Psychologistics

Essays on Psychologistic Themes

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Preface

Besides my treatise «*A theory of the Comprehensive Endosemasiopasigraphic Algebraico-Predicate Organon and its conformal catlogographic interpretations*: *A general analytical solution of trial decision problems for first order predicate calculus*» (Iosilevskii [2016]), being the principal part of Psychologistics, the latter is supposed to include an indefinite number of relatively independent essays, which are called *Psychologistic Essays* or briefly *Essays*, and which form supplementary material to the subject matter of the above treatise – the material to be treated primarily *egocentrically* (like the treatise itself) with one or another degree of thoroughness. The Essays are supposed to be included under the common heading "*Essays on Psychologistic Themes*" (briefly "*EPT*"). The Essays are not formally included into the treatise, because they are not related to the main object of the treatise as straightforwardly as the Appendices of the latter. Consequently, in contrast to a separate Appendix, which has the same status as a *section* of the treatise, a separate Essay has the status of a *chapter of the EPT* and hence the same status as a *chapter* of the treatise. Since separate Essays are relatively autonomous, therefore the EPT is provided with a *list of references of its own*, which is however a certain part of *the list of references of the treatise*. In this case, the treatise will briefly be referred to in the Essays as the *Theory of Trial Logic* or as the *Trial Logic Theory* and also most briefly as the *TTL* or *TLT* respectively. An Essay may, when convenient, contain some pertinent fragments of the TTL or of another Essay, repeated or cited. Separate Essays are arranged in the EPT in the order, in which they are written, and not in the order, in which the associated topics are used or mentioned of the TTL for the first time, for convenience in cross-reference, they are numbered by ordinal numerals in that order. By the moment the EPT comprises ten Essays, the subject matters of which are recapitulated in the following abstract; "Essay" is abbreviated as "E".

Abstract of the current EPT. The subject matter of the current EPT comprises the following topics: E1 addresses various trends of psychology in historical prospect; E2 addresses the complete taxonomy of the senses of a man; E3 is a discussion of native languages and their codes, and also of artificial (contrived) languages; E4 treats special quotations and some relevant topics in greater detail as compared to their treatment in Preface of the TTL; E5 addresses my solution of the problem of universals; E6 addresses taxonomies of bionts, i.e. instances of hierarchal systems of *irregular*, or *proper*, *classes* of bionts as opposed to *regular*, or *small*, *classes* of objects, called also *sets*; E7 deals with basic biochemistry including basic principles of genetic theory (theory of DNA macromolecules); E8 comprises historical remarks on philosophy, logic, and psychology; E9 presents formulation and proof of generalized associative and commutative laws for an abstract binary composition operator; E10 comprises some general remarks on the TTL and Psychologistics.

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Essay 1. Psychology

1. Trends and branches of psychology

In the course of development of the field of study and discourse that is presently called “psychology”, the meaning of this word has been defined in many different ways, each definition having its immediate scope. Therefore, for avoidance of confusion, I shall employ the noun “psychology” and the letter ‘Ψ’ as general names of the above entire field in its historical perspective and distinguish different trends or branches, or particular treatises of Ψ by means of the appropriate added words as qualifiers to “psychology”. In this case, it is, understood that in quoting of or referring to definitions of “psychology” by different writers, the sense of that word will be specified correspondingly. Any discourse on psychology necessarily involves treatment of certain meanings of the nouns “mind” and “consciousness”, being two most fundamental terms of psychology (cf. section 4 below).

Here follows the definition of “psychology” by James [1890; 1950, vol. 1, p. 1], the definition whose immediate scope is his own monograph.

> «Psychology is the Science of Mental Life, both of its phenomena and their conditions. Phenomena are such things as we call feelings, desires, cognitions, reasonings, decisions, and the like; and, superficially considered, their variety and complexity is such as to leave a chaotic impression on the observer.»

Along with the initial fragment of this definition (which is unfortunately cited sloppily), some other definitions of psychology can be found in Atkinson et al [1987, pp. 12, 13]. These writers themselves briefly define psychology as «*the scientific study of behavior and mental processes*» (*ibid.* p. 13), the understanding being that *the immediate scope of this definition is the writer’s monograph*, – just as *the immediate scope of James’ definition is his monograph*.

James’ treatment of psychology is based on the following four principles, which are stated in some other places of his monograph (e.g., *ibidem*, vol. 1, pp. 183–187, 218–219 ff):

i) Psychology is a natural science that studies the minds of distinct sapient subjects and particularly the mind of a psychologist as an object «in a world of other objects» and that therefore psychology has nothing to do with metaphysics and its notion of absolute Mind unattached to a particular body.

ii) Introspection of each individual psychologist, i.e. examination of his own conscious experience, is the main general method of acquiring data and formulating concepts of various aspects of the mind (consciousness) of any sapient subject.

iii) The assumption that the terms "*mind*" and "*consciousness*" are synonyms.

iv) *Cartesian dualism*, an ontological doctrine according to which reality consists of two independent and fundamental principles (primary sources): mind and matter.

James' treatment of psychology and also the entire field of study of the mind and consciousness of a sapient subject, which is based on the first two, but not necessarily on the last two, of the above four principles, will be called *introspective psychology* or *introspectionism*. By contrast, Atkinson *et ale's* treatment of psychology and also the entire modern field of study and discourse, which is based on *mediately* studying various *observable manifestations* of the mind and consciousness of a sapient subject (as his conscious behavior) by *extrospective* (*objective*) methods, will be called *extrospective psychology* or *extrospectionism*. Particularly, an obstinate attempt to study the mind of a sapient subject by studying exclusively his conscious behavior is called *behavioral psychology* or *behaviorism*. Accordingly, an adherent of introspectionism, extrospectionism, or behaviorism is called an *introspectionist*, *extrospectionist*, or *behavioralist*, respectively.

2. Scientific psychology versus metaphysical one

In accordance with the above said, *modern psychology* can most generally be defined as *the scientific study of the mind of a sapient subject*. That is to say, modern psychology is *scientific psychology* – in contrast to *metaphysical psychology*. Particularly, use of the name "the mind of a sapient subject" and not of "mind" alone is designed to emphasize the fact that a mind unattached to the human body is an impossible abstraction that cannot be an object of scientific psychology. By contrast, the main object of *metaphysical psychology* is Mind as such, which is not attached to any material body. James [*ibidem,* vol. 1, p 145] describes some perplexing features of metaphysical approach to psychology, which he rejects, as follows.

> «Metaphysics means nothing but an unusually obstinate effort to think clearly. The fundamental conceptions of psychology are practically very clear

> to us, but theoretically they are very confused, and one easily makes the obscurest assumptions in this science without realizing, until challenged, what internal difficulties they involve. When these assumptions have once established themselves (as they have a way of doing in our very descriptions of the phenomenal facts) it is almost impossible to get rid of them afterwards or to make any one see that they are not essential features of the subject. The only way to prevent this disaster is to scrutinize them beforehand and make them give an articulate account of themselves before letting them pass. One of the obscurest of the assumptions of which I speak is *the assumption that our mental states are composite in structure, made up of smaller states conjoined*. This hypothesis has outward advantages which make it almost irresistibly attractive to the intellect, and yet inwardly it is quite unintelligible. Of its unintelligibility, however, half the writers on psychology seem unaware. As our own aim is *to understand* if possible, I make no apology for singling out this particular notion for very explicit treatment before taking up the descriptive part of our work. *The theory of 'mind-stuff' is the theory that our mental states are compounds*, expressed in its most radical form.»

The main proposition (sense) expressed in the first part of the above passage should be understood as an advice that any scientist, and not only a psychologist, should use his cognitive methods prudently. Keeping to this advice can be called *mental discipline*. The mental discipline is necessary not only in respect to assumptions, i.e. axioms, but also in respect to definitions. It means, that all statements and all technical terms that are used in a given field of study and discourse should be *syntaxic*, i.e. *mutually consistent*. If, particularly, an expression is used as a technical term homonymously (equivocally) in the same field of study and discourse then contradictions are unavoidable. For instance, saying that the name "*entity*" or "*real number*" is a *countable* one, instead of saying that it is a *count* one, and, at the same time, saying that *entities* or *real numbers* are *uncountable* are fraught with a danger of confusion.

Regarding the second part of the above-quoted passage, which expresses James' proposition about the mind-stuff theory, the following remarks will be in order. The mind-stuff theory has nothing to do with the general theoretical principle, according to which, for the purposes of description and study, a single whole complex phenomenon is *mentally divided*, i.e. *mentally analyzed*, into simpler constituent parts, which are provided with proper or common names and are thus identified or classified respectively. This *abstract* (*mental*) *analysis* can alternatively be regarded as a series of mental acts of *prescinding the simpler ingredients from the complex phenomenon*, although these ingredients are actually inseparable from one another. For instance, Newton prescinded his second law of dynamics, which is supposedly valid for a sole material point moving in an inertial frame of reference in a Euclidean affine space, from observation of complex dissipative motions of actual physical bodies, which occurred in a noninertial frame of reference in a Riemann (non-Euclidean space). That is to say, in the conditions existing on Earth, that law *never* holds. Nevertheless, the second law of dynamics is a fundamental law of classical mechanics. Likewise, James' definition of Psychology is based on the assumption that, in spite of the fact the mind (consciousness) of a sapient subject is a *single whole current complex state of consciousness* of the subject, or, more precisely, of his *cerebral cortex*, it is *analyzable* and particularly *self-analyzable* into *simpler states of consciousness* (*mental states*, *psychical states*, *conscious modification*s), some classes of which (as feelings, desires, cognitions, reasonings, decisions) are mentioned in James' definition, while some others should be understood by the reader at his choice. So far, nobody has suggested any *general phenomenological description* of the mind other than that based on its analysis into hypothetical simpler constituent parts. Inconsistence that may result by such an analysis is most often, not due to the analysis itself, but rather it is due the analyzer's ignorance of some facts.

For instance, here follows a passage from Russell [1895; 1956, p. 2]:

> «Now the only mental states whose immediate causes lie in the external world are *sensations*. A pure sensation is, of course, an impossible abstraction – we are never wholly passive under the action of an external stimulus – but for the purpose of Psychology the abstraction is a useful one. Whatever, then, is not sensation, we shall, in Psychology, call subjective. It is in sensation alone that we directly affected by the external world, and only here does it give us direct information about itself.»

In Preface to his essay Russell says that he has «derived useful suggestions from Professor James's "Principles of Psychology"». Still, the above passage is inconsistent because James' advice that any assumption relevant to psychology should be scrutinized before articulating it and letting it pass went unheeded by Russell.

Indeed, a normal (healthy) man has the following *five exteroceptive senses* (*sense functions*), namely ones of *sight*, *hearing*, *touch*, *taste*, and *smell,* also called the *visual*, *auditory*, *tactile*, *gustatory*, and *olfactory senses* in that order. The *sense of touch* comprises *two senses*: the *sense of temperature* (*warmth and cold*) and the *sense of pressure*. The *sense of taste* comprises *five senses*, namely ones of *sweet*, *bitter*, and *umami*, which are mediated by *G-protein-coupled exteroceptors*, and ones of *salty* and *sour*, which are mediated by *ion-sensitive exteroceptors*. Umami is the sense of *taste of meat*, whose receptors have been discovered recently; "G" is an abbreviation for "*globular*". In addition, a man has the following *four* less definite but not less important groups of *interoceptive senses*, upon existence of which the general agreement has been reached among most biologists and most psychologists: *the sense of skeletomuscular motion and coordination*, also called *the kinesthetic sense* or *kinesthesia*; the *organic sense*, also called the *visceral sense*; *the sense of pain*; and *the sense of balance*, or *equilibrium*, also called *the labyrinthine sense*. From the standpoint of the physical agents that are involved in producing labyrinthine sensations of a sapient subject, namely the external mass force (the force of earth gravity or a centripetal force) acting on the *otoliths* of his internal ear, the labyrinthine sense should, more correctly, be classified as a mixed *extero-interoceptive* or *intero-exteroceptive* one. Still, the labyrinthine sensation inform the sapient subject and only the subject about the orientation of his head relative to the direction of the external mass force and it is not available to any other sapient subject, which has the labyrinthine sensation of his own. Therefore, I classify the labyrinthine sense as an interoceptive one. Thus, all the four interoceptive senses are designed to inform the cerebral cortex of the sapient subject about the running state of various *parts of the subject's body*, and not about the external world as Russell states. Consequently, in contrast to the sensations mediated by exteroceptors, which are, according to Russell, *objective*, the sensations mediated by interoceptors should be called *subjective*. By 1897, when the Russell essay was written, the interoceptive sensations have already been known and even the existence of some other, hypothetical, interoceptive sensations (as sensations, or feelings, of innervation) was debated by psychologists (see, e.g., James [1890; 1950, vol. 2, pp. 488, 493] regarding the *kinesthetic* sensations and *ibidem*, pp. 236, 493 regarding the [unnecessity of] feelings of innervation).•

3. The human nervous system

1) For the purposes of description and study, the *nervous system* (*NS*) of a *normal adult man*, which is called also a *sapient* (or *sage*) *subject* (or a *man* or *individual*), is divided into *two complementary subsystems*: the *central*, or *cerebrospinal*, *nervous system* or briefly *CNS* and the *peripheral nervous system* or briefly *PNS* (see, e.g., Campbell [1990, pp. 996–1007] or any textbook on general biology or anatomy). The CNS consists of two parts being bilaterally symmetric each: the *brain*, or *encephalon*, contained within the *cranium*, and the *spinal cord*, or *medulla spinalis*, enclosed in the *vertebral canal* and continuous superiorly at the level of the upper border of the *atlas vertebra* with the most inferior region of the brain called the *medulla oblongata* or *myelencephalon*. The brain is, in turn, divided into the following *three parts* in the direction form the top of the cranium to its base (equivalent to the direction from the front to the rear in quadrupeds): the *forebrain* or *prosencephalon*, also called the *cerebrum* (from the homonymous Latin etymon "*cĕrebrum*" meaning *the brain*"); the *midbrain* or *mesencephalon*; and the *hindbrain* or *rhombencephalon.* The hindbrain and midbrain together appear as a cap on the spinal chord, which is called the *brainstem*. The hindbrain is divided into two parts, namely the lower (inferior) *myelencephalon* or *medulla oblongata,* immediately continuous with the spinal cord, and the upper (superior) *metencephalon*. The cerebrum (forebrain, prosencephalon) is divided into two parts: the upper (superior) *telencephalon* and the lower (inferior) *diencephalon* or *inner brain*. The telencephalon is the largest part of the human brain. It comprises the *two cerebral hemispheres* which are united in the inner region by a thick band of *commissural* nervous fibers, which is called the *corpus callosum*. Each cerebral hemisphere has a cavity called a *lateral ventricle*. The *left* lateral ventricle is called the *first ventricle*, and the *right* lateral ventricle is called the *second ventricle*, the understanding being that there are *four communicating* ventricles in the brain altogether. The *third ventricle* is located in the diencephalon. The *fourth ventricle* is one of the three main divisions of the metencephalon; the two others are the anterior *pons Varolii*, or, briefly, *pons*, and the posterior *cerebellum* (from the homonymous Latin etymon "*cĕrĕbellum*" meaning *a*

small brain). The fourth ventricle is continuous with a narrow longitudinal canal piercing centrally the spinal cord. The cerebral ventricles and the spinal canal are filled with the *cerebrospinal fluid* (*CSF*).

2) The cerebrum is overlaid with a highly folded bark-like superficial layer, which is made of *vesicular gray matter* and which is called the *cerebral cortex* (from the Latin etymon "*cortex*" meaning "bark", "rind", or "shell") or, informally, the *convolutions of the cerebrum*. Vesicular gray matter consists mainly of neuron bodies called *perykaryons*. The cerebral cortex is the largest aggregation of the vesicular gray matter both in the brain and in the entire NS. Being less than 5 mm thick, the human cerebral cortex accounts for more than 0.8 of the entire brain mass of about 1.35 kg, and it has the surface area of about 0.5 m^2. The cerebral cortex is *the ultimate*, or *end*, *organ and seat of the consciousness of the man* and therefore it is alternatively called *the mind of the man.*

3) The cerebral cortex is one of the two most mysterious *nervous centers* found in the CNS, and generally in the entire kingdom of life. The other one is the *hypothalamus* – the small *central neuroglandular organ*, having the mass of about 1/300 of the entire brain mass and the size of an almond, whose function is the *autonomic control and integration of activity of the endocrine system* (*ES*). In other words, the function of the hypothalamus is to incorporate *the ES into the NS at the level of unconsciousness*. The hypothalamus accomplishes its function in cooperation with two compound organs: (i) its *two-lobe appendage*, which is located at its base and which is briefly called the *hypophysis* or *pituitary* and also, in full, the *hypophysis cerebri*, *pituitary body*, or *pituitary gland* and (ii) three highly vascular regions, which are located in a close proximity of the hypothalamus and which are informally called *circumventricular organs* (*CVO's*), because they as if circumvent the *blood-brain barrier* (*BBB*). The CVO's receive hormonal signals with the bloodstream and transduce them into the corresponding nerve signals, which they transmit to the hypothalamus positioned in the interior of the BBB. The full formal names of the three CVO's are: "*the subfornical organ*" ("*the SFO*"), "*the area postema*", and "*the organum vasculosum of the lamina terminalis* ("*the OVLT*"). The pituitary is known as the *master gland* because it secretes a great number of hormones, some of which are so-called *tropic hormones* – hormones that regulate the activity of other *endocrine glands*. To be specific, the *posterior pituitary [lobe]*, called the *neurohypophysis*,

stores and secretes two ordinary (not tropic) peptide hormones (oxylocin and antideuretic hormone – ADH), which are produced by the hypothalamus and which act directly on muscles and on the kidney (being *exocrine glands*). The *anterior pituitary [lobe]*, called the *adenohypophysis*, produces its own peptide hormones, several of which are tropic. Besides the above two hormones, the hypothalamus produces various special hormones, which are called *releasing factors* and which stimulate or inhibit release of specific hormones by the cells of the *anterior pituitary*.

4) The hypothalamus lies at the base of the diencephalon, and hence at the base of the entire cerebrum (prosencephalon), and it is directly and deviously connected by input and output nervous fibers with many parts of the CNS, including the cerebral cortex. Through these fibers, the hypothalamus receives nervous signals originated in various parts of the man's body (particularly those originated in the CVO's) and responds to them both by triggering the appropriate nervous signals targeted at various parts of the brain, particularly at the cerebral cortex, and by secreting the appropriate releasing factors targeted at the anterior pituitary. Although the activity of hypothalamus is *autonomic* (*unconscious*), the nervous signals that it transmits to the cerebral cortex in response to the nervous signals, which it receives from the CVO's and which are mediated by the corresponding hormonal signals, are consciously interpreted in the cerebral cortex and are mentally experienced by the man as his *desires* or *emotions*.

5) Besides the cerebral cortex, another continuous, but much smaller, mass of vesicular gray matter occurs in the brain in the form of the *laminae* on the surface of the *cerebellum* – the part of the brain that concerns with the coordination of movements of the skeletal muscles and with the maintenance of equilibrium. Also, a continuous mass both of vesicular and of *fibrous gray matter* is found in the spinal chord in the form a central core somewhat H-*shaped*, – or, putting it differently, *butterfly-shaped*, – in a cross section and pierced centrally by a narrow longitudinal canal continuous with the cerebral ventricles and filled with the CSF. More precisely, in a cross-section of the spinal chord, vesicular gray matter is arranged in the form of two crescentic masses, which are placed one on each lateral half of the cord and which are united together by a transversal band of *fibrous gray matter*, called the *gray commissure*. The spinal cord serves as a pathway for nervous signals to and from the brain and also as a center for the coordination and integration of reflex actions of the

body independent of the brain. It is noteworthy that, in the brain, the two large masses of vesicular gray matter are organized in layers that are located in the outer region, whereas the *white matter*, consisting mainly of myelinated neuronal axons (nervous fibers), is located mostly in the inner region. In the spinal cord, the situation is reverse: the white matter surrounds the inner H-shaped core of gray matter. The interested reader will find further details regarding anatomic structure and biological functions of the spinal cord, e.g., in Gray [1977, pp. 710–720]) and Campbell [1990, pp. 998–999].

6) In addition to the above-mentioned three large masses of gray matter, there are a great many of relatively small compact aggregations of vesicular gray matter, which are scattered throughout the inner region of the brain and throughout the outer region of the spinal chord. These aggregations of vesicular gray matter are called *cerebrospinal nuclei* ("*nucleus*" in the singular) or briefly *nuclei* if there is no danger of misunderstanding. Discriminatorily, a nucleus is said to be *cerebral* if it is located in the brain and *spinal* if it is located in or associated with the spinal cord.

4. "Mind" and "consciousness"

4.1. Ontological monism versus ontological dualism

1) Most generally, psychology can be defined as *the scientific study of the mind of a sapient subject*. The use of the name "the mind of a sapient subject" and not of "mind" alone is designed to emphasize the fact that a mind unattached to the human body is an impossible abstraction that cannot be an object of psychology. However, under the above definition, the subject matter of psychology depends on a definition of the noun "*mind*" in its relation to the names such as "*consciousness*", "*unconsciousness*", "*subconsciousness*", "*mental entity*" ("*mental state*", "*state of consciousness*", "*conscious modification*"), "*cerebral cortex*", "*brain*", "*nervous system*", etc, which is usually relegated to *philosophical psychology*, also called *philosophy of psychology* or *philosophy of mind*. At the same time, "mind" is the most fundamental term of philosophy and psychology, upon the sense of which a general agreement has not been reached among philosophers and psychologists.

2) *Monism* and *dualism* are two incompatible doctrines that exist in philosophy of mind since ancient time to the present. Greek philosopher *Permenides* (born 515 BC), leader of the Eleatics, proposed the general philosophical principle "*all is one*", and he is therefore credited to be the pioneer both of monism and

metaphysics, although the terms "monism" and "metaphysics" were introduced much later. Dualism of Western philosophy can be traced back to Plato and Aristotle. Still, most explicitly and precisely, dualism was formulated by the French mathematician and philosopher *René Descartes* (1596–1650), also known by his Latin name "*Renatus Cartesius*", and monism by the Dutch philosopher *Baruch* (*Benedict[us]*) *Spinoza* (1632–1677). *Monism* is an ontological doctrine, according to which *reality* consists of a *single fundamental principle* (*primary source*): either mind (pure mental substance) or matter (pure bodily substance), or else neither of the two, i.e. the two in *one biune* (*unanalyzable*) *reality*, which was the answer to the above dilemma by Spinoza (see, e.g., Durant [1926, pp. 134–135]). *Dualism* is an ontological doctrine, according to which reality consists of *two independent fundamental principles*: mind and matter, the understanding being that the two coexist and interact in a common world within the same physical limits, but they are never transduced (metamorphosed) into each other and do not produce each other.

3) Dualism was particularly adopted by James who described it with great clarity thus (*ibidem*, vol. 1, p. 218):

> «*The psychologist's attitude towards cognition* will be so important in the sequel that we must not leave it until it is made perfectly clear. It is a *thoroughgoing dualism*. It supposes two elements, mind knowing and thing known, and treats them as irreducible. Neither gets out of itself or into the other, neither in any way *is* the other, neither *makes* the other. They just stand face to face in a common world, and one simply knows, or is known unto, its counterpart. This singular relation is not to be expressed in any lower term, or translated into any more intelligible name…»

Still, dualism creates the insuperable problem of consistently formulating a relation between the mind and brain of a sapient subject, i.e. actually between the mind and the pertinent matter, thus leading to logical inconsistency. Such inconsistency is particularly found in James' monograph in question.

4) James seems to have been the first psychologist to explicitly define and treat psychology as a natural science rather than a metaphysical psychology. He says (*ibidem*, vol. 1, p. 183):

> «PSYCHOLOGY IS A NATURAL SCIENCE. That is, the mind that a psychologist studies is the mind of distinct individuals inhabiting definite

portions of a real space and of a real time. With any other sort of mind, absolute intelligence, Mind unattached to a particular body, or Mind not subject to the course of time, a psychologist as such has nothing to do. ‘Mind’, in his mouth, is only a class name for *minds*.»

At the same time, by adopting dualism, James prescinded the mind of a man from the brain of the man and thus caused a mental discomfort to his own mind – the discomfort, which he discussed under the heading: “DIFFICULTIES OF STATING THE CONNECTION BETWEEN MIND AND BRAIN” (*ibidem*, pp. 176–179). Particularly, he says (*ibidem*, p. 177):

> «…The consciousness, which is itself an integral thing not made of parts, ‘corresponds’ to the entire activity of the brain, whatever that may be, at the moment. This is a way of expressing the relation of mind and brain from which I shall not depart during the reminder of the book…»

The above quotation explicitly indicates that *James equates mind and consciousnes*, – *just as all early psychologists do*. At the same time, the relation between mind (conscioosness) and brain is blurred by using the verb “corresponds” in an undefined allegoric sense and also by using the vague description “the entire activity of the brain, whatever that may be, at the moment”, which may mean both conscious (mental, psychical) activity and unconscious (material, physical, physiological) activity. Also, somewhat earlier in his book, James says (*ibidem*, p. 66):

> «…*the cortex is the sole organ of consciousness in man*.»:

Using the presently common terminology, the last statement of James, along with his entire analysis of the consciousness of a man (see, e.g., *ibidem*, vol. 1, pp. 185–187), which he identifies with the man’s mind, can be paraphrased thus:

> ***Résumé of James’ concepts of mind and consciousness*:** The cerebral cortex of a sapient subject is the sole organ (effector, creator) and seat (receptacle and interpreter) of his consciousness, i.e. of his mind.

Knowledge, which the mind of a man acquires, comes through his sensations (perceptions, knowledge by acquaintance) or through his conceptions (classes, knowledge by abstraction). Both the sensations and the conceptions *are made* (*created*) and interpreted *by the cerebral cortex* of the man. The latter is material (organic, bodily) reality, and not mental one. Therefore, James’ statement quoted and resumed above contradicts his statement, which has been quoted previously and which says that the «mind knowing» or a «thing known» [to it] «neither gets out of itself or into the other, neither in any way *is* the other, neither *makes* the other.»

5) In textbooks on psychology and in explanatory dictionaries of the English language, there are many different definitions, concise and extended, of the senses of the nouns "mind" and "consciousness" as technical terms. However, the distinction between monistic and dualistic doctrines of a mind is often blurred by the fact that it is neither discussed nor even mentioned, while the phraseology used and the definitions made are eclectic and confusing (cf. Atkinson et al [1987, pp. 109, 110ff]). For instance, A Merriam-Webster® [1981] defines the meaning of the noun "mind" and its synonym "psyche", and also some pertinent adjectives thus:

«**mind** ... *n* ...**3** ... **b** (1) **:** an organized group of events in neural tissue occurring mediately in response to antecedent intrapsychic or extrapsychic events which it perceives, classifies, transforms, and coordinates prior initiating action whose consequences are foreseeable to the extent of the available information (2) **:** the aspect of a biological organism that is not organic in nature ⟨in man ~ is experienced as emotions, imagination, or will⟩ **c :** the sum total of the conscious states of an individual **d :** the sum total of the individual's adaptive activity considered as an organized whole though also capable of being split into dissociated parts ⟨as the conscious and the unconscious ~⟩ **e :** one's capacity for mental activity **:** one's available stock of mental and adaptive responses...

psyche ... *n* ... **1** -s ... **b :** the specialized cognitive, conative, and affective aspects of psychosomatic unity **:** MIND; *specif* **:** the totality of id, ego, and superego including both conscious and unconscious components...

mental ... *adj* ... **1 :** of or relating to mind...

psychical ... *adj* ... **2 :** of or relating to the mind **:** MENTAL – contrasted with *physical*

endopsychic ... *adj* ... **:** arising or existing within the mind

extramental ... *adj* ...**:** existing outside the mind ⟨the ~ world⟩

extrapsychic *or* **extrapsychical** ... *adj* ... **:** being or occurring outside the psyche, the mind, or the personality **– extrapsychically** *adv*

intramental ... *adj* ... **:** INTRAPSYCHIC

intrapsychic *or* **intrapsychical** ... *adj* ... **:** being or occurring within the psyche, the mind, or the personality ⟨~ conflicts⟩ ⟨~ processes⟩ – **intrapsychically** *adv*»

The etymological senses of the qualifiers, occurring in the above quotation, are explicated by the following brief etymological English-Greek and English-Latin dictionaries, which have been compiled in accordance with Simpson [1968] and Pring [1982] respectively (see also the pertinent entries of Dicts A1.1 and A1.2 in the TTL).

Dict. 4.1.

"*endo*"-, comb. form, from the Greek adv. and prep. "ένδον" \énðon, énthon\, meaning *within*.

"*exo*"-, comb. form, from the first of the Greek synonymous prepositions "εξ" \eks\ and "εκ" \ek\, meaning (origin) *from*, and from the first of the kindred comb. forms "εξ"- \eks\, "εκ"- \ek\, and "ξε"- \kse-\, meaning *out*, *off*.

"*psychical*" or "*psychic*", adj., from the counterpart Greek adjective "ψυχικός" \psiçikós, psihíkós\ being in turn a derivative of the noun "ψυχή" (see Dict. A1.1 in the TTL).

Dict. 4.2.

"*extra*"-, comb. form, from the Latin synonymous adjectives "*exter*" and "*extĕrus*" (fem. -"*a*", neut. -"*um*"; compar. "*extĕrĭor*", fem. and neut. -"*ĭus*"; superl. "*extrēmus*" or "*extĭmus*", fem. -"*a*", neut. -"*um*"), meaning *outward*, *foreign*, *strange*, and from the cognate adjective "*externus*" (fem. -"*a*", neut. -"*um*"). meaning *that is outside*, *external*.

"*intra*"-, comb. form, from the Latin adv. and prep. "*intrā*", meaning *within*, *inside*.

"*mental*", adj., from the Latin noun "*mens*" (genitive "*mentis*"), meaning *the mind*, *understanding*, *reason*, *intellect*, *judgment*.

"*cis*"-, comb. form, from the Latin prep. with acc. "*cis*", meaning *on this side of*.

"*trans*"-, comb. form, from the Latin prep. with acc. "*trans*", meaning *over*, *across*, *on* or *to the other side of*.

According to Webster's definitions 3b(1) and 3c of "mind", the mind of a sapient subject is implicitly identified with his consciousness, i.e. with the state of consciousness of his cerebral cortex. By contrast, according to Webster's definitions 3b(2) and 3d of "mind"and also according to the very last definition of "psyche", the mind of a sapient subject consists of both his consciousness and his unconsciousness. Therefore, the latter three definitions are inconsistent with the former two. At the same time, definition 3e of "mind" and all the above definitions of kindred or cognate adjectives are ones of *idem per idem* or even of *obscurum per obscurius*. Particularly, the expressions such as "existing within the mind" and "existing outside the mind", which serve as definientia of the respective adjectives from "endopsychic" to "intrapsychic", are just verbatim etymological translations of those adjectives from Greek or Latin or both in accordance with the pertinent entries of Dicts 4.1 and 4.2 (or A1.1 and A1.2). Those definientia can naturally be understood (interpreted) as abbreviations of the expressions "within the physical limits of the mind" and "outside the physical limits of the mind" respectively. Thus, the count nouns "mind" and "psyche" as defined by any of their Webster's definitions quoted are *nomina nuda* (*mere names*), because their sense is not substantiated and illustrated sufficiently for unambiguously recognizing concrete objects to be conformed to that sense. Also, the entire set of the above definitions leaves the reader in a quandary as to whether a mind is a *physical* (*real*) *spatiotemporal physical object*, i.e. a matter having a certain form [of existence], or whether a mind is a form prescinded from a certain matter, i.e. a purely *imaginary* and hence *immaterial* (*ideal*) *entity* that has, nevertheless, certain *physical limits*.

6) In this exposition, in forming new Anglicized terms and in using established Anglicized dictionary terms, I shall, as far as possible, stick to the following formation rule that I call the *principle of etymological homogeneity of complex Anglicized terms* or, briefly, the *etymological homogeneity principle* (*EHP*):

> A new complex Anglicized monomina (monomial) should, as far as possible, be *etymologically homogeneous* in the sense that each of its formative morphemes should originate from the same language, particularly either from Greek or from Latin, unless of course a morpheme has an etymon in both languages.

According to this principle, the self-explicative complex adjectives "intramental" and "extramental" of the Latin origin or their synonyms "endopsychical" ("endopsychic") and "exopsychical" ("exopsychic") of the Greek origin are acceptable, because they are etymologically homogeneous, whereas the established adjectives "intrapsychic" (“intrapsychical”) and “extrapsychic” (“extrapsychical”) are not. At the same time, in no connection with the EHP, the adjectives “intramental”, “intrapsychic” (“intrapsychical”), and “endopsychic” (“endopsychical”) are *redundant* synonyms of “mental” and “psychical”.

4.2. Physicalistic monism of Psychologistics

Df 4.1. 1) In treating of the mind and consciousness of a sapient subject, dualism results in contradictory wordings. Therefore, in order to avoid logical inconsistency, I adopt the doctrine of *physicalistic monism*, in the framework of which I identify *the mind of a sapient subject* with *his cerebral cortex*. Accordingly, I identify *the conscious mind of a sapient subject* with *his conscious* (*waking*) *cerebral cortex* and *the subconscious* (*sleeping*) *mind of the sapient subject with his subconscious* (*sleeping*) *cerebral cortex*. In this case, the conscious mind and the subconscious mind are two different *hypostases* (*ways of existence*, *matters*) of the mind (cerebral cortex) of the sapient subject, whereas *the consciousness* and *subconsciousness* of the sapient subject are *states* (*forms of existence*) of the above two hypostases of the mind. It is noteworthy that there is no categorical criterion, according to which the mind should be identified with the cerebral cortex, and not, say, with the cerebrum (*forebrain*, *prosencephalon*) or not with the entire brain. If the mind is identified with the above larger part of the brain or with the whole of it, the entire monistic phraseology, in which the words “mind” and “consciousness” occur, does not change.

2) I shall use the name “*mental realm*” as synonym of “*consciousness*”, the understanding being that the noun “*realm*” alone can be used equivocally as a synonym of one or another of the following nouns: “*kingdom*”, “*region*”, “*territory*”, “*sphere*”, “*domain*”, “*range*”, etc is likely akin to “*royal*”, and *not to* “real”. Etymologists consider the following two Latin nouns as etymons of “realm” (cf. Simpson [1968])): “*rĕgĭmen*” (pl. “*rĕgĭmina*”), some meanings of which are the same as those of “the government of a state”, “ruler”, and “governor”, and “*regnum*” (pl. “*regni*”), some meanings of which are the same as those of “kingdom”, “royal power”, and “authority”. In any case, I regard the names such as “*mental realm*”, “*realm of thought*”, “*realm of sensations*”, etc as etymologically correct and therefore as self-explanatory ones.

3) I shall say that the mind of a sapient subject is *a* mind if I do not mentally fix the subject and *the* mind if I mentally fix the subject; and similarly with “cerebral

cortex" and also with "*entelechial mind*" or "*potential mind*" (to be defined in due course) in place of "mind".•

Cmt 4.1. The mind (cerebral cortex) of a sapient subject is a unique mass of *vesicular gray matter*, which consists of an enormous but finite number of perikaryons (neuron bodies), dendrites, synapses, neuroglial cells, and some other biological structural ingredients. At any moment when the mind is *awake* (*waking*, *not sleeping*), the *consciousness* of the mind is its *state of aggregation* just as gaseousness, liquidness (liquidity), or solidness (solidity) is the state of aggregation of gas, liquid, or solid respectively. That is to say, the waking mind is conscious in the same sense as a body of gas, liquid, or solid is gaseous, liquid, or solid respectively; hence, *consciousness is the way of existence of the waking mind.* In this case, a separate neuron, perikaryon, dendrite, synapse, or neuroglial cell comprised in the mind is not conscious, i.e. it has no conscious state of aggregation simply because it is not an aggregate – just as a separate molecule has no physical state of aggregation because it is not an aggregate. In accordance with the above-said, consciousness can be prescinded from the conscious mind and be analyzed as such, – just as liquidness, e.g., can be prescinded from liquid and be analyzed as such. Stating that the consciousness of a sapient subject is the conscious state of aggregation of his mind (cerebral cortex), I mentally put forward *the conscious form of existence of the mind*. Under this mental attitude towards the consciousness, it is consistent to state that the *conscious mind* (*conscious cerebral cortex*) of the sapient subject is the *sole organ* (*effector*, *creator*) and *seat* (*receptacle and interpreter*) *of his entire consciousness* and of any parts of it, into which it is usually analyzed and which are called *mental* (or *psychical*) *states* (or *entities*), *states of consciousness*, *conscious modifications*, *brain*, or *cerebrocortical* (briefly, *cortical*) *symbols*, and also by some other names that will be introduced as required in order to express the pertinent connotations. It would, however, have been logically inconsistent to make the same statement with "mind" or "cerebral cortex" alone, without the qualifier "conscious". Indeed, the statement that the mind (cerebral cortex) of the sapient subject has the above properties of his conscious mind (conscious cerebral cortex) means that the entity that is, in this case, called "the mind" or "the cerebral cortex" is prescinded as a certain *unconscious matter* from the conscious mind (conscious cerebral cortex) by freeing the latter of its consciousness, i.e. of its conscious state of existence. This abstraction is as meaningless as prescinding a certain *non-liquid matter* from liquid by freeing the latter of its liquidness, i.e. of its liquid state of existence.•

5. Entities and coentities

Df 5.1. 1) I use the count noun "entity" (from Greek "οντότης" \ontótis\ *s.f.,* meaning *an entity*, *being*; or, *individuality*) for mentioning *anything that can be treated* (*spoken*) *of*. Since the word "entity" is informal, therefore it is impossible to restrict its use formally. I just avoid using that noun in any context, in which it has no denotatum.

2) A distinct entity, of which I am *conscious* (*aware*) at a *current moment* when I wake is called *my coentity* [*at that moment*] or, more explicitly, *a coentity of mine* [*at that moment*], – to emphasize that this coentity is not the only one that I have or can have. Thus, a coentity of mine is an *ingredient* of my *current universe* at any current moment (see the item 8 below in this definition). The prefix "co", occurring in the count noun "coentity", has a double meaning. First, it is a conventional perfective, associative, and collective prefix, meaning *joint* in this case. Second, it is an abbreviation of the adjective "*conscious*".

3) An entity that I *know by acquaintance* in the sense that it is capable of mediating the adequate distinct *sensation* (*percept*) of mine, thus becoming my sensory object, is called a *sensible entity* or more specifically a *sensible thing*.

4) An entity that I know (apprehend) through my *reason*, particularly by *induction* or *by deduction*, rather than through my *sensations* (*percepts*), *belief*, or *intuition*, as one that exists in my *consciousness* is called an *ideal*, or *psychical*, or *mental*, *entity* (or *coentity*) *of mine*, the understanding being that a mental entity of mine is located within the physical limits of my cerebral cortex (my mind). Any one of the names "*mental state*", "*state of consciousness*", "*conscious modification*", "*thought*" [*sensu lato*], and "*feeling*" [*sensu lato*], due to James [1890; 1950, vol. 1, pp. 185–187], and also "*idea*" [*sensu lato*] will be used synonymously (interchangeably) with the term "*mental entity*", whereas either qualifier "*psychical*" or "*ideal*" can be used synonymously (interchangeably) with "*mental*".

5) An entity, sensible or not, that I know (apprehend) through my *sensations* (*percepts*) and, perhaps, my *reasons*, rather than through my *belief* or *intuition*, as subsisting in space and time in the form of a self-contained spatiotemporal entity,

other than any psychical entity of mine, is called a *physical entity*, and also a *thing*; any of the qualifiers "*real*", "*exopsychical*" (or "*exopsychic*"), "*metapsychical*", "*extramental*", and "*transmental*" will be used interchangeably with "physical".

6) An entity, which I regard as a biune one that comprises a sensible entity (sensible thing) and the sensation (percept), which it mediates in my cerebral cortex, – so that the latter two entities are extensions of each other, – is called a *physopsychical* (*physico-psychical*) or *psychophysical* (*psychico-physical*), *entity* (or *coentity*) *of mine*.

7) My *consciousness*, called also my *mental realm*, is *the most inclusive psychical coentity of mine, through which I know any other coentity of mine*, including *my Self*. *I regard my consciousness*, i.e. *the latter mentally experiences itself*, *as a unique single whole dynamic (mutable*) *state of consciousness of my mind* (*cerebral cortex*) *at any current moment when I wake*. That is to say, *my consciousness is a mental process, in which the entire mental events succeed one another along with time from past through present to future*. Consequently, my state of being conscious and hence *self-conscious* at any current moment when I wake is called *the stream of my consciousness* and also *my current psychical*, or *mental*, *life* at *that moment.*

8) At any *current moment* when I am *conscious* (*waked*), *I* and *not-I* constitute *my natural* (*conscious, waking*) *universe* [*at that moment*], – as contrasted to *my extranatural* (*subconscious*, *sleeping*) *universe*, i.e. *the realm of my dreams*, which sometimes figures in my *subconsciousness* when I sleep. Henceforth, by "my universe" I mean my natural universe unless stated otherwise. *Not-I* is called *my external world*. My state of being conscious and hence *self-conscious* at any current moment when I wake is called *my consciousness* and also *my current psychical*, or *mental*, *life*.

9) I (my Self), my external world, and my universe are my coentities, and each one of the three comprises some less inclusive coentities of mine. That is to say, the count name "coentity of mine" denotes the widest class of entities, which I can conceive (think of) and hence be conscious of, although some of them can be unknown to me at the current moment. This class is *the range of* [*import values of*] *the common name* "*an entity*". Consequently, I may use the latter name for selectively referring to any entity comprised in me or in my external world or in my universe, including the wholes of them.

10) I, my external world, and my universe are *unique* and are different from you, your external world, and your universe respectively. At the same time, my external world and your external world contain *tokens of certain exteroceptive xenonyms (signs)*, by means of which I can communicate with you or you can communicate with me if you wish. Accordingly, the name "an entity" means a coentity of mine or a coentity of any conscious sapient subject. The only fact that I know about coentities of another conscious sapient subject is that they exist and are associated with the subject in the same way as my coentities are associated with me.

11) Consequently, by transcendental extrapolation, the above items of this definition are applicable to any sapient subject in place of me.•

Cmt 5.1. In accordance with Df 5.1(1), I use the count noun "entity" in the same way, in which Aristotle used his term "πράγμα" \práγma\ (pl. "πράγματα" \práγmata\), which is translated by the English count noun "*thing*". Accordingly, the word "thing" is often used in English in the widest sense interchangeably with the term "entity", but I shall not follow this usage. In accordance with Df 5.1(3), I shall use the word "thing" as a synonym of the count name "real entity", because this usage is in agreement with the etymological sense of the adjective "real"; the latter is derived the Latin noun "*rĕs*" (pl. "*rĕs*") that assumes, according to Simpson [1968], the same meanings as "thing", "object", "matter", "affair", or "circumstance".•

Cmt 5.2. 1) Just as the names "space point", "space vector", and "real number", the name (noun) "entity" is a *count* one in the sense that it has a plural and is used with an indefinite article or with other quantifiers (as numerals). At the same time, just as space points, space vectors, and real numbers, entities are *uncountable* in the sense that they cannot be put into correspondence with successive natural numbers. Accordingly, most entities have no individual proper names, – just as most elements of an uncountable set in mathematics being *some entities*. For the set of real numbers, this point is explicated below.

2) Except for *rational real numbers* and *algebraic real numbers*, i.e. the roots of algebraic equations whose coefficients are rational numbers, and also except for *special irrational* (*transcendental*) *numbers* as π or *e*, most *irrational* real numbers have no proper names. Indeed, irrational real numbers are by definition real numbers *not being quotients of integers*. Therefore, an irrational real number is often thought of as a real number that can be expressed by an *imaginary infinite decimal-fraction*

numeral with supposedly *uncountable* number of digits, no finite sequence of which is repeated indefinitely. However, such an *imaginary* numeral can not actually be written down and *it is therefore a mental image of no graphic symbol*, i.e. it is a *fiction*. Therefore, an actual *graphic symbol* that consists of a finite sequence of decimal digits followed by three dots is conventionally used as a makeshift of the above fictitious infinite numeral. However, such a graphic symbol, – say, '3.1415...' that is used as a makeshift of 'π', – is not a proper name of any number. At the best, the expression '3.1415...' can be regarded as a variable, i.e. as a common name, which stands for any real number in the semi-closed interval [3.1415, 3.1416). On the other hand, when the three dots are omitted from the expression '3.1415...', the latter turns into the constant '3.1415' which is a proper name of the specific rational real number 3.1415. Thus, the rational number π is thought of as one that is represented by an as if non-periodic infinite decimal-fraction numeral '3.1415...' with supposedly *uncountable* number of digits, *which is equipollent to the power of continuum*. Any irrational real number has a like paradoxical digital structure.

3) The *class* (*set*) *of real numbers*, can be thought of as an *infinite open interval* $(-\infty,+\infty)$, while any given real number is a certain point of the interval. At the same time, a point is intuitively thought of as a primitive (elementary) entity. The first known definition of a point as given by Pythagoras says that a point is «a monad having position». In Euclid [1956, vol. 1, pp. 153, 154]: «A *point* is that which has no part.» The paradox, according to which a point, being a supposedly elementary object of continuum, turns out to be not namable by any sequence of a countable number of digits, can be called *the problem of continuum*.

4) In modern mathematics, there is no notion of points neighboring to a point, just as there is no notion of *smallness* at all. No matter how small a real number ε is in our intuitive understanding, the transformation $y = \tan\frac{\pi x}{2\varepsilon}$, e.g., maps the interval $(-\varepsilon,\varepsilon)$ of denotata (denotation values) of the variable 'x' into the entire set $(-\infty,+\infty)$ of real numbers, being the domain of values of the variable 'y'. Hence, the intervals $(-\varepsilon,\varepsilon)$ and $(-\infty,+\infty)$ are *equipollent* or, loosely speaking, they have the same number of points; each interval has the power of continuum. This is why the mathematicians have abandoned their concept of *infinitesimals,* as being, supposedly, infinitely small but nonzero real numbers, – just as the physicists have abandoned

their concept of *ether*. The only kind of smallness that exists in mathematics is *comparative* one, which can be defined with the help of the so-called "*ε&δ-language*" and which can therefore be alternatively called "*ε&δ-smallness*". In this language, for instance, the continuity of a real-valued function of one real variable at a given point is defined thus: "*A real-valued function* f *defined on* $(-\infty,+\infty)$ *is said to be continuous at a point* x_0 *if and only if for each* $\varepsilon > 0$ *there exists* $\delta > 0$ *such that for each* $x \in (-\infty,+\infty)$ *such that* $|x - x_0| \leq \delta$*:* $|f(x) - f(x_0)| \leq \varepsilon$" (see any introductory course of higher mathematics). The *ε&δ*-language is the only possible way, in which the notion of convergence of any relevant object of mathematics (as an infinite sequence of numbers, a numerical or functional series, or an improper integral) can be defined. Therefore, *the ε&δ-language should be regarded as the only possible solution of the problem of continuum.* Modern topology and modern theory of metric spaces are based on that language. A *relative smallness as a practical notion*, say that expressed by the factor $\frac{1}{10}$, has nothing to do with the *formal comparative smallness*, which is expressed by the *ε&δ*-language.

5) The evolution of the initial notions of a point and of smallness in mathematics is an illustration of *the triad of motion of thought: thesis-antithesis-synthesis*, which is one of the most general laws of philosophy due to the German philosopher Georg Wilhelm Friedrich Hegel (1770–1831). In this case, (i) *the thesis of the pertinent triad* is the intuitive concept of ancient Greek philosophers that a point in a three-dimensional continuum (e.g.) is an *elementary entity*; (ii) *the antithesis of the triad* is the fact that a 3-point has turned out to be a point in the pertinent three-dimensional affine Euclidean space over the field of real numbers, which should be identified by the ordered triple of irrational real numbers – coordinates of the point relative to a certain coordinate system, but the latter triple has no proper name and is therefore as complicated and transcendental as the three-dimensional continuum itself; (iii) the *ε&δ*-language is *the synthesis of the triad*.

6) Hegel's triad is interpreted as *the dialectic principle of unity*, or *identity*, *of opposites due* to another German philosopher Johann Gottlieb Fichte (1752–1814), a contemporary of Hegel. In a sense, Hegel's triads and its interpretation by Fichte were foreshadowed by the principle of *golden mean*, which was known to ancient Greek philosophers in the form of the dictum that «the knowledge of opposites is one» and

which was also known to medieval scholars in the form of the Latin dictum «In medio stat veritas» – «The truth stands in the middle», which is applicable to any debatable question, and in the form of the more general Latin dictum: «In medio stat virtus» – «Virtue stands in the middle» or «Virtue is in the moderate». Aristotle represents the principle of golden mean and discusses its importance for ethics in his «*Nicomachean Ethics*». The $\varepsilon\&\delta$-language can alternatively be regarded as a *golden mean* of the thesis and antithesis of the pertinent Hegel triad.•

Cmt 5.3. In accordance with Df 5.1(5), an extramental (exopsychic, physical, real) entity of a sapient subject should not necessarily be located outside the physical limits of his cerebral cortex. For instance, when *prescinded from* their function in creating mental states, the perikaryons (neuron bodies), extremities of axons and dendrites, and synapses, of which the *gray matter* of the cerebral cortex of the sapient subject is made up, are extramental (exopsychic, physical, real) entities. By contrast, the mental (psychical, ideal) entities of the sapient subject are ones that are, not only located within the physical limits of his mind (cerebral cortex), but also the ones, of which the mind is *immediately conscious*.•

Cmt 5.4. 1) Regarding the dynamic character of consciousness of a man that is indicated in Df 5.1(7), James [1890; 1950, vol. 1, pp. 224–225] says:

> «*The first fact for us, then, as psychologists, is that thinking of some sort goes on.* I use the word thinking, in accordance with what was said on p. 186, for every form of consciousness indiscriminately. If we could say in English 'it thinks,' as we say 'it rains' or 'it flows,' we should be stating the fact most simply and with the minimum of assumption. As we cannot, we must simply say that *thought goes on.*»

For his use of the word "thinking", see Cmt 5.10(8) below in this essay.

2) A *moment* or *instant*, i.e. an instance of the class of entities denoted by either count noun "moment" or "instant", is a small span of time that is mentally experienced by a sapient subject as a not lasting one. That is to say, a moment thus understood has nothing to do with a point in time continuum when the later is treated topologically as a one-dimensional metric space. If a moment is regarded as a time point then a continuous time sequence of momentary unsteady states of a given entity is called a *process*, or *stream*, *of the entity*. In this sense, the entire state of consciousness of a man and its any constituent mental state are *processes* or *streams*.•

Cmt 5.5. Psychical and physical coentities of mine and psychical coentities of another conscious sapient subject have the following *properties with respect to me*.

1) Every psychical coentity of mine, including the whole of my consciousness, is located within the physical limits of my *cerebral cortex*, called also my mind, – *the sole organ and seat of my consciousness*.
2) *I am the only sapient subject* who is *immediately conscious* (*aware*) of every psychical mental coentity of mine and of its any modification *immediately*, i.e. *here and now and without the intervention of any entity except my cerebral cortex*. Also, *none of my psychical coentities can be duplicated* as a psychical coentity of any other sapient subject.
3) I can be *conscious* (*aware*) *of* a physical (exopsychical) coentity *only through the psychical coentity which the former mediates (stimulates) in my cerebral cortex*, i.e. *mediately*. In other words, my consciousness (awareness) of any physical coentity of mine is always *mediate*. For instance, my sensations are *mediated* by my *sensory end organs* (*SEO's*), whereas the latter are *immediately* activated by certain stimuli.
4) The consciousness of a sapient subject manifests itself in his *conscious behavior*, some aspects of which are observable externally by other sapient subjects (detached onlookers).
5) I can be conscious (aware) of some specific psychical mental coentities of another sapient subject only through *tokens of the exteroceptive xenonyms* (*signs*), by which he denotes or connotes (expresses) those coentities, i.e. *mediately* again.
6) In accordance with the above-said, my psychical coentities are *most intimate coentities of mine*, whereas psychical coentities of another sapient subject are *most intimate coentities of his*.•

The following definition is made in accordance with Df 5.1(11).

Df 5.2. 1) A *state* of an entity is either *the way* (*form*) of existence of the entity or a certain aspect of that way, so that it is a purely immaterial (psychical, imaginary, abstract) entity. Particularly, the *consciousness* (*mental realm*) of a sapient subject is a *single whole complex current state of his mind* (*cerebral cortex*) when it and hence the subject himself are awake. That is to say, the consciousness of a sapient subject is the way (form) of existence of his waking mind (cerebral cortex). Hence, in contrast to the cerebral cortex (mind), *being the seat and ultimate organ of the consciousness*, *the consciousness itself is a purely immaterial* (*not physical, conceptual*) *entity*.

2) A waking mind, i.e. the mind of a waking sapient subject, is alternatively called a *conscious*, or *entelechial*, *mind* and also a *mind in entelechy*. A *single whole faculty* (*power*, *capability*, *inherent function*) of the sapient subject, the *unique realization* (*exercise*) *of which in time* is the stream of the subject's consciousness, is called *his potential mind* or *mind in potency*. Thus, the potential mind *is included in,* thu*s* being an *inseparable part of,* the entelechial mind; the former manifests via the latter and it can be regarded as the *matter* of the latter. Consequently, a potential mind *is not a sleeping* (*subconscious*) *mind*, i.e. it is not the mind of a sleeping sapient subject, and vice versa a sleeping mind is not a potential mind. A man has *exactly one NS* and hence *exactly one mind* (*cerebral cortex*), namely either the *conscious* (*waking, entelechial*) *mind*, when he is awake, or the *subconscious* (*sleeping*) *mind*, when he is asleep. Therefore, no matter how the noun "mind" is defined, it should be used in the scope of its definition in such a way that a sapient subject should have exactly one mind.

3) For the purpose of study and description, the consciousness, i.e. the entire complex mental state, of a waking sapient subject is are postulated to be *divisible* (*analyzable*) into simpler constituent parts of various kinds, which are most generally, but discriminately called partial, or constituent, *mental states* (see Df 5.1(5)), the understanding being that, just as the consciousness itself, a partial mental state is a *mental process* (cf. Df 5.1(7) and Cmt 5.4). Mental states are commonly and quite arbitrarily divided into *cognitions*, *conations*, and *affections*. *Cognitions* are mental processes of gaining knowledge, which include *sensations* (*percepts*), *conceptions* (*thoughts sensu stricto*), *concepts* (*conceptions expressed by permanent abstract symbols, especially by graphic ones*), *reasons, recepts* or *mnemons* (*memory images*), *intuitions*, and *mental attitudes* including *attention. Conations* or *volitions* are conscious drives to perform *volitional acts*, which include *decisions* and *fiats* (*mental cues*) *such as ideo-motor drives controlling the respective ideo-motor actions*. *Affections* include emotions or passions (as wish, anger, fear, hatred, love, libido, pleasure, displeasure, etc), beliefs, and any impulsive mental states of various names swaying cognitions or conations. The predicate "include" that I have used above rather than "consist of" or "comprise" indicates that the above lists of mental entities do no pretend to be complete, so that there are, perhaps, some other mental coentities,

which can be selected out of the concsiousness and be provided with the appropriate specific names.

4) In accordance with the above items 2 and 3, the *potential mind* of a man is a *complex but analyzable faculty* of the man, which is regarded as *a totality of overlapping special faculties* (*powers*, *capabilities*, *inherent functions*) *of his*, each of which is responsible for producing partial mental entities (mental entities, conscious modifications, feelings) of a certain one of the classes mentioned in the item 3. Thus, the potential mind comprises *the faculties of producing cognitions*, *conations*, and *affections*. In turn, the faculty of producing cognitions comprises *the faculties of producing sensations* (*percepts*), *conceptions*, *concepts*, *reasons, recepts* (*mnemons, memory images*), *intuitions*, and *mental attitudes* including *attention*; the faculty of producing conations comprises *the faculties of producing desires* (*volitions*, *wills*) *decisions*, and *fiats* (*mental cues*); the faculty of producing affections comprises *the faculties of producing emotions* (*passions*) as those mentioned in the above item 3. Some of the above special faculties can be provided with alternative concise self-explanatory wordy names. For instance, the faculties of producing cognitions, sensations (perceptions), conceptions, concepts, reasons, mnemons (memory images), attentions, and emotions can alternatively be called the *cognizability* (*cognoscibility*), *sensibility* (*sensitivity*, *perceptivity*), *conceivability*, *conceptuality*, *reasonability*, *memorability*, *attentiveness*, and *emotionality*, respectively. The unique realization (exercise) of the above special faculties in time results in the entelechial mind as *the current terminus ad quem*. In accordance with the item 2, all the above special faculties are at the same time *dramatis personae* of the entelechial mind as well. In addition, the entelechial mind has the *intelligence* – the faculty to learn and to apply the acquired knowledge for optimally coping with new situations and new problems, and it also has the *intellect* (opposed to the will and the emotionality) – the intelligence together with the sum total of knowledge acquired by the man by any current moment.●

Cmt 5.6. 1) “Entelechy” is the Anglicized Aristotelian term “*εντελέχεια*” /enteléçia, entélehia/ – the noun that is composed of the following three etymons: “*εντός*” /entós/, adv. & prep., meaning *inside* or *within,* “*τέλος*” /telós/, noun, meaning *an end*, and “*έχω*” /éxo, écho/, v.t. & i., meaning *to have*, *keep*, or *hold*. That is to say, etymologically, *an entelechy is an entity having* (echo) *its end* (telos) *within* (entos) *itself*. By definition, the Aristotelian entelechy of a being is a *biune* entity that has two distinct *hypostases* (*ways of existence*, *aspects*): the first entelechy and the second entelechy. In other words, in Aristotelianism, the term "entelechy" assumes two distinct senses. This is why "entelechy" is often interpreted differently by different translators and interpreters. The first Aristotelian entelechy of a being is the *full realization of its form-giving cause*, which is called the *energy* ("*ενέργεια*") or *God*. For instance, an egg of a tortoise that has become another tortoise and not a crocodile is the first entelechy of the egg, and some ice that has become water and not oil is the first entelechy of the ice. By contrast, the process of metamorphosis of the egg of a tortoise into another tortoise is the second entelechy of the egg, and the process of metamorphosis of some ice into water is the second entelechy of the ice.

