

# A Note on The Dimensionless Gravitational Coupling Constant

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April 12, 2016

## Abstract

In this paper we are rewriting the gravitational coupling constant in a slightly different form than has been shown before (without changing its value). This makes it simpler to understand what is meant and what is not meant by a dimensionless gravitational coupling constant.

**Key words:** Dimensionless gravitational coupling constant, electron mass, Planck mass, Proton mass.

## The Dimensionless Gravitational Coupling Constant

The gravitational coupling constant, often described as the dimensionless gravitational constant, has been discussed in a series of papers in theoretical physics, see for example Silk (1977), Rozenal (1980), Neto (2005) and Burrows and Ostriker (2013). The gravitational coupling constant  $\alpha_G$  is defined as the gravitational attraction between pair of electrons and is normally given by

$$\alpha_G = \frac{Gm_e^2}{\hbar c} = \left(\frac{m_e}{m_p}\right)^2 \approx 1.7518 \times 10^{-45}$$

where  $\hbar$  is Planck's reduced constant,  $m_e$  is the electron mass, and  $m_p$  is the Planck mass. In the spirit of Haug (2016), we can rewrite this in the following form

$$\begin{aligned}\alpha_G &= \frac{Gm_e m_e}{\hbar c} \\ \alpha_G &= \frac{l_p^2 c^3}{\hbar} m_e^2 \\ \alpha_G &= \frac{m_e^2}{\frac{\hbar^2}{l_p^2} \frac{1}{c^2}} \\ \alpha_G &= \frac{m_e^2}{m_p^2} \\ \alpha_G &= \frac{\frac{\hbar^2}{\lambda_e^2} \frac{1}{c^2}}{\frac{\hbar^2}{l_p^2} \frac{1}{c^2}} \\ \alpha_G &= \frac{\frac{\hbar^2}{\lambda_e^2}}{\frac{\hbar^2}{l_p^2}} \\ \alpha_G &= \frac{\frac{1}{\lambda_e^2}}{\frac{1}{l_p^2}} \\ \alpha_G &= \frac{l_p^2}{\lambda_e^2}\end{aligned}\tag{1}$$

where  $\bar{\lambda}_e$  is the reduced Compton wavelength of the electron. The Compton wavelength of the electron is the rest-mass wavelength of the electron and the reduced Compton wavelength is this length divided by  $2\pi$ , that is  $\frac{\lambda_e}{2\pi}$ . Further  $l_p$  is the Planck length, which actually is the Compton wavelength of the Planck

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mass. The gravitational coupling constant is simply the reduced Compton wavelength of the Planck mass squared, divided by the square of the reduced Compton wavelength of the electron.

$$\alpha_G = \frac{(\text{Reduced Compton length Planck mass})^2}{(\text{Reduced Compton length mass of interest})^2} \quad (2)$$

The meaning of the term dimensionless in the gravitational coupling constant simply infers that its value will not change even if we change our unit systems for length, time, and kg and thereby change the units for mass, the speed of light, and the Planck constant. However, it is important to be aware that the gravitational coupling constant is not the same for different masses; it only holds between two electrons and shows the gravitational force between those two electrons relative to the gravitational force between two Planck masses. For example, the gravitational coupling constant between two protons is

$$\alpha_{G,Proton} = \frac{l_p^2}{\bar{\lambda}_{proton}^2} = \frac{1.61623 \times 10^{-35}}{2.10309 \times 10^{-16}} \approx 5.90595 \times 10^{-39} \quad (3)$$

Further, and more importantly, the gravitational coupling constant between two Planck masses must be

$$\begin{aligned} \alpha_{G,Planck} &= \frac{Gm_p m_p}{\hbar c} \\ \alpha_{G,Planck} &= \frac{\frac{l_p^2 c^3}{\hbar} m_p^2}{\hbar c} \\ \alpha_{G,Planck} &= \frac{m_p^2}{\frac{\hbar^2}{l_p^2} \frac{1}{c^2}} \\ \alpha_{G,Planck} &= \frac{m_p^2}{m_p^2} \\ \alpha_{G,Planck} &= \frac{l_p^2}{l_p^2} = 1 \end{aligned} \quad (4)$$

That the gravitational coupling constant normally is assigned such a low value  $1.7518 \times 10^{-45}$  simply has to do with the low masses of electrons relative to the Planck mass. It would make more sense to say that the fundamental gravitational coupling constant is linked to two Planck masses and that its value is 1. It basically shows that that the gravitational force is very strong between two dense bodies that are very close to each other.

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