v_j)]$

$\sum_{v_i \neq v_j} F_{BU}(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \max\,[F_{AU}(v_i), F_{AU}(v_j)]$

Proof

If G be a self complementary interval valued neutrosophic graph. Then there exist an isomorphism f: $V_1 \to V_1$ satisfying

$\overline{T_{V_1}}(f(v_i)) = T_{V_1}(f(v_i)) \; = \; T_{V_1}(v_i)$

$\overline{I_{V_1}}(f(v_i)) = I_{V_1}(f(v_i)) \; = \; I_{V_1}(v_i)$

$\overline{\overline{F_{V_1}}}(f(v_i)) = F_{V_1}(f(v_i)) \; = \; F_{V_1}(v_i)$ for all $v_i \in V_1$. And

$\overline{T_{E_1}}(f(v_i), f(v_j)) = T_{E_1}(f(v_i), f(v_j)) = T_{E_1}(v_i, v_j)$

$\overline{I_{E_1}}(f(v_i), f(v_j)) = I_{E_1}(f(v_i), f(v_j)) = I_{E_1}(v_i, v_j)$

$\overline{F_{E_1}}(f(v_i), f(v_j)) = F_{E_1}(f(v_i), f(v_j)) = F_{E_1}(v_i, v_j)$ for all $(v_i, v_j) \in E_1$

We have

$\overline{T_{E_1}}(f(v_i), f(v_j)) = \min\,[\overline{T_{V_1}}(f(v_i)), \overline{T_{V_1}}(f(v_j))] - \; T_{E_1}(f(v_i), f(v_j))$

i.e, $T_{E_1}(v_i, v_j)$ = min $[T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(f(v_i), f(v_j))$

$T_{E_1}(v_i, v_j)$ = min $[T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(v_i, v_j)$

That is

$\sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \min [T_{V_1}(v_i), T_{V_1}(v_j)]$

$\sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max [I_{V_1}(v_i), I_{V_1}(v_j)]$

$\sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max [F_{V_1}(v_i), F_{V_1}(v_j)]$

$$2 \sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \min [T_{V_1}(v_i), T_{V_1}(v_j)]$$
$$2 \sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max [I_{V_1}(v_i), I_{V_1}(v_j)]$$
$$2 \sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max [F_{V_1}(v_i), F_{V_1}(v_j)]$$

From these equations, Proposition 3.27 holds

Proposition 3.28. Let G_1 and G_2 be strong interval valued neutrosophic graph, $\overline{G_1} \approx \overline{G_2}$ (isomorphism)

Proof. Assume that G_1 and G_2 are isomorphic, there exist a bijective map $f: V_1 \rightarrow V_2$ satisfying

$T_{V_1}(v_i) = T_{V_2}(f(v_i))$,

$I_{V_1}(v_i) = I_{V_2}(f(v_i))$,

$F_{V_1}(v_i) = F_{V_2}(f(v_i))$ for all $v_i \in V_1$. And

$T_{E_1}(v_i, v_j) = T_{E_2}(f(v_i), f(v_j))$,

$I_{E_1}(v_i, v_j) = I_{E_2}(f(v_i), f(v_j))$,

$F_{E_1}(v_i, v_j) = F_{E_2}(f(v_i), f(v_j))$ for all $(v_i, v_j) \in E_1$

By definition 3.21, we have

$\overline{T_{E_1}}(v_i, v_j)$= min $[T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(v_i, v_j)$

$= \min [T_{V_2}(f(v_i)),\ T_{V_2}(f(v_j))] - T_{E_2}(f(v_i), f(v_j)),$

$=\overline{T_{E_2}}(f(v_i), f(v_j)),$

$\overline{I_{E_1}}(v_i, v_j)= \max [I_{V_1}(v_i),\ I_{V_1}(v_j)] - I_{E_1}(v_i, v_j)$

$= \max[I_{V_2}(f(v_i)), I_{V_2}(f(v_j))] - I_{E_2}(f(v_i), f(v_j)),$

$=\overline{I_{E_2}}(f(v_i), f(v_j)),$

$\overline{F_{E_1}}(v_i, v_j)= \min [F_{V_1}(v_i), F_{V_1}(v_j)] - F_{E_1}(v_i, v_j)$

$= \min [F_{V_2}(f(v_i)), F_{V_2}(f(v_j))] - F_{E_2}(f(v_i), f(v_j)),$

$=\overline{F_{E_2}}(f(v_i), f(v_j)),$

For all $(v_i, v_j) \in E_1$. Hence $\overline{G_1} \approx \overline{G_2}$. The converse is straightforward.