2) "Entelechy' is a magnificent Aristotelian term that, from the viewpoint of modern science, gathered together such notions as *phenotype* and *genotype* in application to any thing, and not just to a *biont* (*living organism*), In presently common usage in English, the noun "entelechy" means *the final end* or *purpose*, thus being an analogue of "phenotype". Accordingly, the postpositive qualifier "*in entelechy*" and the kindred prepositive qualifier "*entelechial*" are *synonyms* of the postpositive qualifiers "*in full realization*", "*in actuality*", and "*in extension*", and also of the prepositive qualifiers "*actual*" and "*extensional*". Hence, the qualifiers "*in entelechy*" and "*entelechial*" are, at the same time, *antonyms* of the postpositive qualifiers "*in potency*" and "*in intension*" and of the kindred prepositive qualifiers "*potential*" and "*intensional*". In Modern Greek, "ενέργεια" means *activity*, *action*, *operation*, *effort*, and also *energy* in the conventional physical interpretation of the word. Therefore, in contrast to "*potential energy*", "*entelechial energy*" can be understood as a synonym of "*kinetic energy*". It is also noteworthy that in Modern Greek, the noun "*εντέλεια*" \entélia\, meaning *perfection*, and the adjective "*εντελής*" \entelís\, meaning *perfect* or *complete*, are cognate with Aristotelian "*εντελέχεια*".●

Cmt 5.7. One of the central and most general doctrines of *Aristotelianism* is *the doctrine* (*principle*) *of opposition and unity of form* (*essence*) *and matter* (*stuff*), which is today called *hylomorphism* or, more specifically, *Aristotelian hylomorphism*. According to hylomorphism, every corporeal entity (being) is a *biune* one that consists of *two inherent principles* (*aspects*), namely a *primordial* (*primary*), *potential* one that is called *matter* and a *secondary, actual* one that is called *form*. That is to say, the matter of a being is its *stuff* or *potency*, whereas the form of the being is its *essence* or *actuality*. In this case, matter and form are two *complementary conceptual* aspects of an entity, which can be distinguished and contrasted, but which cannot be separated from each other. Consequently, the term "hylomorphism" originates from two Greek nouns: "ύλη" \íli\ (pl. "ύλαι" \íle\), meaning *a matter*, and "μορφή" \morfí\ (dual "μορφά" \morfá\, pl. "μορφαί" \morfé\), meaning *a form*. The English nouns "*matter*" and "*form*" are in turn derived respectively from the Latin nouns "mātĕrĭa", meaning *matter*, *material, stuff of which anything is composed* (besides having some other meanings), and "forma", meaning *form*, *figure*, *shape* (see Simpson [1968]). Aristotle derived (induced) the doctrine of hylomorphism from his analysis of changes of *particulars*. Namely, when an entity changes (e.g., from being cold to being hot, or from being hard to being soft, or from being solid to being liquid, or from being green to being yellow), its matter remains unaltered throughout the process of change, while its form differentiates any two distinct successive states of the entity. Thus, the matter (stuff) of a real entity (being) is not that entity, because it needs a certain form (essence) to become so. Consequently, it is often convenient to use the term "*protamatter*" for mentioning (denoting) *the matter of a entity (being) as contrasted to the form of the entity* (as in the latter examples) and to use the term "*metamatter*" for mentioning (denoting) *the entity synonymously as a single whole, including its matter* (*protamatter*) *and its form to complete each other.* For instance, in accordance with the doctrine of *physicalistic monism* of *philosophical psychology*, *the mind* (*cerebral cortex*) *of a conscious* (*waking*) *sapient subject* is *the pertinent metamatter*, *the consciousness of the subject* is *the form (essence) of his mind*, and *certain abstract formless* (*unconscious*) *matter* (*material, stuff*), of which the mind is made, is *the protamatter of the conscious* (*waking*) *mind* (*cerebral cortex*).•

Cmt 5.8. *Desires* (*wills*, *volitions*), *decisions*, and *fiats* of a man are exercises (acts) of his will, whereas *wishes* of the man are exercises of his emotionality (cf. James [1890; 1950, vol. 2, p. 486]).•

Cmt 5.9. In psychology, the names "*faculty*", "*power*", "*capability*", and "*inherent function*" are used synonymously. In accordance with Df 5.2, the expressions "*potential function*" and "*function in potency*" can be added to this list of synonyms. At the same time, a *function in potency* is intuitively analogous to a *function in intension* in logic or mathematics. The set of the ordered pairs of a current argument and of the corresponding value of a function in intension is a *function in extension*. The ordered pair of an argument of the function in intension and of the value of the function at that argument is called the cut of the function in extension at that argument. Accordingly, from the standpoint of a detached onlooker, a mind in potency, is analogous to a function in intension, whereas the mind in entelechy is analogous to the cut of the function in extension at the current argument (current time instant).•

Df 5.3. 1) I say that I am *attending to* a certain coentity of mine if and only if I am *focusing my consciousness on* the coentity in order to observe, contemplate, comprehend, or cognize it. A coentity to which I attend at a certain instant or span of time is said to be an *object of mine* at that instant or in that span respectively. Two or more distinct coentities of mine that I mentally experience as my simultaneous objects are called *simultaneous objects*, or *co-objects*, of mine, the understanding being that the co-objects form a single whole *combined object*. Given two co-objects of mine, if I regard one of them as a part of the other then I shall say that the former is a *sub-object in respect to* (briefly, *of*) the latter and that the latter is a *super-object in respect to* (briefly, *of*) the former. The sub-object is said to be an *attribute* of the super-object if I use it for identified or classifying the latter. Once I *distract from* the coentity, to which I have been attending and which hence has been my object, it ceases to be my object although it can remain my coentity.

2) An object, the awareness of which I attain through my *sensor*s (*sensory end organ*, *SEO*), naked or equipped with the appropriate device (as glasses) or instrument (as a microscope), and through, after all, my sensations mediated by these sensors in my cerebral cortex, is said to be a *sensory* (*sensational, sensorial*) *object*, or *sensum* (pl "*sensa*"), of mine. An object, the awareness of which I attain through my *conceptions* and which is *my conception itself,* is said to be a *conceptional object*, or *conceptum*, of mine. The qualifier "conceptual" in the name "conceptual object" can be used interchangeably with any of the qualifiers "*psychical*", "*imaginary*", and "*ideal*", which mean that I regard a conceptual object as having psychical (immaterial, imaginary) existence in *the psychical* (*mental*) *realm* that I synonymously call *my consciousness.*

3) An object of mine has *properties with respect to me*. The properties are my *coentities* that I call *psychical*, or *mental*, *entities of mine*. A mental entity may

alternatively be called "a *mental coentity*" as opposed to a coentity that is not mental. In contrast to an attribute, a property of an object is not its co-object. An object of mine and a property that the object has with respect to me are my coentities of two different levels of my attention. Namely, an object is my e*xtensional* entity, whereas the property that the object has with respect with respect to me is my *intensional* mental entity, which I, however, often mentally experience as that object. In this case, I say that the property is a *transitive* one or that it is *the generative property of its object*. In alternative phraseology, I say that a property *is used for mentioning the object that it generates, while the property is used but not mentioned*. This property is most strongly pronounced in my *sensations* (*percepts*). Indeed, I always involuntary but consciously mentally experience a sensation (percept) as the sensory object that mediates this sensation, so that the sensation is the generative sensory property of the object. Moreover, the *transitive property of sensations* and the *sensory objects* are *sensationally absolute* (*unconvertible*) in the sense that I cannot mentally convert any one of my sensations into my sensory object and vice versa. A sensory property of a sensory object, which is not the generative property of that object, is necessarily the generative property of a certain attribute of the object. Still, sensations are *conceptually convertible* in the sense that *I can convert my sensations into my conceptual objects*, which is illustrated by this discussion. As a rule, I use my concepts, i.e. conceptual properties and conceptual objects, voluntarily or not but always consciously, in a like manner with the essential difference that a concept is, like a sensation, *conceptually convertible*: at any moment, I can convert a conceptual property into a conceptual object or vice versa. Still, a conception is either a conceptional property or a conceptional object, but not both simultaneously. In accordance with Df 5.1(3), an entity that is capable of mediating the adequate distinct sensation (percept) of mine and of thus becoming my sensory object is a *sensible entity*, i.e. a sensible *thing*.•

Cmt 5.10. 1) In accordance with Df 5.3(3), a mental entity of a sapient subject has typically a *transitive*, or *generative*, *property*, which the sapient subject mentally experiences (*involuntarily* interprets) as his *object*, i.e. as his *extramental*, or *as though extramental*, *entity*, of which he is *conscious* (*aware*), but which is located somewhere outside the subject's head (and hence, outside the subject's cerebral cortex) or even outside the subject's body. This property is most strongly pronounced in the subject's sensations (percepts). But it is also inherent to concepts, and is decisive in *using symbols for mentioning entities*, which are permanently (essentially) or temporarily (accidentally, circumstantially) associated with the symbols, *as relata with their referents*. Accordingly, the transitivity property of a concept determines *the meaning content of a symbol expressing that concept*. I shall repeatedly discuss this property at large in due course later on in this essay and in the TTL. The transitivity property of mental entities, particularly of percepts and concepts, seems to be paradoxical from the standpoint of logical analysis, but it is natural from the standpoint of the biological fact that there are no *sensory end organs* (*SEO's*) in the CNS in general, and in the cerebral cortex in particular. Therefore, there is no mechanism, by means of which the sapient subject could be informed of the actual loci of his mental entities.

2) Owing to the transitive property of mental entities, the later are often called "*brain symbols*", – in analogy with ordinary *communicative* (*linguistic*) *exteroceptive symbols*, as *graphic* (*written*) or *vocal* (*spoken*, *oral*), which *are* habitually *used xenonymously* (*not autonymously*), i.e. which *are used for mentioning* some entities other than the symbols themselves or their *tokens*. Brain symbols can be thought of as some *excitons of the cerebral cortex* – mysterious colonies of streams of frequency-modulated nervous impulses seated in the *cerebral cortex*, whose main distinguishing property is to be conscious. I shall therefore use either the count name "*cerebrocortical symbol*" or its abbreviation "*cortical symbol*" synonymously with the count name "*brain symbol*", and I shall also use the word "*exciton*" synonymously with the word "*symbol*" in any of the above names.

3) I have taken the word "exciton" from semiconductor physics. An *exciton*, also called an *electron-hole pair*, is a *mobile neutral quantum-mechanical quasi-particle* that is formed in a single-crystalline semiconductor due to the Coulomb (electrostatic) interaction between an *electron* displaced from an atom, – either due to an energy quantum of external electromagnetic radiation, called a *photon*, or due to an energy quantum of internal thermal energy of atomic vibrations, called an *phonon*, – and a positively charged *hole* (vacant place) left by the electron. Briefly, an exciton is a bound state of a negatively charged electron and a positively charged hole (ion) in a single-crystalline semiconductor, which is similar to a free hydrogen atom in vacuum. In the presence of external electrostatic field, an exciton *is polarized* and thus

becomes a microscopic quantum-mechanical *electric dipole* subsiding inside the semiconductor material. In general, an electric dipole is a pair of equal and opposite electric charges, whereas a magnetic dipole, or magnet, is a pair of equal and opposite magnetic poles. A polarized exciton is alternatively called a *polaron.*

4) It is understood that the meaning that I have attributed to the word "exciton" in psychology has nothing to do with the meaning that it has in quantum theory of semiconductors. I have just utilized the word in order to characterize the mental states as some *excited states* of the cerebral cortex. Likewise, I shall, in the sequel, use the word "polaron" instead of the word "exciton" in order to connote (express) the transitivity property of percepts and concepts as mentioned in Df 5.3(3) and in the above items of this comment. At the same, the words "phonon" and "photon" can conveniently be used as synonyms of the names "*phonic token-class*" and "*flashlight token-class*" respectively.●

5) Thus, the names:

a) "*mental entity*", "*mental state*", "*state of consciousness*", "*conscious modification*", "*thought*" [*sensu lato*], "*feeling*" [*sensu lato*], "*idea*" [*sensu lato*],

which have been introduced In Df 5.1(4), the names:

b) "*brain symbol*", "*cerebrocortical symbol*", "*cortical symbol*",

which have been introduced above in this comment, and also the variants of the pertinent ones of the above names with either adjective "*psychical*" or "*ideal*" in place of "*mental*" or with the noun "*exciton*" in place of the *appositive* (the second term of an apposition) "*symbol*" are synonyms, so that all of them can be used interchangeably. Still, no matter which one of the various synonymous names is used, it is sealed mystery what the mental entities (mental states, etc, brain symbols, etc) are indeed, how they arise from nervous impulses, and how they become conscious. This mystery is in fact the reason why so many different synonyms of "mental entity" are in use and also why some of the synonyms are often misinterpreted.

6) At the same time, no matter what a brain symbol is, it is a *dynamic and varying entity*, – in contrast to a graphic (written) symbol (e.g.), which is *static and invariable* (*unchangeable*), or in contrast to a vocal (spoken) symbol, which is just *transient*. Therefore, any *functional*, i.e. *single-valued*, correspondence (mapping) from brain symbols to graphic symbols (e.g.) is, in the general case, a *many-to-one*,

i.e. *surjective, and not bijective*, correspondence. Consequently, the inverse correspondence is a *many-valued*, i.e. *not functional*, one. For instance, an interpreter of a certain linguistic graphic symbol (as a word or word group) in a given occurrence can, depending on his mental attitude towards the symbol, use the symbol either in any one of its many *autonymous mental modes* or in any one of its many *xenonymous mental modes*.

7) Any one of the synonymous psychological terms, indicated above in the item 5, except "*thought*" and "*feeling*", is cumbersome and it *has no kindred verb*. Therefore, in treating of brain symbols (mental states), I shall follow James [1890; 1950, vol. 1, pp. 185–187] in using sometimes the word "*feeling*" as the preferred synonym of all other above-mentioned psychological terms including "*brain symbol*" itself. Thus, using the word "feeling" in this broad sense as a generic name (class-name) of any state of consciousness, I may, for instance, assert that a *brain symbol*, or *brain exciton*, is a *feeling*, and vice versa. In general, a cognition, conation, or affection (particularized in Df 5.2(2)) is a feeling and vice versa.

8) Unfortunately, the noun "feeling" is not impartial for using in the above broad sense, because it may, in accordance with the English lexicon, have undesired associations as a synonym of any of the following names: "sensation of touch", "sensation of pain", "organic sensation", and "emotion" (as opposed to "reason"). But on the other hand, the noun "feeling" is extremely convenient as a general term for speaking of mental states of various kinds indiscriminately, because it is, but again in accordance with the English lexicon, often used in this way, and also because it has the kindred verb "to feel", both transitive (active) and intransitive (neuter), – the verb that has, in turn, various expressive derivatives such as "felt", "feelingly", "feelingness", etc, besides "feeling" itself. James (*ibid.*) also suggests to use, and sometimes uses, the word "thought" interchangeably with the word "feeling" in the unusual wide sense. But I shall not follow him in this respect, because it is psychologically difficult to admit that the class of entities denoted by the noun "thought" should cover sensations and emotions as well.

9) In Cmt 5.5(1) and Df 5.2(1), I have characterized the cerebral cortex of a man as the *ultimate*, or *end*, *organ*, and also as the *seat of the consciousness*, *of the man*. It is understood that the cerebral cortex creates its symbols in cooperation with the rest of the brain and first of all in cooperation with the rest of the cerebrum.

Therefore, depending on the mental attitude that I take toward the role that the cerebral cortex or the cerebrum or the entire brain plays in creation of the phenomenon of consciousness, I may, in principle, apply the name “organ of consciousness”, without either of the qualifiers “ultimate” and “end”, for mentioning any of the above three increasingly inclusive parts of the CNS. Still, unless stated otherwise, I shall follow the terminology introduced in Cmt 5.5(1) and Df 5.2(1) closely.•

6. Relations between the consciousness and will

Df 6.1. Any act of a man is said to be:

i) *conscious* if it is performed by exercising the consciousness of the man,

ii) *subconscious* if it is performed without exercising the consciousness of the man,

iii) *voluntary* if it is performed by exercising the will of the man,

iv) *involuntary* if it is performed without exercising the will of the man.

Instead of saying that an act is conscious, or subconscious, or voluntary, or involuntary, it can, concurrently, be said that the act is *performed* (*accomplished, executed, effected*) *consciously*, or *subconsciously*, or *voluntarily*, or *involuntarily*, respectively.•

Th 6.1. Given a man,

i) a conscious act of the man is either voluntary or involuntary.

ii) a subconscious act of the man is involuntary.

iii) a voluntary act of the man is conscious.

iv) an involuntary act of the man is either conscious or subconscious.•

Proof: The will of the man is one of the faculties constituting his consciousness. Hence, the will can be exercised only *consciously*, but a conscious act of the man does not necessarily proceeds from the will. Therefore, the theorem follows from Df 6.1.•

Th 6.2. All acts of a man, and hence all nervous signals of his NS, can be divided into the following three group:

1) voluntary (and hence conscious),

2) involuntary and conscious (or, concurrently, conscious and involuntary,

3) subconscious (and hence involuntary).

Proof: It follows from Df 6.1 that all acts of the man can be divided into *conscious* and *subconscious*, and independently into *voluntary* and *involuntary*. Therefore, from pure combinatorial considerations, it follows that all acts of the man can be divided into the following four groups:

a) conscious and voluntary (or voluntary and conscious),

b) conscious and involuntary (or involuntary and conscious),

c) subconscious and voluntary (or voluntary and subconscious),

d) subconscious and involuntary (or involuntary and subconscious).

By Th 6.1, the group c is empty, whereas items 1, 2, and 3 of the theorem are the corresponding paraphrases of the above items a, b, and d respectively. QED.•

Ex 6.1. Until a sense-datum (sensory nervous signal) reaches the cerebral cortex of the man, it remains subconscious (and hence involuntary). Upon reaching the cerebral cortex, the sense-datum is transduced there into a cortical symbol, which is involuntarily but consciously perceived (mentally experienced) by the man as a sensation (percept) of his.•

Df 6.2. A *bodily act*, or *process*, of a man is said to be performed by the man *reflexly*, and also it is called a *reflex act*, or *process*, or less explicitly a *reflex*, of the man if and only if it is performed by the man *subconsciously.* Accordingly, the *adjective* “*reflex*” means *of, relating to, or concerned in a reflex*.•

7. Psychologistics

1) In spite of the progress that has been made in the study of neuronal networks and biochemical mechanisms underlying functions of the nervous system (NS) of a sapient subject (man), neither the general question “What is the consciousness of a sapient subject?” nor the particular question “How do minds of some sapient subjects produce logic?” can be answered in terms of biochemical processes that have been studied by now. Answers to these questions remain sealed mysteries and will hardly be ever found. Still, the latter question can be answered *pragmatically* and *phenomenologically*, namely by constructing formal methods of logical reasoning on the basis of *the single-minded introspection* that are done by using those same logical methods self-referentially along with some other methods belonging to *applied logic*. The field of study and discourse that I have developed in this way is called “*Psychologistics*” or “*Introspective Logistic*” and also, more

diffusely, "*Psychological foundations of logic and logical foundations of psychology*" ("*PFL & LFP*"). I regard Psychologistics as a *phenomenological psychical theory of logic and of its applications* as analogous in a sense to a *phenomenological physical theory*. In both occurrences, the qualifier "phenomenological" to 'theory' means «*of or relating to phenomena*» with the following difference. The phenomena described by a phenomenological physical theory are *physical* (*real*, *sensible*, *extrospective*), whereas the phenomena described by Psychologistics are *psychical* (*mental*, *insensible*, *introspective*). According to Pring [1982], the noun "phenomenon" is the Anglicized Greek noun "φαινόμενον" \fenómenon\, which has the same meanings (including that of "prodigy") and which is a kindred word of the transitive verb "φαίνομαι" \fénome\, meaning, not only *to appear* or *to be visible*, but also to *seem*, *look*, *be evident*; *prove*, *show oneself*). Therefore, from the standpoint of etymological analysis, the conjoined qualifier "*phenomenological psychical*" is *not a conradictio in adjecto*. It is also noteworthy, that the qualifier "phenomenological" to "theory" is used in physics as a *synonym of* "*macroscopic*" and hence as an *antonym of* "*microscopic*". For instance, thermodynamics was initially developed as a phenomenological theory in no connection with any microscopic theory. However, after emergence of statistical mechanics, the main concepts of thermodynamics were deduced from the concepts of statistical mechanics. Therefore, thermodynamics and statistical mechanics describe the same phenomena from two different viewpoints, macroscopic and microscopic respectively. Still, not to every phenomenological (macroscopic) physical theory there is a microscopic physical theory from which the former can be deduced. For instance, mechanics of continuous media, including theory of elasticity and hydrodynamics, can be deduced neither from quantum mechanics nor from any other conceivable microscopic theory. Needless to say, that it is impossible even to imagine how the *thing in itself* that is called the consciousness, i.e. the entire mental process, or state, of a sapient subject, which is *the form*, or *way*, *of existence* of the subject's cerebral cortex (mind) and which is therefore immaterial (imaginary) entity, can intelligibly be described (particularly, by that same sapient subject) as some biochemical processes in the neuronal network forming the cerebral cortex of the sapient subject.

2) There are many different topics and doctrines within the field of study and discourse that are understood by "*logic*". The foreground of the subject matter of Psychologistics is a system of *trial* (*three-valued*) *mathematical*, or *symbolic*, *logic*

that is denoted by '$\mathcal{A}_1$'. The qualifier "mathematical" to $\mathcal{A}_1$ is descriptive of the fact that, just as any system of dual mathematical logic, $\mathcal{A}_1$ utilizes formal methods of mathematics for expressing logical trains of thoughts based on reason and that, at the same time, mathematics is in turn based on $\mathcal{A}_1$. The background of the subject matter of Psychologistics is a rigorous system of new notions and their terms, which are relevant to *introspective psychology*. Therefore, Psychologistics Sis an interdisciplinary study, which involves, first of all, mathematical logic and introspective psychology, and which may also involve treatment of some aspects of mathematics, physics, biology, and biochemistry.

Essay 2. Taxonomy of the senses of a man and the classification of nyms by adequate senses

It will be recalled that a sapient subject, i.e. a normal (healthy) adult man (unless stated otherwise), has the following well-known *five exteroceptive senses* (*sense functions*): *the sense of sight*, *hearing*, *touch*, *taste*, and *smell,* also called the *visual*, *auditory*, *tactile*, *gustatory*, and *olfactory senses* in that order. The *sense of touch* comprises *two senses*: the *sense of temperature* (*warmth and cold*) and the *sense of pressure*. The *sense of taste* comprises *five senses*, namely, ones of *sweet*, *bitter*, and *umami*, all of which are mediated by *G-protein-coupled exteroceptors* ("*G*" is an abbreviation for "*globular*"), and ones of *salty* and *sour*, which are mediated by *ion-sensitive exteroceptors*. Umami is the sense of *taste of meat*, whose receptors have been discovered recently.

Besides the well-known hereditary distinctions among different persons with regard to visual sensations (cf. various kinds of color blindness and particularly daltonism – red-green blindness occurring in male men as a recessive genetic anomaly), auditory sensations (cf. persons having a good ear to music or absolute pitch and tone-deaf ones), there are like hereditary distinctions among different persons with regard to the gustatory sensations elicited by certain substances. For instance, there is a crystalline compound, which has the molecular formula $C_6H_5NHCSNH_2$ and which is called phenylthiourea or phenylthiocarbamide. The substance is made (is synthesized) from aniline, carbon disulfide, and ammonia. Many people find that phenylthiourea is tasteless, while others find it extremely bitter. The ability to receive one or the other gustatory sensation from phenylthiourea is a hereditary property of each person, which is determined by one pair of genes. The idiomatic English proverbs: «tastes differ», «one man's meat is another man's poison», and «every man to his taste» and also analogous proverbs of other NL's, e.g. the Russian proverb «на вкус и цвет товарищей нет» – «there are no two persons holding the same view towards colors and tastes», intuitively and tersely express complex genetic phenomena.

In addition to his five exteroceptive senses, a man has the following *four* less definite but not less important groups of *interoceptive senses*, upon existence of which the general agreement has been reached among most biologists and most psychologists: *the sense of balance*, or *equilibrium*, also called *the labyrinthine sense*; *the sense of skeletomuscular motion and coordination*, also called *the kinesthetic sense* or *kinesthesia*; the *organic sense*, also called the *visceral sense*; *the sense of pain*. The four interoceptive senses are designed to inform the cerebral cortex of the sapient subject about the running state of various *parts of his body*, and not about the external world. Therefore, I classify all of them as interoceptive ones (see below for greater detail).

The labyrinthine sense (sense of balance) is *the sense of orientation of the subject's head relative to the direction of mass forces* as the field of earth gravity or the centrifugal force caused by rotation of the head. The entire SO of this sense is a bilateral organ, each of the two mirror-symmetrical SEO's of which is located in one the two mirror-symmetrical inner ears of the subject and comprises the three *membranous semicircular canals*, located in three mutually orthogonal planes, and the *membranous vestibule*. All the cavities intercommunicate and are filled with watery liquid called *endolymph*. The mass forces act on the endolymph and also on the *otoliths* (*ear stones*), the small calcium carbonate particles located in the membranous semicircular canals. From the standpoint of physical analysis, the mass forces are the primary stimuli (input agent) that are involved in producing labyrinthine sensations of the sapient subject. Therefore, the labyrinthine sense should, at first glance, be classified more correctly as a mixed *extero-interoceptive*, or *intero-exteroceptive*, one or, alternatively, as an *interactional* one. Still, his labyrinthine sensations inform the sapient subject and only the subject about the orientation of his head relative to the direction of the external mass force and they are not available to any other sapient subject, which has the labyrinthine sensations of his own. Also, the otoliths and endolymph, located in the SEO's of balance, are the actual immediate built-in stimuli of this end organ. Therefore, I classify the labyrinthine sense as an interoceptive one.

The kinesthesia is *the sense of relative positions and relative movements of the different motile parts of the body*. The *kinesthetic* SEO'e are interoceptors (nerve endings), which are scattered throughout the skeletomuscular system and which are stimulated by some *biochemical agents* secreted by the muscle cells when they contract. The organic sense produces various *organic sensations* as those of hunger, thirst, satiety, nausea, filling of the intestine or bladder, etc. The sense of pain is one of the most important senses, because it signals of the danger and, when appropriate, produces a reflex reaction of withdrawal from a dangerous pain-causing object. Pain

SEO's, called *nociceptors* (from the Latin verb "nŏcĕo" meaning *to hurt*, *injure*, or *harm*), are bare dendrites of neurons, which are stimulated by some special chemicals (as histamine or acids) released from damaged, inflamed, excessively heated, or excessively deformed cells of the surrounding non-neural tissue. These chemicals are internal and hence interoceptive stimuli, although some of them are brought into being in the body by an external cause.

Many writers include the restricted sense function of pain, whose values (manifestations) are pain sensations in the epidermis due to excess temperature or excess pressure, into the sense of touch. Accordingly, they consider the nociceptors found in the epidermis as a kind of tactile SEO's. However, pain receptors are scattered throughout the entire body. Particularly, excess movements can cause pain in skeletal muscles and joints, whereas inflammation can cause pain in any part of the body. Therefore, I consider all kinds of pain as manifestations of a *single whole interoceptive sense of pain*. In this case, tactile pain is excluded from the sense of touch and becomes a part of the sense of pain.

Incidentally, *itch* is *mild pain* that is experienced in the epidermis and that elicits an instinctive urge to relieve the affected area by scratching it. Itch can be caused by the same *external precursory stimuli* as those causing pain, but of essentially smaller intensity or concentration. Besides these stimuli, itch sensations can be elicited by irritating skin's nociceptors by some special *internal chemical stimuli of somatogenic* or *psychogenic cause*. Particularly, some products of metabolism can serve as somatogenic stimuli that elicit itch.

The above taxonomy of the senses of a sapient subject is of course an idealization that is bases on the mind-stuff analysis. In fact, however, various sensations, which a sapient subject (as me) can single out of his (correspondingly, of my) consciousness are interrelated. For instance, interrelated are gustatory and olfactory sensations and hence senses or taste and smell are interrelated as well. Similarly, the pressure end organs located in the epidermis of the foot soles serve as auxiliary sense organs of balance and orientation, although these are tactile and hence exteroceptive.

Etymologically, the term "kinesthesia" originates from the following two Greek words: "κίνησις" \kínisis\, being a parasynonym of "movement" or "motion" (action or abstract), and "αίσθησία" \ésthisía\, being a parasynonym of "sense" (bodily faculty) (cf. αναίσθησια" \anésthisía\ meaning *anesthesia*, i.e. lack, or loss, of sensibility). In analogy with “kinesthesia”, the following monomials of Greek origin can be suggested as synonyms of the terms “organic sense”, “pain sense”, and “labyrinthine sense” in this order: “*organesthesia*”, from the noun “όργανον” \órganon\ parasynonymous with “organ”; “*ponesthesia*”, from the noun “πονῶ” \ponó\ parasynonymous with “pain”; and “*isoresthesia*”, from the noun “ισορροπία” \isorropía\ parasynonymous with “balance”. In this case, “organesthetic”, “ponesthetic”, and “isoresthetic” are adjective derivatives “organesthesia”, “ponesthesia”, and “*isoresthesia*”, respectively, – in analogy with “kinesthetic”.•

Definition: ***Classification of nyms by adequate senses.*** In accordance with Preliminary Remark I.1.1(3i) in the TTL, a nym can be classified by the same adjective qualifier or qualifiers as that or those qualifying the sense or senses, with the help of which the nym is perceived and classified. For instance, a nym is said to be exteroceptive, interoceptive, visual, tactile, ponesthetic, visual and tactile, gustatory and olfactory, etc. if it is perceived and identified with the help of the sense or senses of the corresponding type or types.•

Essay 3. Miscellaneous properties of alphabetic and polysyllabic native languages (AbNL’s and PSbNL’s)

1. Definition of the terms “alphabetic native language” (“AbNL”) and “polysyllabic native language” (“PSbNL”)

Df 1.1. 1) For convenience of description and study, I postulate that a given *phonemophonographic native language* (*PmPhgNL*), which was mentioned in Df I.1.19 of the TTL and which will more specifically be called a *basic PmPhgNL* (*BPmPhgNL*) (e.g. *basic written English*)*,* has a certain *primary atomic basis* (*PAB*) – a set of prototypal atomographs, in isotokens of which the PmPhgNL is materialized. By assumption, the PAB can be divided into *three* subsets: the set of *atomophonographs* (*atomic phonographs*) or *phonoatomographs* (*phonic atomographs*), the set of *atomic punctuation marks* (briefly, *atomostixographs* or *stixoatomographs*), and the set of *atomic phonetic marks*. The elements of the last two sets have no phonic paratakens (phonic values of their own) and are therefore called *atomic aphonographs* or in one word *atomo-aphonographs*.

2) A BPmPhgNL is called:

a) a *basic alphabetic native language* (*BAbNL*) if phonic paratokens of *all* its atomophonographs are *atomophons*, i.e. a *speech sounds*;

b) a *basic polysyllabic native language* (*BPSbNL*) if phonic paratokens of *strictly some* (*some but not all*) of its atomophonographs are *vowel speech sounds*, whereas phonic paratokens of all other atomophonographs are *voweled* (*vowelled*) *consonant phononyms next bigger than speech sounds*.

A BAbNL or a BPSbNL is indiscriminately called a *basic WNL* (*BWNL*), but not necessarily vice versa, because there is, e.g., a *basic QMSbNL* (*BQMSbNL*), which is not described here explicitly.

3) The atomophonographs of a BAbNL are called *graphic* (*optographic*), or *written*, *letters*, and also *graphograms* (*optographograms*), *of the BAbNL*. A graphogram is called a *vowel graphogram* if it is a paratoken of a *vowel speech sound* and a *consonant graphogram* if it is a paratoken of a *consonant speech sound*. The conventional set of letters of the BAbHL, which are arranged in a customary order and provided with customary proper names, is called the *alphabet* of the BAbNL.

4) The atomophonographs of a BPSbNL are called *graphic* (*optographic*), or *written*, *syllables* and also *graphosyllables* (*optographosyllables*), *of the BPSbNL*. The conventional set of graphosyllables of the BPSbNL, which are arranged in a customary order and provided with customary proper names, is called the *syllabary* of the BPSbNL. A phonic paratoken of a graphosyllable of the BPSbNL is called a *phonic*, or *spoken*, *syllable* and also a *phonosyllable*, the understanding being that the latter is belongs to the PmNL (SNL) associated with the BPSbNL. A phonosyllable is said to be an *atomic one* if it is a *vowel atomophon* (*vowel speech sound*) and a *molecular one* or a *meriophon* (*meriophononym*), i.e. *molecular phononym*, if it is a *voweled consonant phononym next bigger than speech sounds*.

5) In general, paraphrasing properly the article **«syllable»** in Allen [2003], a *phonosyllable*, i.e. a *spoken syllable*, of a *phonemic native language* (*PmNL*) is an uninterruptible unit of the PmNL that usually consists of one vowel speech sound alone or with a consonant speech sound preceding or following. Accordingly, a graphosyllable of a BPmPhgNL (BAbNL or BPSbNL) that is associated with the PmNL is a graphic paratoken of a certain phonosyllable of the PmNL.

6) An atomic punctuation mark is any atomograph carrying one of the following English names, alone or with a qualifier: "apostrophe", "brace", "bracket" (alone or along with one of the prepositive qualifiers "square", "round", "curly", and "angle") "colon", "comma", "dash", "diagonal" ("virgule", "forth-slashed virgule" or "back-slashed virgule"), "ellipsis", "exclamation point", "full stop" ("period", "point"), "hyphen" (alone or along with the prepositive qualifier "single" or "double"), "parenthesis", "question mark", "quotation mark", "semicolon", "space", and perhaps some others (as "omission points", "blank sign" or "vinculum").

7) Just as atomic punctuation marks, some atomic phonetic marks exist in any BPmPhdNL, alphabetic or polysyllabic. Atomic phonetic marks of a non-Semitic BPmPhdNL are called *atomic diacritical mark*s or *atomic diacritics*. A combination of two or more atomic diacritics that are attached to the same atomophonograph will be called a *molecular diacritic*. An atomic or molecular diacritic will indiscriminately be called a *diacritic*. Thus, a diacritic of the BPmPhdNL is a modifying mark or combination of modifying marks (each of which is a diacritic as well) over, under, near (after or before), or through a phonoatomograph of the BPmPhdNL indicating phonetic and perhaps semantic differences of a phonic paratoken of the marked phonoatomograph from those of the unmarked or otherwise marked

phonoatomograph. Various *accent marks* (*accents*) and the *dieresis* (or *diaeresis*), ¨, are some diacritics. The dieresis is placed over a vowel letter, particularly over the second of two adjacent vowel letters, in order to indicate the phonic paratoken of the letter is pronounced as a separate phonosyllable. Written English has no diacritics except *accents*, whereas written Russian has, besides accents, the dieresis, which can be placed over "e", when appropriate, but which is usually omitted.

8) Instead diacritics, written Hebrew and other Semitic languages have an ingenious system of dots and other marks, which are placed *under* or *after* (*to the left of*) a consonant letter either by ones or in combinations to indicate the vowel speech sound that follows the consonant one represented by the consonant letter. This system of vowel notation is called the *voweling* (*vowelling*) or *pointing* or, in Hebrew, *nikkud* (from Hebrew "נקודה" \nikuda\ meaning *a point*). The positions of some *pointing marks* to the left of a voweled letter are determined by the fact that Semitic languages are *written from right to left*. Accordingly, the adverbial qualifier "*to the left of*" to "letter" is concurrent to the qualifier "*after*". The nikkud is briefly described below in the item 5 of subsection 2. The nikkud is as rule discarded in all Hebrew writings other than Bible, dictionaries, and textbooks of Hebrew.

9) A linguistic form of a BAbNL (as a morpheme or word) that comprises (can be dissected into) exclusively written letters alone or with diacritics but without any punctuation marks is called a *literal*, or *lettered*, *graphonym* or in one word a *grammograph* (*grammographonym*) of the BAbNL. Similarly, a linguistic form of a BPSbNL that comprises exclusively written syllables alone or with diacritics and no punctuation marks is called a *syllabic graphonym* or in one word a *syllabograph* (*syllabographonym*) of the BPSbNL. Once a grammograph of a BAbNL is provided with a *xenovalue* in accordance with Df I.1.19(2) of the TTL, then the graphograms comprised in that *grammograph* are *articulated* so that the latter becomes an *ideograph* and *phonograph* *simultaneously* or, in one word, an *ideophonogrammograph*, *phonoideogrammograph*, *grammographoideograph*, etc, i.e. a *graphic lettered* (*lettered graphic*) *symbol* having *phonic* (*spoken*) *paratokens* (recursive recognizable same phonic values). Similarly, once a *sylllabograph* of a BPSbNL is provided with a *xenovalue* in accordance with Df I.1.19(3) of the TTL, then the graphograms comprised in that graamograph are articulated to become an *ideograph* and *phonograph* *simultaneously* or, in one word, an

ideophonosylllabograph, *phonoideosylllabograph*, *sylllabographoideograph*, etc, i.e. a *graphic syllabic* (*syllabic graphic*) *symbol* having *phonic* (*spoken*) *paratokens*. •

Cmt 1.1. In Df 1.1(4), the graphosyllables of a BPSbNL are straightforwardly defined as elements of the syllabary of the BPSbNL, whereas the respective phonosyllables (spoken syllables) of the associated PmNL (SNL) are straightforwardly defined as phonic paratokens of the graphosyllables. If the graphosyllables are furnished with all necessary diacritics then the correspondence between the graphosyllables and the phonosyllables being their phonic paratokens is one-to-one. On the other hand, if the written counterpart of a PmNL (SNL) is a BAbNL then the correspondence between the letters comprised in a written word and the speech sounds comprised in a spoken paratoken of the word is determined by the orthography of the BAbNL. There are BAbNL's, e.g. Russian or Ukrainian, in which spellings of written words and their letter-by-letter phonations are practically in a one-to-one correspondence. At the same times, there are BAbNL's, e.g. English or French, in which spellings of written words often differ from their letter-by-letter phonations, because some letters occurring in connected writing can be *mute* and also because an articulated phonic paratoken of the juxtaposition of two or more letters often differs from the unarticulated sequence of phonic paratokens of (i.e. from the sequence of separate unarticulated speech sounds corresponding to) the separate letters. One or more letters in a written word that roughly or exactly correspond to a certain phonosyllable (spoken syllable) of a spoken paratoken of the written word is, by Df 1.1(5), called a graphosyllable (written syllable) of the written word. Thus, in the case of a BAbNL, the class that is supposed to be denoted by the count noun "graphosyllable" is not defined univocally, and therefore it is not fundamental, – just as not fundamental is the class denoted by the count noun "phonosyllable".•

Df 1.2. 1) A BPmPhgNL, i.e. a BAbNL or a BPSbNL, allows augmenting itself with *pasigraphs* (*logographs*, *euautographs*, or *both*) so as to become a mixed *phonemophnographic and logographic native language*, which will be called an *advanced PmPhgNL* (*APmPhgNL*) or discriminately an *advanced AbNL* (*AAbNL*) or an *advanced PSbNL* (*APSbNL*) respectively. A BPmPhgNL, i.e. a BAbNL or a BPSbNL, or an APmPhgNL, i.e. an AAbNL or an APSbNL, is indiscriminately called a PmPhgNL, i.e. an AbNL or a PSbNL respectively. To any BAbNL or BPSbNL there is an indefinite number of AAbNL's or APSbNL's and hence an indefinite number of AbNL’s or PSbNL’s, each of which is qualified by the same proper qualifier as the BAbNL or the BPSbNL, e.g. “English” or “Japanese” respectively.

2) Just as in the case of a BPmPhgNL, I postulate for convenience of description and study that an APmPhgNL has a certain *atomic basis* (*AB*) – a set of prototypal atomographs, in isotokens of which the APmPhgNL is materialized. Consequently, the AB can be divided into two subsets, namely, the *primary atomic basis* (*PAB*), which is described in Df 1.1(1), and the *secondary atomic basis* (*SAB*), which is defined with the help of and within the BPmPhgNL. Depending on an APmPhgNL, the SAB can be specified in many different ways. As a quite general example, I shall assume that the SAB comprises the following *lexigraphs* (*atomic logographs*, *atomic aphonographic words*): the ten Arabic digits from “0” to “9”, Roman digits (namely “I” or “i” for *one*, “V” or “v” for five, “X” or “x” for *ten*, “L” or ‘l’ for *fifty,* “C” or “c” for *one hundred*, “D” or “d” for *five hundred,* “M” or “m” for *one thousand*, “$\overline{\text{X}}$” for *ten thousand,* “$\overline{\text{C}}$” for *one hundred thousand*, and “$\overline{\text{M}}$” for *one million*), arithmetical signs, and some special signs as “#” (or “№” in some APmPhgNL’s), “&”, “@”, etc. Accordingly, an APmPhgNL contains particularly the *system of decimal integral numerals*, called also the *decimal numeration system*, the *systems of common* and *decimal fractional numerals* and the *system of Roman numerals*. All the numerals and also all above-mentioned arithmetical and special signs of an APmPhgNL are called *logographs* of the APmPhgNL.

3) An AAbNL or an APSbNL is indiscriminately called an *advanced WNL* (*AWNL*), but not necessarily vice versa because there is, e.g., an *advanced QMSbNL*, (*AQMSbNL*), which is not described here explicitly.•

Cmt 1.2. In accordance with Df 1.1(9), a grammograph of a given BAbNL or a syllabograph of a given BPSbNL is a *juxtaposition*, or *linear assemblage*, of written letters of the BAbNL or written syllables of the BPSbNL respectively. In this case, a grammograph of the BAbNL, e.g., is necessarily a phonograph or, more precisely, a phono*grammograph* and at the same time an *ideo*phonogrammograph because the graphograms, of which the grammograph consists, are articulated thus indicating that the *intended xenonvalue*, which is assigned to the grammograph as a single whole, has nothing to do with any xenovalues that the separate constituent graphograms may have; and similarly with “BPSbNL”, “syllabograph”, and “graphosyllable” in place of

“BAbNL”. “grammograph”, and “graphogram”.[2] If, however, the BAbNL is extended to become an AAbNL of the same name (as “English”) then some grammographs (written letters) of its alphabet can be defined homonymously (equivocally) as *aphonic lexigraphs* (*aphonic atomic logographs*). In this case, a graphonym consisting of such lexigraphs can be an aphonic logograph, whose elements are not articulated phonetically. For instance, if the graphonym “*ab*” denotes the product of numbers *a* and *b* then it is an aphonic logograph and hence an ideograph (graphic symbol) but not an ideophonogrammograph, whereas “*a*” and “*b*” are aphonic lexigraphs (atomic logographs), which can be mentioned only by using their phonographic names “ˈā” and “ˈbē” respectively. As was indicated in Df I.1.8(1) of the TTL, univocal phonographic (wordy, verbal) names of aphonographs (as pasigraphs of punctuation marks) of a WNL are indispensable for mentioning the aphonographs unambiguously in the counterpart SNL.•

2. A syllabary versus an alphabet

1) The difference between a syllabary (other than a lexibary) and an alphabet can be demonstraded by citing as an example the *Modern Japanese Kana*¸ called also *Kana-Majiri* (“*Majiri*” meaning *mixture*) – the standard form of the modern written Japanese language. *Kana* is a Japanese system of syllabic writing, which adapted some monosyllabic Chinese lexigraphs for its own use as names of vowel and consonant sounds somewhere about AD 750. *Kana-Majiri* is the modern form of Kana, which is based on two different but equivalent syllabaries, called *Hiragana* and *Katakana*. Hiragana is a set of cursive Kana characters, whereas Katakana is a set of Kana characters that are squarer and more angular in form than those of Hiragana. More precisely, Kana-Majiri is based on Katakana that is supplemented by Hiragana characters to indicate inflectional endings and function words. Schematically, either

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[2]In principle, an individual can learn his second PmPhgNL (AbNL or PSbNL), actually a certain part of it, as an *aphonic logographic* language, in no connection with the spoken counterpart PmNL of the PmPhgNL. This can be done by learning grammar text-books, consulting first-to-second and second-to-first bilingual dictionaries, also consulting explanatory dictionaries of the second language, and by reading literature in the second language. It can also happen that an individual has a command in a PmNL (phonemic [spoken] NL), the mother one or not, but he has no command in the PmPhgNL, being its written counterpart. Still, such cases are exceptional and they are irrelevant to the general properties of NL’s of various kinds under discussion.

syllabary Katakana or Hiragana is a set of 76 characters, namely, 5 characters for the five vowel sounds *A*, *I*, *U*, *E*, *O*, a character for the sound *UN*, and 70 characters for the 70 consonant sounds obtained by vowelling the 14 consonant sounds *M*, *N*, *S*, *Z*, *P*, *B*, *T*, *D*, *K*, *G*, *Y*, *R*, *H*, *W* with a following vowel sound (14×5=70). In this case, the vowelled sounds *S* and *Z*, *T* and *D*, *K* and *G*, and *P* and *B* are pairs of similar sounds such that the first sound of each pair is voiceless, and the second is voiced. Accordingly, in Katakana, e.g., the character for a voweled sound *Z*, *D*, or *G* is formed by furnishing the character for the respective voweled sound *S*, *T*, or *K* in the top right-hand corner of it with a diacritic mark in the form of a stylized back-slashed double prime for explicitly indicating voicedness. At the same time, the characters for a voweled sound *P* and for the respective voweled sound *B* have the same base character and two different diacritics in the top right-hand corner of it, namely, a stylized empty dot in the case of *P* for explicitly indicating its voicelessness and a stylized back-slashed double prime (as mentioned above) in the case of *B* for explicitly indicating its voicedness. It is understood that the Latin letters do not exactly indicate the pertinent Japanese sounds. For instance, according to Bodmer [1944; 1981, p. 420, Fig. 46], *TI*, *TU*, and *HU* should, more exactly, be read as *CHI*, *TSU*, and *FHU* respectively.

3) The above general way of computing the number of written syllables of a syllabic language is quite accurate (up to few units). For instance, the Amharic speech has 7 vowel and 32 consonant sounds, so that the Amharic syllabary includes 231 primitive signs altogether (7+32×7=33×7=231).

4) The number of letters employed in an alphabetic language is typically less than the number of syllables employed in a polysyllabic language, because the written syllables of the former are formed of letters. Still, some alphabetic languages (as Czech, French, Greek, Polish, and many others) have a system of *diacritics* (some of which are called *accents*) − modifying phonetic marks placed over, under, before, after, or through a letter indicating a changed phonetic value. A letter without a diacritic and the same letter furnished with a diacritic denote two different speech sounds, and should therefore be regarded as two different primitive characters (cf. the Kana syllables with two different diacritics or without any for kindred voiceless and voiced sounds). At the same time, in stating alphabets of some languages, diacritics

are omitted even if they are employed in the writing system of a language. In the result, the number of letters forming an alphabet is apparently decreases.

5) Some languages that are qualified alphabetic have a system of special phonetic marks denoting vowel speech sounds. For instance, written Hebrew, being one of the languages of the Semitic group, has an alphabet consisting of 23 consonant letters and in addition it has an ingenious system of dots and other marks, which are placed *under* or *after* (*to the left of*) a vowelled consonant letter either by ones or in combinations, to indicate the vowel speech sound that follows the consonant speech sound represented by the consonant vowelled letter. This system is called the *Hebrew nikkud* or, briefly, the *nikkud* (from Hebrew “nikuda” meaning *a point*). An individual element of the Hebrew nikkud is equivocally called *a* nikkud. The positions of some nikkuds to the left of a vowelled letter are determined by the fact that Hebrew words and sentences are *read from right to left*. Accordingly, the adverbial qualifier “to the left of” to “letter” is concurrent to the qualifier “*after*”. There is a *short-vowel nikkud* and a *long-vowel nikkud* for each of the five vowel sounds: *AH*, *EH*, *EE*, *OH*, *OO* (cf. the five Japanese vowel sounds). The short-vowel nikkuds for the above vowel sounds are ◌ַ, ◌ֶ, ◌ִ, ◌ָ, ◌ֻ, respectively, so that these are always placed *under* a vowelled consonant letter. The long-vowel nikkud for the sound *AH*, e.g., is the same as the short-vowel nikkud for the sound *OH*, whereas the long-vowel nikkud for *EH* is ◌ֵ. Thus, either one of these two long-vowel nikkuds is also placed under a vowelled consonant, but the three other long-vowel nikkuds are configured differently. The *long*-vowel nikkud for the speech sound *EE* is the combination of a dot, placed under the vowelled consonant, ◌ִ, and of the consonant letter י (*yood*), placed to the left of the vowelled consonant letter. The long-vowel nikkud for the speech sound *OH*, or *OO*, is the consonant letter ו (*vahv*) with an overdot, or with a left midscript dot, respectively, which is placed to the left of the vowelled consonant. Hebrew has one more vowel-like sound, which is denoted by the nikkud ◌ְ, called *shvah*, and which is pronounced like a very short *EH* if it occurs at the beginning or in the middle of a word after a long vowel. In fact the denotatum of the shvah alone does not denote a sound of its own, so that it is most often used after ◌ַ, ◌ֶ, or ◌ָ, i.e. as ◌ֲ, ◌ֱ, or ◌ֳ respectively, to denote the corresponding modified vowel sound. The shvah is analogous to the Russian hard sign ъ and it is indicated by an apostrophe in

transliterations in English. Also, the consonant letters בּ (*beht*), כּ *kahf*), פּ (*peh*), and שׁ (*sheen*) have the alternative forms ב (*veht*), כ (*hahf*), פ (*feh*), and שׂ (*seen*). Incidentally, the root of any Hebrew word comprises three consonant letters, each of which is sometimes called a *radical*. To recapitulate, a nikkud is not a letter, but a *label* (*mark*) that is attached to a vowelled consonant letter. Therefore, a nikkud can alternatively be called a *vowel label* or a *vowel mark*. The nikkud is, as rule, discarded in all Hebrew writings other than Bible, dictionaries, and textbooks of Hebrew. Still, a consonant letter together with a nikkud attached to it, especially with that placed under it, can be regarded as a primitive written syllable. Consequently, written Hebrew can alternatively be regarded either as a *syllabic* (*polysyllabic*) *language* or as a *syllabico-alphabetic* (or *alphabetico-syllabic*) *language*.•

3. Code languages

Df 3.1. A set of prototypal onyms (nyms) of any physical kind, whose elements stand in a one-to-one correspondence to the elements of the atomic basis (AB) of a PmPhgNL, i.e. of an AbNL or PSbNL, is called a *basic code* of the PmPhgNL. The English noun "code" is the Anglicized Greek noun "κωδιξ" \kodiks\. Accordingly, the following *onymological* and *onological terms* can be suggested. An element of the basic code is called an *elemental,* or *primitive*, *codonym* of the PmPhgNL, while the pertinent isotoken-class of a given codonym, especially when it is mentally experienced as its common (general, certain, concrete but not concretized) member, is called an *elemental*, or *primitive, codon* of the PmPhgNL. With the help of a given code, *any given nym of the PmPhgNL* can be converted into another nym, which is called *the codonym of the former nym relative to* (or *with respect to*) *the given code*, whereas the original given nym is called the *ante-codonym*, or *pre-codonym*, of its codonym. In this case, *the isotoken-class of the given nym of the PmPhgNL* is automatically mentally converted into the isotoken-class of that nym, which is called *the codon of the former isotoken-class.* A language whose nyms are the codonyms of nyms of a certain PmPhgNL is called a *code of that PmPhgNL* or less explicitly a *code language.* Any code language will be qualified by the same *proper qualifier* (as "English", "French", etc.) as that qualifying the original NL, which it *encodes*. Consequently, a code language should be distinguished from a *contrived language* (*CL*), particularly from *Esperanto*, *Ido*, and *an esperanto* (see Cmt (I.1.11(1,2) of the TTL).•

Cmt 3.1. The following language codes are most conspicuous ones: the *Braille embossed code* of the blind (named after its inventor Louis Braille 1809–1852, a French blind teacher of the blind), the *Moon embossed code* of the blind (named after its English blind inventor William Moon †1894), the *manual (dactylological) code* of the deaf and deaf-and-dumb, the *Morse written*, *sonic*, and *light-flashed codes* (named after their American inventor Samuel F. B. Morse 1791–1872), *the wigwag codes*. In the following brief description of the language codes, I shall, for the sake of being specific, assume that the encoded PmPhgNL is the respective *basic* written English, unless stated or obviously understood otherwise.

1) The Braille *embossed* code of a given PmPhgNL is a system of basic embossed codonyms, which are made up of raised dots in a 6-dot cell arranged in two vertical columns with 3 dots in each column. The Braille code has been adapted for transcribing various native languages, special logical, mathematical, and scientific symbols, and also for transcribing music. In the Moon *embossed* code of native written English (e.g.), all letters of the English alphabet are represented by nine large *embossed* primitive characters, each of which stands for two or more Latin letters depending on its orientation. Reading Moon characters requires less finger sensitivity than reading Braille ones. Therefore, the Moon code is designed especially for those who blinded late in life. Both Braille and Moon codonyms are tree-dimensional ones that occur *on* (and not *in*) a physical surface, and therefore these are classified, not as written or graphic, but as *embossed*, or *relief*, *codonyms* and also as tangible codonyms or, in one word, *aptocodonyms*.

2) The dactylological (manual) code of native written English represents the letters of the English alphabet and the logographic character "&" by signs made with fingers of one hand. The art of use of this one-hand alphabet in communication with or among deaf or deaf-and-dumb men is called *dactylology* (not to be confused with *dactylography* – the study of fingerprints as a means of identification). Systematic use of dactylology as substitute for spoken English (e.g.) is called *dactylological English*.

3) The international Morse *written,* or *graphic*, code represents the letters of the English alphabet, the English punctuation marks, and the Arabic digits, i.e. the numerals from 1 through 9 and 0, whereas the international Morse sonic, or light-

flashed, code represent the same graphic characters by short and long sounds, or short and long flashes of light, which can be transmitted either in audible form by telegraphy or in visual form by light-flashed signaling respectively.

4) The international *basic wigwag code*, called also the *wigwag semaphore*, consists of 29 primitive figures formed by two flags held one in each hand. 26 primitive figures represent the 26 letters of the English alphabet, whereas 3 figures are *lexinyms* meaning "*error*", "*end of word*", and "*numerals follow*". The Arabic digits 1 through 9 and 0 are represented by the same figures as those representing the letters from "A" through "J" in the alphabetic order.

The qualifier "wigwag" in either name "wigwag code" or "wigwag nym" is a noun that has the homonymous kindred verb. Therefore, a *wigwag nym* can be classified by the act or process of its production as a *wigwaged nym*, – in contrast to a *written*, or *spoken*, *nym*. Also, the noun "wigwag" in either name "wigwag code" or "wigwag nym" is usually used interchangeably with the noun "*semaphore*" which also has the homonymous kindred verb. Therefore, a wigwag nym can, more generally, be classified by the act or process of its production as a *semaphored nym*. At the same time, in contrast to the qualifiers "written", "spoken", and "wigwaged" or "semaphored", which have kindred verbs "to write", "to speak", and "to wigwag" or "to semaphore" respectively, the qualifiers "Morse light-flashed", "Morse sonic", and "dactylological" to "nym" have no kindred verbs that could be used for classifying the respective nyms by the acts or processes of their production. These nyms cannot certainly be classified either as written or as spoken. In order to remedy this situation, I classify them indiscriminately as *semaphored* ones, – just as wigwag ones. Thus, the Morse sonic and light-flashed nyms, the wigwag nyms, and the dactylological nyms are indiscriminately called *semaphored nyms* or *semaphore nyms*. More specifically, the Morse *sonic* nyms will be classified as *echonyms*, and also as *sonic nyms*, whereas the Morse light-flashed will be classified as *photonyms*. In this case, besides Morse sonic nyms, the class of echonyms (sonic nyms) contains phononyms sensu lato, i.e. mylonyms (phononyms sensu stricto) and adonyms. It is also understood that, with respect to a normal individual, a written (graphic), semaphored nym, or light-flashed nym (photonym), each taken individually, is an *optonym*, but not necessarily vice versa. Optonyms and aptonyms can, in turn, be indiscriminately called (classified as) *boobonyms*, i.e. *silent nyms*. The meaning of the combining form "boobo" differs from the meaning of the adjective "surd" (from the Latin adjective *surdus* meaning *deaf*), which is conventionally used of speech sounds as a *synonym of* "*voiceless*" and *antonym of* "*sonant*". A language, ordinary or encoded, will be qualified by the same qualifiers as those qualifying the nyms employed in the language. For instance, a code language which is based on utilizing semaphored nyms will be called a *semaphored language*.•

4. Logographic writing

Like a phonograph (xenophonograph), a logograph is, by Df I.1.28 of the TTL, an ideograph. At the same time, a logograph is, by Df I.1.30 of the TTL, an *aphonograph*, so that the classes designated by the terms "logograph" and "phonograph" are *incompatible* or, in other words, the two terms are *contrary*. Also, an exoiconograph (exopictograph) is a logograph but not necessarily vice versa, because an exoiconograph (exopictograph) is not an ideograph and vice versa. Here follow some examples illustrating logographic writings in the light of the above- said.

1) The Arabic numerals "0", "1", "2", etc, and the signs "&", "@", "$", etc are logographs or, more specifically, aphonic (non-phonetic) logoideographs or ideologographs, whereas the corresponding English numerals "zero", "one", "two", etc, and the corresponding English words "and", "at", "dollar", etc, which can be regarded both as names of the above logographs and as names of denotata of those logographs are phonoideographs or ideophonographs. Also, the ten Arabic digits "0" to "9" and the above-mentioned signs are lexigraphs (atomic logographs).

2) When minuscule or majuscule letters of various alphabets in various fonts, such as '*a*', '*A*', 'A', 'α', 'A', 'א', etc, are utilized as abstract constants or variables, they also become logographs or, more specifically, aphonic logoideographs or ideologographs – in contrast to logo[exo]iconographs or [exo]iconologographs. These logographs can be mentioned orally by using their names, but they cannot be read orally (articulatorily). In general, abstract constants and variables, which are formed of a base letter and some labels on it, alpha-numeric or not, are aphonic logoideographs.

3) *Themis*, the goddess of justice in the Greek mythology, is depicted as a blindfolded woman holding a balance in her left hand, – which is a symbol of the impartial adjustment of conflicting claims, – and also holding a sward in her right hand, which is a symbol of an inevitable punishment when deserved. The entire

picture is an *allegoric atomic* (*indivisible*) *exoiconograph* and hence, in spite of its apparently pictographic character, it is, after all, an *atomic iconoideograph* (*pictoideograph*)*,* or *ideoiconograph* (*ideopictograph*)*, of the concept*, *and not of an instance*, *of justice*.

4) Examples of all kinds of logographic (including [exo]pictographic) writing and of mixed logographic and phonographic writing can be found among the *international traffic signs*. For instance, the "*Cross-roads*" sign, which is designed as two intersecting lines, is an iconograph (pictograph) and hence an iconologograph (pictologograph) or logoiconograph (logopictograph). Likewise, the signs "*Defective road surface*" and "*Pedestrian crossing ahead*" are also iconographs (pictographs) of the same kind, whereas the signs "*Stop! Give priority to traffic on road crossing yours*" and "*No entry to all vehicles*" are ideologographs (logoideographs). The signs "*No left turn*", "*No entry to bicycles*", and "*No entry to all motor vehicles*" are iconoideologographs. In each country, some of the traffic signs are supplemented by alphabetic writings in the local language and, in addition, in English or French. Any such sign is a *mictophonologograph* (*mictologophonograph*), i.e. *mixed phonologograph* (*mixed logophonograph*).

5) A conventional traffic sign as such is a xenograph of the same general category as a common declarative or imperative statement, i.e. as a common declarative or imperative sentence or a sequence of such sentences. That is to say, a traffic sign a *common logographic name* that can, like a common declarative or imperative sentence of a BAbNL, be either asserted (used assertively) or not. Particularly, a traffic sign that is placed in a storehouse or is dumped is not asserted. Accordingly, such a sign is *senseful*, because it *designates* the class of many pertinent indistinguishable conceptual complex objects (states of affairs, cases), which is the *sense* of the sign, but it *does not denote* any concrete object and is, in this sense, *meaningless* (*ineffective*). When the same traffic sign is placed by a specific road, it is thereby *asserted*. In this case, there is a concrete object that matches the sense of the sign, so that the sign *refers to* and thereby *denotes* (*indicates*) that object as its *relatum* (*denotatum*) with respect (in relation) to any interpreter of the sign. Therefore, this sign is *meaningful* (*effective*) and hence *true* and at the same time it serves as a *dictograph*, i.e. *graphic index*, of its relatum (denotatum). Consequently, the combining form "*dicto*" can be adhered (without a preceding combining form "micto") to the taxonym of the sign. If, for instance, the sign in question is the

warning iconographic (pictographic) sign "*Defective road surface*" then this sign is a *dicto[exo]iconograph* (*dicto[exo]pictograph*) that *indicates* that a certain section of the road within the scope of the sign is defective, so that this defective section is *the meaning*, i.e. *the denotatum*, of the sign. It can however happen that by a certain instant of time, the defective section of the road had been repaired, but the sign was not afterwards moved away. Since the sign is as before *asserted*, therefore it is *antitrue* (*false*, *deceptive*, *anti-effective*): it *deceives* a driver, but the deception can be revealed only after passing the scope of the sign. At this same moment, the sign becomes *meaningless* (*ineffective*) *with respect to the driver* in the same sense as meaningless is a token of the sign being in a store house or in a scrap-heap.

Essay 4. Special quotations

1. Special quotations versus ordinary quotations

Df 1.1. A *graphonym* that is obtained by enclosing another graphonym between *quotation marks* is called a *quotation*. The quoted graphonym is called *the interior of the quotation*, while the pair of quotation marks enclosing that graphonym is called *the exterior of the quotation*.•

Cmt 1.1. A Merriam-Webster® [1981] (to be referred to as WTNID) introduces *English quotation marks* and defines the most conspicuous meanings of quotations which are formed with the help of these marks thus:

> «**quotation mark**, *n* **:** one of a pair of punctuation marks “ ”, ‘ ’, "", or '' used to indicate the beginning and the end of a quotation in which the exact phraseology of another or of a text is directly cited – usu. used to enclose the titles of poems, paintings, lectures, articles, and parts of books and sometimes used to enclose technical terms expected to be familiar to the reader, words used in an unusual, ironical, or eye-catching sense, or words (as slang expressions) for which a writer offers a slight apology»

A quotation whose meaning is one of those defined in the above Webster's definition will called an *ordinary quotation* (*OQ*). Philosophical theorizing aimed at categorizing OQ's in depth is irrelevant to formal logic and mathematics, and it is therefore beyond the scope of Psychologistics. The interested reader will have no difficulty in locating and studying various philosophical theories of OQ's elsewhere.

The above quotation of the Webster's definition is also an ordinary one. I have formed it with the help of French quotation marks, « », for the following reason. In Psychologistics, I employ pairs of English quotation marks of various forms, – single and double, light-faced and bold-faced, – and also some other paired characters as *molecular punctuation marks*, or in alternative terminology as *molecular dictographs* (*graphic indices*), *for indicating* a certain *mental attitude* that I take and that any interpreter should take towards the graphonym enclosed between quotation marks of each particular kind. I shall therefore call these quotation marks and also quotations that they form *special*, or *attitudinal*, *quotations*. Thus, special (attitudinal) quotation marks of each form indicate the corresponding kind of value, and hence the value itself, of the graphonym quoted, which should be put forward as the current *denotatum* (pl. "*denotata*"), or *denotation value*, of the graphonym, but they *do not affect* the value such. For avoidance of confusion between special and ordinary quotations, I shall adopt, – in fact, I have already tacitly adopted, – the following radical convention and the subsequent definitions based on that convention.•

Cnv 1.1. In Psychologistics, French double angle quotation marks, « », are employed instead of *ordinary* English single and double quotations marks, whereas the latter are freed of their ordinary meanings and are used only as *special* ones. Accordingly, the quotation marks will hereafter be called *ordinary quotation marks* (*OQ marks*), whereas quotations which are formed with the help of them will be called *ordinary quotations* (*OQ's*).•

Df 1.2. 1) In accordance with Cnv 1.1, quotation marks other then ordinary ones, « », will hereafter be called *special*, or *attitudinal*, *quotation* (*SQ*) *marks*. Consequently, a *graphonym* that is obtained by enclosing another graphonym between SQ marks will be called a *special*, or *attitudinal*, *quotation* (*SQ*). The graphonym quoted will be called *the interior of the quotation*, while the pair of quotation marks enclosing that graphonym will be called *the exterior of the quotation*. The exterior of a SQ is a *molecular punctuation mark* that *is used but not mentioned* and that serves as a *dictograph* (*dictographonym, graphic index*) *indicating* the value of what kind, and hence which one of the values, of the interior of the SQ I put and any interpreter of the SQ should put forward as the denotatum of both the interior and the entire SQ. In this case, the exterior does not affect the value of the interior to which it points as its denotatum. The procedure of using SQ's, which is described below in this definition and in the subsequent pertinent definitions, conventions, and comments, will be called *Special Quotation Method* or briefly *SQM*.

2) There are two kinds of SQ marks: *light-faced* ones and *bold-faced* ones, although some bold-faced counterparts of light-faced SQ marks will be used rarely if ever. An SQ is called a *concrete special quotation* (*CSQ*) if its exterior is light-faced and a *special quotation placeholder* (*SQP*) if its exterior is bold-faced.

3) The interior of a CSQ is regarded as a concrete (ultimate, not place-holding) graphonym, while the *modality* (*form*) of the exterior of the CSQ indicates the value of which kind, i.e. actually which one of the values, of the interior is put forward as its *current* (*circumstantial*) *denotatum*. The exterior of a CSQ *does not affect* the value of its interior, to which it points as denotatum of the CSQ.

4) The interior of an SQP is a placeholder that can be either a *whole* (*unbroken*) *one* or an *interrupted* (*broken*) *one*, i.e. a juxtaposition of alternating concrete and whole placeholders. Accordingly, the exterior of the SQP indicates the kind of value of any concrete graphonym of the range of the place-holding interior, which can be substituted for that interior. Once the place-holding interior of the SQP is replaced with a concrete graphonym, the bold-faced exterior of the SQP should be replaced with the corresponding light-faced one, so that the SQP turns into a CSQ.

5) CSQ's of each given kind will be provided with a taxonomic descriptive name through the *genus* denoted by the word "quotation" and the *differentia* (pl. "*differentiae*") denoted the appropriate *epithet* (*qualifier*), i.e. by the name of the form "— quotation" where "—" is a placeholder (ellipsis) for the epithet. In this case, if **"**— *quotation***"** *is* the taxonym of an SQ then the corresponding SQP, if defined, will be provided with the taxonym **"***quasi-*— *quotation***",** the understanding being that the two tokens of "—" should be replaced alike.

6) An *isolated* (*single*) SQ of any kind is a xenograph. Once the sense of this xenograph is defined, the complex graphonym (context) that includes the SQ and the appropriate *added words* is another, different, *xenograph* that may have a different sense, either narrower or broader, as compared to that of the isolated SQ.•

Besides Cnv 1.1, I have from the very beginning tacitly used English *light-faced curly single and double* quotation marks as special ones in accordance with items 1b and 1c of the following definition.

Df 1.3. 1) Given a *xenograph*, whenever confusion can result, an isotoken of the xenograph will be enclosed between:

a) *slant light-faced single quotation marks*, ′ ′, that are called *kyrioautographic quotation* (*KAQ*) *marks*, if the isotoken *is used self-referentially*, i.e. *for mentioning itself,* and is therefore called a *kyriotychautograph, i.e.* a *proper accidental* (*circumstantial*) *autograph*;

b) *curly* (*decisive*) or *straight* (*indecisive*) *light-faced single quotation marks*, i.e. ' ' or ' ' respectively, called *homoloautographic quotation* (*HAQ*) *marks*, if the isotoken is used for mentioning *the class of it*s *homolographic* (*photographic*, *congruent* or *proportional*) *isotokens* and is therefore called a *homoloautograph* (*photoautograph*);

c) *curly* (*decisive*) or *straight* (*indecisive*) *light-faced double quotation marks*, i.e. “ ” or " ", respectively, called *iconoautographic quotation* (*IAQ*) *marks*, if the isotoken is used for mentioning *the class of it*s *iconographic* (*pictographic*, *analographic*) *isotokens* and is therefore called a *iconoautograph* (*pictoautograph, endoiconograph*, *endopictograph*); the latter can be either a *pure* (*chaste*) *one* or a *combination* (usually a *juxtaposition*) of one or more pure iconoautographs and one or more homoloautographs;

d) *slant light-faced double quotation marks*, *“ ″*, called *phonoautographic quotation* (*PAQ*) *marks*, if the isotoken is a phonograph that is used for mentioning *the class of its phonic* (*vocal*) *paratokens* and is therefore called a *phonoautograph*;

e) a *back-slashed virgule-like quotation mark* and a *forth-slashed one,* \ /, called *enneoxenographic quotation* (*EXQ*) *marks*, if the isotoken is a xenograph that is used *obliquely* (*indirectly*) for mentioning (denoting) *its sense*, i.e. *the biune mental entity* (*process*) of its any interpreter, which comprises *the sense-operation of coordination of the constituent object-classes of the sense* and *the subject class of the sense*, called also *the designatum of the xenograph*, – the mental entity that is *connoted* by the xenograph when it is used *directly* for mentioning (denoting) either a *common* (*general*, *certain, concrete but not concretized*) *member of the designatum* (see section 3 for greater detail) ; the isotoken quoted is called an *enneoxenograph,* i.e. a *sense-valued xenograph.*

An SQ is called: a *kyrioautographic quotation* (*KAQ*) in case a), a *homoloautographic quotation* (*HAQ*) in case b), an *iconoautographic quotation* (*IAQ*) in case c), a *phonoautographic quotation* (*PAQ*) in case d), and an *enneoxenographic quotation* (*EXQ*) and also a *sense*, *or semantic*, *quotation* in case e). In accordance with Df I.1.2(2c) of the TTL, all the above complex Graecized monomials (monomina), having either root “graph” or “phon”, are abbreviated *onym*ological terms, which have been obtained by omission of the root “onym” from the constituent postpositive combining forms “*graphonym*” and “*phononym*” of the corresponding full *onym*ological terms.

2) *An SQ is a xenograph that has the same denotatum as that of its interior*. Namely, a KAQ denotes its interior, an HAQ denotes *the homolographic isotoken-class* (*percept-class*) *of its interior*, an IAQ denotes *the iconographic isotoken-class* (*percept-class*) *of its interior*, a PAQ denotes *the phonographic paratoken-class* (*phonation-class*) *of its interior*, and EXQ denotes *the sense of its interior*, – in agreement with Df I.1.23(3) of the TTL. Accordingly, a KAQ, HAQ, IAQ, PAQ, or EXQ can alternatively be called a *dicto-kyrioautograph, dicto-homoloautograph*, *dicto-endoiconoautograph*, *dicto-phonoautograph*, or *dicto-enneoxenograph*, respectively, where the prefix "*dicto-*" is descriptive of *the indicative function of the exterior of an SQ*, which has been described in Df 1.2(1). It is understood that any of the above "dicto"-terms is regarded as a subterm of "xenograph".

3) In contrast to a KAQ, an HAQ, IAQ, or PAQ, each taken individually, is indiscriminately called a *cenautographic quotation* (*CAQ*). A KAQ or a CAQ, each taken individually, is indiscriminately called a *special*, or *attitudinal*, *autographic*, or *autonymous*, *quotation* (*SAQ*) and also a *syntactic quotation* in contrast to a *semantic quotation*, i.e. an EXQ. Accordingly, HAQ, IAQ, or PAD marks are indiscriminately called *CAQ marks*, whereas KAQ or CAQ are indiscriminately called *SAQ marks*. In accordance with Cmt I.1.27(2,3) of the TTL, the prefix "kyrio" means *proper* or *strict*, while the prefix "cen" means *common* or *lax*.

4) Use of a xenograph after the manner of the interior of EXQ, in which it denotes (puts forward) the mental entity of the interpreter, called the sense of the xenograph, is said to be *oblique* or *indirect* in contrast to its use which is said to be *direct* and in which the xenograph *connotes* the sense and denotes a certain object of the interpreter that matches the sense.

5) In accordance with Df 1.2(6), an SAQ together with the appropriate added words is a xenograph whose denotatum may differ from the denotatum of the SAQ alone.

6) The procedure of using KAQ's or CAQ's or generally SAQ's) in accordance with the above items of this definition and also in accordance the subsequent pertinent definitions, conventions, and comments will be called the *Kyrioautographic Quotation Method* (*KAQM*) or the *Cenoautographic Quotation Method* (*CAQM*) or generally the *Special Autographic Quotation Method* (*SAQM*), respectively.•

Cnv 1.2. 1) A euautograph cannot either have or assume any xenonymous values and hence it cannot be mentally turned into a constant or variable. Therefore, a euautograph *is not, as a rule, enclosed in any SAQ marks*. That is to say, SAQ marks *apply, as a rule, only to xenographs*. Particularly, no SAQ's are used in A_1. At the same time, the xenographic interior of an SAQ may contain some euautographs as its constituent parts. In this case, no harm is done if a euautograph being a constituent part of the xenographic interior of an SAQ is enclosed between HAQ marks.•

Cmt 1.2. In Psychologistics, the OQ marks enclosing a *xenograph* are used exclusively as punctuation marks preventing grammatical incongruities, which might otherwise happen; they do not affect the *xenonymous mental mode*, in which an interpreter of the OQ should use its interior. Particularly, the quotations of fragments of other texts, which preserve the meanings (denotata) of their interiors, and also the quoted titles of articles, journals, or books, which are used as proper names of the articles, journals, or books, are OQ's. By contrast, SAQ's of the above-mentioned titles are proper or common names of the titles themselves.•

Cmt 1.3. 1) In Psychologistics, I use primarily HAQ's and IAQ's and occasionally EXQ's. Psychologistics and particularly the TTL are essentially graphic (written) discourses. Therefore, I have introduced PAQ's mainly for the sake of completeness of the SAQM; I have few occasions to mention or use PAQ's.

2) In fact, a *cen*autographic, i.e. *common* autographic, quotation (CAQ) is a *proper generic name*, i.e. *proper class-name*, whereas a *kyrio*autographic, i.e. *proper* autographic, quotation (KAQ) is a *proper member name*, i.e. a *proper individual name*. Therefore, in the above descriptions through the genus, denoted by the generic name (head word) "quotation", and differentia, denoted by the adjective (qualifier, epithet) "*cen*autographic" or "*kyrio*autographic", it would be more correct to employ "*kyrioclass*autographic" instead of "*cen*autographic" and "*kyriomelo*autographic" instead of "*kyrio*autographic" (see Dict A1.1 in the TTL). Still, in using the adjectives "*ceno*autographic" and "*kyrio*autographic", I assume that the qualifiers "cen" ("common") and "kyrio" ("proper") apply to the adjective "autographic" and not to the noun "quotation".

3) A KAQ and KAQ marks will alternatively be called a *Fregean quotation* (*FQ*) and *Fregean quotation* (*FQ*) *marks* respectively after Frege [1893–1903, vol. 1, p. 4] who was likely the first logician to suggest an impracticable method of

systematically eliminating, – or, from a somewhat different but equivalent viewpoint, of *systematically indicating* (cf. Church, [1956, p. 61]), – *autographs*, which I call proper, or strict, ones and also kyrioautographs, by enclosing them in [curly light-faced] single quotation marks *in all cases*. I shall call this method *Fregean quotation method* (*FQM*). I have introduced KAQ's mainly in order to discuss conveniently the difference between the CAQM or the SAQM in general and FQM, and also in order to illustrate conveniently that, firstly, use of FQ's in the literature has, as a rule, nothing to do with any of their conventional formal definitions and that, secondly, realization of the entire FQM in practice has nothing to do with FQM as a theoretical (hypothetical) construction. In fact, FQM is inconsistent because it is impracticable. At the same time, if used in accordance with their formal definition as Df 1.3(1), separate FQ's, i.e. KAQ's, turn out to be useful mainly as a heuristic means owing to the simplicity of their denotata as compare to the denotata of SQ's of all other kinds.•

Ex 1.1. Here follow simple examples illustrating the difference between HAQ's and IAQ's.

The organon A_1 is *called*, i.e. *phonographically* (*wordily*, *verbally*) *denoted by* "Comprehensive Euautographic Algebraico-Predicate Organon" or briefly "CEAPO". Conversely, CEAPO is *logographically denoted* by 'A_1'. Suppose that instead of the last sentence I state: «CEAPO is logographically denoted by "A_1"». Then in the first sentence of this paragraph I could use, not only 'A_1', but also any of the symbols '$\boldsymbol{\mathsf{A}}_1$', '$\mathsf{A}_1$', '$\mathit{A}_1$', '$\mathrm{A}_1$', '$\mathbf{A}_1$', '$A_1$', '$\mathbf{A_1}$', '$A_I$', '$\mathcal{A}_1$', etc in place of '$\mathsf{A}_1$'. It is understood that this ambiguity is unacceptable. At the same time, when I say, e.g., that A_1 is called "CEAPO", – to use the abbreviation instead of the full name for the sake of brevity, – I mean that I may also call A_1 by, i.e. mention it by using any one of the tokens: 'CEAPO', '**CEAPO**', '*CEAPO*', '*CEAPO*', 'CEAPO', '**CEAPO**', '*CEAPO*', etc.

In accordance with Df 1.3(1c), a cenotych-iconoautograph (cenotych-pictoautograph) that is enclosed in IAQ marks may contain some homolographs or euautographs as its constituent parts. For instance, '**P**' is a homolographic atomic relation-placeholder of $\boldsymbol{\mathsf{A}}_1$, so that **P** *is any relation of* A_1. In this case, the expression "**P** is any relation of A_1", occurring in the previous sentence, is true under the mental attitude of its interpreter, according to which '**P**' *is used for mentioning any relation of* A_1. In the expression "**P** is any relation of A_1", '**P**' and 'A_1' should be set in the given fonts, while the wordy expression "is any relation of" or any part of it can be set in any font, e.g. in italic, so that it has the same sense as any of the expressions: "*is any relation of*", "*is* any relation of", and "is *any relation of*".•

Df 1.4. 1) The xenograph that is obtained by enclosing a *whole* or *interrupted placeholder* between *bold-faced* slant single quotation marks, **′ ′**, is called a *quasi-kyrioautographic*, i.e. *quasi proper autographic*, *quotation* (*QKAQ*), whereas the above quotation marks are called *QKAQ marks*. A QKAQ is a *placeholder for any KAQ* such that once the interior of the former is replaced with a concrete (not place-holding) graphonym, the bold-faced slant single quotation marks should be replaced with light-faced ones.

2) The xenograph that is obtained by enclosing a unbroken (whole) or broken (interrupted) placeholder between curly (decisive) or straight (indecisive) *bold-faced* single quotation marks, **‘ ’** or **' '**, is called a *quasi-homoloautographic quotation* (*QHAQ*), whereas the above quotation marks are called *QHAQ marks*. A QHAQ is a *placeholder for any HAQ* such that once the interior of the former is replaced with a concrete (not place-holding) graphonym, the bold-faced single quotation marks should be replaced with light-faced ones.

3) The *xenograph* that is obtained by enclosing a placeholder in curly (decisive) or straight (indecisive) *bold-faced* double quotation marks, **“ ”** or **" "**, is called a *quasi-iconoautographic quotation* (*QIAQ*), whereas the above quotation marks are called *QIAQ marks*. A QIAQ is a *placeholder for any IAQ* such that once the interior of the former is replaced with a concrete (not place-holding) graphonym, the bold-faced double quotation marks should be replaced with light-faced ones.

4) The *xenograph* that is obtained by enclosing a placeholder in virgule-like *bold-faced* quotation marks, **\ /**, is called a *quasi-enneoxenographic quotation* (*QEXQ*), whereas the above quotation marks are called *QEXQ marks*. A QEXQ is a *placeholder for any EXQ* such that once the interior of the former is replaced with a concrete (not place-holding) graphonym, the bold-faced virgule-like quotation marks should be replaced with light-faced ones.

5) The *xenograph* that is obtained by enclosing a placeholder for phonoghaphs in slant *bold-faced* double quotation marks, **″ ″**, is called a *quasi-phonoautographic quotation* (*QPAQ*), whereas the above quotation marks are called *QPAQ marks*. A QEXQ is a *placeholder for any PAQ* such that once the interior of the former is

replaced with a concrete (not place-holding) phonograph, the bold-faced double quotation marks should be replaced with light-faced ones.

6) In analogy with Df 1.3(1), in contrast to a QKAQ, a QHAQ or QIAQ, each taken individually, is indiscriminately called a *quasi-cenautographic quotation* (*QCAQ*). A QKAQ or a QCAQ, each taken individually, is indiscriminately called a *special*, or *attitudinal*, *quasi-autographic quotation* (*SQAQ*). Accordingly, QHAQ marks or QIAQ marks are indiscriminately called *QCAQ marks*, whereas QKAQ or QCAQ marks are indiscriminately called *SQAQ marks*.

7) A placeholder that is enclosed between SQAQ or QEXQ marks can either be a single whole homogeneous placeholder or some one or some more constitute parts of it can be concrete xenographs or euautographs.

8) In accordance with Df 1.2(6), a SQAQ together with the appropriate added words is a xenograph whose denotatum may differ from the denotatum of the SQAQ alone.•

Cmt 1.4. I shall have few occasions to use QEXQ's and no occasions to use QPAQ's. The latter have been introduced for the sake of completeness of the *SQAQ Method* (*SQAQM*). •

Ex 1.2. The difference between SAQ's and SQAQ's can be illustrated as follows. Any of the graphonyms '$\mathbf{P}_1$', '$\mathbf{P}_2$', etc is an atomic relation-placeholder of A_1, whose range is the entire class of relations of A_1. Let 'n' be a placeholder whose range is the infinite set of the Arabic numerals '1', '2', etc, so that '$\mathbf{P}_n$' is a placeholder for any of the graphonyms '$\mathbf{P}_1$', '$\mathbf{P}_2$', etc. At the same time, '$\mathbf{P}_n$' is the graphonym therein depicted between light-faced single quotation marks, and the same is true of any of the graphonyms '$\mathbf{P}_1$', '$\mathbf{P}_2$', etc. Therefore, one may not assert that '$\mathbf{P}_n$' *is* '$\mathbf{P}_1$' or '$\mathbf{P}_2$' or any other concrete graphonym of those obviously understood by the word "etc" One may however assert that **'**$\mathbf{P}_n$**'** *is* '$\mathbf{P}_1$' or '$\mathbf{P}_2$' or any other concrete graphonym of the set. This example also illustrates that the notion of a concrete graphonym and the notion of a placeholder (place-holding graphonym) are sometimes epistemologically relativistic. Indeed, **'**$\mathbf{P}_n$**'** is a concrete graphonym with respect to the letter '**P**' and a placeholder with respect to the letter 'n'. At the same time, any of the graphonyms '$\mathbf{P}_1$', '$\mathbf{P}_2$', etc is an atomic placeholder of any relation of A_1.•

Cmt 1.5. SQAQ's and SQAQ marks, especially QKAQ's and QKAQ marks, are functionally similar respectively to the so-called *quasi-quotations* and *corner-like* *quasi-quotation marks* of Quine [1951, pp. 33–37]; Quine's quasi-quotations are *patterned placeholders* of Frege's quotations.•

Cmt 1.6. An SQ is a xenograph. But owing to its logographic exterior, an SQ is *not a phonograph*. Therefore, any SQ cannot occur in the exterior of a PAQ or QPAQ. Consequently, the interior of an SAQ or SQAQ of any kind except that of a PAQ or QPAQ can in principle contain or particularly be entirely an SAQ or SQAQ of any kind subject to some self-evident restrictions such as the following.

a) The IAQ of an HAQ (or of a KAQ) and the HAQ of the same HAQ (or, correspondingly, of the same KAQ) are identical.

b) The IAQ of a QHAQ (or of a QKAQ) and the HAQ of the same QHAQ (or, correspondingly, of the same QKAQ) are identical.

c) The QIAQ and the QHAQ of the same QHAQ are identical.

d) Since the interior of a SQAQ has to be a placeholder, it cannot be entirely an SAQ of any kind.•

2. Juxtapositions and concatenations of special quotations

Preliminary Remark 2.1. 1) The words "concatenation" and "juxtaposition" are kindred nouns of the verbs "*to concatenate*" and "*to juxtapose*", which according to WTNID mean «*to place side by side*» and «*to link together*» or «*to unite in a series or chain*», respectively.

2) The interior and the exterior of a KAQ, i.e. the graphonym quoted and the pair of KAQ marks, have two different mental statuses: the interior is mentioned by using the KAQ, while the exterior is thereby used as an integral part of the KAQ but it is not mentioned. That is to say, the exterior of the KAQ is an *operator*, i.e. an *operative* (*operating*, *functional*) graphonym, the interior of the KAQ is the *operatum* of the operator, i.e. the graphonym operated by the operator, and the KAQ is the *operand*, or *scope*, of the operator. A like remark applies, *mutatis mutandis*, to any SQ and to any SQAQ, although the pertinent generalizations of that remark involve more complicated wordings. Likewise, if a given graphonym, – a euautograph or a xenograph, – is a combined one that can be dissected into shorter admissible graphonyms then the constituent graphonyms are not, as a rule, all of the same kind but rather some of them are *operators* forming a *single whole composite operator*, while the others are *operata* that are, together with the operators, united by the

operators so as to form the given single whole graphonym – *the operand*, or *scope*, *of the composite operator*. Therefore, depending on the mental attitude that I take towards the linkage among constituent parts of the combined interior of an SAQ or SQAQ, I adopt either the following concise convention or the convention that will informally be described in Cmt 2.1 below.•

Cnv 2.1: *Principle of Juxtaposition of SAQ's or SQAQ's.* 1) If the interior of an SAQ, or SQAQ, is regarded as a *syntactic juxtaposition* of shorter constituent graphonyms, which are distracted (freed) of all their *xenonymous values* (*xenovalues*) then the SAQ, or SQAQ, has the same autonymous, or correspondingly quasi-autonymous, denotatum as the juxtaposition of the SAQ's, or SQAQ's, of the constituent graphonyms of the interior. To be more specific, let either of the two graphonyms ‘Γ_1’ and ‘Γ_2’ be either a concrete graphonym or a placeholder having a certain range of concrete graphonyms. Let also ‘↔’ be a binary *synonymity*, or *concurrency*, *sign*, defined in Df I.2.19 of the TTL. Then

i) **‘$\Gamma_1\Gamma_2$’**↔**‘Γ_1’‘Γ_2’** if ‘Γ_1’ and ‘Γ_2’ are concrete graphonyms,

ii) ‘$\Gamma_1\Gamma_2$’↔‘Γ_1’‘Γ_2’ if ‘Γ_1’ and ‘Γ_2’ are concrete homolographs,

iii) “$\Gamma_1\Gamma_2$”↔“Γ_1”“Γ_2” if ‘Γ_1’ and ‘Γ_2’ are concrete iconographs,

iv) ‘$\Gamma_1\Gamma_2$’↔‘Γ_1’‘Γ_2’ if ‘Γ_1’ and ‘Γ_2’ are placeholders of graphonyms,

v) **‘**$\Gamma_1\Gamma_2$**’**↔**‘**Γ_1**’‘**Γ_2**’** if ‘Γ_1’ and ‘Γ_2’ are placeholders of homolographs,

vi) **“**$\Gamma_1\Gamma_2$**”**↔**“**Γ_1**”“**Γ_2**”** if ‘Γ_1’ and ‘Γ_2’ are placeholders of iconographs,

vii) ‘$\Gamma_1\Gamma_2$’↔‘Γ_1’‘Γ_2’ if ‘Γ_1’ is a placeholder of graphonym and ‘Γ_2’ is a concrete graphonym,

viii) **‘**$\Gamma_1\Gamma_2$**’**↔**‘**Γ_1**’‘**Γ_2**’** if ‘Γ_1’ is a placeholder of homolographs and ‘Γ_2’ is a concrete homolograph,

ix) **“**$\Gamma_1\Gamma_2$**”**↔**“**Γ_1**”**“Γ_2” if ‘Γ_1’ is a placeholder of iconographs and ‘Γ_2’ is a concrete iconolograph.

The items vii–ix mean that if the interior of a certain constituent SQAQ of the juxtaposition of SQAQ's is a *concrete* graphonym (not a placeholder), i.e. a concrete kyriotychautograph, homoloautograph, or iconoloautograph, then the pertinent SQAQ marks of the constituent SQAQ should be replaced by SAQ marks of the corresponding kind. Consequently, if the interior of a certain constituent SQAQ of the juxtaposition of SQAQ's is a *concrete euautograph* then the pertinent SQAQ marks of the former can be omitted. For instance, on the right-hand side of any one of the synonymity relations vii–ix, one may write Γ_2 in place ‘‘Γ_2’’, ‘‘Γ_2’’, or ‘‘‘Γ_2’’’ respectively.●

Cmt 2.1: *Semantic concatenation of not juxtaposed SAQ’s or QSAQ’s.* Cnv 2.1 comes instead of the conventional principle of juxtaposition of FQ’s as stated, e.g., by Suppes [1957, pp. 125, 126]). However, in some cases, Cnv 2.1 is inapplicable as illustrated in the following typical example.●

1) Let ‘{1,2,...}’ denote the set of strictly positive natural numbers, the understanding being that ‘1’, ‘2’, etc are Arabic numerals denoting the corresponding individual natural numbers. In accordance with Cnv 2.1, under the corresponding mental attitude towards ‘{1,2,...}’, this SAQ is a *denotative synonym* of the juxtaposition ‘{’‘,’‘1’‘,’‘2’‘,’‘...’‘}’. By Df I.2.19 of the TTL, I express the above synonymity relation as

$$‘\{1,2,...\}’\leftrightarrow‘\{’‘,’‘1’‘,’‘2’‘,’‘...’‘\}’, \tag{2.1}$$

All quotation marks occurring in the juxtaposition ‘{’‘,’‘1’‘,’‘2’‘,’‘...’‘}’ are used but not mentioned, whereas an occurrence of the *right* (*closing*) single quotation mark, ’, and, when applicable, the following occurrence of the *left* (*opening*) single quotation mark, ‘, as if *annihilate*. I may, however, take another mental attitude towards ‘{1,2,...}’, under which

$$‘\{1,2,...\}’\leftrightarrow\{‘1’,‘2’,...\}. \tag{2.2}$$

This metamorphosis of ‘{1,2,...}’ can be explicated as follows.

2) The numerals ‘1’ and ‘2’ are *terms* and hence *categoremata* of the *metalanguage* (*ML*), i.e. they are graphonyms that can have self-subsistent denotata in isolation, while the braces, commas, and ellipsis, are *punctuation marks* and hence *syncategoremata* of the ML, i.e. graphonyms which do not have any self-subsistent denotata in isolation. From the standpoint of syntactic analysis, the punctuation marks are *operators* that form a *single whole composite operator* and that unite the categoremata, being *operata* of the composite operator, to produce a new categorem, called the *operand*, or *scope*, of the composite operator. From the standpoint of semantic analysis, the composite operator ‘{ , ,...}’ stands for (expresses) *the mental operations*, which should be performed by an interpreter of the graphonym ‘{1,2,...}’ on the operata ‘1’ and ‘2’ either in connection or in no connection with their

xenonymous senses (*xenosenses*). To be specific, the ellipsis '...' in ' {1,2,...}' stands for the *operation of omission* of the imaginary concatenation of all successive numerals, starting from '3', and of all commas between every two omitted numerals. A comma between '1' and '2' and a comma between '2' and '…' stands for the *mental operation of separation* of '1' and '2' and also for that of every two neighboring numerals, which are obviously understood by '…'. Omission of the comma between '1' and '2', e.g., would result in the string '{12,...}', which differs from '{1,2,...}', because '12' is another numeral. In this case, the ellipsis becomes meaningless, because it does not indicate, which numeral is supposed to follow the string '12'. The comma between '2' and '…' is indispensable for the like reason. Thus, the occurrences of the comma and of the ellipsis in '1,2,...' turn this string into a name of the sequence, or list, of all natural numbers from 1 to infinity. Along with the two commas and ellipsis, the pair of braces in '{1,2,...}' form the single whole operator '{ , ,...}' that can be called a *set-building operator* or briefly a *set-builder* because it stands for a *mental operationI,* in the result of which '{1,2,...}' or {'1','2',...} is turned into a name of the corresponding set. This *mental set-building operation* is the operation of *disengaging* from both the order, in which the numerals are supposed to be written between the braces, and from the number and places of occurrences of each numeral. For instance, each of the strings: '{1,2,3}', '{3,1,2}', '{1,3,3,2}', '{2,2,1,1,1,3}', etc *denotes* one and the same set of three natural numbers 1, 2, 3 taken in any order. That is to say, {1,2,3}, {3,1,2}, {1,3,3,2}, or {2,2,1,1,1,3}, etc *is* the set of natural numbers 1, 2, 3. Likewise, each of the strings: {'1','2','3'}, {'3','1','2'}, {'1','3','3','2'}, {'2','2','1','1','1','3'}, etc *is* one and the same set of three Arabic digits (numerals) '1', '2', '3' taken in any order – the set that *denotes* the set of natural numbers 1, 2, 3. Thus, to say nothing of mental (insensible) objects, *a set of sensible objects is not a sensible collection, or group, of those objects*. Particularly, a set of graphonyms (graphic expressions) is *not a list* of those graphonyms even if that list is finite.

3) A set is a class, but not every class is a set. I shall explicate some properties, by which a set differs from a class not being a set, elsewhere (see, e.g., subsection I.9.3 of the TTL). At the same time, the above examples demonstrate that, like a class in general, *a set cannot be depicted on a material surface* and particularly be written on paper, but rather *it can be represented by any one of its proper names*. A natural

number is also a set, i.e. a class, and therefore it cannot be depicted on a material surface either; it is represented by its Arabic or any other numeral being its proper name. *Both quoted and unquoted numerals are graphonyms*. Accordingly, when I say that '1', '2', etc are Arabic numerals, while 1, 2, etc are natural numbers denoted by those Arabic numerals, I just take two different mental attitudes with regard to the same Arabic numerals. One of the mental attitudes, according to which the numerals are used autonymously, is indicated by enclosing each one of the numerals in single quotation marks. The other mental attitude, according to which the numerals are used xenonymously, is indicated by the absence of single quotation marks. If the set-building operator '{ , ,...}' and the numerals '1' and '2', occurring in the name '{1,2,...}', are used but not mentioned then that name is written without any quotation marks as {1,2,...}, which is said to be *the set of natural numbers from* 1 *to infinity*. If the set-building operator '{ , ,...}', occurring in the name '{1,2,...}', is used but not mentioned, while the numerals '1' and '2' are used for mentioning themselves or their homolographic tokens, then the name '{1,2,...}' turns into {'1','2',...}, which is said to be *the set of Arabic numerals from* '1' *to infinity*.

4) Let

$$\omega_1 \rightarrow \{1,2,...\}, \tag{2.3}$$

so that 'ω_1' is a denotative synonym of '{1,2,...}'. Consequently, ω_1 *is*, just as {1,2,...}, the set of natural numbers from 1 to infinity, whereas

$$\text{'}\omega_1\text{'} \leftrightarrow \text{'}\omega\text{''}_1\text{',} \tag{2.4}$$

in accordance with Cnv 2.1 and in analogy with (2.). Alternatively, in analogy with (2.2),

$$\text{'}\omega_1\text{'} \leftrightarrow \omega\text{'}_1\text{'.} \tag{2.5}$$

In the latter case, 'ω' is a *universal* operator, also called a *kernel-sign*, that comes instead of the composite operator '{ , ,...}', '$_1$' is the operatum of 'ω', 'ω'$_1$'' is the operand, or scope, of 'ω', and ω'$_1$' is the set of all Arabic numerals from '1' to infinity – just as {'1','2',...}. The following definition is based on, and at the same time it illustrates, the universal character of the operator 'ω'.•

Df 2.1. 1) ω_2 is the set of natural numbers from 2 to infinity, ω_3 is the set of natural numbers from 3 to infinity, etc, so that 'ω_2', 'ω_3', etc are logographic proper names of the above-mentioned sets in this order.

2) $\omega_{‘2’}$ is the set of Arabic numerals from ‘2’ to infinity, $\omega_{‘3’}$ is the set of Arabic numerals from ‘3’ to infinity, etc, so that ‘$\omega_{‘2’}$’, ‘$\omega_{‘3’}$’, etc are logographic proper names of the above-mentioned sets in this order.

3) ω_n is any of the sets ω_1, ω_2, etc Consequently, ‘ω_n’ *is a placeholder of* any of the constants ‘ω_1’, ‘ω_2’, etc, while **‘**ω_n**’** *is* any of those constants.

4) $\omega_{‘n’}$ is any of the sets $\omega_{‘1’}$, $\omega_{‘2’}$, etc. Consequently, ‘$\omega_{‘n’}$’ is a placeholder of any of the constants ‘$\omega_{‘1’}$’, ‘$\omega_{‘2’}$’, etc.

5) $\omega_{m,n}$ is the set of natural numbers from m to n subject to $m\leq n$, ‘m’ and ‘n’ are variables having the range ω_1 each.

6) $\omega_{‘m’,‘n’}$ is the set of Arabic numerals from **‘**m**’** to **‘**n**’** subject to **‘**m**’**≤**‘**n**’**, ‘**‘**m**’**’ and ‘**‘**n**’**’ are placeholders having the range $\omega_{‘1’}$ each.•

Preliminary Remark 2.1. If the interior graphonym of an EXQ, to be refer *ad hoc* to as *the enneoxenograph*, is a *syntactic juxtaposition* of shorter constituent sense-producing and hence designatum-producing graphonyms then the designata of the latter are *concatenated* (*coordinated*) *semantically* to produce a single whole sense and hence the designatum of the enneoxenograph. However, a dissection of the enneoxenograph into constituent designative graphonyms is as a rule ambiguous and the enneoxenograph involves *no operators* to indicate any *elemental mental designatum-producing* (*semantic*) *operations of concatenation of the designata of those graphonyms*, so that these operations are *latent* and usually *undefined*. Consequently, concatenation of elemental designative units of the enneoxenograph cannot in the general case be indicated explicitly after the manner of juxtaposition of SAQ’s and QSAQ’s, except few special cases. The most important one of these special cases is a *description of the species through a genus and the difference,* or *differences* (in Latin, *descriptio species per genus et differentiam,* or *differentias*) or briefly a *descriptive specific name* (*DSN*). Under some additional conditions, the sense of a DSN sense can be regarded as the conjoined senses of its *generic name* (GN) and of the qualifiers to the GN as explicated in the following meta-axiom.•

Ax 2.1: ***An analysis of the EXQ of a descriptio species per genus et differentias* (*or differentiam*).** Let ‘Γ’ be a placeholder for a generic name (GN), while ‘E_1’ and ‘E_2’ are placeholder for two commutative prepositive qualifiers (epithets) to the GN. Hence, given a description **“**$E_1E_2\Gamma$**”** of the species $^{\backslash}E_1E_2\Gamma^{/}$

through the genus ${}^{\backslash}\Gamma^{/}$ (denoted by the generic name **“**Γ**”)** and through the differentiae ${}^{\backslash}\mathrm{E}_1{}^{/}$ and ${}^{\backslash}\mathrm{E}_2{}^{/}$ (denoted by the qualifiers **“**E_1**”** and **“**E_2**”)**, so that each of the two qualifiers applies to **“**Γ**”**.Then

$$ {}^{\backslash}\mathrm{E}_1\mathrm{E}_2\Gamma^{/}\leftrightarrow{}^{\backslash}\mathrm{E}_1{}^{//\backslash\backslash}\mathrm{E}_2{}^{//\backslash\backslash}\Gamma^{/}\leftrightarrow[{}^{\backslash}\mathrm{E}_1{}^{//\backslash}[{}^{\backslash}\mathrm{E}_2{}^{//\backslash\backslash}\Gamma^{/}]], \tag{2.6} $$

where the sign $^{/\backslash}$ is formed by the QEXQ marks $^{/}$ and $^{\backslash}$. Once the interiors of the QEXQ’s in the above train of equivalences are replaced with concrete graphonyms, all bold-faced forth-slashed and back-slashed virgules in the superscript line should be replaced with light-faced ones. In the result, the sign $^{/\backslash}$ turns into $^{\wedge}$, which is regarded as the operator denoting the binary operation of intersection of the *classes* or *conceptional masses* (*cmasses*) that are denoted by the EXQ’s standing on both sides of $^{\wedge}$. The form and size of the sign $^{\wedge}$ and locations of its tokens in the superscript line serve as a mnemonic justification of the analysis, which is represented by the train of equivalence relations (2.6) and which reminds the Principle of Juxtaposition of SAQ’s or SQAQ’s. Any other appropriate sign, e.g. $\wedge$ or $\cap$, can be used instead of $^{\wedge}$.•

3. A theory of the meaning content of graphonyms

3.1. The meaning content of euxenographs

Df 3.1. When I *consider* various aspects of a given graphonym, which I prescind from its context and hence from any added words and which I *do not use purposefully*, I shall use the following terminology and phraseology.

1) If I associate the graphonym as *referent* with *one and only one* distinct physical (real) or psychical (mental, ideal) entity as *its principal value* (*relatum*) that differs from any token and any token-class of the graphonym then *ipso facto* the graphonym turns into a *xenograph*, while the above value becomes an *object sui generic* to be called *the denotatum* (*denotation value,* pl. “*denotata*”) of the xenograph. The qualifier “sui generis”, originating from the Latin etymon “sŭi gĕnĕris”, is an adjective that is usually used predicatively or postpositively and that means *constituting a class alone*, *of its own kind*, *of the class of its own*, or *in a class of itself*. The class of a single object is conventionally be called *the singleton of the object* or, more generally, *a singleton*, i.e. *a one-member class*. Thus, the singleton of the denotatum of the xenograph becomes another value of the xenograph. The xenograph thus defined is called a *proper xenograph of its denotatum* or, more generally, a *proper xenograph* (without any postpositive limiting modifier) and also,

in one word, a *kyrioxenograph*. A kyrioxenograph is alternatively called an *identifying xenograph* because it identifies its denotatum. I shall say that a kyrioxenograph *designates* or *connotes* its singleton and that conversely the singleton is *designated*, or *connoted*, *by* and also that it is the *designatum* (*designation value,* pl. "*designata*"), or *class-connotatum* (*connotation value,* pl. "*connotata*") *of the kyrioxenograph*.

2) If I associate the graponym as *referent* with *any one* of two or more *distinct* physical or psychical entities as *its distributive* (*separate*) *values* (*prospective relata*), which differ from tokens and token-classes of the graphonym, then *ipso facto* the graphonym turns into a *xenograph*, while all its distributive values are *prescinded from their differentia* and become *similar* (*conceptually same*) *members of a certain common* (*collective*) *xenonymous value* (*xenovalue*) of the xenograph, which is called *the class of distributive values of the xenograph* or, most generally, *a class*. A *many-member class* will be called a *multipleton*. A class whose members are classes, i.e. a class of classes, will be called a *category*, – from the Greek (Aristotelian) noun "κατηγορία" \kateγoría\. The xenograph thus defined will be called a *common xenograph of its distributive values* or, more generally, a *common xenograph* (without any postpositive limiting qualifier) and also, in one word, a *cenoxenograph*. A cenoxenograph is alternatively called a *classifying xenograph* because it classifies its distributive values. In analogy with a kyrioxenograph, I shall say that a cenoxenograph *designates* or *connotes* its multipleton and that conversely the multipleton is *designated*, or *connoted*, *by* and also that it is the *designatum* (*designation value,* pl. "*designata*"), or *class-connotatum* (*connotation value,* pl. "*connotata*") *of the cenoxenograph*.

3) A xenograph, proper or common, can be a *declarative sentence* (*DS*), *affirmative* or *negative*.

4) An isotoken of a xenograph is called a *euxenograph* if it is used xenonymously and a *tychautograph* if it is use autonymously.•

Df 3.2. 1) When I *consider* a xenograph as a *complex* one that has or is supposed to have a certain designatum in a given domain (say, in a given field of study and discourse) and *analyze* (*divide*) it into *smaller graphonyms,* which I regard, either *ad hoc* or universally, as *single* (*simple*) *designative units* having relevance to the given domain, although *not all of them are necessarily xenographs*, *my mental process of coordination* (*synthesis*) of the classes designated by those units into a single whole class designated by the complex xenograph is called *the immediate sense*, or *sense value*, *of that xenograph*, the understanding being that the immediate sense [value] is one of the xenovalues of the complex xenograph. The xenograph is said *to express* its immediate sense. Henceforth, the term "*immediate sense*" is abbreviated as "*sense*", unless stated otherwise.

2) The designatum of the xenograph is alternatively called *the subject class of its [immediate] sense*, while the classes coordinated by the sense are called *the object classes of the sense*. By a *simple* xenograph I understand any smallest constituent part of the complex xenograph, whose designatum is self-consistently fixed by the layout of the complex xenograph and is known. Accordingly, when I regard a xenograph as a *simple* one then its sense and its designatum, and also the only object class and the subject class of the sense are the same mental entity of mine. In the general case, however, the object classes of the complex xenograph are not of the same *kind*: some of them are *class-operata* (*class-arguments*), i.e. *operated classes,* while the others are *class-operators* (*class-functions*, *class-operations*), i.e. *operating classes*. Also, *the sense-operation of coordination* is, in the general case, a *multilevel* (*composite*) one: some class-operata are united by a certain class-operator into a class, called *the class-operand of the class-operator*, which is then used as a class-operatum of (i.e. is operated by) another class-operator, and so on.

3) In accordance with the above-said, *the [immediate] sense of a xenograph* is a *biune mental* (*psychical*) *substance* (*entity*) *of mine*, one *hypostasis* (*way of existence*) of which is *the sense-operation of coordination on its object classes*, whereas the other hypostasis is *the subject class of the sense*, i.e. *the designatum of the xenograph*. The subject class is *the ultimate result of the sense-operation* and therefore it is the *inseparable dominant aspect* (*part*) of that operation and hence of the entire sense. Therefore, if I perform the sense-operation *fluently*, I *involuntarily but consciously* identify the subject class with the sense.•

Df 3.3. 1) I can use a xenonograph xenonymously (not autonymously), i.e. as a euxenograph (not as a tychautograph), in either one of two different mental modes: *directly*, i.e. as a *direct euxenograph*, or *obliquely* (*indirectly*), i.e. as an *oblique* (*indirect*) *euxenograph*.

2) I say that a xenograph is a *direct kyrioxenograph* (as "Aristotle" or "the founder of logic" or "Aristotle is the founder of logic") if its designatum is a *singleton*

and if I use it as referent in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience the singleton*, being *the generative conceptual property of its only member*, *as the member itself in the hypostasis of my as if extramental* (*exopsychical*) *object* that is *denoted* by the kyrioxenograph. I do *so involuntarily but consciously* – just as I mentally experience the *sensation* (*percept*) *of any given onym* (*sensum*, *sensory object*) *as that onym*, particularly in the case when the onym is the pertinent kyrioautograph (see Df I.1.12(1) in the TTL). Thus, using the appropriate *monistic phraseology*, I may assert that the *denotatum* of the direct kyrioxenograph is just another *hypostasis* (*way of existence*) *of its singleton-designatum*. At the same time, using the appropriate *dualistic terminology*, I may assert that the direct kyrioxenograph is used as referent, along with its connotatum, i.e. connoted singleton-designatum, for mentioning (denoting, referring to, putting forward as its relatum) the only member of the singleton-designatum, while both the kyrioxenograph and its connotatun are used but not mentioned.

3) Likewise, I say that a xenograph is a *direct cenoxenograph* if its designatum is a *multipleton* and if I use it as referent in a certain a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience the multipleton*, being the generative conceptual property of its *common* (*general*, *certain*, *concrete but not concretized*) *member*, *as the common member itself in the hypostasis of my as if extramental* (*exopsychical*) *object* that is *denoted* by the cenoxenograph. I do *so involuntarily but consciously* – just as in the previous case. Thus, using the appropriate *monistic phraseology*, I may assert that the *common member of the multipleton-designatum*, being the *denotatum* of the direct cenoxenograph, is just another *hypostasis* (*way of existence*) *of its multipleton-designatum*. At the same time, using the appropriate *dualistic terminology*, I may assert that the direct cenoxenograph is used along with its connotatum, i.e. connoted multipleton-designatum, for mentioning (denoting, referring to, putting forward as its relatum) the common member of the multipleton-designatum, while both the cenoxenograph and its connotatun are used but not mentioned.

4) In spite of the fact that the above items 2 and 3 are similar, a kyrioxenograph and a cenoxenograph essentially differ from each other in the following respect. In contrast to the designatum of a kyrioxenograph, which is a singleton, the designatum of a cenoxenograph is a multipleton whose members are

prescinded from all differentia and are therefore indistinguishable until the cenoxenograph is attached with some epithets (qualifiers) denoting the differentia that specify or particularize (concretize) some of the members. Consequently, use of the cenoxenograph alone, – i.e. in the absence of any differentia, either sensational (sensory) or conceptual, – for referring to members of its designatum distributively is impossible. Therefore, the only way to turn a cenoxenograph into a direct cenoxenograph that can be used for referring to (mentioning) members of its designatum *as if distributively* is to condense mentally all members of the designatum (as the species man or *Homo sapiens*) into a *single common* (*general, abstract*) *member* (as a man) of the designatum, which is in fact a mental placeholder of concrete but not concretized members of the class. Both the designatum and its common member are *unique* but they are in fact *two different hypostases of a single mental entity*. Thus, in analogy with a kyrioxenograph, a cenoxenograph (as "a man" or "a human being" or "It is raining") is called a *direct cenoxenograph* if it *denotes*, i.e. puts forward as relatum, *the common member* (as a man or a human being or *the common state of affairs*: «It is raining») *of its designatum* (*multipleton*). Thus, the direct cenoxenograph is in fact a *proper xenograph of the common member of its designatum*.

5) A direct kyrioxenograph or a direct cenoxenograph is indiscriminately called a *direct xenograph*. When I use a xenograph as a *direct one* for mentioning its denotatum, I do this via its sense and hence via the subject class of the sense, being the designatum of the xenograph. In this case, the sense is used but not mentioned, – just as the xenograph itself. In order to express this relation between the direct xenograph and its sense, I shall say that the xenograph *connotes* its sense and that conversely the sense is *connoted by* or is *the sense-connotatum* (pl. "*connotata*"), i.e. *connotation sense-value*, or briefly *the connotatum*, *of the xenograph*. The designatum of the direct xenograph is at the same time the subject class of the sense of the xenograph, which either coincides with the sense, if the xenograph is [regarded as] as simple, or is the dominant (ultimate) inseparable aspect of the sense, if the xenograph is [regarded as] as complex. Since it is impossible to separate the sense-operation from the subject class being its ultimate result, I always switch, – voluntarily or not but consciously and sometimes repeatedly, – my attention from the entire sense to the sense-operation or to the subject class. Therefore, I apply the verb "*to connote*" also

for expressing the relation of a direct xenograph to both inseparable aspects of its sense-connotatum: the sense-operation and the subject class. I shall therefore say that the designatum of a direct xenograph is *the class-connotatum*, or *range*, *of the latter*. Hence, the range of the direct xenograph is a singleton, if it is a direct kyrioxenograph, or a multipleton, if it is a direct cenoxenograph.

6) In alternative terminology, I shall say that the [immediate] sense of a direct xenograph is *the immediate class-concept* (*concept of the class*), or *concept*, or *generative conceptual property*, *of the denotatum of the xenograph*, – no matter whether it is a direct kyrioxenograph or a direct cenoxenograph. More generally, I shall use the indefinite article "*a*" instead of the limiting qualifier "the immediate" in any of the above synonymous terms in order to indicate that the same object can be the denotatum of many different direct xenographs having different senses, i.e. different sense-operation, but the same designatum, i.e. the same subject class of a sense.

7) A *conception* (*thought*) of mine is said to be a *concept* if it is a *sense* (sense-connotatum, concept, *class-concept*) of at least one direct xenograph.

8) A xenograph (as "Aristotle", "the founder of logic", "Aristotle is the founder of logic", "a man", "a human being", or "It is raining") is called an *oblique xenograph* and also an *enneoxenograph,* i.e. a *sense-valued xenograph*, if it *denotes*, i.e. again puts forward as relatum, *its sense* (as \Aristotle/, \the founder of logic/, \Aristotle is the founder of logic/, \a man/, \a human being/, or \It is raining/, respectively). In this case, the sense is said to be *the denotatum* of the oblique xenograph. Thus, an oblique cenoxenograph is a *proper xenograph of its sense*.•

Cmt 8. In Dfs 3.1–3.3, and generally in what follows, any one of the nouns "xenograph", "kyrioxenograph", "denotatum", "designatum", "class", "sense", etc. or any one of the predicates "denotes", "is denoted by", "designates", "is designated by", "identifies", etc should, in accordance with Cnv I.1.2 of the TTL, be understood as one that is followed by the limiting qualifier "with respect to me" or, by extrapolation, by the qualifier "with respect to the interpreter".•

Cmt 3.1. Dfs 3.1–3.3 apply, *mutatis mutandis*, with the combining form "*nym*" in place of the suffixed combining form "*graph*" and also with "*phon*" in place of "*graph*" in all occurrences. Still, Psychologistics and particularly SQM (Special Quotation Method) are essentially graphic. Therefore, in application to

Psychologistics and to the SQ's (special quotations), the variants of the above definitions and of all other relevant statements with "nym" in place of "graph" have the same meaning, while their variants with "phon" in place of "graph" are irrelevant.•

Cmt 3.2. 1) I have adopted the verbs "to *denote*" and "to *express*" from Church [1956, pp. 4, 6, footnotes 7, 16], who in turn uses the verbs "to *denote*" and "to *name*" as two synonymous translations of the Frege [1892] verb "*bedeuten*" and who also uses the verb "to *express*" both as a translation of the Frege (*ibid.*) verb "*drückten aus*" and as a close synonym of the Mill [1843] verb "to *connote*" in his original meaning of the verb – the meaning that differs from all other meanings, which the verb has since acquired in common English usage.

2) In the *Frege-Church theory* (*FCT*) *of the meaning of proper names* (Frege [1892], Church [1956, pp. 3–9, 25–28]), *proper propositional* (*truth-functional*) *sentences* are regarded as *proper names* such that a true sentence *denotes* (*names*) *the truth-value truth* and a false sentence *denotes* (*names*) *the truth-value falsity* (*falsehood*). By contrast, in Psychologistics in general and in the TTL in particular, a DS (declarative sentence) is said to be:

a) *true* if it either is *tautologous*, i.e. *universally true* by virtue solely of the *abstract truth-functional validity of its syntactic form* or is *veracious*, i.e. *accidentally* (*circumstantially*) *true*, in the sense that it is a *ttatt-neutral* (*ttatt-indeterminate, neither tautologous nor antitautologous*) *DS*, which *denotes* (*names*, *conforms to*) a certain *nonlinguistic complex object*, called a *state of affairs* and also a *fact*, *case*¸ *event*, *phenomenon*, etc;

b) *antitrue* or *false* if it either *is* or *is equivalent to* the negation of a true DS;

c) *tat-neutral* (*tat-indeterminate*, *neither true nor antitrue*) if it is a *vravr-neutral* (*vravr-indererminate*, *neither veracious nor antiveracious*) *ttatt-neutral DS*.

In this case, *the negation of a ttatt-*, *vravr-*, or *tat-neutral* (*indeterminate*) *DS* is *another ttatt-*, *vravr-*, or *tat-neutral* (*indeterminate*) *DS* respectively. At the same time, in accordance with Df I.1.3) of the TTL, either one he synonymous *generic names* "*name*" and "*name sensu stricto*" is a synonym of the *generic name* "*linguistic form*", either of the allomorphs "onym" and "nym" is *by definition* a synonym of the description "*name sensu lato with respect to me*", where *generic name* "*name sensu* *lato*", i.e. "*name in a broad sense*", is by definition a synonym of the *generic name* "*sensible thing*". Hence, a name [sensu stricto] is an onym (nym), but not necessarily vice versa. Particularly, a *major form class* (*part of speech*) or *its equivalent* and a *sentence* are names [sensu stricto] and hence they are onyms (nyms). Thus, in spite of the fact that *denotative properties of DS's* in Psychologistics are supposed to be completely different from those in the FCT, I will use the verbs "to denote" and "to name" as synonyms, because both of them can predicate a name.

3) Unlike the above two verbs, the verbs "to express" and "to connote" as used in Psychologistics are not synonyms. The verb "expresses" expresses the general relation between a glossonym as referent and its sense as the pertinent relatum, – no matter whether the glossonym is used purposefully as a xenonym or whether it is just considered. By contrast, the verb "connotes" expresses the relation between the glossonym and its sense in case when the glossonym is used together with its sense as a direct xenonym for denoting (mentioning, referring to, putting forward) the pertinent denotatum of the glossonym. Thus, the sense that I attach to the verb "to express" is broader than that attached to it by Church, whereas my use of the verb "to connote" agrees with its use by Mill and Church.

4) I have adopted the term "*class-concept of*" from Russell [1903, §69]) and the term "*concept of*" from Church [1956, p. 6, footnote 17], and my use of the two terms as synonyms in accordance with Df 3.3(6) agrees well with use of the former by Russell and of the latter by Church. However, the sense that I have attached by Df 3.3(7) to the term "concept" without any reference to its possessive relationship to the denotatum of a direct xenonym differs from the sense of the term "concept of" of Df 3.3(6). In this use, "concept" does not have "class-concept" as its synonym.•

Cmt 3.3. There is a doctrine due to Mill [1843] (see also Church [1956, nn. 6, 14, and 16]), according to which, not only a proper name, but also a common, or general, name has the property *to denote*, with the difference that the former denotes only one object, whereas the latter denotes many *distinct* objects simultaneously. In accordance with Mill's doctrine, the improper name "a man", for instance, is said *to denote* Aristotle, Shakespeare, Einstein, a green-grocer, a pedestrian, a cyclist, etc, each taken individually and simultaneously. I have suggested a somewhat different interpretation of common names for the following two reasons. First, *I am unable to attend to an indefinite number of objects simultaneously*. Secondly, in order to mention, say, Aristotle as my conceptual object, I should mentally supplement the species $^{\backslash}$a man$^{/}$ (man, *Homo sapiens*), which is the immediate sense (generative conceptual property, concept, class-concept, range) and at the same time the designatum of the common name “a man”, by the pertinent *differentia* – the characteristic property (class), by which I distinguish Aristotle from all other men, i.e. from all individuals having the class man as their common conceptual property. Formally, such a differentia can be supplemented to a man by adding some words (qualifiers) to the noun “man”. For instance, either of the descriptions (descriptive names): “the man called “Aristotle”” and “the man who founded logic” is a denotative synonym of “Aristotle”. Usually, however, such differentiae are called from the memory of an interpreter of the noun “man” or of the common name “a man” spontaneously and are not anchored down to any additional words. It therefore seems that the common name “a man”, e.g., denotes Aristotle and also an indefinite number of other men. However, $^{\backslash}$Aristotle$^{/}$, i.e. the sense of the *biographical proper name* “Aristotle”, can be thought of as the singleton of Aristotle, i.e. as {Aristotle} in the conventional notation. At the same time, $^{\backslash}$a man$^{/}$, i.e. the sense of the common name “a man”, can be thought of as the species *Homo sapiens*. The two senses are completely different. Accordingly, “*Aristotle*” *denotes Aristotle*, whereas “*a man*” *denotes a man – a featherless biped* (Russell’s definition). Like the isotoken class (percept-class) of the name “a man”, a man being its denotatum is a mental entity that has an indefinite number of specifications and individuations. But any of the individuations is necessary accompanied by attaching a man with the pertinent differentiae, physical (perceptual) or psychical (mental). A *common* name can now be understood as being so, not because it is supposed to distributively denote many different objects simultaneously, but because it can be used together with some added words (qualifiers), particularly be included in the appropriate context, so as to form either a *specific* (*restricted*) *common name* (as “a male man” or “a female man”) or an *accidental* (*circumstantial*) *proper name* (as “the man as mentioned above” or “the man crossing the street”). By contrast, added words do not change the denotatum of a proper name. For instance, either of the appositions: “the philosopher Aristotle” and “Aristotle, the founder of nominalism” denote the same conceptual object [of mine] as the proper name “Aristotle’ alone. Incidentally, the personal names as “John” or “Mary” are *common* personal names, which are converted into accidental (circumstantial) proper names by supplementing them with the appropriate differentiae. Even an outstanding personal name, as “George Washington” or “Abraham Lincoln”, can be used, not as a biographical name, but as its homonym every time when there is a namesake of the known historical personality.

In connection with the procedure of specification of the common name “a man” by the proper name “Aristotle”, as described above, the following two remarks may be in order.

1) The fact that there is an indefinite article in English allows immediately distinguishing between man, i.e. the class (class-concept, concept, generative property) of a man, and a man being the common (general, abstract, conceptual) object of that class. That is to say, the presence of “a” in the name “a man” can be regarded as an indication that that name is used directly, whereas the absence of “a” in the name “man” can be regarded as an indication that that name is used obliquely. However, in the case of proper names such as “Aristotle”, the indefinite article is not available. Therefore, the occurrences of the name “Aristotle”, in which it is used obliquely for denoting the singleton of Aristotle, being the sense of the name, cannot be distinguished from the occurrences of the name “Aristotle”, in which it is used directly for denoting the single Aristotle, unless some appropriate punctuation is used in order to indicate the former occurrences. At the same time, there are native languages, e.g. Greek and Hebrew, that have no indefinite article, and there are also native languages, e.g. Latin and Russian, that have no articles at all. In any indefinite-article-free language, there are no xenographs whose oblique uses can visually be distinguished from their direct uses. Therefore, the only way to distinguish systematically, in any language, between direct and oblique uses of a xenograph is to enclose a token of the xenograph that is used obliquely between some special quotation marks as \ /, which are employed in the Psychologistics.

2) As contrasted to any communicative *exteroceptive symbols*, as *graphic symbols*, which are *immutable* and *static*, *brain symbols*, i.e. mental entities, are *mutable* and *dynamic*. Therefore, the instantaneous spontaneous mental passage, say, from the generative property \a man/ to the generative property \Aristotle/ can not be anchored down to any graphic symbols. Only the initial and terminal brain symbols can be indicated with the help of the names “a man” and “Aristotle”. This fact creates an impression that the former name denotes Aristotle and that at the same time it also

denotes an indefinite number of concrete but not concretized persons. Such an impression likely underlies Mill's doctrine of the meaning of common names.•

Cnv 3.1. Formally, the doctrine of Mill that a common, or general, name *denotes any concrete member of the designatum* (*class-connotatum*, *range*) *of the name distributively* can be adjusted to the interpretation of a common name suggested in Df 3.3(3) by assuming that in this context the qualifier "*any concrete*" to "member" is used as an abbreviation of the qualifier "*any concrete but not concretized*", which is then replaced with "*a certain*" or with "*the common*" or "*the general*", while the adverbial qualifier "*distributively*" is replaced with "*undistributively*".•

3.2. The meaning content of autographs and special quotations

Preliminary Remark 3.1. The *taxonomic division of autographs* can be summarized as follows. An autograph is either a tychautograph or a euautograph. A tychautograph, i.e. an accidental (circumstantial) autograph, is a xenograph that is used autonymously. A tychautograph is either a kyrioautograph, i.e. a proper autograph, or a cenautograph, i.e. a common autograph. A cenautograph is a homoloautograph (photoautograph) or iconoautograph (pictoautograph) or a phonoautograph. A euautograph is homoloautograph, but not necessarily vice versa. The theory of the meaning content of euxenographs, which has been proposed in the previous subsection, can be extended to autographs of the above kinds as stated in the following definition.•

Df 3.4: *The meaning content of an autograph.* 1) The sense of a kyrioautograph is its *visual percept* (*sensation*), whereas its denotatum is the kyrioautograph itself, by Df 1.3(2). Hence, every kyrioautograph is used directly.