4. Complete Interval Valued Neutrosophic Graphs

Definition 4.1. An interval valued neutrosophic graph G= (A, B) is called complete if

$T_{BL}(v_i, v_j) = \min(T_{AL}(v_i), T_{AL}(v_j)), T_{BU}(v_i, v_j) = \min(T_{AU}(v_i), T_{AU}(v_j)),$

$I_{BL}(v_i, v_j) = \max(I_A(v_i), I_A(v_j)),\ \ I_{BU}(v_i, v_j) = \max(I_{AU}(v_i), I_{AU}(v_j))$, and

$F_{BL}(v_i, v_j) = \max(F_A(v_i), F_A(v_j)),\ F_{BU}(v_i, v_j) = \max(F_{AU}(v_i), F_{AU}(v_j))$, for all $v_i, v_j \in$ V.

Example 4.2. Consider a graph G^* = (V, E) such that V $=\{v_1, v_2, v_3, v_4\}$, E$=\{v_1v_2, v_1v_3, v_2v_3, v_1v_4, v_3v_4, v_2v_4\}$. Then G= (A, B) is a complete interval valued neutrosophic graph of G^*.

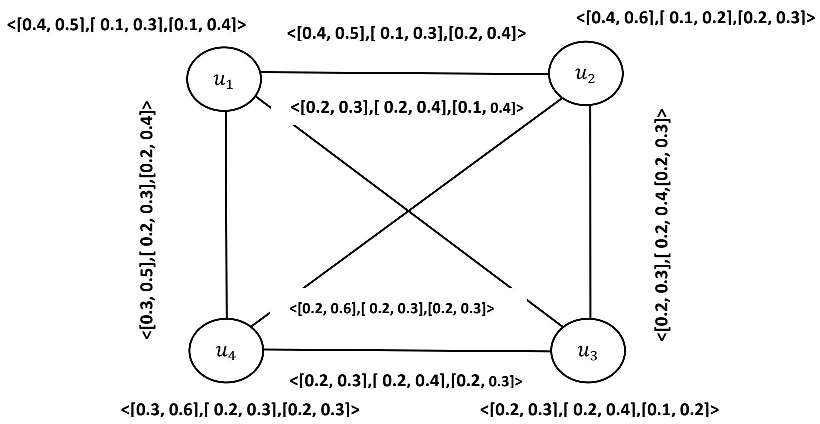

Figure17: Complete interval valued neutrosophic graph

Definition 4.3. The complement of a complete interval valued neutrosophic graph G =(A, B) of G^*= (V, E) is an interval valued neutrosophic complete graph $\bar{G}$= $(\bar{A}, \bar{B})$ on G^*= $(V, \bar{E})$ where

1. $\bar{V}$ =V

2. $\overline{T_{AL}}(v_i)$= $T_{AL}(v_i)$, $\overline{T_{AU}}(v_i)$= $T_{AU}(v_i)$, $\overline{I_{AL}}(v_i)$= $I_{AL}(v_i)$, $\overline{I_{AU}}(v_i)$= $I_{AU}(v_i)$,
$\overline{F_{AL}}(v_i)$ = $F_{AL}(v_i)$, $\overline{F_{AU}}(v_i)$ = $F_{AU}(v_i)$, for all v_j ∈ V.

3. $\overline{T_{BL}}(v_i, v_j)$= min $[T_{AL}(v_i), T_{AL}(v_j)]$ $- T_{BL}(v_i, v_j)$,

$\overline{T_{BU}}(v_i, v_j)$= min $[T_{AU}(v_i), T_{AU}(v_j)]$ $- T_{BU}(v_i, v_j)$,

$\overline{I_{BL}}(v_i, v_j)$= max $[I_{AL}(v_i), I_{AL}(v_j)]$ $- I_{BL}(v_i, v_j)$,

$\overline{I_{BU}}(v_i, v_j)$= max $[I_{AU}(v_i), I_{AU}(v_j)] - I_{BU}(v_i, v_j)$, and

$\overline{F_{BL}}(v_i, v_j)$= max $[F_{AL}(v_i), F_{AL}(v_j)]$ $- F_{BL}(v_i, v_j)$,

$\overline{F_{BU}}(v_i, v_j)$= max $[F_{AU}(v_i), F_{AU}(v_j)]$ $- F_{BU}(v_i, v_j)$, for all (v_i, v_j) ∈ E.