2) The sense of a homoloautograph (and hence that of a euautograph) or the sense of an iconoautograph is *its isotoken-class*. The sense of a phonautograph is *its paratoken-class*. Hence, by Df 1.3(2), the sense of a cenautograph of any one of the above three kinds is its denotatum, so that every cenautograph is used obliquely.•

Cmt 3.4: *The meaning content of a special quotation* (*SQ*). In accordance with Df 1.3(2), an SQ is a euxenograph that has the same denotatum as its interior. However, the sense of an SQ is a coordination of the sense of the interior of the SQ and the sense of its exterior. This coordination has been described by giving the combined names "dicto-kyrioautograph", "dicto-homoloautograph", "dicto-iconoautograph", "dicto-phonoautograph", and "dicto-enneoxenograph" to a KAQ, HAQ, IAQ, PAQ, and EXQ, respectively. In this case, the exterior of an SQ of any kind is a *dictograph*, and hence a xenograph, whereas the interior of the SQ is an autograph if the SQ is a KAQ, HAQ, IAQ, or PAQ and a xenograph if the SQ is an EXQ. Accordingly, the prefix "dicto"- in the combined name of an SQ is descriptive of the sense of the exterior of an SQ, while the second part of the combined name is descriptive of the sense of the interior of the SQ.•

3.3. Further extensions of the theory of the meaning content of a graphonym

In the exclusion of its part relevant to the SQ's, which are *grapho*nyms in principle, the theory of the meaning content of graphonyms that have been proposed in the subsections 3.1 and 3.2 can be extended to nyms in general and to phononyms in particular. In order to do this, it is sufficient to recall how the terms "euxenograph" and "autograph" are incorporated into the pertinent general taxonomy of nyms – the taxonomy, which particularly applies to glossonyms (linguistic nyms).

An occurrence (token) of an isolated nym is called an *autonym* if it is used autonymously, i.e. is used for mentioning either *itself* or its pertinent *token-class* (recept, mental type, memory image), with the help of which an interpreter of the nym (as myself) recognizes the nym and any one of its tokens as *contypical* objects. The above two values of a nym are called *autonymous values*. A nym whose every token is used autonymously in a certain scope, particularly in a certain discourse as this essay, is called a *euautonym*, i.e. *genuine* (*essential*) *autonym*. A nym is called a *xenonym* if, in addition to its autonymous values, it has some other value or values that are called its *xenonymous values* (*xenovalues*) and if the nym *is just considered but is not used* for mentioning any of its values, either autonymous or xenonymous. It is understood that a nym that comprises one or more xenonyms and some autonyms is also a xenonym. A xenonym is called a *strict* one if it does not contain any euautonym and a *lax* one if it contains at least one strict xenonym and at least one euautonym. A [token of the] xenonym is called a *euxenonym*, i.e. *genuine* (*essential*) *xenonym*, if it is used xenonymously, i.e. is used for mentioning one of its xenonymous values, and it is called a *tychautonym*, i.e. *accidental* (*circumstantial*, *nonessential*) *autonym*, if it is used autonymously, i.e. if it is used for mentioning one of its autonymous values. Thus, there are two kinds of autonyms: *euautonyms* and *tychautonyms*. In contrast to a euautonym, whose every token in a given discourse is an autonym, a tychautonym is an autonymous token of a xenononym of the given discourse, while there is in the same discourse at least one xenonymous token of the xenonym, i.e. a euxenonym.

The above terminology applies with “graphonym”, abbreviated as “graph”, or “phononym”, abbreviated as “phon”, in place of “nym”. That is to say, particularly, a *graphic* autonym, euautonym, xenonym, euxenonym, or tychautonym is called an *autograph* (*autographonym*, *graphoautonym*), *euautograph* (*euautographonym*), *xenograph* (*xenographonym*, *graphoxenonym*), *euxenograph* (*euxenographonym*, *graphoeuxenonym*), or *tychautograph* (*tychautographonym*), respectively; and similarly with “phon” in place of “graph” in all occurrences.

A euautograph, e.g., is a euautonym, but not necessarily vice versa. For instance, the chessboard and complete set of chessmen, which are used in a chess game, and also all consecutive *admissible* positions of chessmen on the chessboard are euautonyms but not euautographs. Within the game, each White’s, or Black’s, piece except King and Queen has one homologous token, whereas each White’s, or Black’s, pawn has seven homologous tokens. Outside the game, each chessman has an indefinite number of tokens, both homologous and analogous. By contrast, figures of the chessboard and individual chessmen, and also figures of specific admissible positions of chessmen on the chessboard, which occur in a textbook on chess (as Chernev [1958]), are euautographs.

In accordance with the above remarks and in agreement with Cmt 3.1, all statements of the previous two subsections except those relevant to SQ’s apply, *mutatis mutandis*, with the combining form “*nym*” in place of the suffixed combining form “*graph*” and also with “*phon*” in place of “*graph*” in all occurrences.

The sense of a xenograph (or of a xenophon or generally of a xenonym), as defined in subsection 3.1 (particularly in Df 3.1), and the sense of an autograph (or of an autophon or generally of an autonym), as defined in subsection 3.2 (particularly in Df 3.4), are mental entities of two different levels of the semantico-syntactic net of the interpreter. This fact is made explicit by means of the following definition.

Df 3.5. 1) If a graphonym is a xenograph, i.e. either a logograph or a phonograph, then *its intended sense* as defined in subsection 3.1 (particularly in Df 3.1) is alternatively called the *xenosense both of the xenograph* and *of its isotoken-class* and also *that of the paratoken-class of the xenograph* if the latter is a phonograph; “*ξ-sense*” and “*ksi-sense*” are synonyms of “*autosense*”. At the same time, if a graphonym is either a xenograph or a euautograph then its isotoken-class and, if exists, its paratoken-class are called the *iso-autosense* and *para-autosense of the graphonym*, respectively; "*α-sense*" and "*alpha-sense*" are synonyms of "*autosense*".

2) The above item applies, *mutatis mutandis*, with "phon" in place of "graph". Namely, if a *phon*onym is a xeno*phon*, i.e. either a *[xeno]logophon* (sound having no graphic para-tokens) or a *[xeno]gaphophon* (sound having graphic para-tokens), then *its intended sense* is alternatively called the *xenosense* (*ξ-sense*, *ksi-sense*) *both of the xenophon* and *of its isotoken-class* and also *that of the paratoken-class of the xenophon* if the latter is a graphophon. At the same time, if a phononym is either a xenophon or a *euautophon* (phononym having no xenovalues and no graphic para-tokens) then its isotoken-class and, if exists, its paratoken-class are called the *iso-autosense* (*-α-sense, -alpha-sense*) and *para-autosense of the phononym*, respectively.

3) In general, a *nym* (*onym*, *sensible thing*) *of mine* is either a *xenonym* or an *autonym* (*tychautonym* or *euautonym*). Depending on the mental attitude of an interpreter towards an *autonym*, the *autosense* (*α-sense, alpha-sense*) of the latter is either its isotoken-class or its paratoken-class provided of course that the autonym has paratokens. The *sense of a euxenonym*, i.e. of a certain isotoken of a *xenonym, which is used xenonymously*, is called its *xenosense* (*ξ-sense*, *ksi-sense*), in accordance with the previous two items. •

4. The principle of alternation of opposites

No matter what a *brain symbol*, i.e. *mental state*, of a man is from the standpoint of biophysical and biochemical processes in the aggregate of perikaryons constituting his cerebral cortex, it is a *dynamic and mutable* (*varying*) *entity*, – in contrast, e.g., to a graphic (written) symbol (ideograph), which is *static and immutable* (*invariable*, *unchangeable*), or in contrast to a vocal (spoken) symbol (ideophon), which is just *transient but immutable* as well. Therefore, any *functional*, i.e. *single-valued*, *correspondence* (*mapping*) from brain symbols to graphic ones (e.g.) is, in the general case, *many-to-one*, i.e. *surjective, and not bijective*. Consequently, the inverse correspondence is *many-valued*, i.e. *not functional*. For instance, an interpreter of any given ideograph (graphic symbol), – a logograph or a phonohraph (a word or word group), – in a given occurrence can, depending on his mental attitude towards the ideograph, use it either in any one of his various *autonymous mental modes* or in any one of his various *xenonymous mental modes,* and he can repeatedly change his mental attitude.

In accordance with the above-said, in many cases, which routinely occur in Psychologistics in general and in the TTL in particularly, I use an isotoken of a xenograph in two opposite mental modes as if simultaneously but actually equivocally and intermittently by repeatedly switching from one mental attitude towards the xenograph to the other – just as it happens in my perceiving any one of Escher's *Convex and Concave* pictures, e.g. "*Cube with Magic Ribbons*" (see, for instance, Ernst [1985, p 85f]). I do this involuntary but consciously, so that such a biune hypostasis of the xenograph turns out, not only harmless, but most often useful. This mental phenomenon will be called "*alternation of opposites*", whereas the concept of the phenomenon herewith stated will be called "*the principle of alternation of opposites*". For instance, by Df 3.2, the sense of a euxenograph is *a biune mental entity of mine*, one *hypostasis* of which is *the operation of coordination on its object classes*, whereas the other hypostasis is *the subject class of the sense*, i.e. *the designatum of the xenograph,* being the ultimate result of the operation. In experiencing the sense, I switch my attention, sometimes repeatedly, from one aspect of the sense to the other. Likewise, I may and I often do repeatedly switch from mentally experiencing a given token of a xenograph as a *tychautoghraph* to mentally experiencing it as a *euxenograph*. This alternation of opposites will, more specifically, be called "*tychautograph-euxenograph alternation*" or briefly "*tychauto-euxeno-alternation*" ("*TAEXA*"). The TAEXA is *indispensable* in the TTL in stating *definitions* or executing *valid panlogographic relations of the organon* $\mathbf{A}_1$ as tychautographic definitions or valid tychautographic relations and at the same time as schemata of an infinite number of definitions or of an infinite number of valid euautographic relations of the organon A_1 respectively. Another specific case of TAEXA and a certain associated instance of alternation of opposites of a different kind are explicated in the next section. The specific manifestation of alternation of opposites by Escher will be called "*convex-concave alternation*" ("*CCA*"). This name can also be used as an allegorical name of the entire mental phenomenon of alternation of opposites.

Owing to spontaneous (involuntary) character of the mental phenomenon of alternation of opposites, it is often impossible to restrict it by any formal definitions. For instance, all logographs occurring in Psychologistics are homolographs, whereas all phonographs (grammographs, verbal expressions) occurring in Psychologistics are endoiconographs (endopictographs). Therefore, the interior of an IAQ's is either a phononograph or a combination (most often a juxtaposition) of phonographs and pasigraphs (i.e. logographs or euautographs). If the interior of an IAQ is a phonograph then it is psychologically impossible to use the IAQ for mentioning only the isotoken-class of its interior and not to mention its paratoken-class, which should, by Df 1.3(d), be denoted by the PAQ of an isotoken of that same interior. In order to avoid this artificial conflict, I shall adopt the following convention, which is, in fact, a statement of the pertinent instance of the principle of alternation of opposites.

Cnv 4.1. If the interior of an IAQ is a phonograph then no harm is done if the IAQ is used for intermittently mentioning both the isotoken-class and paratoken-class of the interior.•

It will be demonstrated in section 6 that FQ's (Fregean quotation) are, as a rule, used in the literature, not for mentioning their interiors, as they should be in accordance with any of their conventional definitions, but rather they are used as HAQ's, IAQ's, or PAQ's as defined in Df 1.3. This is another unavoidable manifestation of the phenomenon of alternation of opposites.

In Psychologistics, I do not follow Frege and his followers either in admitting KAQ's as the only kind of SAQ's or in *obstinately attempting to indicate autonymy with the help of the appropriate special quotation marks in all cases* simply because such an attempt is *impracticable* for several reasons, one of which is the phenomenon of TAEXA. Owing to this phenomenon, in some cases, confusion between autonymous and xenonymous uses of xenographs is harmless, while in many other cases such confusion is productive and indispensable. Therefore, I resort to the SAQM only where confusion between autonymous and xenonymous uses of xenographs might otherwise be harmful. For instance, I do not, as a rule, quote assemblages, particularly formulas and operators, of A_1, because these are never used xenonymously.

Various instances of alternation of opposites are extensively employed in the TTL and some of them are explicitly indicated as being so. Accordingly, the entire phenomenon and its specific kinds are illustrated in the TTL sufficiently for recognition of any instance of it even if it is just used and not mentioned as being an instance of the phenomenon.

5. Wordy methods of making definition-statements

In Psychologistics and generally in scientific literature, an informal verbal *definition-statement* is typically a simple or compound affirmative declarative sentence that has a *defining predicate* (*predicate-definer*, *definer-predicate*) such as:

i) "is designated by", "is denoted by", "is called by", "is called", "is termed", etc

or as:

ii) "is", "is said to be", and "is mentioned as",

the understanding being that the copula "is" can, when appropriate or desired, be replaced with "are" or "will be". Any of the above mentioned predicates can be used alone or together with one or more appropriate adverbial modifiers such as:

iii) "properly", "commonly", "collectively", "distributively", "indiscriminately", "ad hoc", etc

that are put at the appropriate place in the pertinent definition-statement, mainly between the link verb "is" or "are" and the notional verb or after the auxiliary verb "will". Such a definition has either a sole *xenographic* definiendum or several synonymous *xenographic* definienda, therefore it will be called a *xenographic definition* (*XGD*). An XGD may have either a sole euautographic or xenographic definiens or many such non-synonymous definientia. Depending on its definiens or definientia, and also depending on the additional modifiers, an XGD is either a *nominal definition* or a *synonymic definition*, which can particularly be an *abbreviative* one. Particularly, an XGD is a synonymic one if its predicate has either one of the adverbial modifiers "*synonymously*" and "*alternatively*" or if the XGD ends with the phrase "*and vice versa*". It is understood that all syntactic (graphic) definientia of an XGD precede its predicate, whereas all syntactic definienda of the XGD follow the predicate.

Any predicate on the list i is supposed to refer either to the only xenographic definiendum or, distributively, to every one of the xenographic definienda, and not to the object to be denoted by a definiendum. Therefore, such a predicate is assumed to be followed, immediately or not, by a linguistic construction of one of the following two kinds:

a) the HAQ or IAQ of a sole tychautographic, – i.e. xenographic but used autonymously, and hence ostensive, – definiendum,

b) a list of the HAQ's or IAQ's of all pertinent synonymous ostensive tychautographic definienda,

the understanding being that the above list can be preceded by a function phrase such as "any of the xenographs:" or "any of the synonyms:". By contrast, any predicate on the list ii is supposed to refer to the object which is determined by the definiens or definientia and which is to be denoted either by the only xenographic definiendum or by any of the synonymous xenographic definienda. Therefore, such a predicate is assumed to be followed, immediately or not, by a linguistic construction of one of the following two kinds:

a′) a sole euxenographic definiendum,

b′) a list of all pertinent synonymous euxenographic definienda.

The defining predicate is also supposed to be preceded by a linguistic construction of one of the following two kinds:

a″) a sole definiens, which can be either an ostensive unquoted euautograph or the HAQ or IAQ of a tychautograph, or else an unquoted euxenograph,

b″) a list of non-synonymous definientia of any of the above three kinds, all items of which are mentioned either distributively or collectively,

the understanding being that the above list can be preceded either by a function phrase such as "any one of the xenographs:" or "any one of the synonyms:", if the list comprises euautographs of special autographic quotations, or by a function phrase such as "any one of the entities:" or "any one of the objects:", if the list comprises euxenographs.

The first two predicates on the list i can be applied to any definiendum or definienda, logographic or verbal (wordy, grammographic, phonographic), whereas any one of the remaining predicates on that list is a specification of the predicate "is denoted by" in the case where the only or each definiendum is verbal. The predicates "designates" and "denotes" can be used instead of "is designated by" and "is denoted by" respectively, while the definiendum construction and the definiens construction are exchanged.

In accordance with the above-said, the predicate “*is denoted by*”, e.g., signifies that it is followed by a xenograph, which is used autonymously, i.e. which is a tychautograph. Therefore, the latter will always be enclosed either in single or in double light-faced quotation marks. Single quotation marks indicate that the definition of the xenograph (tychautograph) quoted is effective only for its *homolographic* (*photographic*) tokens and if hence the xenograph is a logograph (as a formula of $\mathbf{A}_1$). Double quotation marks indicate that the definition is effective for *any iconographic* (i.e. *pictographic*) tokens of the xenograph independent of its font and also, perhaps, for any spoken paratokens of the tychautograph if it is a phonograph (grammograph, verbal expression). At the same time, if the definiendum is a phonograph then any one of the predicates “*is called by*”, “*is called*”, “*is termed*”, and “*is named*” can be used as a synonym of the predicate “*is verbally denoted by*”. Accordingly, any of these predicates should be followed by the IAQ of the phonograph, to which the predicate refers.

In Psychologistics, the predicate “is called” is one that is used most widely in informally stating XGD’s. To be specific, if that predicate is employed in a *simple* affirmative declarative sentence then the subject preceding the predicate is the definiens, and the predicative (direct object), following the predicate, is the definiendum. If, for instance, the definiens is an unquoted xenograph, i.e. is a euxenograph, then the predicate “is called” assigns the xenographic definiendum to the object denoted by the euxenograph as another name. If the definiens is an unquoted euautograph then the predicate “is called” assigns the xenographic definiendum to the homolographic token-class of the euautograph as its verbal name In either case, the definiendum is mentioned, and therefore it should be enclosed in iconographic quotation marks.

In practice, however, use of special autographic marks for introducing two or more synonymous verbal xenographic definienda is often inconvenient, especially in the case when the definienda are descriptions that have the same generic name (headword) and that differ only in synonymous qualifiers (epithets). Use of special autographic quotation marks can be avoided by using any predicate of the list ii instead of “is called”. Alternatively, I shall, most often, just omit quotation marks after the defining predicate “is called” or “are called”, thus employing the pertinent double (biune) sense of the verb “to call”. One aspect of the double sense of the predicate "is called" (e.g.) is that indicated in two previous paragraphs. The other aspect is that the predicate "is called" is used as a synonym of any predicate of the list ii. Use of the predicate "is called" or "are called" in the double sense can be explicated and justified as follows.

The known allegoric idiomatic proverb "*Call a spade a spade*", meaning «*Call things by their right names*», is based on employing the above two senses of the verb "to call" simultaneously. In order to demonstrate this feature of the proverb, I shall, first of all, distinguish between a *written* (*graphic*) *idiom* and a *spoken* (*oral*, *phonic*) *idiom* terminologically. The former will *be called* "an *idiograph*", whereas the latter, being phonic paratoken of the former, will be called "an *idiophon*" (or "an *idiophone*"); "id*i*ograph" and "id*i*ophon" should not be confused with "id*e*ograph" and "id*e*ophon" (or "id*e*ophone") respectively. An idiograph may have some special features as compared to an idiophon, because writing has certain devices that are not available in speech. For instance, in writing, the proverb in question can rigorously be represented as the *allegoric idiograph*: "*Call a spade* "*a spade*"". In this idiograph, the name "a spade" in the first occurrence is used *xenonymously* for mentioning the abstract object that it denotes, while the same name in the second occurrence is used *autonymously* for mentioning its most inclusive token-class. Consequently, the predicate "Call" applies to the two occurrences (isotokens) of the name "a spade" in two different senses. To be specific, the idiograph can be periphrased thus: "*Mention* a spade *by calling it by its name* "a spade"" or, concurrently, thus: "*Mention* a spade *by using its name* "a spade""; either of the two synonymous sentences should, as before, be understood allegorically, and is hence an idiograph as well.

Thus, in writing, it is possible to indicate explicitly the difference between the xenonymous token and the autonymous token of the name "a spade" by enclosing the latter token between light-faced double quotation marks. By contrast, oral tokens of the unambiguous sentence "Call a spade "a spade"" are not distinguishable from oral tokens of the ambiguous sentence "Call a spade a spade", in which the first token of the name "a spade" is used xenonymously and the second one autonymously.

The above analysis of the sense of the idiograph "Call a spade "a spade"" shows that the verb "call" can be used in two mental modes towards the *name-operatum* (*operatum-name*) following it. In one mode "call" applies to *the denotatum of its name-operatum*, whereas in the other mode "call" applies immediately to its name-operatum. I shall say that "call" is used *slidingly* or *non-contactually* or *transitively* in the former mode and *contactually* or *non-slidingly* or *intransitively* in the latter mode. Consequently, the modes themselves will be denoted by (*contactually* called) "*sliding*" or "*non-contactual*" or "*transitive*" and "*contactual*" or "*non-sliding*" or "*intransitive*" in that order.

Thus, the immediate sense of the sentence "Call a spade a spade" is elaborated by using the first token of the name "a spade" xenonymously, and the second one autonymously, and also by applying the predicate [operator] "call" *in the same occurrence* slidingly to the first token of "a spade", and contactually to the second token of the same name. In this case, the immediate sense of the above idiograph, i.e. actually of the sentence "Call a spade "a spade"", is used as an allegoric concept (class-concept) of the denotatum of the sentence.

To recapitulate the above remarks regarding use of the predicates "is called" and "are called" as defining predicates, I shall adopt the following convention.•

Cnv 5.1. The predicate "*is called*" will, as a rule, be used in XGD's in two different hypostases (mental nodes) simultaneously:

a) *contactually* (*non-slidingly*, *intransitively*) and hence as a synonym of either predicates "*is called by*" or "*is termed*";

b) *slidingly* (*non-contactually*, *transitively*) and hence as a synonym of either predicate "*is said to be*" or "*is mentioned as*".

The only definiendum or each definiendum following the predicate "is called" *is quoted* if and only if the predicate is used contactually, i.e. *in the contactual mental mode*. The only definiendum or each definiendum following the predicate "is called" *is not quoted* if and only if the latter is used in *the biune mental mode of alternating between the contactual one and the sliding one*. This mode will briefly be called *the biune alternating predicate mental mode*. In this case, the pair of double quotation marks enclosing the definiendum following the predicate "is called" serves as a *dictograph* (*graphic index*) of the contactual mental mode of using the predicate, whereas the absence of double quotation marks about the definiendum serves as a dictograph of the alternating predicate mental mode. It is understood that the biune alternating predicate mental mode is an instance of alternation of opposites, which is associated with using the pertinent predicative in the TAEXA mental mode but which is not the TAEXA mental mode itself. •

Cmt 5.1. In connection with the above analysis of the allegoric idiograph "*Call a spade* "*a spade*"", it is worthy to recall *the prototype principle* – a general philosophical principle, according to which a *concrete, i.e. most specific*, *instance of a class can represent the entire class* (cf. Hofstadter [1979, p. 352]). Use of allegories evidences that there is another, more refined, philosophical principle – a principle according to which a *concrete instance of a class can represent the whole of another class by an allegoric analogy*. Consequently, this principle can be called *the prototype principle by allegory* or *the allegoric prototype principle*.•

6. Special autographic quotations versus appositional and fontal tychautographic names

In the absence of the SAQM (SAQ Method), a graphonym isolated from any symbolic context can unambiguously be regarded as an autograph if and only if it is a euautograph. Indeed, in these circumstances, the fact that I consider a certain isolated token of a xenograph autonymously, thus mentally turning it into a tychautograph, *is not expressible* until I indicate that fact symbolically by inserting that token into the appropriate assertive context as a sentence or phrase. Here follow some examples of such sentences:

a) man is a monosyllabic word.

b) can is a defective verb.

c) The plural number form of man is men.

Although these sentences are awkward and impractical, it is clear that the words "man" and "can" are subjects of sentences a) and b), "man" is an object and "men" is predicative of sentence c), while the sentences serve as contexts that identify the hypostases of these words as *tychautographs* (*citation forms*, *quotation nouns*). Still, the sentences do not tell that tokens of these tychautographs, which occur elsewhere, should necessarily be tychautographs as well. On the contrary, tokens of these tychautographs will most likely be used xenonymously, unless it is indicated in each case individually that they are used autonymously. That is to say, a specific occurrence of a word in a sentence may determine the hypostasis that the word has in that occurrence, but this hypostasis is not necessarily preserved for tokens of the word occurring elsewhere.

Sentences a)–c) can be paraphrased thus:

a′) The noun man is a monosyllabic word.

b′) The word can is a defective verb.

c′) The plural number form of the noun man is the word men.

In contrast to sentences a)–c), which utilize the tychautographs “man”, “can”, and “men”, sentences a′)–c′) utilize the *appositions* “the noun man”, “the word can”, and “the word men” that serve as *proper names*, i.e. as *synonyms*, of the above tychautographs. Tokens of these *appositional proper names* can be used for mentioning the pertinent tychautographs independently of any additional specific contexts. For convenience in cross reference, the method of naming a tychautonym by adding to it an appropriate *prepositive* word or phrase will be called the *appositional autographic* (or *autonymous*) *nomenclatural method* (*AANM*), whereas a name of a tychautograph formed by this method will be called an *appositional tychautographic name* (*ATAN*).

Another, more popular, method of naming, – or, in fact, *self-naming*, – tychautographs, is to set an autograph in a font different from the current one, usually *in italic*. This method will be called the *fontal autographic* (or *autonymous*) *nomenclatural method* (*FANM*), whereas a *self-name* of a tychautograph formed by this method will be called a *fontal tychautograph* (*FTA*). Thus, for instance, *man* is a proper name, i.e. a synonym, both of “man” and of “the word man”, or, more precisely, of the word “man” because “the word” is a function expression, – just as the double quotation marks; all above three names are independent of any context. Still, the FANM has the following grave shortcomings.

a) The FANM is inapplicable in principle in case when special fonts are used as attributes of terminology or notation.

b) The FANM cannot be applied to a fontal tychautograph repeatedly. That is to say, *a name of a name of a name* cannot be formed by this method.

c) An FTA can be confused with an expression that is italicized just for giving emphasis to it.

Here follow three important cases when italicizing of graphonyms, and actually the FANM in general, cannot be used for unambiguously indicating their autonymous occurrences.

i) Logical and mathematical formulas are conventionally set in some special fonts including italic. In this case, the method of equivocally using formulas both autonymously and xenonymously is the one favored by almost all mathematicians, including writers on set theory (cf. Fraenkel et al [1973, p. 20f]), and also by almost all science writers.

ii) In any one of the *biological taxonomies of bionts* (*BTB's*), particularly in the old *two-kingdom Linnaeus one* and the modern *five-kingdom Linnaeus-Whittaker-one* (see, e.g., Margulis and Schwartz [1987] or Campbell [1990, Unit Five, pp. 505–674, 518–520ff]), *genera* are denoted by *capitalized italicized Latin names*, – e.g. *Populus* (poplar), *Canidae* (dogs), *Felidae* (cats), etc, – whereas *species* are denoted by *italicized two-word Latin names*, – e.g. '*Felis domesticus*' (domestic cat), '*Felis leo*' (lion), '*Felis tigris*' (tiger), etc or '*Homo sapiens*' (man), – which are called *Linnaean* (or *Linnean*) *binomial*s (or *binomina*). The first, *capitalized*, word of a Linnaean binomial denotes the *genus* which the species denoted by the binomial belongs to. The second, *uncapitalized*, word of the binomial is a specific *epithet* (*qualifier*) to the genus name, which denotes the *differentia*, i.e. the *additional conceptual property*, by which a *biont* (*individual living organism*) of the species is distinguished from a biont of any other species of the same genus. The biological categories broader than genera are denoted by capitalized Latin names in a current Roman (upright) font.

iii) The graphonyms of any one of the object logistic systems of this treatise are set in its own distinctive fonts that cannot be changed, i.e. they are homolographs (photographs) of the treatise. In this case, all graphonyms of A_1 are euautographs so that there is no need to indicate that they are used autonymously, – in accordance with Cnv 1.2. Except euautographs that are imported from A_1 into $\boldsymbol{\mathsf{A}}_1$, I_1, and A_1, all other intrinsic (subject) pasigraphs of each of the latter three logistic systems are homolographs (photographs). Also, some subject names of the ML (as 'A_1', '$\boldsymbol{\mathsf{A}}_1$', '$\mathsf{A}_1$', '$\mathsf{I}_1$, '$\mathit{A}_1$') are homolographs. When it is necessary to indicate that an idiohomoloxenograph is used autonymously, I enclose it between light-faced single quotation marks. When it is necessary to indicate that a subject iconoxenograph (pictoxenograph) of the ML is used autonymously, I enclose it between light-faced double quotation marks. At the same time, I shall italicize the interiors of some SAQ's and also some unquoted expressions of the ML in order to give emphasis to them especially in case when they are new terms.

7. Special autographic quotation system versus Fregean quotation system

FQM is described in detail, e.g., by Quine [1951, pp. 23–26] and Suppes [1957, chap. 6]. Particularly, Quine who is the most consistent champion and developer (cf. his quasi-quotations, *ibidem*, pp. 33–37) of FQM, concisely defines an FQ thus (*ibidem*, p. 23):

> «The name of a name or other expression is commonly formed by putting the named expression in single quotation marks; the whole, called a *quotation*, denotes its interior.»

In contrast to its practical realizations, FQM as a theoretical (hypothetical) system has the following two aspects. First, Frege and his followers do not distinguish among various autonymous modes of using xenographs and admit only proper autograph (graphic autonyms). That is to say, they admit only the autonyms in the literal etymological sense of the word "autonym" as meaning a *name of itself*, although many of them, including Frege and Quine, do not employ the name "autonym" at all. The above Quine's definition defines an FQ in this very sense. An attempt to eliminate *every* autonymity by using quotations thus defined amounts to prohibition of the existence of autographs other than proper ones. In practice, however, a xenonograph that Quine calls a quotation and that I call a Fregean quotation (FQ) *does not necessarily denote its interior* (to be illustrated below in this section). In other words, an FQ is most often used not in accordance with any formal definition of it (as Quine's one), but rather it is used informally and intuitively in a completely different sense. Second, *using FQM* does not means using FQ's for indicating autonymy in some cases and not using FQ's in some other cases, but rather it means *making an obstinate effort to use FQ's systematically*, i.e. *in all cases of autonymous use of graphonyms* , which proves to be impracticable.

The word "*autonym*" is due to Carnap [1937, pp. 153–160]. However, the meaning that I attach to the word in Psychologistics essentially differs from its acceptation, so that my term "autonym" is a *homograph* (*graphic homonym*) rather than a *synograph* (*graphic synonym*) of Carnap's one. The names "*citation form*" and "*quotation noun*", as defined, e.g., in WTNID, are two close synonyms of Carnap's term "autonym". Also, the word "*hypostasis*" is, according to WTNID, sometimes used in the special literature in the same sense, but I shall use that word for denoting a *mental status*, or *mental modification*, *of the way of existence of a given entity* (particularly, of a given graphonym).

The method of *equivocally* using graphonyms both autonymously and xenonymously is the one favored by almost all mathematicians, including writers on set theory (cf. Fraenkel et al [1973, p. 20f]), and by almost all science writers (as physicists, biologists, etc). At the same time, writers on logic and metamathematics, all of which are supposedly aware of FQM, can be divided into two groups. Writers of one group do not attempt to employ FQM, although some of them, e.g. Church [1956] and Suppes [1957], occasionally use FQ's. Particularly, Church writes (*ibidem*, p. 62):

> «As the reader has long since observed, Frege's systematic use of quotation marks is not adopted in this book.[136] But we employ quotation marks or other devices from time to time, especially in cases in which there might otherwise be real doubt of the meaning.»

In his n. 136, which is mentioned in the above citation, Church severely criticizes FQ's and FQM. Some points of Church's criticism of FQ's are debatable, so that they can be criticized in turn. However, Church's skepticism regarding the possibility of using FQ's *systematically* and *formally* is certainly justified. Particularly, in support to Church's skepticism, it can be recalled once again that FQ's are not as a rule used in accordance with any of their *formal* definitions (as Quine's one), but rather they are most often used *informally* and *intuitively* in a completely different sense (see below for more details). Writers of the other group follow Frege in attempting to systematically use quotation marks for eliminating or indicating autonymy. Some of these writers follow Frege in using single quotation marks for this purpose in order to reserve double quotation marks for their regular use as conventional punctuation marks, whereas the others use double quotation marks equivocally for both purposes.

Analysis of various ways of using FQ's by different writers, including Quine [1951], shows that a quotation, being supposedly a Fregean one, should, *depending on the context* in which it occurs, be interpreted, not as the quotation of a proper autograph, but as the quotation of a common autograph, i.e. as HAQ, IAQ, or PAQ, defined in Df 1.3 (cf. Church [1956, p. 71, n. 156]). In order to illustrate this fact conveniently, I shall, in making citations from Quine's book, use slant light-faced single quotation marks, ′ ′, in place of curly light-faced single quotation marks, ‘ ’, used by Quine. Consequently, all kinds of SQ marks occurring below are used in accordance with Dfs 1.3 and 1.4.

According to Quine (*ibid.*, p. 24),

ʻBostonʼ designates Boston. (7.1)

But the *interior* of any one of the following FQ's:

ʻBostonʼ, ʻ*Boston*ʼ, ʻ**Boston**ʼ, ʻ*Boston*ʼ, ʻ**Boston**ʼ, (7.2)

etc is *the same name of the same city* rather than to be just the specific token of the name, which is therein depicted between the pertinent single quotation marks. Therefore, besides (7.1), I may also assert that ʻ*Boston*ʼ designates Boston, ʻ**Boston**ʼ designates Boston, etc, and also that ʻBostonʼ designates *Boston*, ʻ*Boston*ʼ designates **Boston**, etc. Moreover, I may state this:

"Boston" designates Boston (7.3)

or, equivalently, this:

Any spoken phonic paratoken of ʻBostonʼ designates Boston, (7.3a)

and similarly with any other of the quotations (7.2) in place of ʻʻBostonʼʼ and with the interior of any one of those quotations in place of ʻBostonʼ. Likewise, in this case, I may also assert that ʻBʼ is *the second capital letter of the English alphabet*, and similarly with ʻʻ*B*ʼʼ, ʻʻ**B**ʼʼ, ʻʻ*B*ʼʼ, ʻʻ**B**ʼʼ, etc in place of ʻʻBʼʼ. Therefore, ʻʻBʼʼ, ʻʻ*B*ʼʼ, ʻʻ**B**ʼʼ, ʻʻ*B*ʼʼ, ʻʻ**B**ʼʼ, etc are *different names of*, whereas ʻBʼ, ʻ*B*ʼ, ʻ**B**ʼ, ʻ*B*ʼ, or ʻ**B**', etc *is*, *the same second capital letter of the English alphabet*. Still, strictly speaking, *this letter is not written or printed anywhere because it is* a *token-class* (*recept*). A class in general and a token-class in particular is a *mental entity* (*mental state, brain symbol*) of an interpreter (as me) of the name of the class, and *not an exteroceptive thing*. Therefore a class cannot be exposed on a material surface but rather it can be represented by any of its graphic or phonic names. In the framework of the SAQM, "Boston", e.g., denotes the percept-class, or token-class, of its interior of this quotation – the class that particularly contains all different names of Boston which are mentioned on the list (7.2).

Incidentally, the fact that the meaning of an FQ may depend on the context, in which it occurs, can be illustrated by the Quine statement (*ibid.*):

ʻBostonʼ has six letters. (7.4)

In contrast to statement (7.1), which implies (7.3) and (7.3a), statement (7.4) applies only to the *graphonym* ʻBostonʼ and to any of its iconographic tokens; i.e. it applies to “Boston”, but it does not apply to any of *its phonic paratokens*. Consequently, the statement that ʺBostonʺ has six letters is false (antitrue).

Quine suggested the phraseology, according to which an autonymous token of a graphonym is *mentioned*, whereas a xenonymous token of the graphonym *is used*, so that the names “*use*” and “*mention*” become *antonyms* (cf, e.g. the heading “§ 4 Use versus Mention” *ibidem*). Moreover, in his «*Mathematical logic*» cited herewith, Quine neither uses nor mentions the word “autonym”, although he was of course aware of its existence and its meaning. For instance, he describes the essence of FQM thus (*ibid*. p. 26):

> «Frege seems to have been the first logician to recognize the importance of scrupulous use of quotation marks for avoidance confusion between use and mention of expressions…»

(cf. the first sentence of Cmt 1.3(3), in which I employ the noun “autograph”). Quine’s “use vs. mention”-phraseology is used by many modern logicians, – Church [1956, p. 61] and Suppes [1957, chap.6] among them. However, the use of the words “use” and “mention” as antonyms is *idiomatic* and confusing because, for instance, a tychautographic token of a xenograph *is* [*mentally*] *used for mentioning* (*referring to*) a certain one of its autonymous values, whereas a euxenographic token of the xenograph *is used for mentioning* (*referring to*) a certain xenonymous value of the xenograph. That is to say, in either case, an isotoken of the xenograph *is used for mentioning its relatum*, while the relatum depends on the metal attitude taken in regard to the isotoken by its interpreter (as me). In fact, either of the synonymous verb-group predicates “*is used for mentioning*” and “*is used for referring to*” is an *antonym* of the predicate “*is considered*” and not of the predicate “*is mentioned*”. For instance, an autograph, euautograph, or euxenograph as such *is used*, whereas a xenograph as such *is considered*. The latter *can be used* either *autonymously,* i.e. as a *tychautograph*, or *xenonymously*, i.e. as a *euxenograph for mentioning the corresponding denotatum*. Besides the verb-group predicate “is used for mentioning”, I shall also use the convenient compound word-group predicate “*is used but not mentioned*” should be understood as an abbreviation of the compound predicate: “*is used for mentioning another but is not mentioned itself*”. But I shall not use either the words “use” and “mention” or their derivatives and kindred words as absolute antonyms, i.e. in the sense of either of the expressions “is used and hence is not mentioned” and “is mentioned and hence is not used”.

To recapitulate the above discussion, the SAQM as compared to FQM has the following general properties.

1) The SAQM utilizes SAQ's of five kinds, which are used in practice in accordance with their formal definitions comprised in Df 1.3. By contrast, FQM utilizes quotations of only one kind, whose uses in practice most often disagree with their formal definition.

2) Just as FQM, the SAQM is an *ad hoc*, i.e. *epistemologically relativistic*, *method* and that hence it *cannot in principle be used systematically*, i.e. *in all cases* where a xenograph is used autonymously. Depending on the type a xenograph that is used autonymously, I enclose it between the respective light-faced quotation marks if there might otherwise be doubt regarding the exact mental mode, in which the xenograph is used, or if some grammatical incongruity might otherwise occur.

3) Graphonyms of A_1 are called "*euautographs*" because they are always used autonymously and also because they have only homolographic (congruent or proportional) isotokens and no phonic paratokens, – like a chessboard and chessmen or, more precisely, like the figures of admissible positions of chessmen on the chessboard, which occur in a textbook on chess. A euautograph cannot either have or assume any xenonymous values and hence it cannot be mentally turned into a constant or variable. Therefore, a euautograph, which is isolated from any xenographic surrounding, is *never enclosed in any SAQ marks*. As a rule, SAQ marks *apply only to xenographs,* the understanding being that a xenograph may contain some euautographs as its constituent parts. At the same time, no harm is done if a euautograph being a constituent part of a xenograph is enclosed between HAQ marks.

8. "Synonym", "parasynonym", and some relevant terms. Metaphonographic quotations

Df 8.1. 1) A *synxenonym* is one of two or more xenonyms, which denote or connote the same class of entities in *the scope of their definition as synxenonyms*, which is called *the scope of synonymity of the xenonyms*; the definition is called a *synonymic definition*. The scope of a synonymic definition is a certain part of the discourse, in which the definition occurs, that follows the definition but does not include it. It is understood that if one of the group of synxenonyms occurs in the scope of their synonymity as a *free* linguistic form, i.e. as a word or word group, then it can be replaced with any other synonym of the group. Accordingly, if all synxenonyms exist only as free linguistic forms they can be used interchangeably in any occurrences in the scope of their synonymity. Synxenonyms that can be used interchangeably in any occurrences in the scope of their synonymity are called *concurrent synxenonyms* or *concurrent xenonyms*.

2) The above definition applies with “xenographonym” or “graphoxenonym”, or briefly with “xenograph”, and also with “xenophononym” or “phonoxenonym”, or briefly with “xenophon”, in place of “xenonym”. Consequently, morphologically, “synxenograph” is an abbreviation of “synxenographonym” or “syngraphoxenonym”, whereas “synxenophon” is an abbreviation of “synxenophononym” or “synphonoxenonym”. Semantically, a *synxenograph* is a *graphic* (*written*) *synxenonym* and vice versa. Likewise, a *synxenophon* is a *phonic* (*spoken*, *oral*) *synxenonym* and vice versa.

3) A *syneuautograph* is one of two or more euautographs which can be used interchangeably in any occurrences in the scope of their definition as syneuautographs. Such a definition is, as before, called a *synonymic definition*, whereas its scope, called *the scope of synonymity of the euautographs*, is defined as indicated in the item 1). Thus, syneuautograps are *concurrent euautographs* and vice versa.

4) A synxenograph or a syneuautograph is indiscriminately called a synograph, i.e. syngraphonym or graphosynonym. A synograph or a synxenophon is indiscriminately called a *synonym*. Synonyms are called *concurrent synonyms* or *concurrent nyms* if they can be used interchangeably in any occurrences in the scope of their synonymity.•

Df 8.2. In linguistic practice, there is the so-called *method of transliteration*, according to which graphonyms of a given alphabetic or syllabic language, which are naturally written with the alphabet or syllabary of the language, can be *transliterated in*, i.e. be *represented or spelled with*, the alphabet or syllabary of another language. The act, process, or instance of transliterating is called a *transliteration*. Usually, the result of transliteration is also equivocally called a *transliteration*, – in agreement with the principle of alternation of opposites, – but I shall, when desired to avoid confusion, designate it by the Latinized noun “*transliteratum*” (“*transliterata*” in the plural).•

Cmt 8.1. In connection with the method of transliteration, the following remarks will be in order. English contains a great number of transliterated words that were loaned from Latin and Greek in the XV century and that retain their original plural number forms. For instance, “calculus”–“calculi”, “datum”–“data”, “definiendum”–“definienda”, “formula”–”formulae”, “radius”–“radii”, etc are transliterated Latin words, whereas, “criterion”–“criteria”, “ellipsis”–“ellipses”, “thesis”–“theses”, “taxon”–“taxa”, etc are transliterated Greek words. An established English expression (combining form, word, or phrase) is said to be *Latinized*, or *Grecized* (*Graecized*), if it is modified in accord with spelling and grammatical forms of the Latin, or correspondingly Greek, language. In this treatise, I often Latinize some established English (Anglicized) words of Latin origin in several different ways and use each Latinized word univocally as a term in contrast to its equivocal original English etymon. For instance, I use “denotatum” (pl. “denotata”) instead of “denotation value”, connotatum” (pl. “connotata”) instead of “connotation value”, “substituendum” (pl. “substituenda”) instead of “substitute”, “substituens” (pl. “substituentia”) instead “replaced expression”, “transliteratum” (pl “transliterata”) instead of “transliteration”, etc. By contrast, an expression (as a word, combining form, or phrase) that is loaned from a foreign language with or without change in form, spelling, or pronunciation and that is *adapted* for use in the English language in accordance with the English grammar is called an *Anglicized*, or *Englishized*, *expression*. An Anglicized expression should not necessarily coincide with the English transliteratum of its foreign etymon, although the two sometimes coincide either partly or completely.•

Df 8.3. Two linguistic forms of two different languages being adequate translations of each other are called *parasynonyms*. In this case, the prefix “*para*”- is a *transliteratum* and at the same time *Anglicized form* of the Greek combining form “παρα”- which is a *parasynonym* of the English adverb “*beyond*” or of the English present participle “*transcending*”.•

Cmt 8.2. Like many other new Anglicized names that I introduce as terms of this treatise, the new Anglicized noun “parasynonym” is my own term – it is not in common usage. This term turns out to be especially useful in stating definitions that introduce some other Anglicized terms. Indeed, instead of stating that so-and-so foreign linguistic form has the same meaning as such-and-such English linguistic form, one may just state that the two are parasynonyms.•

Df 8.4. An ordered set of primitive characters that are used for *phonetic transcription* of phononyms of a certain alphabetic (or syllabic) language will be called the *transcriptional*, or *phonetic*, *alphabet* (correspondingly, *syllabary*) of the language, whereas a separate character of the set is called a *phonetic*, or *pronunciation*, *character*, or *symbol*. The conventional name "*phonetic alphabet*" is commonly used instead of "transcriptional alphabet", but I give preference to the latter because the former is ambiguous (see Cmt 8.5 below). A graphonym that is written with phonetic characters (particularly, a separate phonetic character) is conventionally and equivocally called a *phonetic transcription*, but I shall, when desired to avoid confusion, Latinize the name as "*phonetic transcriptum*" (pl. "*transcripta*") or Grecize it as "*metaphonograph*" ("*metaphonographonym*" in full). Consequently, a metaphonograph denotes any of its *phonetic values* or, more precisely, its *paratoken-class*. That is to say, a metaphonograph is a *phonoautograph* but not necessarily vice versa. Therefore, the term "metaphonograph" thus defined is in fact an abbreviation of the term "*metaphonoautograph*". However, a metaphonograph (phonetic transcriptum) is not, as rule, used xenonymously, which justifies use of the word "metaphonograph" instead of "metaphonoautograph". •

2) A metaphonoautograph is usually put either between back-slashed virgules, \ \, (e.g. in WTNID) or between ordinary forth-slashed virgules, / /, (e.g. in Allen [2003]) which can therefore be called *metaphonoautographic quotation* (*MPAQ*) *marks*. Accordingly, the *xenograph* that is formed by enclosing a metaphonoautograph between virgules is called a *metaphonoaotographic quotation* (*MPAQ*). It is clear that a MPAQ can be regarded as a PAQ whose interior is written with a certain transcriptional alphabet or syllabary, and not with an ordinary one, and whose exterior is replaced accordingly in order to indicate the difference.•

Cmt 8.3. The part of Cmt 3.4 that is relevant to a PAQ applies also to MPAQ. Thus, like a PAQ, an MPAQ a denotative synonym of its interior. In this case, the denotatum of the interior an MPAQ, i.e. the paratoken-class of the interior, is its sense, whereas the MPAQ is a xenograph, whose sense of is a coordination of the sense of its interior and of the sense of its exterior. This coordination can be described by giving to the MPAQ the combined common name "a dicto-metaphonoautograph".•

Cmt 8.4. In Psychologistics and in the TTL in particular, the metaphonograph (transcriptum) of a Greek expression is put after the expression between back-slashed virgules, \ \. Anglicized phonetic symbols of Greek are explained in Pring [1982, pp. xiv–xvi].•

Cmt 8.5. In addition to its use as a synonym of "transcriptional alphabet", the term "*phonetic alphabet*" is equivocally used as a class-name of any system of words that serve for identifying individual letters of the ordinary alphabet of a given alphabetic language in vocal communication (as by radio); and similarly with "syllabary", "syllables", and "syllabic" in place of "alphabet", "letters", and "alphabetic", respectively. For instance, "Alex", "Ben", etc may represent "a", "b", etc in one phonetic alphabet.•

Cmt 8.6. In order to use a transcriptional alphabet, the latter should be known and available typographically. Therefore, use of metaphonographs is often inconvenient or inappropriate or even impossible. In such cases, the appropriate words can be used for identifying individual speech sounds in analogy with using words as elements of a phonetic alphabet. Here follow two typical examples.

a) In Latin, the letter "*s*" is always pronounced like "*s*" in the English word "*sing*" (not like "*s*" in "*rose*").

b) In German, the letter "*s*" before vowel is always pronounced like "*s*" in the English word "*rose*" or like "*z*" in the English word "*zone*".

With the help of PAQ's, the above two statements can also be paraphrased thus:

a′) In Latin, the letter "*s*" is always pronounced like 〃*s*〃 in the English word 〃*sing*〃 (not like 〃*s*〃 in 〃*rose*〃).

b′) In German, the letter "*s*" *before* vowel is always pronounced like 〃*s*〃 in the English word 〃*rose*〃 or like 〃*s*〃 in the English word 〃*zone*〃.

Making statements a) and b) instead of a′) and b′) is possible in agreement with Cnv 5.1. Owing to that convention, use of PAQ's and QPAQ's can be avoided at all.

Essay 5. The problem of universals

Summary

The following three aspects of Aristotle's treatise «*Categories*» have been explicated and scrutinized.

1) I have demonstrated that *Aristotle's four-fold taxonomy of beings is inconsistent* because it *categorically* (*unconditionally*) *asserts* (*assigns*) [*the property of*] *immanence to universals of some kinds* and *categorically denies immanence to universals of some other kinds*. Therefore, characterizing Aristotle as a *moderate realist* as done in some authoritative writings on philosophy and logic, e.g. in the article «**universal (***logic***)**» of *Britannica Online Encyclopedia* (*BOE, Britannica.com*) *is incorrect*. I have suggested replacing Aristotle's inconsistent tetrachotomy of beings with a *consistent dichotomy*, according to which *every being that is defined as a universal turns out to be immanent*.

2) *Aristotle's second, ten-fold taxonomy of predicates* is independent of his four-fold taxonomy of beings. However, experts (respected scholars and philosophers) disagree on many most important and fundamental aspects of *Aristotle's ten kinds of predicates,* called "*categories*", particularly on what the latter are, words or their denotata. Based on the fact that, up to the sixteenth century AD, ancient writings and medieval printings were unpunctuated and free of blanks as word separators, I suggest that Aristotle *involuntarily and unconsciously confused between use and mention of graphic symbols* and I provide his term "*category*" with the appropriate *biune* (*homonymous*) *interpretation*.

3) With allowance for my dichotomy of beings, I have rigorously stated the *problem of universals*, in the framework of which the terms "*moderate realism*", "*moderate nominalism*", and "*conceptualism*" turn out to be synonyms, while *scientism* is as before the *higher, scientific form of conceptualism.* Then I have explicated the solution of the problem of universals in the framework of conceptualism or scientism as a *golden mean between and above extreme realism* and *extreme nominalism*. The solution is illustrated by a few examples, including some *physical laws*.

1. Introduction: Plato's philosophy

1.1. Platonic Universals (Ideas, Forms)

One of the most fundamental and most baffling problems of metaphysics, which philosophers have been trying to solve from ancient times to present, is the so-called *problem of universals*. This problem has stemmed from the teaching of *Universals* or *Forms* or *Ideas*, which was developed by *Plato* (Greek: Πλάτων \Pláton\, Πλάτωνας \Plátonas\, Πλάτωνος \Plátonos\; Latin: Plăton), 428/427–348/347 BC, in his main treatise «*The Republic*» (ca.360 BC) and elsewhere. The original Plato's term, which is rendered into modern English as "Universal", "Form", or "Idea", and also, more specifically, as "Platonic Universal", "Platonic Form", or "Platonic Idea", which are used in various English interpretations of and comments on Plato's teaching, is "είδος" \íðos\ (pl. "είδη" \íði\) meaning *a sort*, *kind*, *species* (see, e.g., Pring [1982]). In compliance with this fact, it is indicated in the article "**idea**" of the English-Latin part of Simpson [1968], that the Latin noun "spěcĭes" was used in the philosophical literature in Latin as a counterpart of the modern English term "Platonic Idea", while, according to the article "**spěcĭes**" of the Latin-English part of Simpson's dictionary, the above Latin noun had, depending on its context, many different meanings, and particularly the following three transferred ones, i.e. ones used in an altered or metaphoric sense, of mental showing: «**a**, *appearance, opp. reality*, *pretext*, *pretence*…; **b**, *notion*, *idea*…; **c**, *a kind*, *species*, *division of a* genus…». In this case, the meanings of the items **b** and **c** are etymologically compatible with the meaning of "Universal" as a counterpart of Plato's "είδος", whereas the meanings of the item **a** is incompatible with (opposite to) with the meaning of the latter. The meanings of the modern English noun "species" are in agreement with those mentioned in the item **c.** In this case, the kindred English adjective "specific" literally (etymologically) means «one of a species of the genus» as opposed to "generic", meaning «one of the genus», although "specific" is often used as a synonym of "particular" ("concrete"). Plato's term "είδος" is often transliterated in English as "eido" (see, e.g., the section "Universal" of the article "Realism" in the *Encyclopaedia Britannica* on *Answers.com*) and is translated in English as "Idea", although the English noun "idea" originates from the Greek noun "ιδέα" \iðéa\, meaning *an idea* or *opinion* (see, e.g., Pring [1982]). Thus, of the three English nouns "Universal", "Form", and "Idea", only the first one is a more or less adequate translation of the original Plato's term "είδος", while the two other are makeshifts, of which "Idea" is misleading, because Plato *does not mean anything mental* (*psychical*) by his είδος but rather something *ultimate real* and at the same time *insensible*. In general, the contemporary terminology that is used in modern English and in other modern European languages for interpreting and classifying teachings of ancient Greek philosophers is often inadequate from the standpoint of etymological analysis and is ambiguous. Unfortunately, it is impossible to avoid using the existing terminology in discussing philosophical questions at issue.

According to Plato's teaching, all *particular sensible* and *insensible* (*conceptual*) *objects* of any particular man and their properties with respect to the man, – such particular objects, e.g., as men, trees, stones, red objects, bright objects, loud sounds, sweet food, pains, just acts, beautiful objects, true sentences, false sentences, triangles, squares, etc, – are imperfect copies (representations, instances) of certain transcendental (abstract) eternal immaterial (incorporeal) unchangeable self-subsistent Universals, – such as Man, Mind, Tree, Stone, Redness, Brightness, Loudness, Sweetness, Painfulness, Justice, Beauty, Truth, Falsehood, Triangularity, Squareness, etc, respectively. Plato regards his Universals as the ultimate *re*al objects (from the Latin noun "*rēs*", singular and plural, meaning respectively *a thing* and *things*) of human knowledge – transcendental things (rēs) occurring beyond *the physical* (*spatio-temporal*) *realm* and existing *independently of the psychical* (*mental*) *realm* of a particular man. Therefore, the word "Idea" in its generally accepted sense seems to be inappropriate as a synonym of "Universal", as was mentioned above. For Plato it was possible to have knowledge only of unchangeable things, i.e. of Universals. In this case, knowledge of Universals can be acquired from reminiscences of them under the stimuli of sense perceptions, whereas the particular sensible things producing those stimuli are changeable and hence they cannot be objects of knowledge. Consequently, a Universal was, for Plato, a *re*al, or *actual*, being **(**ον), i.e. a thing (rēs), whereas a particular (ιδιαίτερος \idiéteros\) being was an *apparent* (φαινόμενος \fenómenos\) one, i.e. a semblance or outward appearance (φαινομενικός \fenómenikos\) of the Universal.

The above discussion can be summarized as the following definition.

Df 1.1. 1) The main doctrine of Plato's philosophy, which is called *Platonic realism* or *the extreme realism*, is the following. The *ultimate beings*, i.e. *ultimate real entities*, are *eternal transcendental entities*, called *Universals* or *Forms* or *Ideas*, which exist beyond space and time, and which a philosopher, i.e. a properly schooled particular man, can know by prescinding them from *pertinent particulars – apparent real entities* that the philosopher knows by acquaintance from his sensations and that are *imperfect appearances* (*phenomena*) of certain Universals.

2) The entire philosophical system (teaching) developed by Plato on the basis of his realism is called *Platonism* or, more precisely, *Platonism sensu stricto*. At present, the term "Platonism", alone or with the appropriate qualifiers, applies to any medieval or modern philosophical system that incorporates Platonic realism and some other doctrines.•

1.2. A particular man versus Universal Man

The rest of Plato's philosophy concerns with the nature of a particular (concrete, not universal) man and the ideal social and political organization of real men. A man, to Plato, has a tripartite soul comprising a preexisting immortal divine (spiritual) constituent (seating in the heart and having the functions of spirit, emotion, ambition, and courage) and two mortal constituents: an intellectual one (seating in the head and having the functions of intellect, thought, reason, and knowledge) and a corporeal one (seating in the loins and having the functions of desire, appetite, impulse, and instinct). In describing the tripartite soul of a man, Plato used the allegory, which became famous: the soul is a chariot, its immortal divine part is the driver, and its mortal parts are two horses. One horse is snow-white, reasonable, and conscientious; the other one is black, impudent, and lustful. According to Plato, concrete flesh and blood men come and go, but Man, i.e. man-Universal (man-Form) goes on forever. The Ideal (Perfect) State should, according to Plato, be made of three classes: the philosopher-rulers, soldiers, and artisans. The ways of building the Ideal State from imperfect human material are suggested in Plato's book «*The Republic*». Still, from pure logical analysis, Plato's political theory is inconsistent. Indeed, Plato comes to the conclusion that the Ideal State cannot be made of concrete men as they are. He therefore suggests a system of upbringing and education of young men, which is designed to make of them the *ideal* (perfect) human material suitable for building the Ideal State. However, the ideal men of any given category are indistinguishable to the extent that they are one and only one man-form of that category. Thus, in order to create the Ideal State, Plato had to destroy the notion of a particular man. At the same time, ideal men do not need any social and political organization. Incidentally, the former USSR is a real example what can be the ultimate result of building a Utopia at the cost of compulsive social organization and of destruction of individuals.

2. The problem of universals

2.1. A preliminary formulation of the problem of universals

Plato's philosophical model of the Universe had created a philosophical or, more specifically, *ontological* problem, which is called the *problem of universals.* This problem can be formulated in many different ways, so that its solution depends on its formulation and particularly on a definition of the noun "universal". For instance, the essence of the problem of universals is described in Allen [2003] thus:

> «**ontology** /on'toləji/ *noun* **:** a branch of philosophy concerned with the nature of being.
>
> Editorial note————————————————————————
>
> By 'the nature of being' (which is the subject of 'ontology') we mean the nature of 'beings'. What sorts of objects are there? Material objects, for one, as occupiers of space-time. Beyond them (or instead of them) there may be such further things as events, states of affairs, persons, numbers, ideas, qualities, waves. As well as asking about beings, we may ask what 'being' itself is. Plato and Heidegger take this question to be very important; Aristotle and many others think it is senseless and so needs no answer – Professor Jonathan Dancy.»

An entity (being) that a man (sapient subject) *knows by acquaintance* via his *sensations* (*perceptions*) is called a *sensible particular real* (*physical, extramental, exopsychical*) *entity*. Therefore, the problem of universals can, preliminarily but quite adequately, be formulated as the following two questions in that order:

i) Beyond *sensible and hence real particular entities* (*beings*), which a given man *knows by acquaintance*, are there some *insensible real universal entities*, of which the man is conscious (aware) *by reason*?

ii) Supposing that such insensible real universal entities exist, then what they are and where they are seated?

In view of the power of properly schooled men to discover various *laws of nature*, which are certainly *real insensible universals* of some kind, to verify the laws by appropriate observations or measurements or both, to make certain predictions on the base of the laws, and to utilize them in various technical devices, the answer to the former question *must be positive*. Therefore, a solution of the problem of universals reduces to answering the latter question, i.e. to deciding, which universals are laws of nature and where they are seated.

2.2. Aristotle against Plato: Different hypostases of Aristotle

It is generally accepted that the uncompromising polemic on Plato's doctrine of his transcendental Universals was initiated by *Aristotle* (Αριστοτέλης \Aristotélís\, Αριστοτέλους \Aristotélus\, Latin: Aristotelēs), 384–322 BC, a student at the Academy of Plato in Athens and the «Greek philosopher and scientist whose thought determined the course of Western intellectual history for two millennia», – as said in the article of the same name in BOE (Britannica.com). The noble Latin dictum: "*Amicus Plato, sed magis amica veritas*", which is translated into English as: «Dear is Plato, but dearer still is truth», «Plato is my friend, but truth is a better friend», «Plato is my friend, but truth is more my friend [than he is]», or «Plato I love, but I love truth more», is attributed to Aristotle, who as though said it to express his uncompromising rejection of Plato's teaching of Universals. At the same time, according to the online «*Dictionary of Phrases and Fable*» of Wikipedia, the above Latin dictum is a very free translation of a phrase of «*Nicomachean Ethics*» (1096a15) by Aristotle, which is literally translated as: «Where both are friends, it is right to prefer truth».

In any case, it is adopted as a historical fact that Aristotle initiated and waged a relentless struggle against the doctrine of transcendental Universals of his teacher Plato, and that this war has been continued by the subsequent generations of philosophers up to the present time. Aristotle's claim to be the initiator of the struggle against Plato's doctrine of Universals rests primarily, if not entirely, on his treatise «*Categories*», although neither Plato nor his philosophy are mentioned in that treatise. Also, from «*Categories*» and from the existing discrepant comments on that and on other Aristotle's works, it is unclear what exactly Aristotle's attitude towards universals was and why the problem of universals has been regarded by many reputable modern and contemporary scholars as an unsolved one. In their comments on Aristotle's opposition to Plato's Universals, the different scholars in the different times characterized his Weltanschauung by characterizing Aristotle himself as the founder and hence as an adherent of various, often incompatible, philosophical currents from *the founder of philosophy of nominalism* to *the founder of moderate realism* and hence from the first *extreme nominalist* to the first *moderate realist.* Here is, for instance, a description of Aristotle in the *former hypostasis* by Durant [1926, pp. 48–49]):

«Passing out from this rear line of logic ⟨the definition through a genus and the difference that Aristotle inaugurated in his «*Posterior Analytics*» –Ya. I.⟩ we come into the great battlefield on which Aristotle fought out with Plato the dread question of "universals"; it was the first conflict in a war which was to last till our own day, and make all medieval Europe to clash of "realists" and "nominalists."[13] A universal, to Aristotle, is any common noun, any name capable of universal application to the members of a class; so *animal, man, book, tree,* are universals. But these universals are subjective notions, not tangibly objective realities; they are *nomina* (names), not *res* (things); all that exists outside us is a world of individual and specific objects, not of generic and universal things; men exist, and trees, and animals; but man-in-general, or the universal man, does not exist, except in thought; he is a handy mental abstraction, not an external presence or *re*-ality.

Now Aristotle understands Plato to have held that universals have objective existence; and indeed Plato had said that the universal is incomparably more lasting and important and substantial than the individual, – the latter being but a little wavelet in a ceaseless surf; *men* come and go, but *man* goes forever. Aristotle's is a matter-of-fact mind; as William James would say, a tough, not a tender, mind; he sees the root of endless mysticism and scholarly in this Platonic "realism"; and he attacks it with all the vigor of a first polemic. As Brutus loved not Caesar less but Rome more, so Aristotle says, *Amicus Plato, sed magis amica veritas* – "Dear is Plato, but dearer still is truth."

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[13]It was in reference to this debate that Friedrich Schlegel said, "Every man is born either a Platonist or an Aristotelian" (in Benn, i, 291) ⟨Benn, *The Greek Philosophers*, London, 1882, vol. i, p. 291 – Ya. I.⟩»

By contrast, here is, for instance, a description of Aristotle in the *latter hypostasis* in the section «*Platonic and Aristotelian realism*» of the article «**universal (***logic***)**» in BOE (Britannica.com):

«Few philosophers now believe in such a "Platonic heaven," at least as Plato originally conceived it; the "copying" theory of exemplification is generally rejected. Nevertheless, many modern and contemporary philosophers, including Gottlob Frege, the early Bertrand Russell, Alonzo Church, and George Bealer are properly called “Platonic” realists because they believed in the existence of universals that are abstract or transcendent.

Aristotle denied that exemplifying a universal is anything like copying it. He parted company with all Platonic realists by affirming that: (1) the properties of material things are “immanent” – i.e., “in” the things that exemplify them, in a nearly literal, spatial way; and (2) properties do not exist independently of the things that exemplify them. Both of these ideas survived in some contemporary theories. Thus, the entities that Alfred North Whitehead called “objects” seem to be universals weaving their way through space-time, numerically the same wherever or whenever they appear. So-called “bundle” theories of universals also construe them as immanent in some sense. According to these theories, an individual thing is nothing more than a bundle of universals in a certain intimate union with one another.»

Many different factors can result in the wide spectrum of Aristotle’s hypostases, some of which seem to be incompatible. Here follow a few of them.

I) *Logic and nominalism.* Aristotle has indeed many different hypostases, especially taking into account that a great many of the works that are credited to him are not his authentic works. Still, for use in his categorical syllogistics, which Aristotle inaugurated in undoubtedly his «*Prior Analytics*» on the base of undoubtedly his «*Categories*» and «*On Interpretations*», he also inaugurated, in the latter two works and in his «*Posterior Analytics*», *nominalism* or, to use the appropriate modern term, *class theory*. Interpretation of *formal* logic by *its matter*, constituting a certain class of *meaningful* (*conformable to facts*) declarative sentences of any given *native language* (*NL*)*, is beyond the scope of formal logic*. Therefore, Aristotle is indeed the founder of extreme nominalism as far as he is the founder of formal logic, and he is indeed an extreme nominalist as far as he is a formal logician (analyst), but it would, in my view, be incorrect to assert that Aristotle practiced extreme nominalism as his Weltanschauung. In this sense, Aristotle as a philosopher can be characterized as a *moderate nominalist*, rather than an *extreme* one. To compare, the fact that Newton discovered the *second law of dynamics* does not mean that he denied existence of friction and dissipation of energy in all cases or that he denied existence of noninertial coordinate systems, i.e. that, in other words, he was an *extreme* «*inertialist*». At the same time, the fact that Newton discovered the *law of gravity* does mean that he had to give up his *law of inertial motion* and that he should be called an *extreme* «*non-inertialist*». A distinguished thinker as Aristotle or Newton does his study for satisfying his curiosity, whereas labels of some -"*isms*" to what he has done and the respective labels of -"*ist*" to him are usually attached by his partial contemporaries or partial successors or both. For instance, Aristotle did not give any indications that he considered as a single whole study his six treatises «*Categories*», «*On Interpretation*», «*Prior Analytics*», «*Posterior Analytics*», «*Topics*», and «*Sophistical Refutations*», which were later compiled in this order under the title «*Organon*» reputably by Theophrastus (Θεόφραστος, c371–c287 BC), the successor to Aristotle as the second scholarch (head) of the Lyceum; Aristotle did not employ either of the terms "Organon" and "logic" as the title or in the title of any of his treatises or their parts. At the same time, Aristotle used the generic name "analytics" in the titles of his two treatises, namely «*Prior Analytics*» and «*Posterior Analytics*», thus regarding them as one work and thus regarding himself as an «*analyst*», and not as a «logician». The word "λογική" \lojikí, loyikí\ *s.f.*, meaning *a way of thinking*, was reputedly used as a name of the science founded by «*Organon»* in writings of the Stoics. However, the Latin version of this name, "*logika*", from which the English noun "logic" is derived, was coined by Marcus Tullius Cicero (106–43 BC), a Roman statesman, orator, and author. Here follows the vocabulary entry of Simpson [1968] that confirms this fact:

> «**lŏgĭcus**, -a, -um (λογικός), *logical*; n. pl. as subst. **lŏgĭca**, - ōrum, *logic*: Cic.».

Aristotle employed the word λογική" occasionally as a close synonym of "*dialectics*" ("διαλεκτική" \ðialectikí\ *s.f.*, dual διαλεκτικά \ðialectiká\, pl. διαλεκτικαί \ðialectiké\) meaning collectively *induction and deduction*.

II) *Punctuation.* Bodmer [1944; 1981, p. 50] writes:

> «…Even when a secular literature spread through the Greek and Roman world, the written language remained a highly artificial product remote from daily speech. Greek writing was never adapted to *rapid* reading, because Greek scribes never *consistently* separated words. The practice of doing so did not become universal among Roman writers. It became a general custom about

> the tenth century of our own era. When printing began, craftsmen took pride in the ready recognition of the written word, and punctuation marks, which individual writers had used sporadically without agreement, came into their own. Typographers first adopted an agreed system of punctuation, attributed to Aldus Manutius, in the sixteenth century. In the ancient world the reader had to be his own *paleographer*.»

Particularly, the ancient *Bible* (from Greek "Βίβλος" *s.f.* \bíblos\ or perhaps from "Τα Βίβλια" \ta bíblia\, meaning *The Books,* – the plural number form of "βίβλιον" \bíblion\ *s.n.*, meaning *a book*) was unpunctuated. Original works of Aristotle, their ancient Greek copies, and their early medieval translations into Latin were set up in the same manner.

III) *Confusion between use and mention of graphic symbols.* Every modern written language has an extensive system of punctuation marks, including the standard faceless En Space for separation words and also including faceful punctuation marks as quotation marks, brackets, commas, etc.In addition, there is a great many of typographical types and fonts, in which different parts of a given text can be printed. However, it was only at the very end of the 19th century when the German logician Gottlob Frege [1893–1903, vol. 1, p. 4] suggested a method of *proper autonymous quotations to systematically* (as he thought) *indicate autonymous* (*self-referential*) *use of graphic signs*. The Fregean quotation method is *epistemologically relativistic¸* so that it cannot be used for indicating all kinds of autonymy and in all case. In spite of this fact, use of special quotation marks is an effective syntactic device for indicating an autonymous use of a graphic sign in cases where there might otherwise be a danger of confusion. Besides the Fregean quotation method and its various modifications, a specific type, particularly *italic*, can be and is often used for indicating autonymy. Nevertheless, the method of *equivocally* using *graphonyms* (graphic expressions) both *autonymously* and *xenonymously* is the one favored by almost all modern logicians and mathematicians, including writers on set theory (cf. Fraenkel et al [1973, p. 20f]), and by almost all modern science writers (as physicists, biologists, etc). Therefore, needless to say that in his works Aristotle did not take care of maintaining, either *syntactically* (with the help of special quotation marks or special fonts) or *semantically* (with the help of added words) the difference *between use and mention* (to use the convenient phraseology of Quine [1951, p. 23]) of a graphonym in cases where there is doubt in its meaning. Moreover, many definitions of Aristotle are *persuasive* ones, and not *real definitions*, i.e. not *definitions of the species through a genus and the difference* (in Latin: *definitiones species per genus et differentiam*), which Aristotle inaugurated in his «*Posterior Analytics*». Therefore, some terms that are employed in the English versions of Aristotle's works are obscure or ambiguous, which reflects obscurity or ambiguity of the Aristotle's pertinent original Greek terminology. Also, it is debatable whether a term that Aristotle introduces in a certain one of his works, say, in «*Categories*» or «*On Interpretation*», preserves its recognizable identity in his other works or even throughout that same work.

IV) *Articles.* Letting aside the problem of punctuation of ancient texts, which has been outlined in item II, it is understood that translation from one language into another of a philosophical (particularly logical) work, which deals with ontological status of universals and with their interrelations, requires elaborating the adequate alternative terminology and phraseology. This problem becomes especially difficult if the two languages have different *vocabularies of articles*. I am familiar with the article problem not by hearsay, because I have written the TTL in English – a language having both the definite and the indefinite article, while the language, which rang at my cradle and which is the first language of my culture and my education, is Russian – a language having no articles. In order to discuss the article problem conveniently, I shall make the following definition.

Df 2.1. 1) A *native language* (*NL*) is called:

a) a *two-article language* (*2AL*) if it has both a definite article and an indefinite article,

b) a *one-article language* (*1AL*) if it has a definite article and no indefinite article,

c) *an article-less*, or *article-free*, *language* (*ALL* or *AFL*) if it has no articles,

d) an *indefinite-article-less language* (*IALL)* if it is either a 1AL or an ALL.

In this case, each article is understood as a *paradigm* that can have many (two or more) allomorphs or many grammatical forms associated with the gender, number form, and case of a *substantive* (a *noun*, *descriptive nounal name*, or *absolute adjective*), which it modifies. It is understood that any substantive of an ALL is an *article-less* (*article-free*) *substantive* (*ALS*), and that any substantive of an IALL, which is not modified with the definite article, is also an *indefinite-article-less* *substantive* (*IALS*). In this case, the terms "*article-less name*" ("*ALN*") and "*indefinite article-less name*" ("*IALN*") are self-explanatory, and therefore these will, when necessary, be used without any further comments.

2) For instance, English, French, German, and Italian are two-article languages, Greek, Hebrew, and Arabic are one-article languages, and Latin, Russian, Ukrainian, and many other Slavic languages are article-less languages.•

Cmt 2.1. Here follows a pertinent entry of Allen [2003]:

«**indefinite article** *noun* in grammar, a word, e.g. *a*, *an*, or *some* in English, that refers to an unspecified person or thing: compare DEFINITE ARTICLE.»

In the same dictionary, there are three vocabulary entries for the word "some", namely, **some**$^{\mathbf{1}}$, **some**$^{\mathbf{2}}$, and **some**$^{\mathbf{3}}$, in which that word is classified as an *adjective*, *pronoun*, and *adverb* respectively, and not specifically as an article. An article is traditionally regarded by grammarians and lexicographers as an *adjective*, but except the above-quoted definition I have never encountered any dictionary definition of "some" as an English indefinite article. The article vocabulary of a native language is one of the most essential expressive means of the language and one of the most essential aspects of philosophy of its grammar. Incidentally, some grammarians posit that there are *three articles in English*: the *indefinite* one, "a" or "an", the *definite* one, "the", and the *null* one. I do not adopt this doctrine for convenience in the three-fold classification of native languages that has been made in Df 2.1.

As far as the adjective "some" is concerned, it is used either as an *indefinite* one or as a *partitive* one. In the first hypostasis, "some" is used as a *makeshift* of the indefinite article and is opposed to any of the *demonstrative adjectives* "this", "these", "that", and "those", whose sense is close to that of the definite article. In the second hypostasis, "*some*" (in Greek: μερικός \merikós\, pl. μερικοί \merikí\) is used as an *existential quantifie*r, denoting a *particular quantity*, i.e. a *strict part or the whole*, of a certain *universal quantity*, denoted by any of the *universal quantifiers* "*all*" (in Greek: όλος, \ólos\, pl. όλοι \óli\), "*every*", and "*each*" (in Greek: κάθε \káθe, káthe\ for both, and also πάς \pás\ for both "all" and "every"). Quantity is one of the ten categories of Aristotle. In this case, it is not clear from Aristotle's treatise «*Categories*» whether his ten categories, which are generally described as *things that are said* (in Greek: "τα λεγόμενα \ta legómena\), are either *the words said* or *the* *things said by the words,* i.e. *the denotata* (*denotation values*) *of the words*, or else, equivocally, both. Accordingly, it is not clear whether Aristotle's quantity is a quantifier or the entity (being) denoted by the quantifier or both (cf. Studtmann [2008, subsection 2.1]). This feature of Aristotle's coinage is essential for interpretation of the ontological status that he assigns to universals (to be discussed in greater detail in the next section).

By extrapolation from the native ALL's and IALL's, with which I am familiar, I may assert that in such a language the duty of a missing indefinite article is *partly* done by the *parasynonym* (*semantic analogue*) of the English indefinite adjective "some", while the duty of a missing definite article is *partly* done by the parasynonyms of the above-mentioned English demonstrative adjectives. Still, semantic properties of substantives of such a language differ from their counterparts of English. Particularly, the *degree of equivocality*, i.e. the *variety of possible mental interpretations*, of a *numerable* (*count*), *but unquantified and particularly indefinite-article-free*, *name*, of a given native language depends on the articles that it has. Therefore, in accordance with Df 2.1 and in contrast to the above-quoted definition of Allen [2003], I shall follow the concept that English has exactly one word, namely, "a" or its allomorph "an", which is entitled to be called the indefinite article, and exactly one word, namely, "the", that is entitled to be called the definite article. The articles are *auxiliary formal* (*syntactic*) *substance-indicative words* that have some features of punctuation marks – words that have no counterparts in some other native languages. By contrast, the indefinite adjective "some" and any of the demonstrative adjectives are ordinary, non-syntactic words, although they can, in some contexts, be used instead of "a" and "the", respectively. I also assume that any given native language has *at most two articles*, and that if the language has one article then the latter is the definite article. In the general case, an article may have not only allomorphs, but also various grammatical forms. For instance, Greek has 24 *grammatical forms* of the definite article and no indefinite article. Any grammatical form of the Greek definite article is often translated into English as "the", but it functions differently from the only form of the English definite article. If the reader knows a native language that has more than two paradigmatic articles then that language should be regarded as an exceptional one.

V) *The operators "is", "is not", "are", and "are not"*.

In ordinary non-technical English, “is” or “are” (“εστί” \estí\ or “είναι” \íne\ in Greek and “est” or “sunt” in Latin, respectively), i.e. the third person singular or plural Present Tense of the copula (link-verb) “to be” (“είμαι” \íme\ in Greek and “esse” in Latin), *along with the indefinite or definite article following it*, if this article is required by the English grammar (Greek has no indefinite article and Latin has no articles at all), may *equivocally* denote any one of a great many different *relations in intension* (*predicates*) such as: the *class-membership* (*member-of-class*, *object-to-property*) *relation*; a *class-inclusion* or *mass-inclusion* (*part-to-whole*, *subclass-to-superclass, species-to-genus*, *submass-to-supermass*) *relation*, an *equivalence relation* and particularly the *identity relation*, the *logical entailment relation*, etc. In this case, a class-inclusion or mass-inclusion relation can, in turn, be either a *strict* (*strong*) one or a *lax* (*weak*) one, because the word “*part*” can be understood either as “*strict part*”, i.e. “*part but not the whole*”, or as “*lax part*” i.e. “*strict part or the whole*”. Accordingly, the class-inclusion or mass-inclusion relation is a strict (strong) one in the former case and a lax (weak) one in the latter case. Like remarks apply to the *parasynonyms* (*counterparts*) of “is” and “are” in any of a great many of phonemic (alphabetic or polysyllabic) languages (as Greek, Latin, Russian, Hebrew, Japanese, etc). Owing to the ambiguity of the copulas “is” and “are” in English and of their counterparts in other languages, a class-membership predicate was confused with a class-inclusion predicate for hundreds of years.

Due to Italian logician and mathematician *Giuseppe Peano* (1858–1932), it was recognized that in English, e.g., either one of the expressions “*is a*” and “*is an*”, which is immediately followed by a *numerable* (*count*) *name*, denotes the *relation of membership in the multipleton* (*many-member class*) *denoted by that name*. Also, after Peano, the *class-membership relation in intension* is denoted by the special *lexigraph* (*atomic logograph*, *atomic logographic symbol*) “$\in$”, which is a stylized abbreviation of the Greek link-verb “εστί” \estí\ meaning *is*. By contrast, the strict (strong) class-inclusion relation [in intension] and the lax (weak) one are, after German logician and mathematician *Friedrich Ludwig Gottlob Frege* (1848–1925), commonly denoted by the signs “$\subset$” and “$\subseteq$”, respectively. Thus, according to Peano, the statement «A man is a mammal», e.g., can be restated as «A man $\in$ mammal» or as «A man $\in$ Mammalia». Likewise, the statement «Socrates is a man» can be restated as «Socrates $\in$ man» or as «Socrates $\in$ *Homo sapiens*». At the same time, using the terms “class” and "subclass" in a broad sense, the statement «Species man is a subclass of class Mammalia» can be restated «Man $\subset$ Mammalia» or as «*Homo sapiens* $\subset$ Mammalia», and also as «Man $\subseteq$ Mammalia» or as «*Homo sapiens* $\subseteq$ Mammalia». Likewise, the statement «The singleton of Socrates is a subclass of species man» can be restated as «Socrates $\subset$ man» or as «Socrates $\subset$ *Homo sapiens*», and also as «Socrates $\subseteq$ man» or as «Socrates $\subseteq$ *Homo sapiens*».

In contrast to Latin, Italian has both the definite article and the indefinite article, although their grammatical properties essentially differ from the grammatical properties of the respective English articles. Like the English indefinite article, "a" or "an", the Italian indefinite article (*l'articolo indeterminativo*) is used with singular nouns and noun constructions, and in addition it denotes the number one. Unlike English indefinite article, Italian one has different forms depending on the gender and initial one or two letters of the word (noun or adjective) that it precedes and modifies, namely,

a) "uno" is used before masculine names beginning either with "s" followed by a consonant or with "z";

b) "un" is used for all other masculine name;

c) "una" is used before feminine names beginning with a consonant;

d) "un'" is used before feminine names beginning with a vowel.

In contrast to the English definite article having only one form, "the", the Italian definite article (*l'articolo determinativo*) has different forms depending on the gender, number, and initial one or two letters of the noun or adjective it precedes, namely:

a′) "lo" (pl. "gli") is used before masculine names beginning either with "s" followed by a consonant or with "z";

b′) "il" (pl. "i") is used before masculine nouns beginning with all other consonants;

c′) "l'" (pl. "gli") is used before masculine nouns beginning with a vowel;

d′) "la" (pl. "le") is used before feminine nouns beginning with any consonant.

An Italian article agrees in gender and number with the name it modifies and is repeated before each name in a conjunction or disjunction. Still, owing to the fact that both Italian and English are two-article languages, Peano's discovery is immediately applicable to English. For instance, one may assert that, according to Peano, "is a" or "is an" denotes the class-membership relation, in spite of the fact that

Peano expressed his discovery in Italian. At the same time, one cannot make such a straightforward assertion in regard to any article-less or indefinite-article-less language as Greek, Latin, Hebrew, or Russian, although Peano's suggestion to employ the character "$\in$" as the class-membership relation is applicable in any language. In application to any of a great many of native languages, Peano's discovery has the following aspects.

i) A count name can be used either as a proper class-name (i.e. as a singular collective name in the terminology of Mill [1843]) or as a common individual name.

ii) If the *grammatical predicate* of an affirmative simple declarative sentence comprises the *copula* analogous to the English copula "is" and a *predicative* in the form of count (numerable) name, which is used as a proper class-name, then that copula is used as the class-membership predicate.

iii) If the language in question has either a definite article or both a definite and an indefinite article, and if an article occurs in the predicative mentioned in the previous item, then that article just serves as an *index* (*indicator*) that the copula and the count name are used in the above-mentioned way.

In application to English, Peano's "is a"-rule can be extended as follows. The expressions "*is the*" or just "*is*" if "the" is omitted in accordance with the pertinent rule of the English grammar (as, e.g., in the grammatical predicate "is captain of the ship"), which is immediately followed by a *numerable* (*count*) *name*, denotes the *relation of membership in the singleton* (*one-member class*) *denoted by the name*. For instance, the statements «Socrates is the husband of Xantippe» and «Aristotle is the founder of logic» can be restated as «Socrates $\in$ husband of Xantippe» and «Aristotle $\in$ founder of logic». At the same time, in accordance with the equivocality of "is", which have been indicated at the beginning of this item, the former two statements can also be restated either as the identities of the individuals: «Socrates = the husband of Xantippe» and «Aristotle = the founder of logic» or as the identities of the singletons of individuals: «Socrates = husband of Xantippe» and «Aristotle = founder of logic» or else as the weak class-inclusion relations between identical singletons: «Socrates $\subseteq$ husband of Xantippe» and «Aristotle $\subseteq$ founder of logic».

After creation of *set theory*, – at first of the naive one by Georg Cantor (1845–1918) during the years 1878-84, and then of the axiomatic one by Ernst Zermelo [1908], – certain special classes were called *sets*, although it took some time with mathematicians and logicians to realize that a set was a class, but not necessarily vice versa. It will be recalled that, in outline, a set is a class, which has permanent member population and which can be linearly ordered in the sense that it can serve as a domain of definition of the linear order relation $\leq$. Therefore, I alternatively call a set “a regular class” and a class that is not a set “an irregular class” (see subsection I.9.3 in the TTL). In the literature, the classes of the two kinds are distinguished by the names “small class” and “proper class” in that order (see, e.g., Fraenkel et al [1973, pp. 128, 134–135, 167] for the former term or the article «**class**» in Wikipedia for both terms).

This and the previous items can be recapitulated as follows. Historically, Latin was the most important cultural language of Western Europe until the end of the 17th century. Therefore, Aristotelianism became known in Western Europe primarily owing to translations of Aristotle’s works into Latin. However, owing to the different article vocabularies of Greek and Latin, the latter is one of the least appropriate languages for treating of Greek-related philosophy, especially the part of it dealing with the problem of universals.

VI) *Partiality of translators and commentators of Aristotle’s works.* About eight and half centuries after Aristotle’s death, his «*Organon*» was translated into Latin by Roman philosopher Anicius Manlius Severinus Boethius (AD ca475–524). Durant [1950, p. 99] writes of the Boethius work:

> «His translation of Aristotle’s *Organon*, or logical treatises, and of Porphyry’s *Introduction to the Categories of Aristotle* provided the leading texts and ideas of the next seven centuries in logic, and set the stage for a long dispute between realism and nominalism.»

Porphyry of Tyre (From Greek: Πορφύριος \porfúrios\, meaning *purple-clad,* AD 234–c305) was a Neoplatonic philosopher, a student of Plotinus. In connection with the above citation of Durant, here follows a pertinent reference from Wikipedia:

> «The ***Isagoge*** (Greek: Εἰσαγωγή) or “Introduction” to *Aristotle’s* “*Categories*”, written by Porphyry in Greek and translated into Latin by Boethius, was the standard textbook on logic for at least a millennium after his death. It was composed by Porphyry in Sicily during the years 268–270… The

> work includes the highly influential hierarchical classification of genera and species from substance in general down to individuals, known as the *Tree of Porphyry*, and an introduction which mentions the problem of *universals*. *Boethius*' translation of the work, in Latin, became a standard medieval textbook in European schools and universities, setting the stage for medieval philosophical-theological developments of logic and the problem of universals. Many writers, such as Boethius himself, *Averroes*, *Abelard*, *Scotus*, wrote commentaries on the book. Other writers such as *William of Ockham* incorporated them into their textbooks on logic.»

To say nothing of the essential contributions to «*Organon*» that have been made by Scholastics, many translations of «*Organon*» and *Isagoge* from Greek and Latin into the main European languages and into Arabic have been made since Porphyry's and Boethius' times and a great many of comments have been written on these translation in various languages.

Every translation of a work of any ancient writer in general and of Aristotle in particular from one language into another, – say, from Greek into Latin or from Greek or Latin into English, – unavoidably includes translator's covert or overt interpretation of the work. In addition, there are a great many of comments on Aristotle's works in English, especially on his «*Organon*» and «*Metaphysics*». Many terms that are employed in the English version of Aristotelianism are obscure or equivocal, which reflects obscurity or equivocality of Aristotle's pertinent original terminology. Also, as I have already pointed out in the item III, it is debatable whether a term that Aristotle introduces in a certain one of his works, say, in «*Categories*» or «*On Interpretation*», preserves its recognizable identity in his other works (cf. Studtmann [2008]). Therefore, in treating of the subject matter of Aristotle's works, I shall use modern English terminology and phraseology or that of my own, while Aristotle's original terms or, most often, their alternative translations into English will be mentioned and used as object ones.

In accordance with the above-said, the character of Aristotle's opposition to Plato's doctrine of Universals might often have been misinterpreted by the later translators of his works or by the later scholars commenting on those works or by both. No matter whether or not the standpoint of Plato or the standpoint of Aristotle in their debate was interpreted adequately by a certain interpreter, that standpoint appears as its interpretand (interpretation) by the interpreter and sometimes either as the standpoint of the interpreter himself or as his preference. Sometimes, the interpreter pinpoints (defines more clearly from his viewpoint) the contradictory attitudes of the debaters in order to emphasize their uncompromising character. For instance, in the apparently impartial Editorial note by Prof. Jonathan Dancy to the above-cited article «**ontology**» of Allen [2003], Platonic doctrine of Universals is understood as one that deserves re-examination from the standpoint of certain contemporary concepts, whereas Aristotle and his followers are obviously (although implicitly) understood (interpreted) as *extreme nominalists*. At the same time, in his above-cited comment, Durant explicitly expresses his agreement and his sympathy with his own interpretation of Aristotle's world outlook, which he implicitly interprets as *extreme nominalism*. Lastly, the author of the above-cited comment of BOE explicitly expresses his agreement and his sympathy with his completely different interpretation of Aristotle's world outlook, which he explicitly interprets as *moderate realism*.

VII) ***Inconsistencies of Aristotle's immanence.*** However, in the next section, I shall unambiguously demonstrate that, besides concepts, which *categorically* (*unconditionally*) *affirm immanence* (*immanent property*) *of universals of some kinds*, Aristotle's «*Categories*» involves concepts, which *categorically deny immanence of universals of some other kinds*. Therefore, Aristotle cannot be characterized either as the founder of moderate realism or even as a moderate realist. In general, except for the fact that he had *categorically rejected the doctrine of Plato of transcendental Universals,* no consistent comprehensive constructive viewpoint of Aristotle on the ontological status of universals is known. Consequently, in order to solve the problem of universals in agreement with modern scientific practice, it is necessary to develop a certain pertinent self-consistent system of notions that should include a rigorous definition or definitions of the generic name "*universal*" versus the generic name "*particular*" and that should also involve a general axiomatic *immanence principle for a universal of any kind.*

2.3. Terminological problems

2.3.1. Ancient Greek and traditional Latin terminologies versus modern English terminology

Voltaire (François Marie Arouet) said, «If you wish to converse with me, define your terms». This dictum is especially relevant to any discussion of the problem of universals, which unavoidably involves the problem of adequate translation of the pertinent ancient Greek and traditional Latin terms into modern English. Particularly, the main Plato's term "είδος" \íδos\ that is translated into modern English as "Universal" and also unfortunately as "Form" or "Idea" has been discussed in the subsection 1.1. Here follow some other examples of serious terminological inconsistencies.

1) "Το ον" \to on\ (*s.n.*, pl. "τα όντα" \ta ónta\), meaning *a being* or *creature*, is one of many fundamental *undefined* Aristotelian terms. Circularly, "being" means *everything that exists*; substantially, a being *is anything* that can be *treated* (*spoken*, *predicated*) *of* as one that is located in the φύσις \físis\, i.e. in *the nature* or *physical world*; "φύσις" is Homer's term *of The Odyssey* that was employed by Plato in the sense of "*nature*". Therefore, the Aristotelian term "ον" is alternatively translated into English by either substantive "*thing*" or "*real entity*". For instance, a word is a being (thing, real entity), but an import value of a word is not necessarily a being. In this case, the Aristotelian term "πράγμα" \práγma\ (pl. "πράγματα" \práγmata\), which is also translated by the English noun "*thing*", means *anything that can be treated* (*spoken*) *of*, including beings and also including anything *supra-natural* as Aristotelian God, which is not located in the physical world and which is not, therefore, a being. Thus, "πράγμα" ("thing") in Aristotle's philosophy is a more general and vaguer term than "όν" ("being"). At the same time, in the Late Ancient Greek philosophy of Neo-Platonism founded by *Plotinus* (Πλωτῖνος \plotínos\, AD ca.204/5–270), the noun "ὀντότης" \ontótis\ *s.f.*, i.e. "*entity*" in English spelling, meant *reality* (*Platonic Universals*).

2) In contrast to the ancient Greek terminology, in modern English, there is a tendency to use the noun "entity" for mentioning anything that can be treated (spoken) of, i.e. to use it in analogy with the Aristotelian term "πράγμα", and at the same time to use the noun "thing" in analogy with Aristote's term "όν" ("being") and Plotinus' term "ὀντότης" ("reality"). In this usage, the noun "thing" is *parasynonym* of the Latin noun "rēs" (pl. "rēs"), which, according to Simpson [1968], means *a thing,*

object, matter, circumstance, and especially *a real thing, fact,* or *truth*. The English words "*reality*" and "*real*" have been derived from that Latin etymon.

2.3.2. Some elements of the pertinent modern terminology and of my own one

Df 2.2. In translations of Aristotle's works into English, the count noun "thing" is used equivocally for mentioning *anything* (*any thing*) that can be treated (spoken) of, except perhaps for *nothing* (*no thing*). I shall alternatively call *nothing* "*the empty entity*" and also "*the empty individual*", "*the empty class*", or "*the empty mass*". Accordingly, I shall use the count noun "*entity*" for mentioning *anything* that can be treated (spoken) of *without any exception*s, while using "thing" for mentioning *any entity* (*anything*) *except the empty one* (*except nothing*). Whenever it is necessary to distinguish between *real* and *ideal* (*nominal*) things, I shall introduce the pertinent special terms.•

Cmt 2.2. The above usage of the nouns "entity" and "thing" allows particularly avoiding the inconsistencies in using the noun "nothing", which occurs in the literature. Foir instance, A Merriam-Webster® [1981] (abbreviated as "WTNID" – "Webster's Third New International Dictionary of the English Language") defines one of the meanings of the noun "nothing" in this manner:

«[3]**nothing** … *n* -s …**1 a :** no thing at all **:** something that does not exist …»

This definition is a *persuasive one* and *not a real one,* i.e. *not* a traditional *definition through a genus and the difference* (*definitio per genus et differentiam*). Also, the second part of this definition is a *contradictio in adjecto*, and hence the noun "nothing" thus defined is a *nomen nudum* (*mere name*, *naked name*).•

Df 2.3. 1) A distinct *entity*, of which I am *conscious* (*aware*) at a *current moment* when I wake, is called *my coentity* [*at that moment*] or, more explicitly, *a coentity of mine* (to emphasize the fact that I may have more than one coentity simultaneously), no matter whether or not somebody else is also conscious of the entity. Thus, a coentity of mine is an *ingredient* of my *universe* at any current moment. The prefix "co", occurring in the noun "coentity" has, a double meaning. First, it is a conventional perfective, associative, and collective prefix meaning, in this case, *joint*. Second, it is an abbreviation of "*conscious*". The name "coentity of a *sapient subject*", i.e. "coentity of a *man*", is defined likewise by extrapolation.

2) A coentity of mine is called:

a) a *real*, or *physical*, or *extramental*, or *exopsychical*, *one* – briefly a *res-coentity,* if it occurs in *the external or internal material world of mine*;

b) an *ideal,* or *mental*, or *psychical*, *one* if it occurs in *the mental* (*psychical*) *realm of mine*;

and similarly, by the *pertinent transcendental extrapolation*, with "*a sapient subject*" or "*a man*" in place of "*mine*". A sensible and hence real coentity of mine, especially one that occurs in my external world, can be a coentity of some other men (sapient subjects). By contrast, an ideal coentity of mine is seated in my mind (my cerebral cortex) and therefore I am the only sapient subject who is *immediately conscious* (*aware*) of it.•

Cnv 2.1: ***The rule of an*** "***I***"***-game.*** For the sake of simplicity and rigor, in order to treat of coentities of a certain sapient subject, I shall most often use the first person singular *egocentric phraseology* with respect to me after the manner of Df 2.3, while any concrete reader of this essay is expected to interpret that phraseology as relevant to him. (For the name of the convention and for its implications, see Cnv I.1.1 and Cmt I.1.2 in the TTL.)•

Df 2.4: ***The consciousness of a sapient subject.*** 1) For the purpose of study and description, the *consciousness*, i.e. the *entire mental state*, of a waking sapient subject is divided into simpler constituent parts of various kinds, which are most generally, but discriminately called *partial mental states*, the understanding being that, just as the consciousness itself, a partial mental state is a *mental process* (cf. James [1890; 1950, vol. 1, pp. 224–225]). "*Mental entity*", "*psychical entity*", "*ideal entity*", "*brain symbol*", "*idea*" [*sensu lato*], "*thought*" [*sensu lato*], and "*feeling*" [*sensu lato*] (cf. James [1890; 1950, vol. 1, pp. 185–187]) are some of many names that are used as synonyms of "mental state". Mental states are commonly and quite arbitrarily divided into *cognitions*, *conations*, and *affections*. *Cognitions* are mental processes of gaining knowledge, which include *sensations* (*percepts*), *conceptions* (*thoughts sensu stricto*), *concepts* (*conceptions expressed by abstract artificial symbols, especially by linguistic ones*), *acts of reasoning, recepts* (*mnemons*, *memory images*), *intuitions*, and *mental attitudes* including *attention*; *sensations* (*percepts*) are *cognitions by acquaintance*, whereas *concepts* are *cognitions by dialectics*, i.e. by *induction* or *deduction. Conations* or *volitions* are conscious drives to perform *volitional acts*, which include *decisions* and *fiats* (*mental cues*) *such as ideo-motor* *drives controlling the respective ideo-motor actions*. *Affections* include emotions or passions (as wish, anger, fear, hatred, love, libido, pleasure, displeasure, etc), beliefs, and any impulsive mental states of various names swaying cognitions or conations. The predicate "include" that I have used above rather than "consist of" or "comprise" indicate that the above lists of mental entities do no pretend to be complete.

2) A sapient subject, i.e. a normal (healthy) adult man (unless stated otherwise), has the following well-known *five exteroceptive senses* (*sense functions*): *the sense of sight*, *hearing*, *touch*, *taste*, and *smell,* called also the *visual*, *auditory*, *tactile*, *gustatory*, and *olfactory senses* in that order. The *sense of touch* comprises *two senses*: the *sense of temperature* (*warmth and cold*) and the *sense of pressure*. The *sense of taste* comprises *five senses*, namely, ones of *sweet*, *bitter*, and *umami*, all of which are mediated by *G-protein-coupled exteroceptors* ("*G*" is an abbreviation for "*globular*"), and ones of *salty* and *sour*, which are mediated by *ion-sensitive exteroceptors*. Umami is the sense of *taste of meat*, whose receptors have been discovered recently. In addition to his five exteroceptive senses, a man has the following *four* less definite but not less important groups of *interoceptive senses*, upon existence of which the general agreement has been reached among most biologists and most psychologists: *the sense of balance*, or *equilibrium*, also called *the labyrinthine sense*; *the sense of skeletomuscular motion and coordination*, called also *the kinesthetic sense* or *kinesthesia*; the *organic sense*, also called the *visceral sense*; *the sense of pain*. The four interoceptive senses are designed to inform the cerebral cortex of the sapient subject about the running state of various *parts of his body*, and not about the external world. Therefore, I classify all of them as interoceptive ones. Particularly, the labyrinthine sense (sense of balance) is *the sense of orientation of the subject's head relative to the direction of mass forces* as the field of earth gravity or the centrifugal force caused by rotation of the head. t

3) A *sensation* (*perception*) *of a man* is a *projective* (or *polarized*) *mental* (*psychical*) *coentity of the man*, i.e. one, which is located within the physical limits of the specialized part of his cerebral cortex, called the *sensorium*, and hence within the physical limits of the cerebral cortex, and which is *always involuntarily but consciously mentally* (*psychically*) *experienced* by the man as *his real* (*physical, extramental*, *exopsychical*, *res-*) *object*, which has stimulated the sensation and which I generally called an *onym*, or *nym¸ of the man*. Since a sensation of a man is his ideal (mental, psychical) entity, therefore it seems that he can *mentally* (*psychically*)

experience the sensation only as his *as if* (*as though, hypothetical*) real (physical, extramental, exopsychical, res-) object. However, there is a certain additional *inherent mechanism of immediate verification of authentity of the stimuli mediating concrete sensations* – a mechanism, owing to which the man *involuntarily* mentally experiences all his sensations *as his real objects*, *and not as his as if real objects*, without any *a posteriori voluntary verification*, although *spurious* (*as if*, *as though*) *real objects* may be detected afterwards in some cases.•

Df 2.5: ***Onymology (nymology).*** 1) Etymologically, both allomorphs "onym" and "nym" that are have been mentioned in Df 2.4(5), originate from the Greek noun "όνομα" \ónoma\ that assumes (takes on) the same meanings as the English nouns "*name*" and (gram.) "*noun*". Accordingly, the *etymological sense* of either one of the allomorphs "onym" or "nym" will be expressed by the *substantive* (*noun equivalent, noun construction*, *nounal name*) "*name sensu stricto*", i.e. "*name in a narrow sense*", while the *lexical sense* of either allomorph as used in this essay will be expressed by the *two denotatively synonymous* substantives "*sensible thing*" and "*name sensu lato*", i.e. "*name in a broad sense*". Henceforth, the substantive "*name sensu stricto*" will be abbreviated as "*name*", while the *lexical sense* of either of the two substantives is supposed to be the same as that of the substantive "*linguistic form*". That is to say, the three substantives "*name*", "*name sensu stricto*", and "*linguistic form*" are hereafter supposed to be synonyms.

2) A *complex monomen*, comprising either allomorph "*onym*" or "*nym*" *as its root* (*generic name*) and one or more prepositive prefixes or combining forms of Greek origin as the *qualifiers to the root*, is a description of the *species* through the *genus* denoted by the above root and through the *differences* denoted by the above qualifiers, i.e. it is a traditional *descriptio sprecies per genus et differentias*. Either allomorph "*onym*" or "*nym*" standing alone or any complex monomen of the above kind is indiscriminately called an *onymological*, or *nymological*, *monomen* (or similarly with "*term*" or "*noun*" in place of "*monomen*") and also an "*onym*"-*noun* or "*nym*"-*noun.* The constituent *graphonyms* "*graphonym*" and "*phononym*" of *onymological* (*nymological*) *terms* are abbreviated respectively as "*graph*" and as *"phon",* which are used as the pertinent *effective roots*. The system comprising onymological monomina and their abbreviations of the above kind and also

comprising kindred adjectives and adverbs of the full and abbreviated onymological monomina is a terminological *esperanto* that will be called *omymology* or *nymology*.

3) In the framework of the egocentric phraseology that is used in the essay, any element of onymology and also any one of the above three synonymous names "name", "name sensu stricto", and "linguistic form" is supposed to be followed either with the postpositive qualifier "*with respect to me*" (or "*in relation to me*") subject to Cnv 2.1 or with the postpositive qualifier "*with respect to a given* (*fixed*, *concrete and concretized*) *sapient subject*", by the corresponding transcendent extrapolation.•

Df 2.6: *The basic dichotomy of exteroceptive nyms by genesis.* All *exteroceptive* nyms are divided into two classes (kinds): *artificial* ones and *natural* ones. An *artificial*, or *man-made*, *nym* is one that is produced through the art, skill, and will (mental effort) of a man or group of men. By contrast, a *natural*, or *nature-made, nym* is one that is produced by *animate or inanimate nature*, without any *purposeful* agency of a man or group of men. An artificial nym can briefly be called a *technonym* and a natural nym a *physonym*.•

Df 2.7. 1) In accordance with Dfs 2.3, 2.4(1), and 2.6, a coentity of mine is called:

a) a *linguistic*, or *conceptual*, or *nominal*, *coentity of mine*, and also briefly a *nom-coentity of mine* if it is a concept of mine;

b) a *nonlinguistic coentity of mine* if otherwise, i.e. if it is *not linguistic.*

In this case,

i) a *linguistic* (*nom-*) *coentity of mine* is necessarily an *ideal* (*mental*, *psychical*) *coentity of mine*;

ii) a *real* (*physical*, *extramental*, *exopsychical*, *res-*) *coentity of mine* is necessarily a *nonlinguistic coentity of mine*;

iii) a *nonlinguistic coentity of mine* either is a *real coentity of mine* or an *ideal coentity of mine* other than any one of my concepts.•

Df 2.8: *Substantives*. 1) A *noun* or *noun equivalent* together with all pertinent limiting or unlimiting modifiers, *except a predicate*, which is one of the limiting nounal modifiers, is called a *substantive*. A substantive in the nominative (common) case of a singular number form is called an *unlimited singular substantive* (*ULSgS*) or briefly a *basic substantive* (*BS*) if it does not involve any limiting modifier, although it may involve any appropriate limiting modifiers, and a *limited singular substantive*

(*LSgS*) if it involves at least one limiting modifier and none or some unlimiting modifiers. An ULSgS or a LSgS is indiscriminately called a *singular substantive* (*SgS*). A BS, i.e. ULSgS, is called a *simple BS* (*SBS*) or *simple ULSgS* (*SULSgS*) and also an *unlimited simple SgS* (*ULSSgS*) if it is a *simple* (*mono-morpheme*) *singular noun* or a *simple singular noun equivalent* having the form of a *simple* (*mono-morpheme*) *word* other than a simple singular noun. A SgS is called a *simple* one (*SSgS*) if it is an *ULSSgS*, i.e. *SBS* (*SULSgS*), and a *complex*, or *descriptive*, one (*CSgS* or *DSgS*) if it is a complex (poly-morpheme) word or a word group. Consequently, an LSgS is unavoidably a CSgS (DSgS), whereas an ULSgS (BS) is either an SSgS (ULSgS, SULSgS, SBS) or a CSgS (DSgS). The denotatum of an SgS is called a *being*. The denotatum of an *SBS* is called an *elementary being*.

2) An ULSgS (BS) is called a *count singular substantive* (*CtSgS*), *individual* or *collective*, if it has a *plural number form*, which is called an *unlimited*, or *count, plural substantive* (*ULPlS* or *CtPlS*), and if hence the former can be used with the prepositive *numeral* (*numeric quantifier*) "1" ("one") as its *limiting modifier*, while the latter can be used with any of the prepositive *numerals* (*numeric quantifiers*) "2" ("two"), "3" ("three"), etc as their *limiting modifiers*, thus forming *dimensional numerals* that denote the corresponding *dimensional natural numbers*. I shall therefore use the qualifier "count" to "noun" or "substantive" interchangeably with "*numeralable*" meaning *capable of being modified with a numeral* and also meaning *capable of serving as a dimension of a numeral*. Thus, "*count singular substantive*" ("*CtSgS*") and "*numeralable singular substantive*" ("*NSgS*") are synonyms and therefore "*count plural substantive*" ("*CtPlS*"), and "*numeralable plural substantive*" ("*NPlS*") are also synonyms. It is understood that an NSgS can also be modified with the *indefinite article* as another *limiting modifier* if it is available in the pertinent native language (as in English, but not in Greek, Latin, Hebrew, or Russian), whereas an NPlS can also be *limited* (properly modified) with either of the prepositive *unspecific* quantifiers "*many*" and "*few*", denoting *unspecified numerable* (*numeric*) *quantities* and being two more limiting modifiers. A substantive, which comprises an NSgS or NPlS and an appropriate limiting modifier, is called a *limited NSgS* (*LNSgS*) or a *limited NPlS* (*LPlS*) respectively.

3) The fact that an NSgS has a plural number form signifies that the NSgS *designates* [with respect to me] a certain *multitudinous class,* i.e. a class that has strictly more than one member. This class is alternatively called a *multipleton*, in

analogy with the conventional term "*singleton*", denoting a class of a single member. Irrespectively to the mental mode, in which I use the NSgS, I say that the multipleton that it designates is the *designatum* (*designation value*, pl. "*designata*"), or redundantly *multipleton-designatum* or *class-designatum*, *of the NSgS* [with respect to me]. Accordingly, the NPlS designates *the power class of the multipleton-designatum of the NSgS*. Once I put the multipleton forward as the intended value of the NSgS, I say that the multipleton is *denoted by* the NSgS and also that it is the *multipleton-denotatum*, or less explicitly *class-denotatum*, of the NSgS, while the NSgS is called an *unlimited proper name* (*ULPrN*) *of the multipleton* or less explicitly an *unlimited proper multipleton-name* (*ULPrMnN*). In this case, the NPlS is said to *denote*, or to be an *unlimited proper name* (*ULPrN*) *of*, *the power class of the multipleton-denotatum* (*class-denotatum*) *of the NSgS*. Incidentally, an *unlimited proper singleton-name* (*ULPrSnN*), i.e. an *ULPrN of a singleton*, is not an NSgS, but an *unlimited non-numeralable singular substantive* (*ULNNSgS*) that has no plural number form (see the next item).

4) An ULSgS (BS) is called an *unlimited non-numeralable singular substantive* (*ULNNSgS*) if it is *unpluralizable*, i.e. if it *that has no plural number form* either universally or in a given circumstance, although in some other circumstances it may have a *numeralable homograph*, which can be pluralized. Accordingly, "non-numeralable" means *incapable of being modified either with the numeral* "1" ("*one*") *or with the indefinite article* (if the latter is available in the pertinent NL), and hence it also means *incapable of serving as a dimension of the numeral* "1". The fact that an ULNNSgS has no plural number form signifies that it *designates* [with respect to me] either *a certain singleton*, i.e. *the singleton of a certain entity*, or a *certain mass*, *material* or *abstract*. In the former case, where a given ULNNSgS designates a singleton, once I mentally put the singleton forward as the intended value of the ULNNSgS, I say that the singleton is *denoted by* the ULNNSgS or that it is the *singleton-denotatum*, or less-explicitly *class-denotatum*, of the ULNNSgS, while the ULNNSgS is called an *unlimited proper name* (*ULPrN*) *of the singleton* or an *unlimited proper singleton-name* (*ULPrSnN*). In the latter case, where a given ULNNSgS designates a mass, once I mentally put the mass forward as the intended value of the ULNNSgS, I say that the mass is *denoted by* the ULNNSgS or that the mass is the *mass-denotatum* ULNNSgS, while the ULNNSgS is called an *unlimited proper name* (*ULPrN*) *of the mass* or an *unlimited proper mass-name* (*ULPrMsN*).

5) Since an ULNNSgS, i.e. either an ULPrSnS or an ULPrMsN, has no plural number form, therefore it cannot be limited by any modifier, which is associated with counting. In English, an ULNNSgS can, depending on what it is, be limited by the definite article. Also, an ULPrMsN is limited in indefinite singular constructions by the prepositive limiting modifier "*some*" – in contrast to the indefinite article "a" or "an" limiting an NSgS in the like constructions. Besides "some", an ULPrMsN can, when appropriate, be limited either by some other prepositive *unspecific mass quantifier* such as "*much*", "*a lot of*", "*a little of*", or "*plenty of*" (e.g. "some water", "much money", "a lot of time", "a little of space", "plenty of trouble", etc), thus becoming a *limited common mass name* (*LCmnMsN*), or it can be limited by a prepositive *specific mass quantifier* (*possessive dimensional numeral*) such as "*a bottle of*", "*two bottles of*", "*three bottles of*", etc (applied, e.g., to "water", "juice", or "wine"), thus becoming a *limited proper name of a common member of the respective class*. Consequently, the last case should more correctly be interpreted as follows. The string of an ULPrMsN and a preceding preposition in that order, which follows a limited or unlimited NSgS or NPlS, is a *postpositive qualifier* to the NSgS or NPlS, which has nothing to do with various roles that the ULPrMsN may play in some other occurrences.

6) A DBS (CBS), i.e. a descriptive (complex) BS (ULSgS), is more specifically called in English a *description*, or even more explicitly a *description of the species*, *through a genus and the difference*, or *differences*, – briefly *DcTrG&D*, *DcSTrG&D*, *DcTrG&Ds*, or *DcSTrG&Ds* in that order, and also in Latin *descriptio*, or *descriptio species*, *per genus et differentias*, or *differentiam*, respectively. A *definition* whose definiens is a DcTrG&D or DcTrG&Ds is a traditional *definition through the genus and difference* (*differentia*), or *differences* (*differentiae*), – briefly a *DfTrG&D* or *DfTrG&Ds*, in Latin *definitio per genus et differentiam*, or *differentias*, which was inaugurated by Aristotle [350 BCE, *Posterior Analytics*] and which is often called a *real*, or *explicative*, *definition*. Any of the above terms that contain an occurrence of the word "differences" ("Ds") is supposed to be applicable also in the case, where there is a single difference to the genus, i.e. it is supposed to include its variant with "difference" ("D") in place of "differences" ("Ds"). The plural number forms of the terms and of their abbreviations will, when necessary, be made by replacing the nouns "description" and "definition", abbreviated as "Dc" and "Df", with "descriptions" and "definitions", abbreviated as "Dcs" and "Dfs", respectively. A

DcTrG&Ds (DcSTrG&Ds) or particularly DcTrG&D (DcSTrG&D) is briefly called a *descriptive name* (*DN*), and also a *description* if there is no dander of misunderstanding. In this case, the abbreviation "*Dc* is used equivocally for "*description*" or "*descriptive*". A DcSTrG&Ds (particularly a DcSTrG&D) is more precisely called a *description of the species through the intersection of the genus, designated by the pertinent generic substantive* ("*GS*"), called also "*generic name*" ("*GN*"), *and through the differences* (correspondingly, *the difference*)*, designated by the pertinent qualifiers* (correspondingly, *qualifier*). To be recalled, a DcSTrG&Ds or DcSTrG&D has been briefly called a *DBS* (*descriptive basic substantive*), while in contrast to a *GS* (*GN*), it will briefly be called a *descriptive specific name* (*DSN*). A *qualifier* (*Ql*, pl. "*Ql's*") to a GN can be either prepositive (as a prefix, combining form, adjective, or adjective equivalent) or postpositive (as an adjective equivalent). However, every qualifier occurring in an *onymological* (-"*onym*") or *onological* (-"*on*") *monomial DSN* is a prepositive one (namely an Anglicized prefix or combining form).

7) A DSN (DBS) is either a *count DSN* (*CtDSN*), called also a *numeralable* (capable of being modified by a numeral or by the indefinite article) *DSN* (*NDSN*), or a *non-numeralable* (incapable of being modified either by a numeral or by the indefinite article) *DSN* (*NNDSN*). An NDSN is a DSN of a *multitudinous* (*many-member*) *class-species* (*specific class*), which is alternatively called a *multipleton-species* (*specific multipleton*), and therefore the NDSN is alternatively called a *descriptive specific multipleton-name* (*DSMN*). An NNDSN is either a DSN of a *one-member class-species* (*specific class*), which is alternatively called a *singleton-species* (*specific singleton*), or a DSN of a *mass-species* (*specific mass*). Accordingly, the former NNDSN is alternatively called a *descriptive specific singleton-name* (*DSSN*) and the latter NNDSN a *descriptive specific mass-name* (*DSMsN*). A DSMN or a DSSN is indiscriminately called a *descriptive specific class-name* (*DSCsN*). The GN of a DSCsN is a *count*, or *numeralable*, *GN* (*CtGN* or *NGN*), which is alternatively called a *generic class-name* (*GCsN*) or *generic multipleton-name* (*GMN*), because it necessarily denotes a *generic multitudinous class*, which is called a *class-genus* (*generic class*) or alternatively *multipleton-genus* (*generic multipleton*); and vice versa. Likewise, the GN of a DSMsN is a *mass GN* (*MsGN*), which is alternatively

called a *generic mass-name* (*GMsN*), because it necessarily denotes a *mass-genus* (*generic mass*); and vice versa.•

3. Aristotle's «*Categories*» («*Κατηγορίαι*»)

3.1. Preliminary remarks

«*Categories*» (see [ACE] or [ACO]) is one of the most important and influential and at the same time one of the most controversial, equivocal, and enigmatic Aristotle's writings. In this connection, Studtmann [2008] says:

> «Aristotle's *Categories* is a singularly important work of philosophy. It not only presents the backbone of Aristotle's own philosophical theorizing, but has exerted an unparalleled influence on the systems of many of the greatest philosophers in the western tradition. The set of doctrines in the *Categories*, which I will henceforth call *categorialism*, provides the framework of inquiry for a wide variety of Aristotle's philosophical investigations, ranging from his discussions of time and change in the *Physics,* to the science of being *qua* being in the *Metaphysics*, and even extending to his rejection of Platonic ethics in the *Nicomachean Ethics*. Looking beyond his own works, Aristotle's categorialism has engaged the attention of such diverse philosophers as Plotinus, Porphyry, Aquinas, Descartes, Spinoza, Leibniz, Locke, Berkeley, Hume, Kant, Hegel, Brentano and Heidegger (to mention just a few), who have variously embraced, defended, modified or rejected its central contentions. All, in their different ways, have thought it necessary to come to terms with features of Aristotle's categorial scheme.
>
> Plainly, the enterprise of categorialism inaugurated by Aristotle runs deep in the philosophical psyche. Even so, despite its wide-reaching influence — and, indeed owing to that influence — any attempt to describe categorialism faces a significant difficulty: experts disagree on many of its most important and fundamental aspects. Each of the following questions has received markedly different answers from highly respected scholars and philosophers. What do the categories classify? What theory of predication underlies Aristotle's scheme? What is the relationship between categorialism and hylemorphism, Aristotle's other major ontological theory? Where does matter fit, if at all, in the categorial scheme? When did Aristotle write the *Categories*? *Did* Aristotle

write the *Categories*? Is the list of kinds in the *Categories* Aristotle's considered list, or does he modify his views elsewhere? Is Aristotle's view of substance in the *Categories* consistent with his view of substance in the *Metaphysics*? Is there some method that Aristotle used in order to generate his list of categories? Is Aristotle's categorialism philosophically defensible in whole or in part? If only in part, which part of categorialism is philosophically defensible? »

According to the article «**category**» of Wikipedia, categories are «perhaps the single most heavily discussed of all Aristotelian notions». In the following brief review of the subject matter of «*Categories*», I shall make explicit only those debatable aspects of that book, which are, in my view, most immediately relevant to Aristotle's viewpoint on universals and reality.

3.2. Aristotle's inconsistent axiomatic four-fold taxonomy of things

In general outline, in his treatise «*Categories*» Aristotle *axiomatically* establishes two different semantic taxonomies (classifications) of words and thereby he establishes the respective taxonomies *of the denotata* (*denotation values*, sing. "*denotatum*") *of*, i.e. *of the things* (*entities*) *denoted by*, *the words*. The first taxonomy is a *four-fold taxonomy of words* and hence of their denotata from the standpoint of capability or incapability of the words, or of their denotata, to serve as *predicates* of some other words or word groups, or of their denotata, respectively. The second taxonomy is a *ten-fold taxonomy of categories*, i.e. it is actually a *list of one-word names of ten classes of predicates,* while the *taxonyms* (*taxonomic names*) or the *taxa* (*taxons, taxonomic classes*) or both are called "*categories*". In order to explicate conveniently the implications of the fact that both Aristotle's taxonomies deal with separate *simple words*, and neither with *complex* (*composite*) *words* nor with *word groups*, I shall henceforth say that a thing is an *induced*, or *basic*¸ one if it is denoted by a single simple word and a *deduced*, or *derivative*, one if it is denoted either by a complex word or by an expression consisting of two or more words. It is also noteworthy that by "*a predicate*" Aristotle means *a grammatical predicative*, i.e. *the substantive complement of the link-verb in a compound grammatical predicate*. For instance, the predicative "mammals" in the sentence "All men are mammals" and the predicative "men" in the sentence "Some mammals are men" are Aristotelian *predicates* of those sentences.

Aristotle's first taxonomy proceeds from the following persuasive definitions [ACE, Part 2]:

> «Forms of speech are either simple or composite. Examples of the latter are such expressions as 'the man runs', 'the man wins'; of the former 'man', 'ox', 'runs', 'wins'.
>
> Of things themselves some are predicable of a subject, and are never present in a subject. Thus 'man' is predicable of the individual man, and is never present in a subject.
>
> By being 'present in a subject' I do not mean present as parts are present in a whole, but being incapable of existence apart from the said subject.
>
> Some things, again, are present in a subject, but are never predicable of a subject. For instance, a certain point of grammatical knowledge is present in the mind, but is not predicable of any subject; or again, a certain whiteness may be present in the body (for colour requires a material basis), yet it is never predicable of anything.»

The grammatical predicates "*is predicable of*" and "*is capable of being said of*", i.e. "*can be said of*", are synonyms. Consequently, the first taxonomy is tacitly based on the assumption (axiom) that a basic thing denoted by a certain word can always be put into a correspondence with another thing, basic or derivative, according to which the former thing either (a_1) is *predicable*, or *said*, *of* the latter thing or (b_1) is *not predicable*, or *not said*, *of* the latter thing and at the same time the former thing either (a_2) is *present in* the latter or (b_2) is *not present in* the latter. That is to say, from pure *combinatory considerations*, the former thing always stands to the latter thing in exactly one of the following *four* correspondences: (i) the former *is not said* (*predicable) of* and i*s not present in the latter*; (ii) the former *is not said* (*predicable*) *of and is present in the latter*; (iii) the former *is said (predicable) of and is not present in the latter*; (iv) the former *is said* (*predicable*) *of* and *is present in the latter*. In this case, the expressions "*said of*" and "*present in*", often hyphenated as "*said-of*" and "*present-in*", and hence their negations, are in fact undefined technical terms, so that *said-of* and *present-in* are undefined *conceptual predicates* (*relations in intension*), regarding to which Studtmann [2008] says:

«Because these are technical notions, one would expect Aristotle to have defined them. Unfortunately, he does not define the said-of relation; and his definition of the present-in relation is either circular or rests on an undefined concept of being in. He says: 'By "present in a subject" I mean what is in something, not as a part, and cannot exist separately from what it is in' (1a24-5). Notice that the word 'in' occurs in this definition of present-in. So, either 'in' means the same as 'present-in', in which case the definition is circular; or 'in' is itself in need of a definition, which Aristotle does not give. Hence, Aristotle's first system of classification rests on technical concepts whose precise characterization is not settled by anything Aristotle says.»