Proposition 4.4: The complement of complete IVN-graph is a IVN-graph with no edge. Or if G is a complete then in $\bar{G}$ the edge is empty.

Proof

Let G= (A, B) be a complete IVN-graph.

So $T_{BL}(v_i, v_j)$ = $\min(T_{AL}(v_i), T_{AL}(v_j))$, $T_{BU}(v_i, v_j)$ = $\min(T_{AU}(v_i), T_{AU}(v_j))$, $I_{BL}(v_i, v_j)$ = max $(I_{AL}(v_i), I_{AL}(v_j))$, $I_{BU}(v_i, v_j)$ = max $(I_{AU}(v_i), I_{AU}(v_j))$ and $F_{BL}(v_i, v_j)$ = max $(F_{AL}(v_i), F_{AL}(v_j))$, $F_{BU}(v_i, v_j)$ = max $(F_{AU}(v_i), F_{AU}(v_j))$, for all $v_i, v_j \in$ V

Hence in $\bar{G}$,

$\bar{T}_{BL}(v_i, v_j)$= $\min(T_{AL}(v_i), T_{AL}(v_j)) - T_{BL}(v_i, v_j)$ for all i, j,…..,n

= $\min(T_{AL}(v_i), T_{AL}(v_j)) - \min(T_{AL}(v_i), T_{AL}(v_j))$ for all i, j,…..,n

= 0 for all i, j,…..,n

$\bar{T}_{BU}(v_i, v_j)$= $\min(T_{AU}(v_i), T_{AU}(v_j)) - T_{BU}(v_i, v_j)$ for all i, j,…..,n

= $\min(T_{AU}(v_i), T_{AU}(v_j)) - \min(T_{AU}(v_i), T_{AU}(v_j))$ for all i, j,…..,n

= 0 for all i, j,…..,n.

and

$\bar{I}_{BL}(v_i, v_j)$= $\max(I_{AL}(v_i), I_{AL}(v_j)) - I_{BL}(v_i, v_j)$ for all i, j,…..,n

= $\max(I_{AL}(v_i), I_{AL}(v_j)) - \max(I_{AL}(v_i), I_{AL}(v_j))$ for all i, j,…..,n

= 0 for all i, j,…..,n

$\bar{I}_{BU}(v_i, v_j)$= $\max(I_{AU}(v_i), I_{AU}(v_j)) - I_{BU}(v_i, v_j)$ for all i, j,…..,n

= $\max(I_{AU}(v_i), I_{AU}(v_j)) - \max(I_{AU}(v_i), I_{AU}(v_j))$ for all i, j,…..,n

= 0 for all i, j,…..,n

Also

$\bar{F}_{BL}(v_i, v_j)$= $\max(F_{AL}(v_i), F_{AL}(v_j)) - F_{BL}(v_i, v_j)$ for all i, j,…..,n

= $\max(F_{AL}(v_i), I_{AL}(v_j)) - \max(F_{AL}(v_i), F_{AL}(v_j))$ for all i, j,…..,n

= 0 for all i, j,…..,n

$\bar{F}_{BU}(v_i, v_j)$= $\max(F_{AU}(v_i), F_{AU}(v_j)) - F_{BU}(v_i, v_j)$ for all i, j,…..,n

= $\max(F_{AU}(v_i), F_{AU}(v_j)) - \max(F_{AU}(v_i), F_{AU}(v_j))$ for all i, j,…..,n

= 0 for all i, j,…..,n

Thus ([$\bar{T}_{BL}(v_i, v_j), \bar{T}_{BU}(v_i, v_j)$], [$\bar{I}_{BL}(v_i, v_j), \bar{I}_{BU}(v_i, v_j)$],
[$\bar{F}_{BL}(v_i, v_j), \bar{F}_{BU}(v_i, v_j)$]) =

= ([0, 0], [0, 0], [0, 0]).

Hence, the edge set of $\bar{G}$ is empty if G is a complete IVN-graph.

5.Conclusion Interval valued neutrosophic sets is a generalization of the notion of fuzzy sets, intuitionistic fuzzy sets, interval valued fuzzy sets, interval valued intuitionstic fuzzy sets and single valued neutrosophic sets. Interval valued neutrosophic models gives more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy, intuitionistic fuzzy and single valued neutrosophic models. In this paper, we have defined, for the first time, certain types of interval valued neutrosophic graphs, such as strong interval valued neutrosophic graph, constant interval valued neutrosophic graph and complete interval valued neutrosophic graphs. In future study, we plan to extend our research to regular interval valued neutrosophic graphs and irregular interval valued neutrosophic.

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