Therefore, Aristotle's first taxonomy is an *axiomatic* one and it remains far-fletching and unintelligible until any consistent interpretation of it is found. However, such an interpretation does not likely exist. In any case, the *interpretand* of Aristotle's first taxonomy, which is described in the following quotation of Studtmann [2008] as one that is accepted by many scholars¸ is *easily refutable* and is therefore *not defendable*:

«Despite the lack of helpful definitions of these two concepts, there is a fairly straightforward, though certainly not uncontroversial, characterization of them that many scholars have adopted. By focusing on Aristotle's illustrations, most scholars conclude that beings that are said-of others are universals, while those that are not said-of others are particulars. Beings that are present-in others are accidental, while those that are not present-in others are non-accidental. Now, non-accidental beings that are universals are most naturally described as essential, while non-accidental beings that are particulars are best described simply as non-accidental. If we put these possibilities together, we arrive at the following four-fold system of classification: (1) accidental universals; (2) essential universals; (3) accidental particulars; (4) non-accidental particulars, or what Aristotle calls *primary substances*. This system maps readily onto Aristotle's own terminology, given at 1a20: (1) Said-of and present-in: accidental universals; (2) Said-of and not present-in: essential universals; (3) Not said-of and present-in: accidental particulars; and (4) Not said-of and not present-in: primary substances. A brief discussion of each of these classes should suffice to bring out their general character.»

The above interpretand has various aspects, owing to which it is not self-consistent and is therefore unacceptable – just as Aristotle's original first taxonomy, being its *interpretans* (pl. "*interpretantia*"). Here is the most conspicuous one of those aspects. Whatever it is, a universal is the respective common property of every particular (instance, specimen), which exemplifies it. At the same time, any thing is the only carrier of its every property, so that all the properties must *be present in the thing*, i.e. *be immanent* (*inherent*) *to it* and hence *be physically inseparable from it*. However, in accordance with the above interpretand, there are *two kinds of universals* such that any universal of one kind is present in every particular carrying it as its property, while any universal of the other kind is not present in any particular carrying it as its property. For instance, Aristotle explicitly asserts that «'man' is predicable of the individual man, and is never present in a subject» (see the quotation of [ACE] at the beginning of this subsection). On the other hand, it may be asked how a certain property can be present in the subject if it is not predicable of the latter? I cannot put my finger on any specific passage either of Aristotle's «*Categories*» or of any other of his works, where his *undefined* "present in"-term is explicitly used as an argument against Plato's doctrine of transcendental *real* Universes, i.e. against Plato's *extreme realism*. At the same time, the various mutually inconsistent uses of that term in «*Categories*» indicate that Aristotle's world outlook cannot be interpreted as *moderate realism.* The inconsistencies of Aristotle's terms "*predicable of*" ("*said of*") and "*present in*" are immediately eliminated if Aristotle's artificial axiomatic four-fold taxonomy of things is replaced with a natural *axiomatic dichotomic immanence principle* for things as stated below in Ax 3.1.

3.3. An alternative axiomatic dichotomy of things: Immanence principles

Ax 3.1: ***The general immanence principle for things.*** 1) An entity is called:

a) the *empty entity*, *empty individual*, or *empty primary particular*, and also *nothing* if and only if it is *present in* any other entity and in itself;

b) a *nonempty entity* and also *a thing*, *some thing*, or *something* if and only if it is *not empty*, i.e. if and only if it is *not present in* some, i.e. in strictly some or in all, other entities (some things).•

2) A thing (nonempty entity) *is present in,* i.e. *is immanent* (*inherent*) *to, another thing* if and only if the former is *predicable* (*said*) *of the latter.*•

Cmt 3.1. In accordance with Ax 3.1(2), I shall not use the antonymous qualifiers "accidental" and "non-accidental" in the sense defined in the previous quotation of Studtmann [2008], but I can use them in some other senses.•

Cmt 3.2. In accordance with the examples, by which Aristotle illustrates his four-fold taxonomy, the latter is supposed to apply primarily to *simple basic substantives SBS's* as defined by Df 2.8(1). I do not intend to revise the whole of Aristotle's «*Categories*». However, the notion of immanence of things is decisive for solving the problem of universals. I shall therefore supplement that notion with some additional aspects in order to make it perfectly clear.

Here is immediate corollary of Ax 3.1 and Df 2.8, which comes instead of Aristotle's four-fold taxonomy of things.•

Crl 3.1: *The primary first dichotomy of the denotata of SgS's*: *said-of and present-in things versus not said-of and not present-in things.* Given two things denoted by *SgS's*, one of them either (a) *is predicable* (*said*) *of and is present in the other one* or (b) *is not predicable* (*not said*) *of and is not present in the other one*.•

Df 3.1: *The secondary first dichotomy of the denotata of SgS's*: *universals versus particulars.* 1) The denotatum of a SgS is called:

a) a *nonempty primary*, or redundantly *primary proper*, *particular* or *nonempty individual* if it *is not predicable of and hence is not present in any other thing*;

b) a *secondary proper particular* if it *is predicable of and hence is present in exactly one other thing*;

c) a *universal* if it *is predicable of and hence is present in two or more other things*.

2) The *empty primary particular* (*empty individual*) or a *nonempty primary particular* (*nonempty individual*) is indiscriminately called a *primary particular* (*individual*). A *primary* [*proper*] *particular* (*empty individual*) or a *secondary proper particular* is indiscriminately called a *proper particular*. The denotatum of a SgS, which *is predicable* (*said*) *by a universal*, is called a *particular*, so that *a proper particular is a particular* but not necessarily vice versa. A particular that is not proper is said to be an *improper,* or *common*, or *general,* or *universal, particular*, and also a *particular universal,* because it is a *universal*, i.e. it is in turn *predicable of and hence is present in two or more other things*. A *particular universal* is called a *relative*

xuniversal and vice versa, while a *universal that is not particular*, i.e. a *universal of a highest kind, which is not supposedly predicable by any other universal and which does not therefore contain any other universal as its property*, is called an *absolute universal* or a *category.*

3) Here follow some examples and also some alternative or more specific names of the particulars and universals defined in the above items 1 and 2.

a) Socrates, Xantippe, Aristotle, a man, or a horse is a *nonempty primary particular* or *nonempty individual*, in accordance with the item 1a. By contrast, the *empty primary particular* or *empty individual*, called also the *empty class* or *empty set*, which has been introduced by Ax 3.1(1a), is conventionally denoted by '∅', e.g. in Halmos [1960, p. 8], although some other logographs are also in use instead of '∅', for instance 'Λ' in Whitehead and Russell [1910; 1925; 1962, pp. 216, 217, **∗24.02**] or 'O' in Fraenkel et al [1973, p. 39].

b) A unique thing, of which the pertinent secondary particular is predicable in accordance with the item 1b and in which it is present, is an *object sui generis* and therefore it *ipso facto* produces the *singleton of its own*, being that very secondary particular. For instance, "husband of Xantippe", "wife of Socrates", or "founder of logic" is an ULSgS (BS), or more precisely an ULPrN (unlimited proper name), of the *singleton of Socrates*, *Xantippe*, or *Aristotle* respectively. Now, I can always use a singleton in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental* (*psychical*) *mode*, in which I *mentally experience it as my as if extramental* (*exopsychical*) *object* being the member of the singleton (cf. Ax 3.2(2)). In English (e.g), in order to indicate that I use the pertinent ULSgS (ULPrN) of a singleton in the above projective mode, I attached that name with the definite article thus turning it into the descriptive LDNSg that serves as a proper name of the member of the singleton. Therefore, each one of the following three sentences is *veracious*:

"Socrates is the husband of Xantippe", (3.1)

"Xantippe is the wife of Socrates", (3.2)

"Aristotle is the founder of logic". (3.3)

In accordance with the pertinent terminology of the TTL, "veracious" means *accidentally true*, i.e. *true but neither valid nor antivalid* or briefly *true vav-neutral* or *true vav-indeterminate* – in contrast to "*tautologous*" meaning *universally true*, i.e. *true valid.* Thus, "Socrates" and "the husband of Xantippe", "Xantippe" and "the wife of Socrates", and "Aristotle" and "the founder of logic" are three pairs of *denotative synonyms*. That is to say, the husband of Xantippe, the wife of Socrates, and the founder of logic are *nonempty individuals* – the same as Socrates, Xantippe, and Aristotle respectively. At the same time, by Ax 3.1(2), it follows from (3.1), e.g., that the singleton *husband of Xantippe is predicable* (*said*) *of Socrates* and *is hence present in Socrates*. Likewise, it follows from (3.3) that the singleton *founder of logic is predicable* (*said*) *of Aristotle* and *is hence present in Aristotle*. Incidentally, when I mention (denote) any concrete individual (primary particular) in this discussion – the empty one ∅ or a nonempty one, e.g. Aristotle, etc, I unavoidably use but do not mention its proper name '∅' or "Aristotle" along with its singleton $^{\backslash}\varnothing^{/}$ ($\{\varnothing\}$) or $^{\backslash}$Aristotle$^{/}$ ({Aristotle}) as its *connotatum* (*connotation value*, pl. "*connotata*").

c) A *universal* as defined in the item 1c is called:

i) a *class-universal* or *many-member class* and also briefly a *multipleton* if it is denoted by an *individual* or *collective CtSgS* (*NSgS, ULPrMnN*), in accordance with Df 2.8(2,3)*;*

ii) a *mass-universal* or briefly a *mass* if it is denoted by a *material* or *abstract ULNNSgS* (*ULPrMsN*), in accordance with Df 2.8(4,5).

4) The empty class (empty individual, empty primary particular), a singleton (secondary particular), and a multepleton are classes. However, the empty class is the memberless class, a singleton is a class of exactly one object, called its *member* or its *element*, while a multipleton is a class of many (two or more) objects and it is therefore called a *class-universal*. Therefore, the empty class or a singleton is called the *empty class-particular* or a *nonempty class-particular* respectively and also an *empty class-particular* indiscriminately, while a multipleton is alternatively called a *class-universal*. The self-explanatory *hyphenated appositional name* "*singleton-particular*" can be used synonymously (interchangeably) with "*secondary particular*".

5) Besides itself as its *whole part*, a singleton has its member as its only *nonempty instance*. A multipleton has nonempty instances of two kinds, namely its *members*, called also its *elements*, and its nonempty *parts*, called also its nonempty *subclasses.* By contrast, a mass-universal (mass) has nonempty instances of one kind, namely its nonempty *parts*, called also its nonempty *submasses.*•

Cmt 3.3: ***The secondary first dichotomy of the denotata of SBS's.*** Here follows the instance of Df 3.1(1) with "SBS" ("simple basic substantive") in place of "SgS". The denotatum of an SBS is called:

a) a *particular* if it *is not predicable of and hence is not present in any other thing*;

b) a *universal* if *it is predicable of and hence is present in at least one and unavoidably some more other things*.●

Cmt 3.4. 1) In accordance with the item V of subsection 2.2, by asserting the sentence:

$$\text{“}x \text{ is a real number”,} \tag{3.4}$$

the CtSgS (count singular substantive) "*real number*" *is predicated* (*said*) *of* the logographic variable '*x*' and hence the entity (my coentity) *real number*, denoted by the CtSgS, *is predicated* (*said*) *of* the denotatum (my coentity) *x* demoted by '*x*'. Hence, by Df 3.1, the entity *real number* is a *universal*, called also a *multipleton* (*many-member class*), while the entity *x* is its *particular*, called also *a* real number, so that *the universal should be present in x.* In what follows, I shall, for illustration, make explicit the most conspicuous aspects that the relation *present in* has in this case.

2) Except for *rational real numbers* and *algebraic real numbers*, i.e. the roots of algebraic equations whose coefficients are rational numbers, and also except for *special irrational* (*transcendental*) *numbers* as π or *e*, most *irrational* real numbers have no proper names. Indeed, irrational real numbers are by definition real numbers *not being quotients of integers*. Therefore, an irrational real number is often thought of as a real number that can be expressed by an *imaginary infinite decimal-fraction numeral* with supposedly *uncountable* number of digits, no finite sequence of which is repeated indefinitely. However, such an *imaginary* numeral can not actually be written down and *it is therefore a mental image of no graphic symbol*, i.e. it is a *fiction*. Therefore, an actual *graphic symbol* that consists of a finite sequence of decimal digits followed by three dots is conventionally used as a makeshift of the above fictitious infinite numeral. However, such a graphic symbol, – say, '3.1415...' that is used as a makeshift of 'π', – is not a proper name of any number. At the best, the expression '3.1415...' can be regarded as a variable, i.e. as a common name, which stands for any real number in the semi-closed interval [3.1415, 3.1416). On the other hand, when the three dots are omitted from the expression '3.1415...', the latter turns

into the constant '3.1415' which is a proper name of the specific rational real number 3.1415. Thus, the rational number π is thought of as one that is represented by an as if non-periodic infinite decimal-fraction numeral '3.1415...' with supposedly *uncountable* number of digits, *which is equipollent to the power of continuum*. Any irrational real number that can be understood by '*x*' occurring in (3.4) has a like paradoxical digital structure.

4) Alternatively, the variable '*x*' occurring in (3.4) can be understood as a *constant* that denotes the *common* (*general*, *universal*) *member of the multipleton*, which I mentally experience as *my as if extramental* (*exopsychic*) *object that is just another hypostasis* (*way of existence*) *of the multipleton*.

5) The multipleton *real mumber,* i.e. the *class* (*set*) *of real numbers*, can be thought of as an *infinite open interval* (-∞,+∞), while any given real number is a certain point of the interval. At the same time, a point is intuitively thought of as a primitive (elementary) entity. The first known definition of a point as given by Pythagoras says that a point is «a monad having position». In Euclid [1956, vol. 1, pp. 153, 154]: «A *point* is that which has no part.» The paradox, according to which a point, being a supposedly elementary object of continuum, turns out to be not namable by any sequence of a countable number of digits, can be called *the problem of continuum*.

6) In modern mathematics, there is no notion of points neighboring to a point, just as there is no notion of *smallness* at all. No matter how small a real number ε is in our intuitive understanding, the transformation $y = \tan\frac{\pi x}{2\varepsilon}$, e.g., maps the interval $(-\varepsilon, \varepsilon)$ of denotata (denotation values) of the variable 'x' into the entire set $(-\infty, +\infty)$ of real numbers, being the domain of values of the variable 'y'. Hence, the intervals $(-\varepsilon, \varepsilon)$ and $(-\infty, +\infty)$ are *equipollent* or, loosely speaking, they have the same number of points; each interval has the power of continuum. This is why the mathematicians have abandoned their concept of *infinitesimals,* as being, supposedly, infinitely small but nonzero real numbers, – just as the physicists have abandoned their concept of *ether.* The only kind of smallness that exists in mathematics is *comparative* one, which can be defined with the help of the so-called "*ε&δ-language*" and which can therefore be alternatively called "*ε&δ-smallness*". In this language, for instance, the continuity of a real-valued function of one real variable at a given point

is defined thus: "*A real-valued function* f *defined on* $(-\infty,+\infty)$ *is said to be continuous at a point* x_0 *if and only if for each* $\varepsilon > 0$ *there exists* $\delta > 0$ *such that for each* $x \in (-\infty,+\infty)$ *such that* $|x - x_0| \leq \delta$*:* $|f(x) - f(x_0)| \leq \varepsilon$" (see any introductory course of higher mathematics). The $\varepsilon\&\delta$-language is the only possible way, in which the notion of convergence of any relevant object of mathematics (as an infinite sequence of numbers, a numerical or functional series, or an improper integral) can be defined. Therefore, *the* $\varepsilon\&\delta$*-language should be regarded as the only possible solution of the problem of continuum.* Modern topology and modern theory of metric spaces are based on that language. A *relative smallness as a practical notion*, say that expressed by the factor $\frac{1}{10}$, has nothing to do with the *formal comparative smallness*, which is expressed by the $\varepsilon\&\delta$-language.•

Cmt 3.5. In forming or using verbal predicates, whose predicatives are CtSgS's (NSgS's), one should remember and take into accounts the remarks that have been made in the items IV an V of the subsection 2.2 (cf. Df 3.1(3b)). Using predicates, whose predicatives are LNNSgS's or ULNNSgS's is much less ambiguous. For instance, the substance *water,* denoted by the noun "water", is *predicable* (*said*) *of some*, i.e. *of the whole or of strictly some, water* occurring in a given bottle and *of some water of* (*occurring in*) a given river and of *some water of* a given sea, etc, and at the same time *water is present in* each of the above objects. Here is a somewhat different example. Both the adjective "*white*" and the color denoted by this adjective is *predicable* (*said*) *of some white* (*not dirty*) *snow* and *of some white* (*not colored*) *paper,* – i.e. *of some white entity* (*white being*) in general, – and at the same time *white* [*color*] or *white entity is present in* each object of the above two kinds. In this case, the adjective "white" can be regarded as an *absolute adjective* in the sense of either *mass noun equivalent* "*white color*" or "*white entity*", so that the grammatical predicate "*is white*" is equivalent to either grammatical predicate "*is of white color*" or "*is a white entity*".•

Ax 3.2: *The second dichotomy of denotata of SgS's: linguistic (and hence ideal) things* (*nom-things*) *versus real (and hence nonlinguistic) things* (*res-things*).

1) In accordance with Df 2.7, the denotatum of a SgS is called:

a) a *linguistic* (*conceptual*, *nominal*, *nom-*) and hence *ideal thing* if it is a like coentity of mine;

b) a *real* (*physical*, *extramental, exopsychical*) and hence *nonlinguistic thing* if it is a like coentity of mine.

2) To any *real* (*physical, extramental, exopsychical*, *res-*) *thing*, there always corresponds, or can be put into correspondence, a substantive standing for a certain *linguistic* (*conceptual*, *nominal*, *nom-*) *thing*, but not necessarily vice versa; e.g., the substantives “a centaur”, “the 16^{th} president of the USA in the years 1913–21”, and “the capital of the USSR in AD 2000” are *contradictiones in adjecto* and therefore they have *neither linguistic nor real denotata*. In order to *refer to* (*mention*, *denote*) the res-thing, I use the pertinent counterpart nom-thing along with the substantive designating it in the respective *projective* (*polarized*, *extensional*) *mental* (*psychical*) *mode*, in which I *mentally experience the nom-thing as my* ***as if*** (***as though, hypothetical***) *real* (*physical, extramental*, *exopsychical*, *res-*) *object.* I do *so involuntarily but consciously* – just as I mentally experience the *percept* (*sensation*) *of any given nym* (*sensum*, *sensory object*) *as* **that** *nym*, particularly in the case when the nym is a graphic one, i.e. a graphonym. Thus, the fundamental difference between a percept (sensation) of mine and a concept of mine is that I always mentally experience the former as a *certain sensible* and hence *real* (*physical, extramental, exopsychical, res-*) *object of mine*, while I always mentally experience the latter as an ***as if*** (***as though, hypothetical***) *real coentity of mine*, which *can be* either *an* ***as if*** *sensible one* or *an insensible real one*, but which *can never be a* **sensible one** *indeed.* This is self-evident because a man has no mental mechanism, either inherent or acquired, by which he could metamorphose some of his concepts into his percepts. Any statement of a man in the projective mental mode, in which he mentally experiences his certain concept as his as if real object, is in fact an implicit hypothesis postulating real existence of that object. If it is known from other sources that the object exists then the statement made is consistent. Otherwise, real existence of the object should be subject to the appropriate *a posteriori* verification.•

Ax 3.3: ***The detailed immanence principle for linguistic*** **(*nom-*)** ***things and for real*** **(*res-*)** ***things.*** In accordance with Ax 3.2, Ax 3.1 applies separately with “*linguistic thing*” and with any of its synonyms (as “*nom-thing*”) or with “*real thing*” and with any of its synonyms (as “*res-thing*”) in place of “*thing*”.•

Crl 3.2: ***The detailed primary first dichotomy of things*****:** ***said-of and present-in linguistic*****,** ***or real*****,** ***things versus not said-of and not present-in linguistic, or***

correspondingly real, things. In accordance with Ax 3.3, Crl 3.1 applies with "*linguistic thing*" and with any of its synonyms (as "*nom-thing*") or with "*real thing*" and with any of its synonyms (as *"res-thing*") in place of "*thing*".•

Crl 3.3: *The detailed secondary first dichotomy of things*: *linguistic* (*nom-*) *universals versus linguistic* (*nom-*) *particulars and real* (*res-*) *universals versus real* (*res-*) *particulars.* 1) In accordance with Axs 3.2 and 3.3, depending on my mental attitude towards the meaning of the expession "*the denotatum of a SgS*", Df 3.1 applies separately either with "*linguistic particular*" and "*linguistic universal*" and with any of their synonyms (as "*nom-particular*" and "*nom-universal*") or with "*real particular*" and "*real universal*" and with any of their synonyms (as "*res-particular*" and "*res-universal*") in place of "*particular*" and "*universal*" respectively.

2) In general, when appropriate and necessary, any one of the generic nouns

"*particular*", "*universal*", "*class*", "*mass*", "*singleton*", "*multipleton*", (3.5)

occurring in the terms introduced in Df 3.1, can be adhered either with any one of the prepositive synonymous qualifiers:

"*linguistic*", "*conceptual*", "*nominal*", "*nom-*" (3.6)

of the prepositive synonymous qualifiers:

"*real*", "*physical*", "*extramental*", "*exopsychical*", *"res-*" (3.7)

in a grammatically congruous way. The substantives thus obtained stand with one another in a certain system of binary semantic relations with respect to any given man (sapient subject), for instance, in the following semantic relations *with respect to me*.

3) By Ax 3.2, to *any real* (*physical, extramental*, *exopsychical*, *res-*) *coentity of mine* of the range of a substantive that is formed by adhering a certain one of the generic names of the list (3.5) with any one of the prepositive qualifiers of the list (3.7), there *corresponds a linguistic* (*conceptual*, *nominal*, *nom-*) *coentity of mine* of the range of a substantive that is formed by adhering the *same* generic name of the list (3.5) with any one of the prepositive qualifiers of the list (3.6) as follows. I use the latter linguistic coentity along with the substantive designating it in the respective projective (polarized, extensional, connotative) mental mode, in which I *mentally experience it as my as if real coentity* that I associate with the corresponding former real coentity. In accordance with Df 2.3, a sensible and hence real coentity of mine occurs in my real (external or internal) world and therefore it can be a coentity of some other men (sapient subjects). By contrast, an ideal and particularly linguistic

contity of mine is seated in my mind (my cerebral cortex) and therefore I am the only sapient subject who is *immediately conscious* (*aware*) of it. At the same time, an insensible real coentity of mine is *conformable to* a certain linguistic coentity of mine, which I mentally experience as an *as if* real (extramental, exopsychical) coentity. Consequently, the above correspondence (association) between a real coentity of mine and the counterpart linguistic coentity of mine is self-evident (unquestionable) if the real coentity is supposedly a sensible one that is known from another source, and it requires the appropriate *a posteriori* verification, including a certain *interpersonal verification*, if the real coentity is supposedly insensible.•

Cmt 3.6. 1) By the pertinent transcendental extrapolation, Crl 3.3(3) applies, *mutatis mutandis*, to any given man (sapient subject) in place of me. Namely, to *any real* (*physical, extramental*, *exopsychical*, *res-*) *coentity of a given man* (*sapient subject*) of the range of a substantive that is formed by adhering a certain one of the generic names of the list (3.5) with any one of the prepositive qualifiers of the list (3.7), there *corresponds a linguistic* (*conceptual*, *nominal*, *nom-*) *coentity of the man* (*sapient subject*) of the range a substantive that is formed by adhering the *same* generic name of the list (3.5) with any one of the prepositive qualifiers of the list (3.6) as follows. The man uses the latter ideal coentity along with the substantive designating it in the respective projective mental mode, in which he *mentally experiences it as his as if real coentity* that he associates with the corresponding former real coentity. In accordance with Df 2.3, a sensible and hence real coentity of the man occurs in his real (external) world and therefore it can be a coentity of some other men (sapient subjects). By contrast, an ideal and particularly linguistic contity of the man is seated in his mind (his cerebral cortex) and therefore he is the only sapient subject who is *immediately conscious* (*aware*) of it. At the same time, an insensible real coentity of the man is *conformable to* a certain linguistic coentity of his, which he mentally experiences as an *as if* real (extramental, exopsychical) coentity. Consequently, the above correspondence (association) between a real coentity of the man and the counterpart linguistic coentity of his is self-evident (unquestionable) if the real coentity is supposedly a sensible one that is known from another source, and it requires the appropriate *a posteriori* verification, including a certain *interpersonal verification*, if the real coentity is supposedly insensible.

2) The correspondence (association) between a real (res-) coentity of a man (as me or you) and the counterpart linguistic (nom-) coentity of the man, which has been described above in Crl 3.3(3) and in the above item of this comment, allows reconciling *nominalism* and *immanence* of universals and it is therefore the basis for the subsequent rigorous statement and rigorous solution of the problem of universals, which will be done in section 4.•

Cmt 3.7. Any *linguistic* (*nom-*) *universal,* e.g. *nom-man* is an *ideal* (*mental*, *psychical*, *nom-*) *coentity of a given particular* (*concrete*) *real* (*nonlinguistic, res-*) *sapient subject* (*res-man,* as. *me* or *you*), i.e. a coentity, which is seated in his mind (cerebral cortex) and which *is predicable of and present in every selected particular linguistic* (*nom-*) *man,* e.g. *linguistic* Socrates, Xantippe, Plato, Aristotle, Cleopatra, Napoleon, Lord Randolph Churchill, Russian Premier V. V. Putin, etc – a nom-man, which is also seated in the mind (cerebral cortex) of the given sapient subject as his coentity. Particularly, in reference to me as the given sapient subject, I always habitually (involuntarily but consciously) mentally (psychically) experience each one of the above *linguistic* (*nom-*) and hence *mental* (*psychical*) *objects of mine* in the *projective mental mode* as *the corresponding as if real* (*extramental, exoipsychical, res-*) *objects of the same names*. In the same way, I habitually mentally experience the nom-universal nom-man, being another mental coentity of mine, in the like projective mental mode as *my unique as if real* (*extramental, exopsychical, res-*) *common member of the nom-man, which is just another as if real* (ditto) *hypostasis of the non-man –. the corresponding as if real* (*res-*) *universal*, i.e. *as if real* (*res-*) *man.* It is self-evident that the latter *is predicable of and present in every selected particular as if real* (*res-*) *man,* i.e. *as if real* (*res-*) Socrates, Xantippe, Plato, Aristotle, Cleopatra, Napoleon, Lord Randolph Churchill, Russian Premier V. V. Putin, etc. In this case, any *as if real* (*res-*) *object, universal* or *particular,* is habitually but of course *paradoxically* called a *real* (*res-*) *one*.•

3.4. Aristotle's ten-fold taxonomy of categories

3.4.1. The statement of the ten-fold taxonomy

In no connection with his unfortunate four-fold taxonomy of *elementary beings*, Aristotle introduced *ten basic classes*, or *kinds*, *of one-word predicates* (*universals*), which he calls *categories*, as follows:

«Expressions which are in no way composite signify substance, quantity, quality, relation, place, time, position, state, action, or affection.» [ACE, Part 4]

or

«OF things incomplex enunciated, each signifies either Substance, or Quantity, or Quality, or Relation, or Where, or When, or Position, or Possession, or Action, or Passion.» [ACO, Chapter IV]

Cmt 3.8. *The English taxonyms* (*taxonomic names*) *of Aristotle's categories* depend on the translation of *the corresponding original Aristotelian taxonyms* by a particular interpreter. For instance, instead the categorical taxonyms that are used in the above quotation, the following ones are suggested in some other sources: (a) "possession" in place of "state" in the article **category** of WTNID; (b) "situation", "condition", and "passion" in place of "position", "state", and "affection" in the article «**Aristotelian logic**» of Wikipedia; (c) "substance", "quantity", "qualification", "a relative", "where", "when", "being-in-a-position", "having", "doing", "being-affected" in the article «**Categories**» of Wikipedia; (d) "substance", "quantity", "quality", "relatives", "somewhere", "sometime", "being in a position", "having", "acting", "being acted upon" in Studtmann [2008]. Thus, there is no conventional list of English taxonyms for separate categories. Aristotle's remark that the category taxonyms «*are in no way composite*» can, perhaps, serve as the criterion for choosing most appropriate expressions. At the same time, no matter which expressions are used as the *category taxonyms*, *a being* (*όν*) *is any entity* (*anything*) *that can be described in terms of some of the 10 categories*. Accordingly, a *category taxon*, i.e. *the universal denoted by a category taxonym*, *is not a being.* Therefore, *it cannot be defined as a species by a definition through being as the pertinent genus and a difference* – a traditional *definitio per genus et differentiam* that has been inaugurated by Aristotle in his «*Posterior Analytics*», [APsAM].•

Cmt 3.9. 1) Aristotle's original parasynonym of the English noun "*substance*" is "ή ουσία" \í usía\ *s.f.* (pl. "ουσίαι" \usíe\). In Aristotle's categorialism, *substances*, – e.g. a mineral, a biont, an animal, a plant, a man, or a mammal, and also mineral, biont, animal, plant, man, or mammal (without any article)[3], i.e. the class of animals, plants, minerals, men, or mammals, – play a special role as compare to members of other nine categories, because substances are independent entities having existence on their own, whereas non-substances are inseparable aspects of substances, which can exist only *in* substances.

2) Aristotle's ten-fold taxonomy is in principle independent of his four-fold taxonomy and therefore the former remains unaltered when the latter is replaced with the dichotomy that has been established in the previous subsection. However, Aristotle utilizes some concepts of his four-fold taxonomy in order to define his terms "*primary substance*" and "*secondary substance*". Consequently, under the above dichotomy of things, these definitions should be revised somewhat. To be specific, using the taxonym "*substance*" of his ten-fold taxonomy of categories, Aristotle terms *a thing, which is denoted by an SBS and which is not predicable of and is not present in any other thing* "*a primary substance*". He says:

> «Substance, in the truest and primary and most definite sense of the word, is that which is neither predicable of a subject nor present in a subject; for instance, the individual man or horse. But in a secondary sense those things are called substances within which, as species, the primary substances are included; also those which, as genera, include the species. For instance, the individual man is included in the species 'man', and the genus to which the species belongs is 'animal'; these, therefore – that is to say, the species 'man' and the genus 'animal', – are termed secondary substances.» [ACE, Part 5]

The wording of the corresponding passage of [ACO, Chapter V] is very close to the above. In accordance with the dichotomy of things and also in accordance with pertinent modern terminology, the range of Aristotle's term "*primary substance*" can be extended to include *the empty individual* and *all nonempty individuals*, whereas the range of Aristotle's term "*secondary substance*" can be extended to include *all secondary particulars* (*singleton-particulars*, *singletons, one-member classes*), *all class-universals* (*multipletons*, *many-member classes*), and *all mass-universals* (*masses*).•

[3] It will be recalled that Greek, both Ancient and Modern, has no indefinite article, whereas Latin has no articles at all, either definite or indefinite.

3.4.2. What are Aristotle's categories?

According to Wiktionary, the Ancient Greek noun "κατηγορία" \katiɣoría\ *s.f.* (*pl.* "κατηγορίαι" \katiɣoríe\) has two meanings: (1) *charge* or *accusation* and (2) (in logic) *predication* or *category* and hence, more specifically, *a type of predication* or *an act or instance of predicating*. In connection with its first meaning, the noun "κατηγορία" is etymologically derived from the verb "κατηγορέω" \katiɣoréo\, which means «*I accuse*, *speak against*» and which is in turn derived by combining the preposition "κατά" \katá\, meaning «*against*», and the verb "ἀγορεύω" \agorévo\, meaning «*I speak*». At the same time, in connection with the second meaning of the noun "κατηγορία", its kindred verb "κατηγορέω" means «*I predicate*» in the active or «*is predicated*» in the passive. Derived from "κατηγορία" are the nouns "το κατηγορούμενον" \katiɣorúmenon\ (*s.n.*, *pl.* "τα κατηγορούμενα" \ta katiɣorúmena\) and "το κατηγόρημα \to katiɣórima\ (*s.n., pl.* "τα κατηγορήματα" /ta katiɣórimata/), which mean *the predicate* (*what is predicated*) each. As was already pointed out in the first paragraph of subsection 3.2, in the English version of Aristotle's terminology, the noun "predicate" is actually used in the sense of the noun "*predicative*", the understanding being that *a predicative is the complementary part of the link-verb in a compound grammatical predicate*. When it is, e.g., stated that (to use the appropriate alternative English phraseology) *something* (*κάτι \káti*, *pron.*), e.g. a *being* (*ον* \on\), *is said of* (*υποκείμενου* \ipokímenu\) *something else*, e.g. *of another being*, then the former is called "*the predicate*" ("*το κατηγορούμενον*" \to katiɣorúmenon\ *s.n.*, *pl.* "τα *κατηγορούμεντα*" \ta katiɣorúmenonta\) and the latter "*the subject*" ("*το υποκείμενον*" \to ipokímenon\ *s.n.*, *pl.* "τα υποκείμενοντα" \ta ipokímenonta\). In this case, however, it remains unclear what the predicate and subject are, – namely, whether *they are written or spoken words,* which are beings themselves, or whether *they are the beings denoted by the words*.

In accordance with the above etymological sense of "category" ("κατηγορία"), Studtmann [2008. subsection 2.1] provides Aristotle's tenfold division with the following apparently natural but in fact obscure interpretation:

> «Aristotle divides what he calls *ta legomena* (τᾶ λεγόμενα), i.e. *things that are said*, into ten distinct kinds (1b25). Things that are said according to Aristotle, are words (*De Int* 16a3), and so it is natural to interpret his second system as a classification of words. And because the English word 'category' comes from the Greek word for *predicate*, one might naturally think of the second system

as a classification of distinct types of linguistic predicates. There is, however, considerable debate about the subject matter of the second system of classification.»

The expression “τᾶ λεγόμενα” \ta legómena\ is the plural number of “τό λεγόμενον” \tó legómenon\, whereas “λεγόμενον” is, according to GWST, the singular nominative neuter present participle form of the verb “λέγω” \légo\ meaning *to say* or *speak*.

The last sentence of the above quotation means that it is unclear what the predicate and the subject are, – namely, whether *they are written or spoken words, which are beings themselves*, or whether *they are the beings denoted by the words*. Consequently, it is unclear whether a category is *a class of words* or *a class of beings denoted by the words*. Consequently, one may interpret the nouns “predicate”, “subject”, and “category” and all other relevant words and expressions (as “said-of” and “present-in”) as he pleased in no connection with Aristotle’s interpretation, which is unknown. The interpretation of the expression “τᾶ λεγόμενα” (“the things that are said*”*) will be called the *nominalistic*, or *syntactic*, or *linguistic*, one if it is understood as *words*, and the *semantic*, or *extra-linguistic*, one if it is understood as the *denotata* (*denotation values*) *the words*. The interpretation of Aristotle’s categorealism will be qualified by the same respective qualifier. The above two interpretations of “τᾶ λεγόμενα” and the respective two interpretations of Aristotle’s categorealism will be called *extreme* ones.

In continuation of his remarks given in the previous quotation, Studtmann is, nevertheless, inclined to the semantic (extra-linguistic) interpretation of Aristotle’s categorealism as follows (*ibid.*):

«There are three reasons to think that Aristotle is not primarily interested in words but rather in the objects in the world to which words correspond. First, his locution *ta legomena* is in fact ambiguous, as between ‘things said’ – where these might or might not be words – and ‘things spoken of’ – where these are more naturally taken to be things referred to by means of words. Second, Aristotle's examples of items belonging to the various categories are generally extra-linguistic. For instance, his examples of substances are an individual man and a horse. Third, Aristotle explicitly accepts a doctrine of meaning according to which words conventionally signify concepts, and concepts naturally signify objects in the world (*De Int* 16a3). So, even if he is

> in some sense classifying words, it is natural to view his classification as ultimately driven by concerns about objects in the world to which our words correspond.»

This interpretation is then supplemented by the following important feature (*ibid.*):

> «Aristotle gives pride of place in this scheme to primary substances. He says that were primary substances not to exist then no other entity would exist (2b6). As a result, Aristotle's categorialism is firmly anti-Platonic. Whereas Plato treated the abstract as more real than material particulars, in the *Categories* Aristotle takes material particulars as ontological bedrock – to the extent that being a primary substance makes something more real than anything else, entities such as Socrates and a horse are the most real entities in Aristotle's worldview.»

The whole of the above semantic interpretation of Aristotle's categories is of course one of Studtmann and not one of Aristotle himself – in agreement with the item VI of subsection 2.2. Particularly, the polemic between Aristotle and Plato on the nature of *universals* and *reality* is commonly regarded (interpreted) as *uncompromising*, so that the expressions such as "more real than" and "the most real" can hardly be literal translations of anything that Aristotle might apply to any beings. Also, supposing that categories are denotata of the pertinent words, and not the words themselves, a category should be a *universal*. However, this fact is blurred in «*Categories*», – just as blurred is the difference between the category and any particular (instance) exemplifying it. Owing to the systematic uncertainty of Aristotle's categorialism, either one of its extreme interpretations unavoidably involves some persuasive definitions or obscure expressions and turns out to be unconvincing. Consequently, arguing for or against either extreme interpretation of Aristotle's categorialism would have been counterproductive. Instead, in the next subsection, I shall attempt to explain why in my view Aristotle treats his categories in that obscure and unintelligent way. That explanation of mine can of course be regarded as another interpretation of Aristotle's categorialism. But it is not certainly the one that Aristotle had underlain «*Categories*» *voluntarily and consciously*.

3.4.3. Categories as syntactico-semantic homonyms

If a graphonym that is intended to be *used xenonymously,* i.e. as a *euxenograph* (*euxenographonym*), *for denoting a class* (as a number, vector, or function), i.e. an *abstract* and hence *insensible* object of a given interpreter of the graphonym, then the interpreter can *involuntarily but consciously* change his mental attitude towards the graphonym and use it as a *tychautograph*, i.e. as an (*accidental,* or *circumstantial*, *autographonym*, and thus to confuse the *class-denotatum*, or *class-connotatum*, *of the euxenograph* with its *isotoken-class*, being the *class-denotatum*, or *class-connotatum*, *of the tychautograph*. At the same time, a class is a mental (psychical) entity of the interpreter and therefore it cannot be exhibited on a material surface (as that of a sheet of paper or of the screen of a computer monitor) but rather it can only be represented by an appropriate graphonym (graphic expression). It is therefore not accidental that, for instance, the count (numerable) noun "number" is equivocally used for mentioning both a *numeral*, i.e. a *proper name*, *phonographic* (*wordy*) or *logographic* (*aphononographic*), *of a number*, and *its class-denotatum*, i.e. *the number itself*. Accordingly, the concrete *numerals* "one", "two", etc, or "1", "2", etc, and *their xenonymous class-denotata* are often *equivocally called numbers*. In this case, using quotations for mentioning numerals and their interiors for mentioning the respective numbers is just an *epistemologically relativistic* devise because both a quotation and its interior are graphic symbols, and not classes. Analogously, either one of the nouns "vowel" and "consonant" is equivocally used for mentioning vowel, or consonant, speech sounds and the letters denoting those sounds.

I have derived the prefix "*tych*"- or "*tycho*"- from the following Greek etymons: the noun "τύχη" \tíxi, tíhi\ meaning *chance*, *fortune*, *fate*, or *good luck*; the adjective "τυχαι͡ | ος" \tixéos, tihéos\ meaning *fortunate* or *lucky*; the homophonic adverb "~ως" meaning *by chance*.. Also, according to the English-Greek part of Pring [1982], the English vocabulary entry "*accidental*" is translated into Greek by a token of the above Greek adjective with ˜ in place of ⁀.

Like the equivocal use of the noun "number", the noun "*taxon*" is also used equivocally for mentioning both a *taxonomic name*, and *the class denoted by the name*. Authoritative explanatory dictionaries of the English language and of other languages legitimize such equivocal usage of many names – usage that originates from confusion between autonymous and xenonymous uses of glossonyms (linguistic onyms) in general and of graphonyms in particular. For instance, here follows a definition of WTNID:

> «**taxon** *n, pl* **taxa** *also* **taxons 1 :** a taxonomic group or entity **2 :** the name applied to a taxonomic group in a formal system of nomenclature»

Equivocal usage of the noun “taxon”, which is legitimized by the above Webster’s definition is psychologically explicable because, once any of a given group of individuals is called by the same name, the individuals become *ipso facto* members of the same class thus called. However, the ambiguity of the word “taxon” is often confusing because this word is, as a rule, used in both senses in the same field of study and discourse. A like remark applies to the words “number” and “numeral”. For instance, using the nouns “number” and “numeral” *univocally*, I can make the following statement:

> A numeral is a symbol whose denotatum is called a number. Accordingly, a *numeral is used for mentioning the number*, *which it denotes*, provided of course that it is not used *autonymously,* i.e. *for mentioning either itself or its tokens*.

With “number” in place of “numeral”, the above italicized clause reduces to the nonsense: “*a number is used for mentioning the number which it denotes*”.

In the TTL, I eliminate the ambiguity of the noun “number” by not using it interchangeably with of the noun “numeral” in any occurrence. That is to say, a number is a class and therefore it cannot be exposed (depicted) on any material surface but it can only be represented by a numeral denoting it and be mentioned by using that numeral. Likewise, I *partly* eliminate the ambiguity of the noun “taxon” by using it for denoting only a *taxonomic class*, while the name of a taxon is called a *taxonomic name* or, briefly, a *taxonym*. Nevertheless, the nouns “numeral” and “taxon” are still ambiguous because, given a *numeral token* (e.g.), “numeral” can mean either a *token* (*the given one or another one*) *of the numeral* or a *token-class* (*memory image*, *recept*) *of the numeral*, while a *token* can mean either an *isotoken* (e.g. *a graphic token of a graphic numeral*) or a *paratoken* (e.g. *a phonic token of a graphic numeral*); and similarly with “taxon” in place of “numeral”.

Df 3.2. I shall give the name “*syntactico-semantic homonym*” to a common or proper name of a class, which equivocally denotes, i.e. is used for mentioning, the class. A syntactico-semantic homonym is called a *syntactico-semantic homograph* if it is a graphonym and a *syntactico-semantic homophon* if it is a phononym. The practice (phenomenon) of employing syntactico-semantic homonyms is called *syntactico-*

semantic homonymy, and similarly with "*graph*" (in full "*graphonym*") or "*phon*" (in full "*phononym*") in place of "*nym*". The occurrence of the qualifier "*syntactico-semantic*" in any of the above terms can be used interchangeably with an occurrence of the qualifier "*semantico-syntactic*" without altering the meaning of the term. Thus, for instance, the nouns "vowel", "consonant", "taxon", and "number" are syntactico-semantic homonyms and also syntactico-semantic homographs (homographonyms).•

The origin of the linguistic phenomenon of syntactico-semantic homonymy is *confusion between use and mention of symbols*. It is impossible to distinguish between use and mention of *onyms* (or *nyms*)*,* i.e. *graphonyms* or *phononyms*, in all cases, but it is possible and necessary to do so in all particular cases where there is real doubt in the meaning. At the same time, based on the unpunctuated character of all ancient scripts, which has been described in the item II of subsection 2.2, I have concluded in the item III of that subsection that Aristotle did not formally distinguish between use and mention of graphonyms even in the cases where there is real doubt in the meaning. Moreover, it is likely that he *systematically confused between use and mention of graphonyms*. Since, however, I have not read Aristotle's original works in Ancient Greek, I would more correctly have said that in the translations of Aristotle's works into English and in the comment on them in English, use of graphonyms is not as a rule formally distinguished from their mention even in the cases where there is real doubt in the meaning, and this property of the translations and comments supposedly reflects the like property of Aristotle's original works. At the same time, the phenomenon of confusion between use and mention of linguistic expressions that occurs in any native language and particularly in modern English does not, of course, have any connection with Aristotelianism.

In accordance with the above said, it is most likely that Aristotle just used his term "category" *involuntarily and unconsciously* or, perhaps, *subconsciously* (*intuitively*) as a *three-range syntactico-semantic homonym,* the syntactic range of which is the set of 10 pertinent words, while the ultimate semantic range of which is the set of 10 real (physical, extramental, exopsycical, res-) *absolute universals* that are denoted by the 10 words via the set of the respective 10 linguistic (conceptual, nominal, nom-) *absolute universals* (subject to Df 3.1(2)) that Aristotle used in the pertinent projective (polarized, extensional, connotative) mental mode, in which he mentally experienced them as his *as if* real (physical, extramental, exopsycical, res-)

objects. Since *projective use of his own conceptions*, which is similar to *the projective usage of his own sensations* (*perceptions*), is an inherent property of the nervous system of any man, therefore any reader of Aristotle's «*Categories*» *involuntarily* interprets (mentally experiences) Aristotle's term "category" in a like equivocal way, although the reader can do so *unconsciously* or *subconsciously* (like Aristotle) or *consciously* (like me). Thus, putting it in the appropriate *semantic phraseology*, any one of the 10 categories: substance, quantity, quality, relation, place, time, position, state, action, and affection *is a word of the respective self-referential name* (cf. a vowel, consonant, taxon, or number), which *denotes* a real (res-) absolute universal of the same name via a linguistic (nom-) absolute universal *connoted* by the word.

4. A rigorous formulation of the problem of universals and its solution

4.1. Preliminary remarks

In modern philosophy, the count noun "class" is often used as a synonym of the count noun "universal". However, in accordance with subsection 3.3, the class-denotata of, i.e. the classes (multipletons) denoted by, the above two count nouns are distinct, particularly in the following respects.

i) A universal is either a many-valued class, called also a multipleton, or a mass, and vice versa.

ii) In accordance with Ax 3.2 and Crl 3.3, a universal is either a *linguistic* (*conceptual*, *nominal*, *nom-*) *universal* or a *real* (*physical*, *extramental*, *exopsychical, res-*) *universal*, and similarly with "class", "multipleton", or "mass" in place of "universal". Therefore, the two instances of the previous item, in which each of the four generic names "universal", "class", "multipleton", and "mass", occurring on the list (3.5), is attached either with any one of the synonymous qualifiers of the list (3.6) or with any one of the synonymous qualifiers of the list (3.7), are semantically sound.

iii) Any *linguistic* (*nom-*) coentity of a sapient subject¸ of the genus denoted by a certain generic name of the list (3.5) is mentally experienced by the sapient subject in his pertinent *projective* (*connotative*) *mental mode* as his *as if* (*as though*) *real* (*physical*, *extramental*, *exopsychical, res-*) *coentity of the same genus*, which is in turn conformable to the pertinent real coentity if it exists.

The following definition is designed to demonstrate how the above properties of universals arise straightforwardly by their induction from the pertinent particulars and not pure grammatically by predicating some of the particulars in accordance with the revised concepts of Aristotle's «*Categories*». For the sake of simplicity and rigor, I shall use the appropriate *egocentric phraseology* that is relevant to me, while any concrete reader of this essay can immediately interpret that phraseology as relevant to him.•

Df 4.1: *Induction of universals from particulars.* 1) In agreement with Aristotelian doctrine of *nominalism,* if two or more, i.e. finitely or infinitely many, *mutually* (*pairwise*) *distinct* (separate) particular coentities of mine, i.e. entities, having or not having proper names, of which I am conscious (aware), have a certain single or cumulative *conceptual property in common* with respect to me and if therefore I provide every one of the coentities with the same *denotative common name* then all those particular coentities become *ipso facto instances* (*specimens*) of a single whole *mental* (*psychical*) *universal coentity* of mine, which is called a *linguistic* (*conceptual*, *nominal*, *nom-*) *universal of all particular coentities in question*. Consequently, the nom-universal is alternatively called *the range*, or *designatum, of the denotative common name of its instances* (*specimens*). The above *main* (*primary*) mental process of genesis of the nom-universal is accompanied by the following *concomitant* (*secondary*) mental processes.

a) Each original distinct particular coentity of mine turns out to be devoid of all its individual properties, by which it is distinguished from any other original distinct particular coentity, so that it is left only with the properties that it shares with the other coentities. In the result, each original distinct particular coentity turns into a *common* (*general*, *indistinct, certain*, *particular but not particularized*, *concrete but not concretized*) *instance* (*specimen*) *of the nom-universal*, *which represents the whole nom-universal*, thus being just *another hypostasis* (*way of existence*, *aspect*) *of the latter*. Consequently, *the common instance of the nom-universal is a nom-universal itself, and not a nom-particular*, *and it is unique.*

b) The above denotative *common* name of the original distinct particular coentities of mine becomes a *proper name of the common instance of the nom-universal* and at the same time it becomes a *connotative proper name of the nom-universal itself.* In order to mentally metamorphose the former name into either of the latter two name, I *habitually* and hence *involuntarily but consciously* use the denotative common name along with its range in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* the range as *my as if* (*as though*) *real* (*physical, extramental*, *exopsychical, res-*) *object* (other than the common name itself) – just as I *always* use the *sensation* (*percept*) of any nym (sensum, sensory object) and particularly that of the common name itself. The above *as if* extramental object is the very object that I call the *common instance* of the nom-universal. In this case, I say that both *the common name and its* [*original*, *unpolarized*] *range are used for mentioning the common instance of the range* or that, less explicitly, *they are used but not mentioned*, whereas the range is said to be *connoted by*, or to be *the connotatum* (*connotation value,* pl. "*connotata*") *of, the common name.* At the same time, the common instance of the range, which is a metamorphosed (polarized, externalized) hypostasis of the latter, is said to be *denoted by*, or to be the *denotatum* (*denotation value,* pl. "*denotata*") *of,* the common name. Thus, the denotative common name of any one of the original distinct particular coentities of mine has indeed two hypostases as indicated at the beginning of this psragraph.

c) In accordance with the above point b, I *tacitly* (*involuntarily*) *but consciously hypothesize* that *to the as if real common instance of the nom-universal* and hence *to the nom-universal itself* there exists a certain *res-universal* (*real universal*) *that is mentioned by using that common instance*, the understanding being that in all doubtful cases this hypothesis should, if possible, be subject to the appropriate *a posteriori* verification.

d) When the proper name of the common instance of the non-universal is detached from the projective mental mode and is just *considered*, the connotatum of the proper name is impartially called its *designatum* (*designation value*, pl. "*designata*").

2) In accordance with the above item 1a, the common instance of the nom-universal was decided to be unique because it is regarded as another hypostasis of the nom-universa. This decision is supported by the following two most general philosophical principles.

a) There is a general philosophical principle of "saving thoughts", the original version of which is known under the name "*Ockham's razor*" or "*The Ockham's razor principle*", after the English Scholastic philosopher William of Ockham or Occam

(AD ca1300–ca1349). This principle says that *entities should not be multiplied unless necessary*. Consequently, postulating the existence of more than one mutually indistinguishable common instances of a nom-universal would contradict Ockham's razor.

b) Two and half centuries after William of Ockham, the German philosopher and mathematician Gottfried Wilhelm Leibniz (1646–1716) proposed another general philosophical principle under the Latin name "*principium identitatis indiscernibilium*", which is translated into English as: "*The Principle of Identity of Indiscernibles*", and which is also called "*Leibniz's Law*". This principle says that *no two objects have exactly the same properties.* That is to say, if supposedly *two objects have exactly the same properties,* – such objects, e.g. as two common instances of a nom-universal, – *then they are a single object.*

3) The Aristotelian doctrine of nominalism can be extended to the case of a single unique coentity of mine as follows.

a) If a unique coentity of mine, i.e. an entity that has unique distinguishing properties with respect to me, is provided, as the *relatum*, with a *proper name* as its and only its *referent*, then the coentity is said to be *denoted by* the proper name and accordingly it is called the *denotatum* (*denotation value*, pl. "*denotata*") *of* the proper name, with respect to me. Since the denotatum of the proper name is an *object sui generis*, therefore *ipso facto* it automatically produces the *linguistic* (*conceptual*, *nominal*, *nom-*) *singleton of its own*, which becomes another value of the proper name – the value that is said to be *connoted* by that name and that is accordingly called the *singleton-connotatum*, or impartially *singleton-designatum* (or *singleton-range*), of the proper name. In this case, I *use* the proper name along with its singleton-connotatum *for mentioning* (*denoting*, *putting forward*) its denotatum, while both the proper name and its singleton-connotatum *are used but not mentioned*. The above mental phenomenon of using the proper name can theoretically substantiated in analogy with similarly using a common name of an instance of a nym-universal (described in the previous item 1) as follows.

b) The singleton-connotatum (singleton-designatum) of the proper name is a *mental* (*psychical*) *coentity* of mine. However, when I use the proper name for mentioning (denoting, putting forward) its denotatum, I use the singleton-connotatum in a certain *projective* (*polarized*, *extensional*, *connotative*) *mental mode*, in which I *mentally experience* it as its member, i.e. as the denotatum of the proper name, *in the* *hypostasis of my as if real* (*physical, extramental*, *exopsychical*, *res*-) *particular object* (other than the proper name itself). I do so *habitually* and hence *involuntarily but consciously* – just as I *always* use the *sensation* (*percept*) of any nym (sensum, sensory object) and particularly that of the proper name itself. In this case, I *use* the proper name along with its singleton-connotatum *for mentioning* (*denoting*, *putting forward*) its denotatum, while both the proper name and its singleton-connotatum *are used but not mentioned*. Thus, the member of the singleton-connotatum is put forward as the intended import value, i.e. as the denotatum (meaning) of the proper name, while the singleton-connotatum itself is *as if* put backward. In fact, however, the singleton-connotatum *is*, to use the appropriate *monistic phraseology*, involuntarily *mentally transduced into another hypostasis* (*way of existence*, *aspect*) *in the form of its only member*. In order to describe this mental phenomenon in the appropriate alternative *dualistic phraseology,* I say that the member of the singleton-connotatum of the proper name *represents* the singleton-connotatum, so that the two entities as if coexist as a single biune entity.

e) Just as in case indicated in the item 1c, in accordance with the above point b, I *tacitly* (*involuntarily*) *but consciously hypothesize* that *to the as if real nom-member of the nom-singleton* and hence *to the nom-singleton* (*nom-particular*) *itself* there exists a certain *res-particular* (*real particular*) *that is mentioned by using that nom-singleton*, the understanding being that in all doubtful cases this hypothesis should, if possible, be subject to the appropriate *a posteriori* verification.

d) When the proper name of the unique coentity of mine is detached from the projective mental mode and is just *considered*, the singleton-connotatum of that name is impartially called its *singleton-designatum* and also less explicitly its *class-designatum* or simply its *designatum* (*designation value*, pl. "*designata*").

e) The proper name can *denote a singleton* so that it has a *singleton-denotatum*. In this case, the *singleton-connotatum* of the proper name is the *singleton of its singleton-denotatum*, i.e. a *two-fold* (*repeated*) *singleton*.

4) The form of a common name of particular coentities of mine to be collectivized by that name depends on the coentities and hence on the kind of their intended nom-universal and also on the pertinent expressive (semantic) properties of the native language or of the system of notation (nomenclature), to which the common name belongs. Here follow some most conspicuous cases.

a) If some particular coentities of a man are substances that are supposed to become *members* (*elements*) and perhaps, at the same time, *parts* of their intended nom-universal, – such coentities, e.g., as natural numbers, which are members and at the same time parts (subsets) of their entire set (regular class) (see, e.g., Burrill [1967, p. 14]), – then the nom-universal is, in accordance with the item V of subsection 2.2 and with Df 2.8(3), called the *linguistic* (*conceptual*, *nominal*, *nom-*) *universal class of all particular coentities in question* or, more generally (less explicitly), a *linguistic* (ditto) *multipleton,* i.e. *many-member linguistic* (ditto) *class, of mine* – in analogy with a *one-member linguistic* (ditto) *class* that is called a *linguistic* (ditto) *singleton*. In this case, no matter whether the native language, which is used for forming a common name of the members of a multipleton and the pertinent proper name of the multipleton, has or does not has an indefinite article, the former name can always be formed as an *unlimited* (and hence *article-free*) *count substantive*, i.e. *unlimited count noun* or *noun equivalent*, in a singular number form (e.g., as any one of the English substantives: "animal", "man", "biont", "living organism", "mortal [being]", "natural number", "author of Principia Mathematica", etc), which automatically becomes the pertinent equivocal proper name of the multipleton. If, however, the language used has an *indefinite article* then the former name is conventionally formed as a *singular count name limited by the preceding indefinite article*.

b) If some particular coentities of a man are substances that are supposed to become *parts*, *and not members*, of their intended nom-universal, – such coentities, e.g., as samples (instances) of water, sand, dough, beauty, courage, etc, – then the nom-universal is called the *universal linguistic* (*conceptual*, *nominal, nom-*) *mass* (briefly *u-nom-mass*) *of all particular coentities in question*, – in agreement with the previous uses of the generic name "mass" in the item V of subsection 2.2, Dfs 2.8(4,5,7) and 3.1(3,5), Cmt 3.5, and Crl 3.3(2)). In a native language, a u-nom-mass is denoted by the appropriate *unlimited proper mass name* (*ULPrMsN*), i.e. by an *unlimited non-numerable singular substantive* (*ULNNSgS*). Accordingly, a *limited mass name* having the prepositive *indefinite mass quantifier* "*some*" to a certain ULPrMsN (e.g. "some water", "some money", "some time", etc) denotes a *common particular nom-mass*, i.e. a common instance (specimen) of the u-nom-mass denoted by the pertinent ULPrMsN.

c) If some particular coentities of a man are *states of affairs* (*facts*, *events*, *phenomena*, etc) that are supposed to become *members* (*elements*) of their intended nom-universal then the latter is the *nom-multipleton of all particular coentities in question of the man*, which is denoted by the appropriate *common declarative sentence*, a verbal one (e.g. "It is raining", "The sky is blue", "The night is moonlit", etc) or a logographic one (e.g. '$m\boldsymbol{a}=\boldsymbol{F}$', '$\nabla\cdot\boldsymbol{J}+\partial\rho/\partial t=0$', '$\neg[x\in\varnothing]$', etc).

5) I avoid using *contradictiones in adjecto* either in the form of substantives as those mentioned in Ax 3.2(2) or the form of declarative sentences containing such substantives as their constituent parts and being therefore *contradictiones in adjecto* themselves. For instance, the sentences "A centaur is a mammal", "A centaur is not a mammal", and "The capital of the USSR in AD 2000 was in Europe" are inacceptable. Consequently, in accordance with the item 1c, I *tacitly* (*involuntarily*) *but consciously hypothesize* that *to the as if real common instance of a nom-universal class*, or *mass*, and hence *to the nom-universal class*, or *mass*, *itself* there exists a certain *res-universal* (*real universal*) *class*, or *mass*, respectively *that is mentioned by using that nom-universal*, the understanding being that in all doubtful cases this hypothesis should, if possible, be subject to the appropriate *a posteriori* verification.

6) Thus, in accordance with the previous items of this definition and in agreement with the subsection 3.3, each one of the count generic nouns of the list (3.5), either alone or together with any one of the prepositive synonymous qualifiers of either list (3.6) or (3.7), can be used projectively. In this case, each one of the count generic nouns of the list (3.5) alone can be used *equivocally* in two different projective modes. In one of them,

i) a *universal* (e.g.) is either a *linguistic* (*conceptual*, *nominal*, *nom-*) *universal* or a *real* (*physical*, *extrameintal*, *exopsychical*, *res-*) *universal,*

while in the other one, subject to the items 1c, 3e, and 5,

ii) a *universal* is a *real* (ditto) *universal*;

and similarly with any other noun of the list (3.5) in place of "universal".•

Cmt 4.1. Any one of the entities that have been mentioned in the previous definition can be the member of the respective singleton. Also, any singleton can be the member of another, repeated singleton. In this case, if a properly schooled man (sapient subject) involuntarily but consciously uses any one of the above singletons, along with its name, in the pertinent projective (polarized, extensional, connotative) mental mode then he mentally experiences the singleton as its member in the hypostasis of his as if extramental (exopsychical) object – just as he always use the sensation (percept) of any nym (sensum, sensory object) and particularly that of the proper name itself. For instance, the man does not have in his mental realm (consciousness) any entity that is called “Aristotle”, but he does have in his consciousness the singleton that is called “\Aristotle/” or “{Aristotle}”. When the sapient subject uses the singleton \Aristotle/ (or {Aristotle}) *projectively*, he mentally experiences \Aristotle/ as Aristotle, i.e. as his as though extramental sensible object. Likewise, when the sapient subject projectively uses the singleton \man/ (or {man}), he mentally experiences it as the extramental *multipleton* (*species*, *specific class*) *man* (*Homo sapiens*). Analogously, when the sapient subject projectively uses the multipleton man, he mentally experiences it as an *extramental common member of the multipleton* that is denoted by the common name “a man”. In contrast to English, Greek and Hebrew, e.g., have no indefinite articles, whereas Latin and Russian, e.g., have no articles at all. Therefore, in any one of these languages, equivocal use of the pertinent parasynonym of the word “man” is unavoidable. To be specific, the count noun “άνθρωπος” \ánthropos\ in Greek, “אדם” \adam\ in Hebrew, “hŏmo” in Latin, or “человек” \chelovek\ in Russian is a homonym that is equivocally used both as a parasynonym of the English count noun (class-name) “man” and as a parasynonym of the English common name “a man”.•

Cmt 4.2. Here follows a summary of various definitions that have been made earlier in this essay (including the previous one), which are supplemented by some minor explicative definitions for more clarity. It is understood that the summary is *veracious* under either of the above two mental attitudes of any given sapient subject (as me or you), which have been indicated in Df 4.1(6), unless stated otherwise.

i) An entity is called:

a) the *empty entity*, *empty individual*, *empty primary particular*, *empty* (or *memberless*) *class* (or *set*), or *empty mass*, and also *nothing* if and only if it is *a part of* (*present in*) any entity including itself;

b) a *nonempty entity* and also *a thing*, *some thing*, or *something* if and only if it is *not the empty individual*, i.e. if and only if it is *not a part of* (*not present in*) some, i.e. in strictly some or in all, other entities (some things).

ii) An individual, particular, class (particularly set), or mass is said to be a *nonempty* one if it is *not empty*, i.e. if it is a *nonempty entity.*

iii) A *nonempty class* is either a *one-member class* (*set*), called also a *singleton,* or a *many-member class*, called also a *multipleton* or *universal class.*

iv) A nonempty class or the empty (memberless) class (empty individual) is indiscriminately called a *class*.

v) A *multipleton* (*many-member class, universal class*) or a *universal mass* is indiscriminately called a *universal*.

vi) A member of a nonempty class or a part of a nonempty class or mass is called a *particular.,*

vii) A member of a nonempty class is called an *element*, so that an element is a *particular*, but not necessarily vice versa.•

Cmt 4.3. In a one-individual class (or set) theory, in which the only individual is the empty one, i.e. the empty class, conventionally denoted by '$\varnothing$', every member (element) of any nonempty class, i.e. the member of a singleton or every member of a multipleton, is a class (or, correspondingly, a set). In this case, '$\varnothing$' can stand on both sides of any one of the signs '$\in$', '$\subseteq$', and '$\subset$', whereas the emptiness of $\varnothing$ is expressed as $\neg[x\in\varnothing]$ or as $\varnothing\subseteq x$ or as $\neg[x\subset\varnothing]$, where '$x$' is a class-valued (or particularly set-valued) variable. In a many-individual class (or set) theory, nonempty individuals can be mentioned only by using *nonempty-individual-valued variables,* so that they are *indistinguishable*. In this case, any nonempty-individual-valued variable, say 'ξ', *can stand only to the left of any one of the signs* '$\in$', '$\subseteq$', and '$\subset$'; i.e. occurrences the strings such as '$\in\xi$', '$\subseteq\xi$', and '$\subset\xi$' are prohibited.•

Cmt 4.4. An entity that I *know by acquaintance* via its sensations is called a *sensible entity* or *sensible thing*, and also a *sensible particular* or a *concretum* (*concrete object*, pl. "*concreta*"). An insensible entity that *I know either by inducing* (*prescinding*) *it from sensible entities or by deducing* (*detaching*) *it from other insensible entities* is called an *abstractum* (*abstract object*, pl. "*abstracta*"). Thus, a coentity of mine is either a concretum or an abstractum, the understanding being that a coentity not being an abstractum is a concretum and that conversely a coentity not being a concretum is an abstractum. However, I may, paradoxically, *mention* (*refer to*) *a concretum as an abstract relatum* by using its *proper name* as the *pertinent concretum*, along with the pertinent nom-particular, which is an *abstractum*. The property of a coentity of mine is called: *thisness* if it is a concretum, *haecceity* (*hecceity*) or *individuality* if it is an individual, and *quiddity* or *universality* if it is a universal. Haecceity, thisness, or quiddity is indiscriminately called *essence*; “haecceity”, “thisness”, “quiddity”, and “essence” are class-names.•

4.2. The dialectic principle of unity of opposites

One of the most general laws of philosophy is *the triad of motion of thought: thesis-antithesis-synthesis* due to the German philosopher Georg Wilhelm Friedrich Hegel (1770–1831). The above triad is interpreted as *the dialectic principle of unity*, or *identity*, *of opposites due* to another German philosopher Johann Gottlieb Fichte (1752–1814), a contemporary of Hegel. In a sense, Hegel’s triads and its interpretation by Fichte were foreshadowed by the principle of *golden mean* of ancient Greek philosophers and also by the following two Latin dicta: «In medio stat veritas» – «The truth stands in the middle» and «In medio stat virtus» – «Virtue stands in the middle» or «Virtue is in the moderate». Aristotle represents the principle of *golden mean* and discusses its importance for ethics in his «*Nicomachean Ethics*».

The evolution of the initial notions of a point and of smallness in mathematics is an illustration of *Hegel’s triad*. In this case, (i) *the thesis of the pertinent triad* is the intuitive concept of ancient Greek philosophers that a point in a three-dimensional continuum (e.g.) is an *elementary entity*; (ii) *the antithesis of the triad* is the fact that a 3-point has turned out to be a point in the pertinent three-dimensional affine Euclidean space over the field of real numbers, which should be identified by the ordered triple of irrational real numbers – coordinates of the point relative to a certain coordinate system, but the latter triple has no proper name and is therefore as complicated and transcendental as the three-dimensional continuum itself; (iii) the $\varepsilon\&\delta$-language is *the synthesis of the triad*. At the same time, the $\varepsilon\&\delta$-language can alternatively be regarded as a *golden mean* of the thesis and antithesis of Hegel’s triad.

In what follows, I shall make explicit a straightforward solution of the problem of universal, which can be regarded as a *golden mean of* the antagonistic doctrines of *extreme realism* and *extreme nominalism.*

4.3. A rigorous formulation of the problem of universals

I) Previously, I have already mentioned that the problem of universals can be formulated in many different ways, so that its solution depends on its formulation. Having defined the notions of *linguistic* (*conceptual*, *nominal*, *nom-*) and *real* (*physical*, *extramental*, *exopsychical*, *res-*) universals, the problem of universals can be stated as the following series of interrelated questions raised for academic inquiry.

1) Besides *nom-universals*, i.e. besides both nom-universal classes (universal nom-classes, many-valued nom-classes, nom-multipletons) and nom-universal masses (universal nom-masses), are there any *real* and hence *nonlinguistic universals*, briefly called *res-universals*, which exist independently of and outside the interpreter's mind and which are projected by the mind onto certain nom-universes, thus being responsible for the phenomenon of similarity of the members of each pertinent nom-multipleton and of similarity of the parts of each pertinent universal nom-mass?

2) If res-universals exist then where do they seat: (a) in a separate transcendental realm beyond space and time prior to any sensible entities or (b) in the sensible and hence real (res-) particulars exemplifying them?

In general outline, depending on the answers to the first two of the above questions, philosophers have classified each other or themselves as follows. A philosopher is called an *extreme nominalist* if his answer to the first question is negative and a *realist* if his answer to the first question is positive. A realist is called an *extreme realist* if he answers positively to the question 2a and a *moderate realist* and at the same time *a moderate nominalist*, or in one word a *conceptualist*, if he answers positively to the question 2b.

II) A conceptualist is by definition an adherent of *conceptualism* – any one of various philosophical doctrines, which positions itself as a *golden mean between and above extreme realism* and *extreme nominalism*. Here follow some typical definitions of the acceptations (presently common meanings) of "conceptualism".

«**conceptualism** *n* -s…*philos* **:** a theory that is intermediate between nominalism and realism and holds that universals exist in the mind as subjects of discourse or as predicates which may be properly affirmed of reality» (A Merriam-Webster® [1981])

«**conceptualism** <is> the theory of *universals* that sees them as shadows of our grasp of concepts. Conceptualism lies midway between out-and-out *nominalism*, holding that nothing is common to objects except our applying the same words to them, and any *realism* which sees universals as existing independently of us and our abilities.» (*Oxford Dictionary of Philosophy* on *Answers.com*)

«**conceptualism**, in philosophy, position taken on the problem of *universals*, initially by *Peter Abélard* in the 12th cent. Like nominalism it denied that universals exist independently of the mind, but it held that universals have an existence in the mind as concept. These concepts are not arbitrary inventions but are reflections of similarities among particular things themselves, e.g., the concept *male* reflects a similarity between Paul and John. This similarity shows that universals are also patterns in God's mind according to which he creates particular things. Slightly modified, this view becomes the position of moderate *realism,* the classical medieval solution to the controversy. For a modern statement of conceptualism, see C. I. Lewis, *Analysis of Knowledge and Valuation* (1946, repr. 1962).» (*Columbia Encyclopedia* on *Answers.com*)

Comparison of the above three definitions with one another and with my definition of "conceptualist" given in the above item I shows that, in accordance with all the four definitions, the three terms "*moderate realist*", "*moderate nominalist*", and "*conceptualist*" or the kindred three terms "*moderate realism*", "*moderate nominalism*", and "*conceptualism*" are synonyms, – in agreement with both *Ockham's razor* and *Leibniz's Law*.

4.4. Science and scientism

The most effective method to conceive nom-universals, which most probably have counterpart res-universals, is the natural science, in accordance with the following definition.

Df 4.2. 1) The *natural science* is the entire field of study and discourse, which is divided into the conventional branches, called physics, chemistry, astronomy, biology, psychology, logic, mathematics, etc, and which deals with investigation of cause-and-effect (antecedent-and-consequent) relations in inanimate and animate nature by objective (interpersonally verifiable) qualitative and quantitative methods, including extrospection, introspection, and measurement and also including *conceptualization* (particularly, *classification*), i.e. expression in intelligible exteroceptive symbols, of the results of application of the above three methods.

2) A version of conceptualism, according to which the natural science should, as far as possible, be used in conceiving nom-universals is called *scientific conceptualism* or *scientism*. A conceptualist being an adherent of scientism is called a *scientific conceptualist* or a *scientificist* – in contrast to a *scientist*. Thus, a scientificist is a conceptualist but not necessarily vice versa.•

Cmt 4.5. Here follows a typical definition of "scientism".

«**scientism** *n* -s…**2 :** a thesis that the methods of the natural science should be used in all area of investigation including philosophy, the humanities, and the social sciences **:** a belief that only such methods can fruitfully be used in the pursuit of knowledge» (A Merriam-Webster® [1981])•

4.5. A solution of the problem of universals on the base of scientism or conceptualism

1) Consider, for instance, the equalities (graphic symbols):

$$‘\, F = -\frac{\gamma m m'}{R^2}\,’, \tag{4.1}$$

$$‘\, E = mc^2\,’, \tag{4.2}$$

the first of which *expresses Newton's law of gravitation*, whereas the other one *expresses Einstein's law of equivalence* of two different measurable characteristics of a distinct portion of matter, namely, its *energy E* and its *gravitating mass m*. The recepts (memory images) and the meanings of the above equalities exist in the cerebral cortex of a properly schooled man in the form the corresponding mental (psychical) entities (brain symbols, cortical symbols), which are called *concepts*. However, equality (4.1) explains relative motion of the Sun and all celestial bodies that are held by it to form the Solar System, whereas equality (4.2) explains all known transformations of matter into energy and vice versa. Therefore, at least in our spatio-temporal part of the Universe, called the Solar System, equalities (4.1) and (4.2) *express universal laws of nature,* which are certain *aspects* (*forms*) *of the ways of existence of all real things in relation* (*with respect*), e.g., *to me* (or *my mind*, i.e. *my cerebral cortex*). Therefore, I should admit that each one of these laws exist in two forms (hypostases): (i) the *pertinent nom-universal*, i.e. the *pertinent name* (4.1) or (4.2) or, more precisely, the *token-class of the name, along with its sense* (*class-concept*), and (ii) the *corresponding res-universal* in the hypostasis of an *immanent* (inseparable and insensible) aspect of existence of certain real things. Consequently, I should also admit that the pure nominalistic mental attitude with regard to universals amounts to the denial of my ability to cognize, at least partly, any properties of any things of my Universe except for the sensations, which those things mediate in my cerebral cortex. By extrapolation (generalization), I may then assert the variant of the previous sentence with “the ability of any man”, ”his Universe”, and “his cerebral cortex” in place of “my ability”, ”my Universe”, and “my cerebral cortex” respectively.

2) Either of the laws of nature (4.1) and (4.2) designates a certain *class* [*of equivalence*], i.e. a certain *similarity*, of distinct natural phenomena (states of affairs, complex objects), which *are prescinded from their differentia so as to become recognizably same*. Like either of the graphic symbol (4.1) and (4.2), any one of the graphic symbols “a man”, “a chimpanzee”, “a tree”, “a stone”, etc is an individual common name, which expresses (connotes) a *morphological similarity* of distinct things comprised in the range of the name. This similarity reflects however a certain *insensible similarity in depth* of those things. For instance, the genomes, i.e. the complete sets of DNA, of two concrete men, not being identical twins, are not identical, but they are similar as compared, e.g., with the genome of a concrete chimpanzee. Likewise, the genomes of two concrete different chimpanzees, not being identical twins, are not identical, but they are similar as compared, e.g., with the genome of a concrete man. Therefore, there are some universal laws of nature, according to which all individual men are classified into the species *Homo sapiens*, or *man* (without any article), while all individual chimpanzees are classified into the species *Pan troglodytes*, or *chimpanzee*. These universal laws of nature can be regarded as res-universals, i.e. universals belonging to things, which underlie the nom-universals, i.e. classes denoted by the nouns “man” and “chimpanzee” respectively. Thus, for instance, the res-universal man or the res-universal chimpanzee, i.e. *the abstract man* or *the abstract chimpanzee*, does exist in each concrete specimen (member) of the respective species in the form of its individual genome, which determines the immediately recognizable morphological similarity of any two specimens of the same species, and which also determine the morphological differences between any specimen of one species and any specimen of the other species.

3) Assume that by his Universals, i.e. res-universals, Plato understood universal laws of nature. These laws are intangible. Therefore, a man can frame res-

universals only in the form of nom-universals such as denotata of certain graphic symbols. Assume then that a scientific method, i.e. a method that is based on logic, and that is hence based on nominalism, is found to make sure that the way of existence of some one or some more spatio-temporal objects agrees with a certain nom-universal, i.e. with the class denoted by a certain graphic symbol (nomen, name). In this case, one should admit that the mind (cerebral cortex) of a properly schooled man, that is a part of nature, has the *epistemic* (*epistemological*) *property to project* at least some res-universals, i.e. physical (spatio-temporal, exopsychic, extramental) universals, onto certain ones of its nom-universals, i.e. psychic (mental, conceptual) universals.

4) It is understood that some nom-universals are not projections of res-universals, so that they do not express any laws of nature. For instance, the mathematicians have abandoned their concept of *infinitesimals* as being supposedly infinitely small but nonzero real numbers, – just as the physicists have abandoned their concept of *ether* (cf. Cmt 5.2(4) in E1 and Cmt 3.4(5) in this Essay).

Essay 6. Taxonomies of bionts

1. Preliminaries: The major trichotomy of natural beings

1.1. A statement of the trichotomy

Df 1.1. Natural *matter* of the Earth can be classified into two major partial *matters*: **cellular**, or **living**, **matter** and **acellular** (**not cellular**), or **nonliving**, **matter**. A cellular matter can in turn be divided into two *matters*: **viral matter** and **mineral matter**. A portion of matter, *not being its part*, that is an indivisible (self-consistent) distinct and hence independent (self-subsistent) in a given circumstance or universally is called an *individual being* or briefly an *individual.* An individual of cellular matter is said to be a **cellular**, or **living**, **organism** and also a **biont**. An individual of viral matter, i.e. an intact extracellular viral particle, is said to be a **viral organism**, or **virus**. A distinct portion of mineral matter, not necessarily indivisible, is said to be a **mineral**. Bionts, viruses, and minerals are collectively called **natural beings**. Thus, in accordance with the above *trichotomy* of natural matter, all natural beings are also divided into *three material universals*, which will be called *material realms* in order to emphasize their all-embracing character, namely: (1) *the realm of bionts* (*living organisms*) or *the biontic,* or *life*, *realm*, (2) *the realm of viruses* or *the viral realm*, and (3) *the realm of minerals* or *the mineral realm.* The former two realms are usually treated as *material universal* (*many-member*) *classes*, while the third one is primarily treated as a *material universal mass* subject to the following general definitions.•

Df 1.2. 1) An entity is called:

a) the *empty entity*, *empty individual*, or *empty primary particular*, and also *nothing* if and only if it is *present in* any other entity and in itself as *the empty part*;

b) a *nonempty entity* and also *a thing*, *some thing*, or *something* if and only if it is *not empty*, i.e. if and only if it is *not present in* some, i.e. in strictly some or in all, other entities (some things).

2) The empty individual is alternatively called *the empty class* or *the empty mass.* Consequently, a class or a mass or an individual that is not empty is called a

nonempty one, the understanding being that a *nonempty individual* is neither a class nor a mass.

3) A unique entity, i.e. the empty individual, a unique nonempty individual, or a unique nonempty class or mass, is an *object sui generis*, i.e. an *object of its own class* (*kind*), and therefore it *ipso facto* produces the *singleton*, i.e. *one-member class, of its own*.

4) A universal is called a *multipleton* or *many-member class* if it is exemplified by its *two or more members* (*elements*) and a *universal mass* if it is exemplified by its *two or more parts,* none of which is empty. "*Matter*" is a synonym of "*mass*".

5) A singleton or a multipleton is indiscriminately called a *nonempty class* and conversely a nonempty class is either a singleton or a multipleton. Thus, a class is either the empty class or a nonempty class, but not both at one time.

6) Independently of the above dichotomy of classes, there are *two kinds* (*classes*) *of classes*: *regular*, or *small*, *classes*, called also *sets*, and *irregular*, or *proper*, *classes.* The *empty class* and a *singleton* are sets, i.e. regular (small) classes, but not necessarily vice versa. Broadly speaking, a *multipleton*, i.e. a class of finitely or infinitely many number (two or more) of *members* (*elements*), which *persistently coexist* (*exist simultaneously*) either in *the mind* (*cerebral cortex*) *of an interpreter* (as *me* or *you*) *of any one of the proper names of the class* or in *his real* (*extramental*, *exopsychical*) *world*, is also called a set, i.e. a regular (proper) class. A class that is not regular, i.e. that is not a set, is called an irregular (proper) class, the understanding being that an *irregular class* is necessarily *nonempty*. For instance, the biont realm and the viral realm are *irregular classes*, and *not sets*.

7) In ordinary language. a multipleton is denoted by a *count* (*numeralable*) *name*, whereas *mass* is denoted by a *mass* (*non-numeralable*).

8) Most immediate subdivisions of the life realm will be called *kingdoms*, either without any qualifiers or with the appropriate qualifiers, such as the prefixes "super"- and "sub"- or such as adjoined adjective equivalents "of first rank" and "of second rank" (as specified). In this case, the noun "*superkingdom*" will informally be used as an *ad hoc* name in order to emphasize the superiority of a class thus called over its *subclasses* (*parts*) in the case where the latter are called *subkingdoms*,

formally or not, or, generally, in the case where the latter have the word “kingdom” in their taxonyms.•

1.2. Minerals

A mineral has either *a molecular or a crystalline pattern of organization of ions and electrons*. In other words, a mineral is an *aggregation* of ions and electrons, which can be organized into atoms or into molecules, or else into crystals. In the last case, a mineral is either a *single crystal* or a *polycrystal*, i.e. an unorganized aggregation of single crystals. There are two classes of minerals: *inorganic minerals* and *organic minerals*. Organic minerals are remains or excrements of bionts or substances that result from chemical (non-metabolic) processes occurring in those remains or excrements in nature.

1.3. Viruses

Outside of a living cell, a virus is, like a mineral, an inert particle, which is unable either to drive metabolism or to replicate. That is, a virus isolated from a living cell lacks two major properties, which characterize a biont (living organism), namely metabolism and reproductivity. This is why a virus is classified as a non-living organism. At the same time, inside of a living cell, a virus is capable of utilizing the metabolic resources of the cell to reproduce. Making use of the appropriate etymological senses of the prefix “para”- and of the combining form “quasi”- (see Dictionary 3.1 in subsection 3.5 below), I shall therefore say that a virus has a *paracellular pattern of organization of ions and electrons* and also that it is a *quasi-living organism* or a *quasi-biont*.

A virus is a *parasite of a cell*, although it can also exist outside of a cell. Depending on the type of cells, on which viruses parasitize, the latter are divided into three kingdoms: *bacterial viruses* or *bacteriophages*, *plant viruses*, and *animal viruses*. Accordingly, some viruses are *bacterial pathogens*, some are *plant pathogens*, and the others are *animal pathogens*. *No viruses are known to parasitize either on protists or on fungi*. Bacterial and animal viruses are also called *virions,* whereas plant viruses are divided into two categories, namely *virions* and *viroids*. Thus, besides the above mentioned trichotomy of viruses, there is the dichotomy of viruses into *virions* and *viroids*. Specific structure and basic properties of virions have been studied quite well, whereas viroids are debatable pathogens of some plants, many basic properties of which remain puzzling.

The necessary attribute of any virus is its *genome*, i.e. its complement of *genetic material.* A viral genome is usually a single linear or circular, single-stranded or double-stranded, *polymer* (*macromolecule*) of *DNA* (*deoxyribonucleic acid*) or *RNA* (*ribonucleic acid*). A virion may have one or two genome copies. Besides its genome, a virion has *a capsid* – a shell that is built of a large number of protein polymers (macromolecules) of one or several types. Some animal virions (as those of influenza) have an additional *membranous envelope* covering the capsid. A virion that is not wrapped in an envelope is said to be *naked* though it has a capsid. Virions are rigid or flexible particles of various linear sizes in the range 20–350 nm and of various shapes. Typically, a virion capsid consists of a *head* and of distinct *protein* or *glycoprotein* (protein covalently bonded to carbohydrate) *accessory structures* (as various spikes), with the help of which the virion attaches to the surface of a *host cell*, pierces the cell's wall (when the cell is a plant one) and membrane, and infects the cell with its genome. The mechanism of attachment of a virus to a cell is based on a *key-to-lock-like fit* between proteins of the accessory structures of the virus on the one hand, and proteins of certain *receptor cites* on the surface of the host cell on the other hand. Therefore, there is a certain range of species of cells, which can be infected by viruses of a given type. Virions of each kingdom are informally divided into various categories in accordance with the type of nucleic acid, of which their genomes are made or in accordance with the structure of their capsids. Particularly, it is possible to distinguish among the following four major types of virion capsids:

1) a helical naked head with the overall appearance of a rigid rod, typically of about 20–30 nm in diameter and of about 300–350 nm in length, and with no distinct spikes (the structure of a tobacco mosaic virus);
2) a flexible helical head wrapped in an outer membrane that is studded with glycoprotein spikes, typically of about 70–300 nm in length (the structure of an influenza virus);
3) a naked or enveloped regular icosaheral head, typically of 50–350 nm in diameter, with a protein spike at each vertex (the structure of as an animal adenovirus, which is a naked one);
4) a naked regular icosahedral head with a flexible rodlike tail (the structure of a λ-virus that infects the Escherichia coli bacterium). It will be recalled that

a regular icosahedron is a 20-sided polyhedron with equilateral triangle facets.

It has been suggested that pathogens of some plants (as chrysanthemum, coconut palms, potato, and tomato) are viroids – viruses different from virions both in structure and in mechanisms of interaction with host cells. Particularly, as contrasted to virions, a viroid is a non-capsulated tiny macromolecule of RNA that consists of several hundreds of nucleotides. Thus, a viroid is a pure genome. Still, it remains questionable how this genome penetrates the host cell and how it violates the natural genetic control of metabolism of the cell.

1.4. Cells and bionts

In contrast to a nonliving being, a *biont,* i.e. a *living organism*, consists of at least one cell. A biont is said to be *single-celled* or *one-celled* or *unicellular* if it consists of exactly one cell and *many-cellular* or *multicellular* if it consists of many (two or more) cells.

A cell is a smallest structural and functional unit that manifests *the phenomenon of life*. A unicellular organism is said to have *the cellular level* (*pattern*) *of structural organization of ions, electrons, atoms, and molecules*. The cells constituting a multicellular organism are, not only structural units, but also functional units of the organism, so that the vital activity of the organism as a whole consists of the coordinated vital activities of the constituent cells. A multicellular organism is said to have *the tissual level* (*pattern*) *of structural organization* if it is composed of integrated groups of cells with a common structure and function, called *tissues*. A multicellular organism is said to have *the organic level of structural organization* if it consists of structural and functional units, which are composed of various tissues, and which are called *organs*. A multicellular organism is said to have *the systemic level of structural organization* if it consists of systems of organs.

There are a great variety of cell types. Some structural similarities and differences of various cells are made explicit below.

i) A cell has a singly-connected *plasma membrane*, which encloses *cytoplasm* and, depending of the cell’s type, none, one, or many *membrane-enclosed nuclei* and none or some *membrane-enclosed organelles*. A cell is called an *enucleate* (*enucleated*) *cell* if it has no nucleus, a *nucleate* (*nucleated*) *cell* if it has one or more nuclei, a *single-nucleate* (*single-nucleated*, *single-nuclear*) *cell* if it has one nucleus, and a *multinucleate* (*multinucleated*, *multinuclear*) *cell* if it has many (two or more) nuclei. For instance, a *bacterium*, or *protist*, is an enucleated unicellular biont, and a mammalian erythrocyte is an *enucleate* (*enucleated*, *not nucleated*) *mammalian cell*. By contrast, a vertebrate muscle fiber is a *multinucleate* (*multinucleated*, *multinuclear*) *cell*. The plasma membrane and any of the internal membrane of a cell is impermeable to macromolecules and selectively permeable to small inorganic and organic ions and neutral molecules.

ii) Depending on its type, a new living cell is naturally formed either by *division of a preceding cell* or *by metamorphosis of a preceding cell* or by *fusion of two preceding cells*, or else new cells are created (as erythrocytes) in some other cells.

iii) Chemical composition and major biochemical processes are similar in all cells. A living cell is built of organic polymeric compounds belonging to the following four families: *proteins*, *carbohydrates*, *lipids,* and *nucleic acids*. In addition, a cell contains free or bound small inorganic and organic ions and also small neutral inorganic and organic molecules as *intermediates*. In contrast to a virus that is built of a nucleic acid and proteins only, the interior of a cell contains *protoplasm* − the irritable *living matter* consisting of organic and inorganic substances, e.g. proteins, carbohydrates, and salts in solution. Particularly, the entire content of a cell inside the plasma membrane and outside the nucleus, or nuclei, and outside the organelles, is protoplasm that is called *cytoplasm*. At the same time, a cell can be dismantled to the same atoms as those occurring in minerals and viruses.

iv) A cell manifests its existence as a living being, i.e. it manifests a phenomenon of life, by continually and simultaneously driving thousands of chemical reactions (processes) of two mutually reverse kinds: *synthesis of*, i.e. *assembling*, some large organic molecules from smaller organic or inorganic ones and *analysis of*, i.e. *disassembling*, some other large molecules into smaller organic or inorganic constituents. A chemical reaction of the first kind is said to be *anabolic* (meaning *ascending* or *constructive*), whereas the totality of all anabolic chemical reactions occurring in a cell is said to be *the anabolism of the cell*. A chemical reaction of the second kind is said to be *catabolic* (meaning *descending* or *destructive* or *degradative*), whereas the totality of all catabolic chemical reactions occurring in a cell is said to be *the catabolism of the cell*. Either an anabolic or a catabolic chemical reaction is indiscriminately called a *metabolic*, or *biochemical*, reaction. The totality of all metabolic chemical reactions occurring in a living cell is said to be *the* *metabolism of the cell.* The totality of all metabolic chemical reactions occurring in a biont is said to be *the metabolism of the organism*, and similarly with “ana” or “cata” in place of “meta”. If a biont is multicellular, then the metabolism of the organism is the integrated metabolism of all its constituent cells. The class of the metabolisms of diverse living cells and diverse bionts is said to be *metabolism*. Metabolism is the way of existence of any living cell and of any biont, by which the latter differ from the way of existence of any nonliving being (as a mineral or virus). In other words, metabolism is a necessary and sufficient property of living matter.

v) A living cell drives its metabolism with the help of *enzymes* − *protein catalysts* that the cell produces by itself. A specific enzyme *catalyzes* a specific metabolic reaction. This means that the enzyme brings about the reaction at the cell’s temperature and maintains it with an optimal rate without being consumed by the reaction. In the absence of the enzyme, the reaction would be impossible under the given conditions.

Most cellular enzymes are proteins. In addition, a cell synthesizes and contains special enzymatic nucleotide molecules of the ester called *adenosine triphosphate* or, briefly, *ATP*, − small organic molecules that carrying chemical energy from one molecule to another in cellular metabolism. In this case, among the protein enzymes that a cell synthesizes is *adenosine triphosphatase*, or *ATP-ase*, − the enzyme driving *endergonic* chemical reactions of hydrolysis of ATP, which supply energy required both for driving *exergonic* chemical reactions and, when applicable, for performing the mechanical work of muscular contraction. Nucleotides are organic compounds, which enzymes link as monomers into polymers called *polynucleotides* or *nucleic acids*. By contrast, ATP comprises modified nucleotides, which exist as free molecules.

vi) A living cell has a certain *genome* − the complete single complement of DNA (deoxyribonucleic acid) macromolecules that are comprised in threadlike structures called *chromosomes*, *one individually structured DNA macromolecule per each chromosome*. A DNA macromolecule comprises discrete units of hereditary information called *genes*. Besides a DNA polymer (polynucleotide) as genetic material, a chromosome contains some protein polymers as supporting material. All biochemical processes in a cell are controlled by its *genome*.

1.5. A dictionary of term-forming morphemes

The meaning and etymological sense of the some English morphemes (mainly, prefixes and combining forms), both established and new ones, which occur in the terms to be used or mentioned in this essay and generally in *Psychologistics* are given in the following brief *etymological dictionary* (compiled from or in accordance with A Merriam-Webster® [1981], Pring [1982], Simpson [1968]).

Dictionary 1.1

"*aut*"- or "*auto*"- means *self* or *same*, from the Greek comb. form "αυτ"- \aut\ taking on the same denotata.

"*chemo*"-, comb. form, means *chemical* ⟨chemoautotroph⟩; originates from the Greek noun "χημ|εία" \ximía, chimía\ meaning medieval *alchemy* or modern *chemistry*; ~"ικός" \ikós\, adj., means *chemical* or as a noun means *a chemist*.

"*ec*"- or "*eco*"-, pref., means *habitat* or *environment* ⟨ecosystem, ecad, ecology⟩; originates from the Greek noun "οικ|ία" \ikía\ meaning *a house*; ~"ιακός" \iakós\, adj., means *domestic*.

"*eu*"-, pref., means *true* or *genuine* ⟨Eukaryotae, Eumetazoa, eukaryote, eumetazoan⟩; originates from the Greek adv. and comb. form "ευ", \ev\ before voiced sounds or \ef\ otherwise, meaning «well» each, or from the Latin interj. "eu" having the same meaning as "good!" or "well done!".

"*hetero*"-, pref., means *other*, *different*, *other than auto-*, *hom-*, or *iso-*; originates from the Greek comb. form "έτερο"- \étero\ meaning *other* or *different*" and from the Greek pronoun "έτερος" \éteros\ meaning (*an*)*other*, *one* or *the other* (*of two*).

"*kell*"- or "*kello*"-, comb. form, means *cellular*, esp. *of or relating to classification of cells*; originates from the Greek synonymous nouns "κελλί" \kellí\ and "κελλίον" \kellíon\ meaning *cell*; should be distinguished from the comb. form "cell"- or "cello"- meaning *cellulose* ⟨*cello*biose⟩.

"*meta*"-, comb. form, means *later or more highly organized* ⟨Metazoa, metazoan⟩ or *of higher logical type* ⟨metalanguage⟩; originates from the Greek adv. "μετα" \meta\ meaning *afterwards*, or from the homonymous prep. meaning (with an accusative noun) *after*.

"*mes*"- or "*meso*"-, pref., means *in the middle*, *intermediate* ⟨Mesozoa, mesozoan⟩; originates from the Greek adj. "μεσος" \mesos\ meaning *middle*, *mid*.

"*micto*"-, pref., means *mixed* ⟨mictotroph⟩; originates from the Greek adj. "μικτος" \miktos\, having the same meaning as "*mixed*".

-"*on*" or -"*n*", noun suffix, means *a creature of the class specified by the preceding adherent morpheme* ⟨taxon, proton, phonon, photon, metazoan, parazoan⟩; originates from the Greek noun "όν" \ón\ having the same meaning as either of the nouns "being" or "creature";

"*par*"- or "*para*"-, pref., means *beside*, *alongside*, *beyond* ⟨Parazoa, parazoan⟩; originates from the Greek comb. form "παρα"- \para\, having the same meaning as "*near*", "*beyond*", "*contrary*", "*excess*", etc.

"*phot*"- or "*photo*"-, comb. form, means *light*, originates from the Greek noun "φως" \fos\ having the same meaning as "*light*" or as "*sight*" (in the sense of "faculty of sight").

"*phyt*"- or "*phyto*"-, comb. form, means *plant* ⟨phytoplankton, phytoplankter⟩, originates from the Greek noun "φυτόν" \fitón\ (pl. "φυτά" \fitá\) meaning *a plant*.

-"*phyta*" or -"*phyte*", comb. form that occurs in the names of taxa and their specimens ⟨e.g. "Chlorophyta", "Saprophyta", "a chlorophyte", "a saprophyte"⟩ and that means *plants* or *plant*, respectively, – in accordance with the previous vocabulary entry.

"*pro*"-, pref., means *earlier than*, *prior to*, *before* ⟨Prokaryotae, protist⟩; originates from the Greek prep. and comb. form "προ" \pro\ or from the Latin adv. and prep. "pro", having the same meaning as "*before*" each.

"*proto*"- or "*prot*"-, comb form., means *lowest in organization*, *status*, or *in a series* ⟨Protista, protist, Protozoa, protozoan⟩; originates from the following Greek etymons: "προτοῦ"\protú\, conj. and adv., having the same meaning as "*before*"; "πρῶτα" \próta\, adv., having the same meaning as "(*at*) *first*" or "*before*"; "πρωτο"- \proto\, comb. form, denoting *first;* "πρώτος" \prótos\ adj., means *first*, *foremost*.

quasi", adv., adj. comb. form, originates from and takes, as an adverb and partly as a comb. form, on the same denotata as the homonymous Latin adv., namely, *as if*, *just as*, *as it were*, or *a sort of*; as an adjective and partly as a comb. form, takes on the denotata: *resembling*, *seeming*, *virtual*.

"*taxo*"-, comb. form, means *being*, *of*, *or relating to a taxonomy, taxonym* (*taxonomic name*)*, ortaxon* (*taxonomic categor*y), originates from the noun "ταξις" \taxis\ having the same meaning as "*order*" (the quality or state of being ordered or tidy), and also, homonymously, as "*class*" or "*grade*".

-"*troph*" or -"*trophe*", root, means *taker nutrients* [*in a specified way or of a specified type*] ⟨autotroph, heterotroph⟩; originates from the following Greek etymons: "τροφη" \trofi\, noun, having the same meaning as "*food*" or "*nourishment*".

"*trophic*", adj., a synonym of "*nutritional*", means *relating to or functioning in nutrition*.

-"*trophic*", comb. form, means *of or relating to a specified mode of nutrition peculiar to a pertinent -troph*.

-"*tropho*", comb. form, means *of or relating to nutrition or to modes of nutrition* ⟨trophotaxon, trophocategory⟩.

"*zo*"- or "*zoo*"-, comb. form, means *animal* ⟨zooplankton, zooplankter⟩, originates from the Greek noun "ζώον" \zóon\ (pl. "ζώα" \zóa\) meaning as *animal*.

-"*zoa*" or -"*zoan*", comb. form, means *animal* or animals, respectively – in the names of taxa and their specimens ⟨Parazoa, Metazoa, parazoan, metazoan⟩•

Cmt 1.1. The new English combining form "prota" is the transliteratum of the Greek adverb "πρῶτα", which has the same meaning as the conjunction and adverb "προτοῦ", namely *before*. At the same time, "prota" is consonant with both "meta" and "para". It is therefore convenient to use the morpheme "prota" interchangeably with or instead of "proto" as a *complimentary antonym of* "meta".•

2. Comprehensive biological taxonomies of bionts

2.1. Introduction

The first comprehensive *biological taxonomy of living organisms* was developed by Swedish botanist *Carolus Linnaeus* (the Latinized form of the name "*Carl von Linne*"), 1707–1778. A *living organism* is synonymously called a *biont*, from the Greek etymons: "βίος" \bíos\ (*s.m.*, pl. "βίου" \bíu\) meaning *life* and "όν" \ón\ (*s.n.*, *pl.* "όντα" \ónta\) meaning *a being* or *creature.* Consequently, the generic names "*biological taxonomy of living organisms*" and "*biological taxonomy of bionts*" ("*BTB*", pl. "*BTB's*") are synonyms. Linnaeus adopted the Aristotelian division of all bionts into *two kingdoms*: **Tá Φitá** /tá fitá/ (*sg.* "τό φυτόν" \tó fitón\, *s.n.*) and **Tá Ζώα** /tá fitá/ (*sg*. "τό ζώον" \tó zóon\, *s.n.*), – in Latin: **Plantae** (*sg,* "planta") and **Animalia** (sg. "animal") or **Animantia** (sg. "animans"), and in English: **The Plants** and **The Animals**. Incidentally, the Greek noun "ζώον" is, etymologically, a descendant of "ζωή" \zoí\ (*s.f.* "ή" \í\, *pl.* "αί ζωαί" \é zoé\) meaning *life*, – just as "βίος".

The above *dichotomy* of bionts was made in accordance with two different major modes of their nutrition or, more specifically, in accordance with two different ways in which bionts obtain *energy* and *carbon*. Namely, a plant is either an *autotroph* (i.e., more specifically, a *photoautotroph* or a *chemoautotroph*) or an *absorptive heterotroph*, whereas an animal is an *ingestive heterotroph*. Accordingly, the *Linnaean taxonomy*, or briefly the *LT,* comprises two separate taxonomies: the taxonomy of plants and the taxonomy of animals, which were presented in two separate Linnaeus' reports: "*Species Plantarum*" (1753, 1st edition) and "*Systema Naturae*" (1758, 10th edition). Still, both taxonomies are made and organized in accordance with the same principles, so that they form the single whole LT, which will alternatively be called the *two-kingdom BTB*.

Cmt 2.1. Etymologically, the English nouns "plant" and "animal" originate from the Latin etymons "*planta*" (pl. "*plantae*") and "*ănĭmăl*" (pl. "*ănĭmălia*"). The Latin noun "planta" means *a plant* in general, or *a green twig*, *cutting*, *graft* in particular. At the same time, the Latin noun "ănĭmăl" (or "ănĭmans", *pl.* "ănimantia") denotes, not *an animal* in the narrow sense, which the name has in English, but rather it denotes *a living being* (*creature*, *organism*) sometimes including a man (often contemptuously) and sometimes excluding a man. An animal [in English] other than a man is, depending on a context, denoted in Latin by the nouns “bēlŭa”, “bestĭa”, and “pĕcus”. The above meanings of the Latin nouns planta” and “animal” have been cited from Simpson [1968, p. 45, 46, 74, 76, 429, 472, 658]). It was in the Middle Ages, after the works of Aristotle on biology were translated into Latin, when the Latin word “animal” acquired its presently common meaning.•

Cmt 2.2. 1) The numeralable (count) noun “taxon” (without any article) is often used equivocally as a synonym of “taxonomic class” or “taxonomic category” and as a synonym of “taxonomic name” (see, e.g., A Merriam-Webster® [1981] or Allen [2003]). In the light of Aristotelian philosophy of nominalism, such an equivocal usage of the word “taxon” is explicable because, once every one of a given group of individuals is called by the same name the individuals become *ipso facto* members of the same class thus called. Still, I use the word “taxon” only as a synonym of either expression “*named taxonomic class*” or “*named taxonomic category*”, while the new noun “*taxonym*” will be used as a synonym of “*taxonomic name*”.

2) The taxonyms of genera, species, and subspecies of any BTB are, after the manner of the LT distinguished from the higher rank taxonyms by setting the former in italic, while all taxonyms are capitalized. This means that a BTB *is not straightforwardly expressible orally.* Therefore, it is, in this case, etymologically more correct to use the term “*taxograph*” (“*taxographonym*”) instead “taxonym” and to replace the noun “taxonomy” in the name “biological taxonomy of bionts” with “*taxography*” (“*taxographonymy*”).•

Linnaeus was a natural theologian, and he believed that all species of bionts were created by God in accordance with the taxonomic schema, which he, Linnaeus, managed to reveal. The God’s creatures were unchangeable, and therefore no concepts of evolution of life either influenced the LT or were implied by it. Still, a century later, Darwin made use of that taxonomy as the main argument in his theory of “*descent with modification*”, which has been formulated in his book “*The Origin of Species*” (1859) and which is now known as *the theory of evolution by natural selection*. Thus, it has turned out that the Linnaean hierarchy of increasingly wide taxa of animals from species to phyla or those of plants from species to divisions, – the hierarchy, which was based on similarities and dissimilarities of the *morphological* (*anatomic*) *patterns* of individual organisms of the diverse categories, – reflected in the main the *evolutionary genealogy* of the categories. *The genealogical tree of a species*, or *of any other taxonomic category above species and below phyla (divisions)*, which descends from a certain phylum (division) as the evolutionary ancestor, is said to be *the phylogenetic tree*, or *phylogeny*, *of the species*, or *of the pertinent descendant category*, respectively. The comparative morphology of species of bionts reflects the evolutionary history of the species. Therefore, the LT survived Darwin's evolutionary theory. Moreover, Darwin used the LT as the basis for tracing the historical development of the life nature in his theory.

The original LT was revised from time to time in the light of the achievements brought about in biology since the Linnaeus time, especially in the light of Darwin's evolution theory and genetic theory. Particularly, the LT was supplemented by some additional taxa. Still, the LT had remained basically unchanged and had been in common use as an exclusive one for more than two centuries until Whittaker [1969] (Robert Harding Whittaker, 1920–1980) suggested a *five-kingdom taxonomic schema* comprising **Monera** or **Prokaryotae**, **Protista**, **Fungi**, **Plantae**, and **Animalia**. This schema with some important modifications of Lynn Margulis is discussed in detail in Margulis and Schwartz [1987]. The five-kingdom taxonomy, to be called the *Linnaeus-Whittaker taxonomy* (*LWT*), is substantiated and followed closely as a general frame of reference in Campbell [1990, pp. 8–9, 505–674, 518–520ff]. The LT served as the frame of reference of any textbook on general biology published before the advent of the LWT. Therefore, the interested reader will find a description of most general taxonyms of the LT in any of such textbooks, of which *Biology* by Villee [1957] is likely the best one.•

Cmt 2.3. 1) The LT or the LWT is indiscriminately (commonly) called a *biological* taxonomy of bionts. The adherent quantifier "biological" in the above common name is not redundant because it suggests that any taxonomy that the name denotes *relates to biology* and is therefore *formal* and *technical*, and not an *ad hoc* or *informal* one. At the same time, the are many different taxonomies, which are entitled to be qualified *biological* in the above sense of this word, e.g. a general taxonomy of biochemicals or separate taxonomies of hormones, enzymes, organic polymers, polynucleotides (amino acids), nucleotides (monomers of polynucleotides), etc, and also taxonomies of cells, tissues, organs, or systems of organs of an individual multicellular organism, etc, etc. Therefore, the adjoined qualifier "bof living organisms" in the common name in question is also indispensable, although it is of course replaceable with the synonymous qualifier "of bionts".

2) It should be remembered that the terms "*dichotomy* of bionts" and "*two-kingdom taxonomy* of bionts" are not synonyms. The former denotes the act or result of dividing the realm of living organisms into two categories (e.g. the autotrophs and the heterothrops or the prokaryotes and the eukaryotes) and is in fact an abbreviation of the term "*major dichotomy* of bionts", whereas the latter denotes a *comprehensive hierarchical system of classification* of bionts in the two complementary kingdoms of the given dichotomy. A like remark applies, *mutatis mutandis*, with "five-fold" (or "pentachomy") and "five-kingdom" in place "dichotomy", and "two-kingdom" respectively.●

2.2. The LT

The main features the LT are explicated below (cf. Campbell [1990, pp. 484–492]) (cf. Campbell [1990, pp. 461, 485]).

2.2.1. A hierarchy of taxa of the LT

The LT and any other BTB is a *hierarchy* of increasingly broad (inclusive) *taxa* (*taxonomic categories, taxonomic classes* [*sensu lato*]), which are provided with the appropriate *proper Latin taxonyms* (*taxonomic names*) and whose *principal ranks* are denoted by numeralable (count) nouns of the following list.

List 2.1: ***The principal rank-names (hierarchal meataxonyms) of increasingly broad taxa of a BTB, after the LT:*** "species","genus", "family", "order", "class" *sensu stricto*, "division" *for plants or* "phylum" *for animals*, "kingdom".●

That is to say, all similar species, either of plants or of animals, are united into the same genus, all similar genera are united into the same family, all similar families are united into the same order, and all similar orders are united into the same *class* [*sensu stricto*]. Then all similar classes of plants are united into a division of plants, whereas all the divisions are united into the kingdom of plants, called **Plantae.** Likewise, all similar classes of animals are united into a phylum (*pl.* "phyla"), whereas all the phyla are united into the kingdom of animals called **Animalia**. In this case, taxon **Fungi** is *one of the divisions of* **Plantae**.

In some cases, named subdivisions of species that are ranked as *subspecies*, *microspecies*, *races*, or *varieties* are used, while some of the above rank-names are often used synonymously. Also, there are *taxa of intermediate ranks*, such as:

subgenera or *superspecies* (*macrospecies*), ranking between species and genera; *suborders* or *superfamilies*, ranking between orders and families; *subclasses* (*infraclasses*) or *superorders*, ranking between classes and orders; *subphyla* or *superclasses,* ranking between phyla and classes (in kingdom Animalia); *subdivisions* or *superclasses*, ranking between divisions and classes (in kingdom Plantae), *subkingdoms*, ranking between kingdoms and phyla or divisions.

The names given on List 2.1, and also the names of intermediate ranks that are used but not mentioned in the previous paragraph will be called *rank-names*, *rank-terms*, or *metataxonyms*, of the pertinent taxa. The noun "class" is used in the LT in the *narrow sense* (i.e. synonymously with the name "class sensu stricto") as the rank-name of taxa ranking between the orders and the divisions of plants or between the orders and the phyla of animal. Therefore, as the reader has long since observed, I use the word "category" as a synonym of the word "class" in its usual broad sense, i.e. as a synonym of the name "class sensu lato", – for avoidance of confusion. All *"super"-metatataxonyms* (*"super"-terms*) and some "*sub*"-*metatataxonyms* ("*sub*"-*terms*) are informal (unconventional). Accordingly, all taxonyms, having such a rank are also informal (unconventional). For instance, some taxonomists divided kingdom Animalia into two subkingdoms Protozoa and Metazoa. Some others divided Metazoa into three *sub-subkingdoms*: Parazoa, Mesozoa, and Eumetazoa, so that Animalia was effectively divided into four subkingdoms: Protozoa, Parazoa, Mesozoa, and Eumetazoa.

2.2.2. The morphological method

The LT is based on the so-called *morphological method* – the method, according to which the criteria of similarity of bionts for uniting them into a taxon of a given rank are prescinded from *anatomical observations and measurements* of the bionts; "anatomical" is understood as "*histological*" in the case of unicellural bionts. The morphological (anatomical) criteria of similarity of bionts are the only possible criteria of similarity for forming taxa of any rank higher than that of species (cf. subsection 2.4). Still, they are applicable for describing species as well. Thus, the morphological method is indispensable and universal, but at the same time it is intuitive and hence subjective and ambiguous to a great extent. The stricter are criteria of morphological similarity, the narrower is the corresponding taxon.

For instance, one may posit that two triangles are *similar* if they satisfy any one of the known criteria of the triangle similarity of Euclidean geometry.

Accordingly, any two triangles, which do not satisfy to those criteria, are said to be *dissimilar*. However, one may *extend* the criteria of similarity so as to say that all triangles are mutually similar in contrast, e.g., to a polygon having four or more vortices. *Similarly* (in the completely different sense of this derivative of the word 'similar'), one may posit that all polygons are mutually similar in contrast, e.g., to a plane geometrical figure bounded by a smooth closed curve. *Similarly*, all two-dimensional geometrical figures are similar as compared to any three-dimensional figure. And so on. Thus, the very intelligible use of the common names such as "a triangle', "a quadrangle", "a polygon", "a plane figure", etc is possible only because each of them *connotes the respective class* and hence *expresses the respective property of morphological similarity of members of the class*. At the same time, one may *restrict* the criteria of similarity so as to posit that any two plane figures are similar if and only if they are congruent tokens of each other and to distinguish this kind of similarity by calling it "*congruity*" or "*identity*".

Owing to the morphological method on which the LT is based, the LT can be called a *morphological taxonomy* or, briefly, *morphotaxonomy,* while any taxon of the LT can be called a *morphotaxon* (*morphocategory*). Consequently, Linnaean species, genera, families, orders, classes, divisions, phyla, and kingdoms are *morphospecies, morphogenera*, *morphofamilies*, *morpho-orders*, *morphoclasses*, *morphodivisions*, *morphophyla*, and *morphokingdoms*, respectively. The term "morphospecies" is a conventional one (cf. Campbell [1990, p. 461]), whereas the rest of the above "morpho"-terms are ones of my own – they are not in common use. It goes without saying that Linnaeus and its followers defined the morphological criteria so as to guarantee that all morphotaxons of the same rank should be *mutually* (*pairwise*) *disjoint*. In analogy with the established general adjectives "conspecific" and "congeneric", any two or more members of a morphospecies, or of a morphogenus, can be qualified as *conmorphospecific*, or *conmorphogeneric*, respectively, and vice versa.

At first glance, morphotaxa seemed to describe the diversity of life adequately. However, it turned out that not all members of morphokingdom Plantae were autotrophs and not all members of morphokingdom Animalia were digestive heterotrophs. For instance, genera *Chlamydomonas* and *Euglena* were morphologically classified as Animalia. However, from the standpoint of their nutrition modes, the former should have been relegated to Plantae, while the latter could, on the sufficient grounds, be relegated either to Plantae or to Animalia.

Cmt 2.4. To be specific, Animalia of the LT is divided into two subkingdoms (see, e.g. Villee [1957, chap. XIII, § 126]): Protozoa (the protozoans) and Metazoa (the metazoans). In this occurrence, the combining form "*proto*" means *earliest in time or lowest in organization*, *status*, *or in a series*, whereas "*meta*" means *later in time or higher in organization*, *status*, *or in a series*. The two combining forms are derived from the following Greek etymons:

«**μετά** \metá\ 1. *adv.* afterwards. 2. *prep.* (*with acc.*) after.
προτού \protú\ *conj. & adv.* before.
πρώτα \próta\ *adv.* (at) first; before.
πρωτο- \proto\, *comb. form*, denoting *first.*
πρώτος \prótos\ *adj*. first, foremost.»

Thus, "Protozoa" means the *lowest animals*, and Metazoa the *higher animals.*

Subkingdom Protozoa is at the same time the only *phylum* of itself. A protozoan is a *unicellular eukaryotic organism*. Most protozoans are aquatic organisms, while the others (as the sporozoans) parasitize in blood and in tissual liquids of plants and higher animals. Protozoa is divided into five *classes* [*sensu stricto*] *ibid,* § 127): Sarcodina (the sarcodinian organisms, the amoebas and ameba-like organisms), Ciliata (the infusoria), Suctoria (the suctorians, the suctorial infusoria), Sporozoa (the sporozoans), and Flagellata (Flagellatae, the flagellates). The protazoans of the first four classes are *ingestive heterotrophs*, and hence they are animals, in accordance with the nutrition-mode criterion. However, the flagellates contain chlorophyll. Particularly, the flagellates of genus *Chlamydomonas* are solitary biflagellated *photoautotrophs*, whereas the flagellates of genus *Euglena* inhibit dark ponds and are versatile in their nutrition. In the light, a euglena lives as an autotroph by driving photosynthesis, while in the dark it lives as an ingestive heterotroph by ingesting particles of food by phagocytosis. The heterotrophic phases in the life of euglena are indispensable, because it requires tiny amounts of amino acids and vitamin B_{12}. Therefore, a euglena is a. photoautotroph and ingestive heterotroph simultaneously. Thus, as stated, *Chlamydomonas* should be related to Plantae, while *Euglena* can be related either to Plantae or to Animalia. Consequently, the taxonym

"Protozoa" of the LT turns out to be is a *misnomer*, because some protozoans are not zoans (not animals) at all.•

2.2.3. Formation rules of taxonyms of the LT

The taxonyms of the LT are formed in accordance with the following two general formation rules (cf. Campbell [1990, p. 485]).

I) A species is denoted by an *italicized two-word* Latin name that is called a *Linnaean* (or *Linnean*) *binomial* (or *binomen,* pl. *"binomina*"). The first, *capitalized*, word of a Linnaean binomial denotes the *genus* which the species denoted by the binomial belongs to. The second, *uncapitalized*, word of the binomial is a specific *epithet* (*qualifier*) to the genus name, which denotes the *differentia*, i.e. the *additional conceptual property*, by which a biont of the species is distinguished from a biont of any other species of the same genus. Both the genus and the differentia are *classes* [*sensu lato*], whose *intersection* is the species, thus being a *class* [*sensu lato*] as well. Thus, the species that is *denoted* by a given Linnaean binomial is uniquely *described* by the binomial *through the* [*intersection of the*] *genus and the differentia*. In other words, a Linnaean binomial is a *description through the genus and the differentia*, or, using the appropriate Latinized expression, a *descriptio per genus et differentiam* (cf. "*definition through the genus and the differentia*", in Latin: "*definitio per genus et differentiam*"). Incidentally, "*species*" is a Latin noun that, among a great many of its meanings, takes on the same denotata as the English names "*kind*", "*species*", "*division of* a genus" (see Simpson [1968]).

By way of example, here follow some Linnaean binomials:

a) *Populus* (poplar), *Populus tremuloides* (American ~), *Populus angulata* (angel-twig ~), *Populus diversifolia* (Asiatic ~), *Populus balsamifera* (balsam ~), *Populus deloides* (berry-bearing ~), *Populus nigra* (black ~), etc;

b) *Canidae* (Dogs): *Canis familiaris* (dog), *Canis lupus* (wolf), *Canis latrans* (praire dog), *Canis jubatus* (maned wolf), *Canis niger* (red wolf), etc;

c) *Felidae* (cats): *Felis domesticus* (domestic cat), *Felis sylvestris* (European wild cat), *Felis leo* (lion), *Felis tigris* (tiger), *Felis pardus* (leopard), etc;

and also *Pan troglodytes* (chimpanzee), *Simia satyrus* (orangutan), *Homo sapiens* (human being, man), etc.

Likewise, a *Linnaean trinomial*, or *trinomen*, is a Latin three-word taxonym whose first term denotes a genus and whose second term denotes the differentia of a species, while the third term denotes the differentia of a certain subspecies; e.g. *Capitata sphaerica alba* (white cabbage). A highly specific taxon of bionts lower than a subspecies, which has no formal rank, can be indicated by a taxonym, not necessarily Latin or Latinized, which consists of *more than three words* and which is therefore called a *polynomial*, or *polynomen*, although it is not qualified Linnaean.

II) As has already been indicated in sub-subsection 2.2.1, the LT is made by filing species into the hierarchy of increasingly broad (inclusive) taxa, i.e. named taxonomic categories. As proper names of the taxa, the appropriate *capitalized Latin monomials* (*monomina*, *one-word names*) are used; the monomials of genera are italicized, whereas the monomials of the broader taxa are set in *roman* (*upright*) *font*. For instance, Chordata is a phylum, i.e. a taxon (taxonomic category, taxonomic class sensu lato) at the phyla level, which is termed by the taxonym (taxonomic name) "Chordata" and which comprises three subphyla: Cephalochordata, Urochordata, and Vertebrata. Consequently, Vertebrata is a subphylum, i.e. a taxon at the subphyla level, which is termed by the taxonym "Vertebrata" and which comprises seven classes [sensu stricto], Mammalia being one of them.

To recapitulate, there are *three general formation rules* of taxonyms of the LT:

1) The taxonym of a species is a Latin binomial, whose first term is the capitalized and italicized taxonym of the genus, into which the species is included as its part, and whose second term is an uncapitalized and italicized epithet denoting the diffrentia, which distinguishes the given species from any other species of the same genus.
2) The taxonym of a genus is a capitalized and italicized Latin monomial, no matter whether it stands alone or as the first term of the binomial of a species.
3) All taxa at any rank higher than genus are capitalized roman (upright) Latin monomials.

2.3. The LWT

The LWT is, in the first place, based on recognizing two fundamentally different types of cells: *prokaryotic cells* and *eukaryotic cells* and also on recognizing the fact that *prokaryotic cells exist only in the form of unicellular bionts* called *prokaryotes* or *bacteria*. Thus, the LWT is, from the very beginning, based on the *microscopic* distinguishing properties of bionts. *Prokaryotes are bacteria, and vice versa*. These are placed in their own kingdom, **Prokaryotae** or **Monera**, apart from all eukaryotic organisms. Accordingly, Monera is the most unambiguous taxon of the LWT. Depending on modes of their nutrition, prokaryotes are divided into three categories: *the photoautotrophic prokaryotes*, *the chemoautotrophic prokaryotes*, and *the absorptive heterotrophic prokaryotes*. Particularly, Monera includes *cyanobacteria* – photoautotrophic bionts formerly known as blue-green algae and related to plants. Prokaryotes are considered as the earliest bionts that lived and evolved on the Earth for two or three billion years.

The eukaryotic organisms are classified in the remaining four kingdoms on the grounds of their *biontic* (individual, opposed to *phyletic*) organizations, modes of nutrition, types of oxygen metabolism, ways of reproduction, and evolutionary history. **Fungi**, **Plantae**, and **Animalia** are kingdoms of multicellular eukaryotes. In this case, fungi are absorptive heterotrophs, plants are photoautotrophs, and animals are ingestive heterotrophs. **Protista** is described in detail in Campbell [1990, pp. 540–563]. In general outline, Protista is mostly the kingdom of unicellular eukaryotes, although some multicellular eukaryotes, which, according to certain criteria, are more closely related to unicellular ones than to fungi, plants, or animals, are also placed in this kingdom. Protista is *semi-formally* divided into three subkingdoms: **Protozoa** comprising animal-like ingestive heterotrophic unicellular protists, **Algal Protists** comprising plant-like photoautothropic protists, and **Unicellular Molds** comprising fungus-like absorptive heterotrophic protists (cf. Campbell [1990, pp. 540–563]).

I have already pointed out in Cmt 2.4 that the taxonym "Protozoa" is a misnomer in the LT. Protozoa of the LWT essentially differs from Protozoa of the LT. Still, the taxonym "Protozoa" is a misnomer in the LWT as well, because the new denotatum of "Protozoa" also contradicts its etymological sense. Indeed, since the new Protozoa is a subkingdom of kingdom Protista, which is disjoint of the kingdom Animalia, therefore a protozoan, i.e. a member of Protozoa, is a protist and not a zoan (not an animal) in general and not a protozoan (not a lowest animal) in particular. Instead of "Protozoa", I may in this case suggest to use the noun "**Prot*a*zoa**" meaning «*before animals*», – in accordance with the Greek adverb "πρώτα" \próta\ meaning *before*. Yet I shall, when needed, use the conventional term "Protozoa" for avoidance of confusion. Incidentally, genus *Euglena*, which is problematic in the LT, belongs now to phylum **Euglenophyta** of Algal Protists. According to Campbell (*ibid.* pp. 549, 550), «some members of the phylum Euglenophyta lack chloroplasts and depend exclusively on heterotrophism.»

The general hierarchal schema of the LT and particularly the rank-names of progressively broader taxa of the LT as those given on List 2.1, alone or with a prefix "sub" or "super" (when applicable), are also adopted in the LWT. However, the variety of *major* (*higher-rank*) *taxa* of the LWT at the ranks (levels) of kingdoms, subkingdoms, divisions, phyla and subphyla and their matters (populations) essentially differ from those of the LT. The *major taxonyms* of the LWT are listed, e.g., in Campbell [1990, pp. A3-A4]. In general outline, the nomenclature of the LWT has the following features. Kingdom Monera, or Prokaryotae (prokaryotes, bacteria) is divided into two subkingdoms: **Archaebacteria** and **Eubacteria**, which comprise respectively 3 and 10 homogeneous (undivided) taxa of *unspecified ranks*. There is not yet a consensus on how to divide this kingdom and on how to name its taxa. Therefore, the taxonyms of Monera and their number are informal. The rank-name "phylum" is used in kingdoms Protista and Animalia, while the parallel rank-name (i.e. the name of taxonomic categories of the same rank) "division" is used in kingdoms Fungi and Plantae. Particularly, kingdom Protista is divided into 16 phyla having neither subphyla nor classes. The division of Protista into three subkingdoms; Protozoa (Protazoa), Algal Protists, and Unicellular Molds, which have been mentioned earlier, is informal. Therefore, this division is not indicated on the Campbell list, although it is indicated and discussed in Campbell [1990, chap. 26].

One of the most recent suggestions in the field of taxonomies of bionts is that of Woese et al [1990] to divide the *superkingdom of bionts* (opposed to the *superkingdom of minerals*) into three *domains*: **Archaea**, **Bacteria**, and **Eukarya**.

Concepts of the modern genetic theory of inheritance and mutation have resulted in a revision of putative evolutionary relationships mainly among the widest Linnaean morphotaxa, namely, at the level of phyla and divisions, to say nothing of kingdoms. Therefore, there are in the LWT higher-rank taxonyms, which are absent in the LT, while the others are *homonyms* of those of the LT, which essentially differ from the latter semantically. At the same time, all taxa of the LWT are defined by Linnaean *morphological method*. That is to say, just as the LT, the LWT is a *morphotaxonomy,* while any taxon of the LWT is a *morphotaxon* (*morphocategory*) so that its kingdoms, phyla, divisions, classes, orders, families, genera and species are *morphokingdoms*, *morphophyla*, *morphodivisions*, *morphoclasses*, *morpho-orders*, *morphofamilies*, *morphogenera*, and *morphospesies* respectively. In this case, the lower-rank and middle-rank taxonyms of the LT have basically remained untouched by the above revision both syntactically and semantically. Particularly, most of the Linnaean morphospecies and their binomials are also used in the LWT.

By way of example, here follow a full biological definition of the term "man" in the framework of the LWT.

> **Df 2.1.** *A man,* or *human being*, is a specimen (member) of kingdom Animalia, [subkingdom Eumetazoa, – optionally and informally], phylum Chordata, subphylum Vertebrata, class Mammalia, subclass Eutheria, order Primates, family Hominidae, genus *Homo*, species *sapiens*.•

This definition is also effective in the framework of the LT, provided that "Animalia" is interpreted respectively.

The LWT has become popular with the majority, if not with all, of the contemporary biologists. At the same time, most people, not being biologists, who once learned the LT, are still persistent in their intuitively understanding the homonymous taxonyms, formal (as "Plantae", "Fungi", or "Animalia") or informal (as "plant", "fungus", or "animal"), occurring in both the LT and the LWT, in the sense that they have in the LT. A discussion of the meanings of concrete taxonyms of the LWT or LT is beyond the scope of this exposition. However, the following remarks regarding most general relationships between the two taxonomies may be in order.

Many taxonyms of the LWT have homonyms among the taxonyms of the LT, which satisfy the following general rule.

Rule 2.1. If a taxonym of the LWT has a homonym in the LT then the taxon denoted by former taxonym is *laxly* (*weakly*) *narrower*, i.e. is *either equal or strictly narrower*, than that denoted by the latter homonym. In this case, the rank of the former taxon is *laxly* (*weakly*) *higher*, i.e. *either the same or strictly higher*, than that of the latter. •

For instance, kingdoms Plantae, Animalia, and Fungi of the LWT are strictly narrower than homonymous kingdoms (i.e. taxa of the same rank) Plantae and Animalia, and homonymous division (i.e. a taxon of a strictly lower rank) Fungi of the LT, respectively.

The abstract object, which a certain one of the common names such as "a plant", "an animal", "a fungus", "an alga", etc denotes, i.e. the general denotatum of the name, is determined by the category of bionts, which the name connotes, i.e. which is the connotatum of the name. At the same time, the connotata of the above mentioned common names coincide with the denotata of, i.e. with the categories denoted by, the corresponding numeralable (count) nouns "plant", "animal", "fungus", "alga", etc, without any modifiers, while the latter names are synonyms of the taxa "Plantae", "Animalia", "Fungi", "Algae", etc, respectively. Therefore, the general denotatum of a common name of a biont, whose connotatum is narrower than that of the common name "a biont" (the connotatum of the latter name coincides with the denotatum of the count name "biont" without any modifier) depends on the definition of the corresponding taxonym within a given taxonomy of bionts.

However, among champions and ordinary users of either one of the two taxonomies, a consensus has not yet been reached regarding the boundaries of many specific taxonomic categories *above the level of species*, whereas all species are, basically, same in both taxonomies. The taxonyms, which are not generally accepted, turn out to be tentative. Any hierarchical taxonomy of objects of a certain class is an order relation on that class. In this case, establishing a universal order relation in the diversity of life is probably an unachievable object. In spite of this fact, the two taxonomies stand in the following general relation.

Cnv 2.1. Whenever there is a danger of confusing a taxonym of the LT and the homonymous taxonym of the LWT, the former will, as I have already done above, be suffixed with the qualifier "*sensu lato*" and the latter with the qualifier "*sensu stricto*". The same qualifiers will be attributed to the common names denoting the general individuals of the corresponding categories.•

Cmt 2.5. 1) For instance, kingdoms Plantae and Animalia and *division* Fungi (of Plantae) of the LT can unambiguously be called "Plantae sensu lato", "Animalia sensu lato", and "Fungi sensu lato", whereas the homonymous kingdoms Plantae, Animalia, and Fungi of the LWT can unambiguously be called "Plantae sensu stricto", "Animalia sensu stricto", and "Fungi sensu stricto". Accordingly, individuals of the first three categories can be called "a plant sensu lato", "an animal sensu stricto", "a fungus sensu lato", whereas individuals of the last three categories can be called “a plant sensu stricto”, “an animal sensu stricto”, and “a fungus sensu stricto”, respectively.

2) The adjoined qualifiers “sensu lato” and “sensu stricto” to a Latin or English name are epistemologically relativistic ones, which mean that that name is used in a relatively broad sense and in a relatively narrow sense, respectively. Therefore, the exact sense of any name containing either of the two qualifiers should be defined explicitly. For instance, the qualifier “sensu lato” in the name “animal sensu lato” has nothing to do with the broad sense, which the etymon “animal” had in Latin (cf. Cmt 2.1).

3) With the help of Cnv 2.1, the difference between the two taxonomies can in general outline be characterized as follows.

a) Protists, i.e. specimens of kingdom Protista, are either aquatic unicellular or simplest multicellular eukaryotes, which can be divided into three categories: (i) photoautotrophic protists, (ii) absorptive heterotrophic protists, (iii) protozoans, i.e. ingestive heterotrophic protists, with the understanding that specimens of each one of the three categories fit some but not all definitions of plants sensu stricto, fungi sensu stricto, and animals sensu stricto, respectively (see below). Therefore, Protista includes the whole of Protozoa of Plantae sensu lato.

b) Fungi sensu stricto, i.e. specimens of the kingdom Fungi, are multicellular eukaryotes which have the tissual level of biontic organization and the absorptive heterotrophic mode of nutrition. A fungus lives embedded in its nourishing environment, and it digests food externally by secreting organic acids and hydrolytic enzymes into the food. The secreted digestive agents decompose polymeric constituents of the food into the small organic molecules that the fungus absorbs and utilizes as its immediate nutrients. Depending on the type of nutrient medium on which a fungus lives, all fungi are divided into three types: *saprophytes*, *parasites*, and *simbionts*. Fungi reproduce both asexually and sexually.

c) Plants sensu stricto, i.e. specimens of the kingdom Plantae sensu stricto, are photoautotrophic eukaryotes having the organic level of biontic organization, but possessing neither locomotive nor nervous organs. A plant cell has an outer wall built mainly of **cellulose**. Plants reproduce sexually, but are capable of propagating asexually. Plants are terrestrial organisms which have aerial parts as steams and leaves. Plantae sensu lato includes Monera, Fungi sensu stricto, Plantae sensu stricto, and all species of absorptive protists.

d) Animals sensu stricto, i.e. specimens of kingdom Animalia sensu stricto, are ingestive heterotrophic eukaryotes which have the organic level of biontic organization. Animalia sensu stricto is divided into two subkingdoms: Parazoa and Eumetazoa. Parazoa is the single phyla Porifera (sponges). Eumetazoa is divided into two branches: Radiata, consisting of the animals sensu stricto that have radial symmetry (as hydras, jellyfishes, and the like), and Bilateria, consisting of the animals sensu stricto that have bilateral symmetry. At the same time, Animalia sensu lato is divided into four subkingdoms: Protozoa, Parazoa, Mesozoa, and Metazoa, of which Parazoa is the same category as that included in Animalia sensu stricto. All phyla of Radiata are phyla of Metazoa, while all phyla of Parazoa are included in Bilateria. Thus, Animalia sensu lato includes Animalia sensu stricto and Protozoa.•

Cmt 2.6. Any term has its scope. Accordingly, within the scope of any one of the two BTB's, the LT and the LWT, i.e. in using it in practice or in discussing it alone in no connection with the other BTB, all taxonyms can and will be used without any additional qualifier "sensu lato" or "sensu stricto". The new taxonyms, which can be or have already been introduced in accordance with Cnv 1.1, belong to this metalanguage, and not to the LT or LWT, and that they will be used only in comparing the two taxonomies. With the same purpose, homonymous taxonyms of the two taxonomies can be renamed *ad hoc* in any other convenient way, for instance as follows.

Overt similarity in properties of objects, particularly in their form or functions, is called *analogy*. Covert similarity in properties of objects that is attributed to their common origin is called *homology*. For instance, the wings of a butterfly and the wings of a bird are analogous in function, but they are not homologous. By contrast, the hands of a man, the wings of a bird, and the thoracic fins of a whale are homologous organs, but they are not analogous in form and function.

I have already pointed out previously that the LT is based on the morphological method, i.e. primarily on likeness of specimens of the same taxon by the pertinent analogies. By contrast, the LWT is based primarily on likeness of specimens of the same taxon by the pertinent homologies. This remark applies particularly to kingdoms Plantae and Animalia of the LT and to the *homonymous* kingdoms of the LWT. Therefore, whenever confusion can result, I may use the taxonyms "**Analoplantae**" ('the analoplants") and "**Analoanimalia**" ('the analoanimals") or "**Morphoplantae**" ('the morphoplants") and "**Morphoanimalia**" ('the morphoanimals"), instead of "**Plantae sensu lato**" ("the plants sensu lato") and "**Animalia sensu lato**" ("the animals sensu lato"), for mentioning kingdoms **Plantae** (the plants) and **Animalia** (the animals) of the LT, while using the taxonyms "**Homoloplantae**" ('the homoloplants") and "**Homoloanimalia**" ('the homoloanimals"), instead of "**Plantae sensu stricto**" ("the plants sensu stricto") and "**Animalia sensu stricto**" ("the animals sensu stricto"), for mentioning the respective kingdoms of the LWT. The former two kingdoms are broader than the respective latter two.•

2.4. Euspecies

From the standpoint of evolutionary and genetic theories, two populations of the so-called *sexual*, or *gamic*, *bionts* that interbreed or are capable of interbreeding belong to the same specific category (class sensu lato) of bionts, which is conventionally called a *biological species* or, briefly, a *biospecies*. Here follows, e.g., the definition of the term of Campbell [1990, p. 461]:

> «A **biological species** is a population or group of populations whose members have the potential to interbreed with one another to produce fertile offspring, but cannot successfully interbreed with members of other populations… In other words, a biological species is the largest unit of population in which gene flow is possible, and which is genetically isolated from other such populations. Put still another way, each species is circumscribed by reproductive barriers that preserve its integrity as a biological package by blocking genetic mixing with other species. Members of a species, said to be conspecific, are united by being reproductively compatible, at least potentially. A businesswoman in Manhattan has little possibility of sharing offspring with a dairyman in Outer Mongolia, but if the two should get together, they could have viable babies that develop into fertile adults. All humans belong to the same biological species. In contrast, humans and chimpanzees remain distinct species even where they share territory, because the two species cannot successfully interbreed and produce hybrid offspring.
>
> It is important to remember that biological species are determined by their reproductive isolation from other species in *natural* environments. It is

often possible in the laboratory to produce hybrids between two species that do not interbreed in nature.»

However, in accordance with Cmt 2.3, the qualifier "biological", and hence the combining form "bio" being its abbreviation, means *of or relating to biology*. In this sense, any morphotaxon of the LWT or LT and particularly a morphospecies can be qualified *biological* – just as the LWT and LT themselves. Therefore, instead of the term "biological species" or "biospecies", I shall use "*euspecies*" – a new term of my own.

The criteria of morphological similarity of bionts, on the grounds of which the latter are placed into a certain morphospecies, do not include the *genetic morphology* of the members of the morphospecies. Therefore, the morphological method does not explain the uniqueness and integrity of a morphospecies of gamic bionts. Consequently, in order to turn a morphospecies of gamic bionts into a euspecies, the morphological criteria defining the morphospecies should be supplemented by certain *criteria of reproductive compatibility of its members*, – to be also called the *criteria of inbreeding* or the *inbreeding criteria*, – and by certain criteria of reproductive incompatibility of members of two different morphospecies of gamic bionts. Still, these criteria depend on the morphospecies and are not therefore universal. By way of example, the interbreeding criteria for some wide categories of bionts, including humans, are established in the following definition.

Df 2.2. Two gamic bionts of any one of the following kinds are said to be *reproductively compatible if they are able to interbreed with each other* ***in nature*** *and to produce fertile offspring*:

1) Two mature *isogamic bionts*, i.e. bionts producing indistinguishable *gamets*.
2) Two mature *euhermaphrodites* (*true hermaphrodites*), conventionally called *hermaphrodites* (cf. Campbell [1990, pp. 932, 933]), i.e. bionts producing sperms and eggs simultaneously during a certain period of their lifetime.
3) Two mature *sequential hermaphrodites* (*ibid,* p. 933) *of opposite sex,* i.e. bionts being first a male and then a female, or vice versa, sequentially during their lifetime – to be also called *quasi-hermaphrodites*.
4) Two mature *unisexual bionts of opposite sex.*

5) Two mature *quasi-hermaphroditic,* or *unisexual, bionts of the same sex*, each of which is *reproductively compatible with one and the same quasi-hermaphroditic*, or correspondingly *unisexual*, *biont of the opposite sex*, as indicated above in the item 3, or 4, respectively.

6) Two immature bionts that are expected to become mature ones of any one of the kinds indicated in items 1–5.

Two gamic bionts that are not reproductively compatible, i.e. ones that do not satisfy the above criteria, are said to be *reproductively incompatible*.•

Under Df 2.2, Campbell's definition of a biological species definition can briefly be paraphrased thus:

Df 2.3. The population of all *reproductively compatible gamic bionts* is called a *euspecies* (*true species*) or, conventionally, a *biospecies* (*biological species*). It is understood that all euspecies are *mutually* (*pairwise*) *disjoint* because a member of one euspecies is *reproductively incompatible* with a member of another euspecies.•

Cmt 2.7. 1) Just as "euspecies", the nouns "euhermaphrodite" and "quasi-hermaphrodite" are my own terms, which I have introduced in Df 2.2 as synonyms of the conventional terms "hermaphrodite" and "sequential hermaphrodite" respectively (see, e.g., Campbell [1990, pp. 461, 933]). Thus, a euhermaphrodite is, by definition, a biont that can fertilize itself, but it can also mate with another member of the same euspecies. In the latter case, each euhermaphrodite donates and receives sperms simultaneously. I employ the noun "euhermaphrodite" ("true hermaphrodite") instead of "hermaphrodite" because the *specific* meaning of the latter as s synonym of my term "euhermaphrodite" is incompatible with the *generic* meaning, which it has in the description "sequential hermaphrodite" through the genus and the differentia. Once "euhermaphrodite" is used in the former (conventional) sense of "hermaphrodite", the latter becomes a generic name such that a hermaphrodite is either a euhermaphrodite or a sequential hermaphrodite. At the same time, I have introduced the term "quasi-hermaphrodite" as a synonym of "sequential hermaphrodite" because the former has, in contrast to the latter, a natural adjectival derivative "quasi-hermaphroditic" and is less cumbrous. The combining form "*quasi*" originates from the homonymous Latin adverb "quăsī", meaning *as if*" or *a sort of* (see Simpson [1968]). Although this meaning is less specific than that of "sequential", it is unambiguous in the given context.

2) A morphospecies of sexual organisms should not necessarily coincide with one of the euspecies (biospecies) of such organisms. Particularly, a morphospecies can turn out to be a collection of several disjoint euspecies, and conversely, a euspecies can turn out to be a collection of several disjoint morphospecies. The morphospecies of a single euspecies biospicies of animals are called *races*. For instance, *Homo sapiens* is a euspecies (biospecies). *Euspecies are the only taxa of bionts, which can be defined unambiguously and which have a definite justification at the genetic* (*biomolecular*) *level*.

3) The whole of the terminology which has been introduced previously in connection with morphospecies applies with "*eu*" in place of "*morpho*". Particularly, in accordance with Df 2.2 and in analogy with the adjective "conmorphospecific", any two or more bionts of a euspecies can be qualified as *coneuspecific* and vice versa.

4) A biont that is not sexual is said to be *asexual*, or *nongamic*. However, the criterion of interbreeding is inapplicable to asexual (nongamic) living organisms (as bacteria or some protists). The only way to divide these organisms into species is to apply either the *pure* morphological method as described above or a *combined* morphological method, i.e. one that consists of a pure morphological method supplemented by some *biomolecular*, i.e. actually *micromorphological*, *criteria* or by some *evolutionary criteria*, or else by both. Application either of the pure morphological method or of a combined morphological method and biomolecular method is, also, the only way to define any taxonomic categories of living organisms at the level of genera or higher.•

3. Four major taxonomies of bionts: the foundation of the comprehensive biological taxonomies of bionts

3.1. The major dichotomy of living cells and the relevant dichotomies of bionts

Ax 3.1: ***The major dichotomy of cells.*** Depending on the morphology of a cell as revealed with the help of both the light microscope and the electron microscope, and also depending on the functions of a cell as revealed by the methods of biochemical analysis, all cells are divided into two major *types* (*classes*): *the eukaryotic* (*eucaryotic*) *cells* and *the prokaryotic cells*.•

Cmt 3.1. A eukaryotic cell is usually a nucleated cell containing certain membrane-enclosed bodies, which perform functions analogous to those of the organs of a multicellular organism, and which are therefore called *organelles*. For instance, the so-called *mitochondria* (*mitochondrion* in the singular) are organelles serving as cites of cellular respiration. Except for mammalian erythrocytes, all eukaryotic cells are nucleated. In this case, all chromosomes of a nucleated eukaryotic cell are located in its nucleus and are thus separated from the rest of the cell by the nucleus membrane. The cell's proteins are synthesized in the organelles, called *ribosomes*, with the help of the *RNA* (*ribonucleic acid*) *polymers* whose structure is determined by the *DNA* (*deoxyribonucleic acid*) *polymers*. The ribosomes consist of RNA and proteins and they are synthesized in the *nucleolus* – a properly specialized structure located in the nucleus.•

Ax 3.2. Prokaryotic cells are found only as *unicellular organisms*, which are called *prokaryotes*, or *bacteria* ("*bacterium*" in the singular); i.e. *a prokaryote is a bacterium and vice versa*. A prokaryotic cell lacks a membrane-enclosed nucleus and membrane-enclosed organelles, so that its chromosomes are not separated from the rest of the cell by any membrane.•

Cmt 3.2. A biont other than bacteria is composed of one or more *eukaryotic cells* and, perhaps, of some *modified eukaryotic cells*, which are specialized to perform certain functions for the entire organism. For instance, like prokaryotes, mammalian erythrocytes (red blood cells) are *enucleated*, and they lack *mitochondria* – organelles that serve as sites of respiration in all other mammalian cells. Erythrocytes are specialized to transfer oxygen from the lungs to the rest of the mammal's body through the cardiovascular system. It is therefore natural that erythrocytes are not respirable and that metabolic processes in erythrocytes are *anaerobic*, – like those in some bacteria. Still, the fact that mammalian erythrocytes are enucleated is an anomaly, because erythrocytes of non-mammalian vertebrates are nucleated. Also, in contrast to bacteria, which reproduce asexually by *binary fission*, erythrocytes are produced by the *bone marrow*. In order not to violate the convenient and otherwise universal dichotomy of cells into prokaryotic cells and eukaryotic cells, erythrocytes can be classified as *degenerate eukaryotic cells*.•

Df 3.1. A biont consisting of one or more eukaryotic cells and, perhaps, of some degenerate eukaryotic cells is called a *eukaryote* (*eucaryote*).•

Ax 3.3: ***The major kellodichotomy (cellular dichotomy) of bionts.*** In accordance with types of constituting cells, all bionts are divided into two *kellokingdoms* (*cell kingdoms*) *of first rank*: **Prokaryotae** and **Eukaryotae**, i.e. *the prokaryotes* and *the eukaryotes*.•

Df 3.2. A eukaryote is called a *euprotist* (a derivative of "prota"- or "proto"-, cf. Cmt 3.2) if it is *unicellular*, and a *eumetist* (a derivative of "meta"-) if it is *mulicellular*. That is to say, my new term "*euprotist*" is, by definition, a synonym of "*unicellular eukaryote*", while my new term "*eumetist*" is a synonym of "*multicellular eukaryote*".•

Cmt 3.3. Just as many other words that I form and employ in this essay, the words "euprotist" and "eumetist" are my own terms of this metalanguage – they are not in common usage. Instead of or along with the two words, I might have used the names "*protist sensu stricto*" and "*metist sensu stricto*" respectively. I have given preference for the former because they are monomial (one-word) ones, and are therefore more convenient.•

Ax 3.4: ***The major kellodichotomy of Eukaryotae.*** In accordance with Df 3.2, Eukaryotae, i.e. the eukaryotes, being one of the two kellokingdoms of first rank, is divided into *two kellokingdoms* (*cell kingdoms*, *kingdoms of cells*) *of second rank*: **Euprotista** and **Eumetista**, i.e. *the euprotists* and *the eumetists*.•

Ax 3.5: ***The major kellomonochotomy (cellular monochotomy) of Prokaryotae.*** Prokaryotae, being another one of the two kellokingdoms of first rank, is simultaneously *a kellokingdom of second rank*, thus being *the weak*, or *lax* (*not strict*), or *monochotomic*, *subkingdom*, i.e. *the whole part*, *of itself*.•

Th 3.1: ***The major kellotrichotomy (cellular trichotomy) of bionts.*** All bionts are divided into three *kellokingdoms of second rank*: **Prokaryotae**, **Euprotista**, and **Eumetista**.

Proof: The theorem follows from Ax 3.3 and Ax 3.4 by *combinatorial considerations*.•

Cmt 3.4. In order to deduce corollaries from divisions (dichotomies, trichotomies, etc) of a given class of entities, a simple rule of substitution can be laid down.

I) To start with, let us consider the simplest instance of the categorical syllogism of "Barbara"-type:

Every w is *v*. *Every u* is *w*. Hence, *u* is *v*.

("Barbara" is one or the catch words of 19 different types of categorical syllogisms.) The conclusion of this syllogism can be regarded as the result of substitution of the predicative '*v*' of the first (major) premise for the predicative '*w*' of the second (minor) premise via the subject '*w*' of the first premise. Therefore, the above syllogism can be restated as the following rule of substitution.

The rule of PSP-substitutions. Let S_1 *and* S_2 be two *veracious* (*accidentally true*) affirmative simple declarative sentences, whose predicates consist of the link-verb "is" and of a nounal predicative. Let the subject of S_1 and the predicative of S_2 be same. Then the sentence, which results by substitution of the predicative of S_1 for the predicative of S_2, *is veracious.* The letters "PSP" in the head of this rule are an abbreviation of the expression "the predicative of the sentence S_1 via its subject for the predicative of the sentence S_2".•

II) Let us now postulate as valid the following complex syllogism that is not among the basic nineteen Aristotelian categorical syllogisms.

Every w is *v* or *v′*. *Every u* is *w* or *x*. Hence, *u* is *v* or *v′* or *x*.

The conclusion of this syllogism can be regarded as the result of substitution of the disjunction '*v* or *v′*' forming the predicative of the first premise for the disjunct '*w*' of the disjunctive predicative '*w* or *x*' of the second premise via the subject *w* of the first premise. Therefore, the above syllogism can be restated the following rule of substitution

The rule of DPSPD-substitutions. Let S_1 *and* S_2 be two *veracious* affirmative contracted declarative sentences, each of which has one simple nounal subject and a compound predicate consisting of the link-verb "is" and contracted predicative in the form of the disjunction or of two or more simple nounal predicatives. Let the subject of S_1 be one of the disjuncts of the contracted predicative of S_2. Then the sentence, which results by substitution of the contracted predicative of S_1 into the sentence S_2 for the token of the subject of S_1, being a disjunct of the contracted predicative of S_2, *is veracious*. The letters "DPSPD" in the head of this rule are an abbreviation of the expression "the disjunctive predivative of the sentence S_1 via its subject for the predicative disjunct of the sentence S_2".•

Either of the established nouns "*substituendum*" (pl. "*substituenda*"), after the manner of "*definiendum*" (pl. "*definienda*"), and "*substituend*" (pl. "*substituends*") is a synonym of the description (descriptive name) "*replaced expression*", whereas the new noun "*substituens*" (pl. "*substituentia*"), after the manner of "*definiens*" (pl. "*definientia*"), is a synonym of the description "*substituted expression*",

By "combinatorial considerations" in the proof of Th 3.1, I mean the rule of DPSPD-substitutions. In this case, that rule can be particularized as follows. Axs 3.4 and 3.3 can respectively be paraphrased thus:

1) Every eukaryote is either a euprotist of a eumetist.

2) Every biont is either a prokaryote or a eukaryote.

Replacement of the disjunct "a eukaryote" in the predicate of statement 1 with the disjunction "a euprotist or a eumetist", predicating the subject "a eukaryote" in statement 2, yields:

3) Every biont is a prokaryote or a euprotist or a eumetist.

This statement can be paraphrased as Th 3.1.

The rule of DPSPD-substitutions will widely be used in the sequel for establishing various taxonomic theorems (derived taxonomic relations). In this case, some taxonomic theorems will be proved by repeated DPSPD-substitutions, so that, in contrast to Th 3.1, they will not be self-evident.•

3.2. The trophotaxonomy of bionts

Df 3.3. A biont is called an *autotroph* (*autotrophe*) if and only if it synthesizes *all* necessary *organic* nutrients by itself in the form of *monosaccharides*, mainly in the form of *glucose*, from inorganic molecules of *carbon dioxide* (CO_2) and *water* (H_2O), which it *absorbs* from its environment. The synthesis reaction is *endergonic*, i.e. requiring expenditure of energy. Therefore, depending on species, an autotroph synthesizes monosaccharides either with the help of the *absorbed sunlight energy* or with the help of the *energy of the exergonic*, i.e. *liberating energy, chemical reaction* between some inorganic substances, which the autotroph absorbs from its immediate environment. The autotroph is called a *photoautotroph* in the first case and a *chemoautotroph* in the second case. Synthesis of organic compounds from inorganic substances is called *photosynthesis* if it is made by energy of light, and *chemosynthesis* if it is made by energy derived from exergonic chemical reactions.•

Cmt 3.5. Cells of all photoautotrophs contain numerous *chloroplasts* – organelles, which are specialized to drive the photosynthesis of monosaccharide, mostly in the form of glucose, from carbon dioxide, water, and sunlight photons with the help of *chlorophyll*. “Chlorophyll” is a collective name for *green protein pigments* of various kinds, which are located within chloroplasts and which serve as enzymes (catalysts) of the photosynthesis reactions. Chlorophyll has green color because it partly reflects and partly transmits green light and also because it completely absorbs red and blue light.•

Df 3.4. 1) A biont is called a *euheterotroph* (*euheterotrophe*) if and only if it obtains *all* organic and inorganic nutrients, which it needs as a source of carbon skeletons, minerals, and energy for synthesizing the vital organic compounds, from its exterior into its interior with food.

2) A biont is called a *mictotroph* (*mictotrophe*) if and only if, in addition to some nutrients (e.g. glucose), which it obtains with food and which it uses as a source of carbon skeletons and minerals for making some vital organic compounds, the biont utilizes either *light energy* or *chemical energy* for synthesizing *ATP and ATP-ase* and, perhaps, for synthesizing some other organic compounds. A *mictotroph* is called a *photomictotroph* if it utilizes light energy and a *chemomictotroph* if it utilizes chemical energy.

3) A biont is called a *heterotroph* (*heterotrophe*) if and only if it is either a *euheterotroph* or a *mictotroph*.•

Cmt 3.6. Like cells of a photoautotroph, cells of a photomictotroph contain chlorophyll, by means of which the photomictotroph drives enzymatic reactions of photosynthesis.•

Df 3.5. 1) Either one of the nouns “*autoheterotroph*” and “*heteroautotroph*” is a synonym of “*mictotroph*”.

2) “*photoheterotroph*” is a synonym of “*photomictotroph*”.

3) “*chemoheterotroph*” is a synonym of “*chemomictotroph*”.•

Cmt 3.7. 1) Df 3.5 is in agreement with the etymological sense of “micto”- (see Dictionary1.1). I shall have no occasion to use the morphemes “autoheterotroph” and “heteroautotroph”, introduced in Dfs 3.5(1), in the sequel. The terms

“heterotroph”, “photoheterotroph”, “chemoheterotroph”, as defined in Dfs 3.4(3) and 3.5(2,3), are conventional ones that have the conventional meanings.

2) The item 3 of Df 4.4 and the entire Df 4.5 can be disregarded. Consequently, all subsequent taxonomic statements, in which any one of the nouns (4.1), alone or with some qualifiers, occurs as the subject, can be omitted. At the same time, all occurrences of any one of the above three displayed conventional terms, together with all its qualifiers, in the predicates of all other subsequent taxonomic statements can be omitted. In the taxonomy, which is thus freed of the above three terms, the word "euheterotroph" can be replaced with the word "heterotroph", provided that the latter is defined as a term by the variant of Df 4.4(1) with "heterotroph" in place of "euheterotroph". The term "heterotroph" thus defined will evidently be a homonym of the conventional term "heterotroph".•

Df 3.6. 1) A heterotroph is called an *absorptive heterotroph*, or *nutrient absorber*, if it *digests* food, i.e. converts it into absorbable form, outside its body by secreting into the food acids and hydrolytic enzymes that decompose complex organic molecules to the simple ones that the heterotroph absorbs through its surface.

2) A heterotroph is called an *ingestive heterotroph*, or *nutrient eater*, if it *ingests food* and then *digests* it in a specialized cavity.

3) It is understood that items 1) and 2) apply particularly with "euheterotroph" or "mictotroph" in place of "heterotroph".•

Df 3.7. 1) An absorptive, or ingestive, mictotroph that utilizes *light energy* is called an *absorptive*, or, correspondingly, *ingestive*, *photomictotroph* (*photoheterotroph*).

2) An absorptive, or ingestive, mictotroph that utilizes *chemical energy* is called an *absorptive*, or, correspondingly, *ingestive*, *chemomictotroph* (*chemoheterotroph*).•

Df 3.8*: The basic modes of nutrition of bionts.* The mode of nutrition of a biont is said to be

1) *autotrophic* if the biont is an autotroph:

1.1) *photoautotrophic* if the biont is a photoautotroph,

1.2) *chemoautotrophic* if the biont is a chemoautotroph;

2) *heterotrophic* if the biont is a heterotroph:

2.1) *euheterotrophic* if the biont is a euheterotroph:

a) *absorptive euheterotrophic* if the biont is an absorptive euheterotroph,

b) *ingestive euheterotrophic* if the biont is an ingestive euheterotroph;

2.2) *mictotrophic* if the biont is a mictotroph:

i) *photomictotrophic* (*photoheterotrophic*) if the biont is a photomictotroph (photoheterotroph),

ii) *chemomictotrophic* (*chemooheterotrophic*) if the biont is a chemomictotroph (chemoheterotroph);

iii) *absorptive mictotrophic* if the biont is an absorptive mictotrootroph:

a) *absorptive photomictotrophic* (*photoheterotrophic*) if the biont is an absorptive photomictotroph (photoheterotroph),

b) *absorptive chemomictotrophic* (*chemoheterotrophic*) if the biont is an absorptive chemomictotroph (chemoheterotroph);

iv) *ingestive mictotrophic* if the biont is an ingestive mictotrootroph:

a) *ingestive photomictotrophic* (*photoheterotrophic*) if the biont is an ingestive photomictotroph (photoheterotroph),

b) *ingestive chemomictotrophic* (*chemoheterotrophic*) if the biont is an ingestive chemomictotroph (chemoheterotroph).

2.3) *absorptive heterotrophic* if the biont is an absorptive heterotroph;

2.4) *ingestive heterotrophic* if the biont is an ingestive heterotroph.•

Ax 3.6*: The basic trophotaxonomy (nutritional taxonomy) of bionts.*

1) A biont is either an autotroph or a heterotroph.

2) An autotroph is either a photoautotroph or a chemoautotroph.

3) A heterotroph is either an absorptive heterotrop or an ingestive heterotroph.

4) A heterotroph is either a euheterotroph or a mictotroph.

4.1) A euheterotroph is either an absorptive euheterotroph or an ingestive euheterotroph.

4.2) A mictotroph is either an absorptive mictotroph or an ingestive mictotroph.

4.3) A mictotroph is either a photomictotroph (photoheterotroph) or a chemomictotroph (chemoheterotroph).

4.3a) A photomictotroph (photoheterotroph) is either an absorptive photomictotroph (photoheterotroph) or an ingestive photomictotroph (photoheterotroph).

4.3b) A chemomictotroph (chemoheterotroph) is either an absorptive chemomictotroph (chemoheterotroph) or an ingestive chemomictotroph (chemoheterotroph).•

Cmt 3.8. Item 3 of Ax 3.6 is identical with item 3 of Df 3.4. The former has been included in Ax 3.6 for the sake of completeness.•

Cmt 3.9. The *basic* taxonomy of bionts, which is determined by Ax 3.6, is based on *nutrition modes only*. Therefore, that taxonomy is called a *trophotaxonomy*, i.e. a *trophic*, or *nutritional*, *taxonomy*. Accordingly, the taxonyms which occur in Ax 3.6, i.e. the *count names*

“autotroph”, “photoautotroph”, “chemoautotroph”, “heterotroph”, “euheterotroph”, “mictotroph”, “absorptive heterotroph”, etc, “absorptive chemomictotroph” (“absorptive chemoheterotroph”), “ingestive chemomictotroph (“ingestive chemoheterotroph”),

without the indefinite article, and also any names, which have or will be defined in terms of these names (particularly any synonyms of the names), are called *trophotaxonyms* (*trophic taxonyms*, *nutritional taxonyms*) *of bionts*, whereas the *taxon* (*category*, *class*), which is denoted by a trophotaxonym, is called a *trophotaxon* (*trophocategory*, *trophic category*, *nutritional category*) *of bionts*. In this connection, it is worthy to recall that all taxonyms and hence all categories of the *Linnaean taxonomy* (*LT*) of bionts are qualified *morphological*, i.e. they are *morphotaxonyms* and *morphotaxa* (*morphocategories*), respectively. The most natural immediate synonym of any of the above count names that are *used as taxonyms* is *that same name in the plural number form together with the definite article*. For instance, “the autotrophs” is a class-synonym of the *trophotaxonym* “autotroph” and “the ingestive chemoheterotrophs” is a class-synonym of the *trophotaxonym* “ingestive chemoheterotroph”. Still, the reader has already observed, the above straightforward method of naming categories (classes) is used in the exposition generally, and not only for naming nutritional categories. Particularly, it was used in the subsection 3.1.•

Df 3.9. 1) A trophocategory of the basic trophotaxonomy is said to be *divisible* if it can be divided (partitioned) into less inclusive trophocategories, and *indivisible* or *ultimate* otherwise. An indivisible trophocategory is synonymously called a *trophospecies*. The trophotaxonym denoting a trophospecies is called a *trophospecific taxonym.*

2) There are the following *eight trophospecies* of the basic trophotaxonomy of bionts:

1) the photoautotrophs,

2) the chemoautotrophs,

3) the absorptive euheterotrophs,

4) ingestive euheterotrophs,

5) the absorptive photomictotrophs (absorptive photoheterotrophs),

6) the absorptive chemomictotrophs (absorptive chemoheterotrophs),

7) the ingestive photomictotrophs (ingestive photoheterotrophs),

8) the ingestive chemomictotrophs (ingestive chemoheterotrophs).•

Cmt 3.10. The following statements are paraphrases of the respective statements of Ax 3.6.

1′) The bionts are divided into *two trophokingdoms*: the autotrophs (autotrophes) and the heterotrophs (heterotrophes).

2′) The autotrophs are divided into *two trophospecies*: the photoautotrophs and the chemoautotrophs.

3′) The heterotrophs are divided into *two trophocategories*: the absorptive heterotrophs (nutrient absorbers) and the ingestive heterotrophs (nutrient eaters).

4′) The heterotrophs are divided into *two trophocategories*: the euheterotrophs and the mictotrophs.

4.1′) The euheterotrophs are divided into *two trophocategories*: the absorptive euheterotrophs and the ingestive euheterotrophs.

4.2′) The mictotrophs are divided into *two trophocategories*: the absorptive mictotrophs or the ingestive mictotrophs.

4.3′) The mictotrophs are divided into *two trophocategories*: the photomictotrophs (photoheterotrophs) and the chemomictotrophs (chemoheterotrophs).

4.3a′) The photomictotrophs (photoheterotrophs) are divided into *two trophospecies*: the absorptive photomictotrophs (photoheterotrophs) and the ingestive photomictotrophs (photoheterotrophs).

4.3b′) The chemomictotrophs (chemoheterotrophs) are divided into *two trophospecies*: the absorptive chemomictotrophs (chemoheterotrophs) and the ingestive chemomictotrophs (chemoheterotrophs).

Thus, the heterotrophs are divided into two trophocategories in two independent ways, namely 3′ and 4′. Likewise, the mictotrophs are divided into two trophocategories in two independent ways, namely 4.2′ and 4.3′. Ax 3.6(1) or the statements 1′ will be called *the first trophodichotomy of bionts*; Ax 3.6(2) or the statements 2′ will be called *the trophodichotomy of autotrophs*; Ax 3.6(3) or the statements 3′ will be called *the first trophodichotomy of heterotrophs*; Ax 3.6(4) or the statements 4′ will be called *the second trophodichotomy of heterotrophs*. The proper names of the rest of the above statements and of their counterparts of Ax 3.6 can be formed analogously.•

Df 3.10. 1) An *ingestive heterotroph* (*nutrient eater*) is alternatively (synonymously) called an *animal sensu lato*.

2) A biont other than an animal sensu lato, i.e. either an *autotroph* or an *absorptive heterotroph* (*nutrient absorber*), is indiscriminately called a *plant sensu lato*.•

Cmt 3.11. Replacement of "heterotroph" in both items of Df 3.10 with the disjunction "a euheterotroph or an ingestive mictotroph" as follows from Ax 3.6(4) yields:

1^*) A plant sensu lato either is an autotroph or an absorptive euheterotroph, or else an absorptive mictotroph, and vice versa.

2^*) An animal sensu lato either is an ingestive euheterotroph or an ingestive mictotroph, and vice versa.

Statements 1^* and 2^* are evidently theorems that follow from Df 3.10 and Ax 3.6(4) by the rule of DPSPD-substitutions.•

Th 3.2: *The second, or Aristotelian, trophodichotomy of bionts.* The bionts are divided into *two trophokingdoms*: **Plantae sensu lato** and **Animalia sensu lato**, i.e. *the plants sensu lato* and *the animals sensu lato*.

Proof: Replacement of "heterotroph" in Ax 3.6(1) with the disjunction "an absorptive heterotroph or an ingestive heterotroph" occurring in the predicate of Ax 3.6(3) yields:

1^*) A biont is an autotroph or an absorptive heterotroph or an ingestive heterotroph.

By Df 3.10, this statement becomes:

1°) A biont is either a plant sensu lato or an animal sensu lato.

Along with the pertinent self-evident contextual definitions, the last statement can be paraphrased in the form of Th 3.2. QED.•

Cmt 3.12. The English names "the plants sensu lato" and "the animals sensu lato" and also their pure Latin synonyms "Animalia sensu lato" and "Plantae sensu lato" are my own terms, which belong exclusively to this metalanguage (this essay), and which are not *object* terms of any biological taxonomy of bionts. In connection with these terms the following remarks should be made.

1) Any one of the triple of class-synonyms "*plant sensu lato*", "*the plants sensu lato*", "Plantae sensu lato" and any one of the triple of class-synonyms "*animal sensu lato*", "*the animals sensu lato*", "Animalia sensu lato", which are defined in Df 3.10 and Th 3.2 are *trophotaxonyms of bionts*, *and not morphotaxonyms*.

2) In accordance with two different modes of nutrition, Aristotle divided all bionts into two kingdoms: *the plants* and *the animals*. The English nouns "plant" and "animal" have the same meanings as the Greek nouns "φυτον" \fiton\ (pl. "φυτα" \fita\) and "ζωον" \zoon\ (pl. "ζωα"\zoa\), respectively (cf. Dictionary1.1). Etymologically, however, the English nouns "plant" and "animal" originate from the Latin etymons "*planta*" (pl. "*plantae*") and "*animal*" (pl. "*animalia*"). The Latin noun "planta" means *a plant* in general, or *a green twig*, *cutting*, *graft* in particular. At the same time, the Latin noun "animal" (or "animans", pl. "animantia") denotes, not *an animal* in the sense that the name has in English, but rather it denotes *a living being* or *living creature*, sometimes including a man (often contemptuously) and sometimes excluding a man. An animal [in English] other than a man is, depending on a context, denoted in Latin by the nouns "belua", "bestia", and "pecus". The above meanings of the Latin nouns planta" and "animal" are cited from Simpson [1968, p. 45, 46, 74, 76, 429, 472, 658]). It was in the Middle Ages, after the works of Aristotle on biology were translated into Latin, when the Latin word "animal" acquired its presently common, Aristotelian meaning.

3) The adjoined qualifier "*sensu lato*", or "*sensu stricto*", to an English name means that that name is used in a wide sense, or in a narrow sense, respectively. Still, the qualifier "sensu lato" in the name "animal sensu lato" has nothing to do with the broad sense, which the etymon "animal" had in Latin; i.e. that qualifier *does not mean*

that an animal sensu lato is any living creature. In this metalanguage, the count names “plant sensu lato” and “animal sensu lato” denote *the same classes* (*kingdoms*) *as those denoted by the English count nouns* “*plant*” *and* “*animal*” *alone in accordance with the Aristotelian dichotomy of bionts*. That is to say, I postulate that, according to Aristotle, *a necessary and sufficient condition* for a biont to be called *a plant* [in English] is that the biont should be *either a photoautotroph or an absorptive heterotroph*, whereas *a necessary and sufficient condition* for a biont to be called *an animal* [in English] is that the biont should be *an ingestive heterotroph*.•

Th 3.3: *The basic trophotrichotomy of bionts.* The bionts are divided into three *trophic subkingdoms*: the *autotropths*, *the euheterotrops*, and *the mictotrophs*.

Proof: By of Ax 3.6(4), Ax 3.6(1) becomes:

1″) A biont is an autotroph or a euheterotroph or a mictotroph.

The theorem is a periphrasis the last statement.•

Cmt 3.13. In statement 1′ of Cmt 3.10, the trophocategory of autotrophs has been ranked as a trophokingdom, i.e. *trophic kingdom*. In Th 3.3, the same trophocategory is ranked as a *trophic subkingdom*. This contradiction is immediately eliminated by assuming that the trophokingdom of autotrophs is *the weak*, or *lax* (*not strict*), or *monochotomic*, *subkingdom*, i.e. *the whole part*, *of itself* (cf. Ax 3.5).•

Cmt 3.14. Ax 3.6 presents the *basic* trophotaxonomy of bionts as a sequence of dichotomic statements, whose subjects *connote* decreasingly narrow trophocategories. Given a divisible trophocategory that is connoted by the subject of a given itemized statement of Ax 3.6, except for statements 3.3a and 3.3b, either trophotaxonym occurring in the predicate of that statement either occurs as the subject of one or two of the succeeding statements or not. In the former case, the trophotaxonym in question can be replaced with the binary disjunction of the pair of trophotaxonyms predicating it as the subject of another statement, in accordance with the rule of DPSPD-substitutions (see Cmt 3.4). Continuing this way, any given divisible trophocategory mentioned in Ax 3.6 can be divided into an increasing number of decreasingly narrow disjoined trophocategories and, after all, it can be presented as the union of disjoined trophospecies. This plan is realized below.•

Th 3.4: *The ultimate trophotetrachotomy of mictotrophs.* The mictotrophs are divided into the following *four trophospecies*:

1) the absorptive photomictotrophs (photoheterotrophs),

2) the absorptive chemomictotrophs (chemoheterotrophs),

3) the ingestive photomictotrophs (photoheterotrophs),

4) the ingestive chemomictotrophs (chemoheterotrophs).

Proof: In accordance with statements 3.3a and 3.3b of Ax 3.6, replacement of "photomictotroph" ("photoheterotroph") and "chemomictotroph" ("chemoheterotroph") in the predicate of statement 3.3 of Ax 3.6 with the disjunctions

"an absorptive photomictotroph or an ingestive photomictotroph"

("an absorptive photoheterotroph or an ingestive photoheterotroph"),

"an absorptive chemomictotroph or an ingestive chemomictotroph"

("an absorptive chemomictotroph or an ingestive chemomictotroph"),

– in accordance with statements 3.3a and 3.3b of Ax 3.6, yields two synonymous (concurrent) statements, which can be written simultaneously thus:

3.3°) A mictotroph is an absorptive photomictotroph (photoheterotroph) or an absorptive chemomictotroph (chemoheterotroph) or an ingestive photomictotroph (photoheterotroph) or an ingestive chemomictotroph (chemoheterotroph).

The theorem is a periphrasis of 3.3°. QED.•

Th 3.5: *The ultimate trophohexachotomy of heterotrophs.* The *heterotrophs* are divided into the following *six trophospecies*:

1) the absorptive euheterotrophs,

2) the ingestive euheterotrophs,

3) the absorptive photomictotrophs (photoheterotroph),

4) the absorptive chemomictotrophs (chemoheterotrophs),

5) the ingestive photomictotrophs (photoheterotroph),

6) the ingestive chemomictotrophs (chemoheterotrophs).

Proof: Replacement of "heterotroph" in both occurrences in the predicate of statement 3 of Ax 3.6 with the disjunction "a euheterotroph or a mictotroph" occurring in the predicate of statement 4of Ax 3.6 yields:

4^*) A heterotroph is an absorptive euheterotroph or an ingestive euheterotroph or an absorptive mictotroph or an ingestive mictotroph.

Replacement of "euheterotroph" and "mictotroph" in the predicate of statement 4) of Ax 3.6 with the disjunctions "an absorptive euheterotroph or an ingestive euheterotroph" and "an absorptive mictotroph or an ingestive mictotroph" – in accordance with statements 3.1 and 3.2 of Ax 3.6, yields the above statement 4^* again.

Of the four taxonyms "absorptive euheterotroph", "ingestive euheterotroph", "absorptive mictotroph", "ingestive mictotroph", occurring in the predicate of statement 4^*, the first two are *trophospecific* taxonyms, i.e. they denote trophospecies. The two other taxonyms can be reduced by replacing the noun "mictotroph" in each of them with the predicate of statement 3.3° occurring in the proof of Th 3.4. The adjectives "absorptive" and "ingestive" are *complementary* (*antipodal*, *antithetic*) *antonyms*. Therefore, they cannot qualify a noun simultaneously. Hence, combination of statements 4^* and 3.3° in accordance with the rule of the DPSPD-substitutions yields:

4°) A heterotroph is an absorptive euheterotroph or an ingestive euheterotroph or an absorptive photomictotroph (photoheterotroph) or an absorptive chemomictotroph (chemoheterotroph) or an ingestive photomictotroph (photoheterotroph) or an ingestive chemomictotroph (chemoheterotroph).

The theorem is a periphrasis of 3.3°. QED.•

Th 3.6: ***The ultimate tropho-octachotomy of bionts.*** The bionts *are divided* into *the eight trophospecies* mentioned in Df 3.9(2), the understanding being the trophospecies pairwise disjoint.

Proof: The theorem follows from Ax 3.6(1) by statements (lemmas) 3.3° of the proof of Th 3.4 and 4° of the proof of Th 3.5.•

3.3. The basic tropho-kellotaxonomy of bionts

Ax 3.6, Ths 3.2–3.6, subject to Dfs 3.9 and 3.10, and also all theorems that have informally been stated in Cmts 3.9–3.14 hold independently of the basic kellotaxonomy elaborated in subsection 3.1. Consequently, *each kellocategory of bionts can be divided into certain trophocategories, and each trophocategory of bionts can be divided into certain kellocategories.* A taxonomy that obtained in the former way will be called a *tropho-kellotaxonomy of bionts*. A taxonomy that obtained in the latter way will be called a *kello-trophotaxonomy of bionts*. Particularly, the kellocategories of bionts established in subsection 3.1 can be divided into trophocategories established in subsection 3.2 by means of the following two-step procedure.

1) In accordance with Df 3.8, to all *substantive* trophotaxonyms occurring in Ax 3.6 there correspond the adjective derivatives or adjective equivalent which are formed by attaching to each trophotaxonym the suffix "-*ic*", namely:

"autotrophic", "photoautotrophic", "chemoautotrophic", "heterotrophic",

"euheterotrophic", "mictotrophic", "absorptive heterotrophic", ...,

"absorptive chemomictotrophic" ("absorptive chemoheterotrophic"),

"ingestive chemomictotrophic ("ingestive chemoheterotrophic")

(cf. Cmt 3.9). Attaching any one of these adjectives or adjective equivalents as an adherent *epithet* (*qualifier*} to any one of the *three kellotaxonyms*: "prokaryote", "euprotist", "eumetist", defined in subsection 3.1, one obtains a well-formed *description* (*descriptive name*) *through the kellocategory, denoted by the given kellotaxonym, and the differentia, denoted by the given one-word or two-word epithet*, or, briefly, a *description through the kellocategory and the differentia* (cf. *descriptio*, or *definitio*, *per genus et differentiam* – *a description*, or *a definition, through the genus and the differentia*).

2) Given a descriptive name, one should resort to the nature, via the pertinent literature (e.g., via Campbell [1990, Unit Five by William Schopf, pp. 505–674,]), in order to verify whether the tropho-kellocategory denoted by that descriptive name, i.e. the intersection of the category (class), denoted by the given kellotaxonym, and of the category, denoted by the given epithet, is empty or not. If the tropho-kellocategory is empty then the descriptive name denoting it should be disregarded. If, however, the tropho-kellocategory is not empty then the descriptive name denoting it should be adopted as a tropho-kellotaxonym of the combined tropho-kellotaxonomy of bionts.

Division of Prokaryotae, Euprotista, and Eumetista in the above way results in the following three axioms and one definition, which express the basic tropho-kellotaxonomy of bionts.

Ax 3.7: *The ultimate trophotetrachotomy of Prokaryotae.* Prokaryotae is divided into the following *four trophospecies*:

1) the photoautotrophic prokaryotes,

2) the chemoautotrophic prokaryotes,

3) the absorptive photomictotrophic (photoheterotrophic) prokaryotes,

4) the absorptive chemomictotrophic (chemoheterotrophic) prokaryotes.•

Ax 3.8: ***The ultimate tropho-octachotomy of Euprotista.*** Euprotista is divided into *the eight trophospicies* in accordance with Th 3.6, namely:

1) the photoautotrophic protists,

2) the chemoautotrophic protists,

3) the absorptive euheterotrophic protists,

4) the ingestive euheterotrophic protists,

5) the absorptive photomictotrophic (photoheterotrophic) protists,

6) the absorptive chemomictotrophic (chemoheterotrophic) protists,

7) the absorptive photomictotrophic (photoheterotrophic) protists,

8) the absorptive chemomictotrophic (chemoheterotrophic) protists.•

Df 3.11. 1) A *photoautotrophic eumetist* is alternatively called a *plant sensu stricto*.

2) An *ingestive euheterotrophic* eumetist is alternatively (synonymously) called an *animal sensu stricto*.

3) An *absorptive euheterotrophic eumetist* is alternatively called a *fungus sensu stricto*.•

Ax 3.9: ***The basic trophotrichotomy of Eumetista.*** Eumetista is divided into the following *three trophospecies*, being by definition *tropho-kelokingdoms*:

1) **Plantae sensu stricto**, i.e. *the plants sensu stricto*, or the photoautotrophic eumetists,

2) **Animalia sensu stricto**, i.e. *the animals sensu stricto*, or *the ingestive euheterotrophic eumetists*,

3) **Fungi sensu stricto**, i.e. *the fungi sensu stricto*, or *the absorptive euheterotrophic eumetist*.•

Cmt 3.15. "**Euplantae**", "**Euanimalia**", and "**Eufungi**" are other names that can be used instead of or interchangeably with "Plantae sensu stricto", "Animalia sensu stricto", and "Fungi sensu stricto" as synonyms of "the photoautotrophic eumetists", "the ingestive euheterotrophic eumetists" and "the absorptive euheterotrophic eumetist", respectively (cf. Cmt 3.3). This time, I give preference to the "sensu stricto"-terms because the first two of them are opposed in form to the respective "sensu lato"-terms.•

Th 3.7. 1) Prokaryotae, Plantae sensu stricto, and Fungi sensu lato are *strict part*s, or, more specifically, *strict trophic subkingdoms of* [*trophokingdom*] Plantae sensu lato.

2) Animalia sensu stricto is a *strict trophic subkingdom of* [*trophokingdom*] Animalia sensu lato.

Proof: The theorem follows from Dfs 3.10 and 3.11 and Axs 3.7 and 3.9 by the rule of DPSPD-substitutions.•

Th 3.8: *The basic trophodichotomy of Prokaryotae.* Prokaryotaeis divided into the following two *trophocategories*:

a) the autotrophic prokaryotes,

b) the absorptive mictotrophic (absorptive heterotrophic) prokaryotes.

Proof: By Ax 3.6(1), the category a of the theorem is the union of the categories 1 and 2 of Ax 3.7. At the same time, by Ax 3.7, *there are neither ingestive prokaryotes nor euherotrophic prokaryotes*. Therefore, items 3, 3.3a, and 3.3b of Ax 3.6 are specified for prokaryotes thus:

1) A heterotrophic prokaryote is a mictotrophic prokaryote and vice versa.

2) A photomictotrophic (photoheterotrophic) prokaryote is an *absorptive* photomictotrophic (photoheterotrophic) prokaryote and vice versa.

3) A chemomictotrophic (chemoheterotrophic) prokaryote is an *absorptive* chemomictotrophic (chemoheterotrophic) prokaryote and vice versa.

Hence, item 3.3 of Ax 3.6 is specified thus:

4) A mictotrophic (heterotrophic) prokaryote is either an absorptive photomictotrophic (photoheterotrophic) prokaryote or an absorptive chemomictotrophic (chemoheterotrophic) prokaryote.

That is to say, the category b of the theorem is the union of the categories 3 and 4 of Ax 3.7. QED.•

Cmt 3.16. Since there are neither ingestive prokaryotes nor euherotrophic prokaryotes, the count names “mictotrophic prokaryote”, “heterotrophic prokaryote”, “chemomictotrophic prokaryote”, “chemoheterotrophic prokaryote”, “absorptive chemomictotrophic prokaryote”, “absorptive chemoheterotrophic prokaryote”, “absorptive mictotrophic prokaryote”, “absorptive heterotrophic prokaryote” denote the same trophocategory of prokariotes, i.e. they are synonyms.

The photoautotrophic prokaryotes include *cyanobacteria,* formerly known as *blue-green algae*, and some other homologous or analogous organisms. Along with the entire kellokingdom Prokaryotae, all photoautotrophic prokaryotes were included in the kingdom Plantae of the Linnaean taxonomy (LT) of bionts, which is basically coincide with Plantae sensu lato (cf. Th 3.7).•

Cmt 3.17. The following passage is quoted from Campbell [1990, Unit Six, by Ruth Satter, pp. 733, 734]):

> «Living in acid bogs and other habitats where soil conditions are poor", especially in nitrogen, are plants that fortify themselves by occasionally eating meat. These carnivorous plants make their own carbohydrates from photosynthesis, but they obtain some of their nitrogen and minerals by killing and digesting insects. Various kinds of insect traps have evolved by the modification of leaves (Figure 33.15). The traps are usually equipped with glands which secrete digestive juices.»

In accordance with Dfs 3.3 and 3.4, the carnivorous plants are unambiguously classified as autotrophic eumetists, i.e. as plants sensu stricto.•

Cmt 3.18. By Ax 3.4, Euraryotae is the union of Euprotista and Eumetista. Therefore, Ax 3.8 applies with "Eukaryotae" and "eukaryotes" in place of "Euprotista" and "euprotists", respectively.•

3.4. The major ecotaxonomy of bionts

The major trophotaxonomy is the classification of bionts by modes of nutrition as such. The taxonyms of this taxonomy are not descriptive of any ways in which various bionts of any given trophocategory reach the nutrients, which they consume in accordance with their inherent modes of nutrition. Any biont takes nutritients from its environment, which includes both biotic (living) and abiotic (nonliving) factors (beings) and which is called the *ecosystem* (*ecological system*) *of the biont*. In accordance with Dictionary1.1, "ecosystem" has the same meaning as "*habitat*", "*environment*", or "*common house*". Accordingly, without any explicit reference to a separate biont, an ecosystem is any relatively self-contained geographical area, which is shared by various biospecies (biological species). The totality of bionts of the same biospicies that inhibit a certain ecosystem is called *the population of that species in that ecosystem*, or, less explicitly, *a biopopulation of the ecosystem*. The totality of bionts of different biospicies that inhibit a certain ecosystem, i.e. the totality of all biopopulations of the ecosystem, is called *the biotic community* (briefly, *biocommunity* or, simply, *community* if there is no danger of misunderstanding), or *biotic component* (*constituent*, *part*), *of the ecosystem*; "*biocenosis*", "*biocoenosis*", "*biocenose*", and "*biocoenose*" (pl. "*biocenoses*" or "*biocoenoses*)" are some other synonyms of "biotic community". The remaining, *physico-chemical* part of the ecosystem, which consists of *minerals and viruses*, is called *the abiotic component of the ecosystem*. The totality of all ecosystems, i.e. the entire part of the Earth that is inhibited by bionts is called, *the biosphere*. The scientific field of study of ecosystems is called *ecology*. In this case, the ecological terms "population", "community", "ecosystem", and "biosphere" are descriptive (connotative) of various increasingly broader levels of organization of bionts as they understood by ecologists. The interested reader can read more about ecology as an inseparable part of biology in Campbell [1990, Unit Eight, pp. 1045–1165, by Jane Goodall].

In order to reach required nutrients, a biont established certain relations with the appropriate factors of its ecosystem. Accordingly, individual bionts belonging to a given trophocategory or tropho-kellocategory can be classified further in accordance with the above ecological relations, i.e. in accordance with the different ways in which they obtain necessary nutrients from the ecosystem. A taxonomy of bionts that is based on ecological criteria will be called an *ecotaxonomy* (*ecological taxonomy*). Accordingly, a taxonym of an ecotaxonomy will be called an *ecotaxonym* (*ecological taxonym*), whereas the category (class) which is denoted by an ecotaxonym will be called an *ecocategory* (cf. Cmt 2.2(1)).

A taxononomy that is obtained by dividing strictly some or all trophocategories of the trophotaxonomy of subsection 3.2 into echocategories will be called an *eco-trophotaxonomy of bionts*. A taxononomy that is obtained by dividing strictly some or all tropho-kellocategories of the tropho-kellotaxonomy of subsection 3.3 into echocategories will be called an *eco-tropho-kellotaxonomy of bionts*.

Ecological relations are very complex, and therefore an ecological taxononomy of bionts is unavoidably selective. In what follows I shall make explicit that some terms that are widely used either as substantive or as qualifiers together trophotaxons or tropho-kellotaxons are ecotaxons.

Df 3.12. 1) A biont is called *a symbiont* if it lives within or on the larger biont, called *the host of the simbiont* or, less explicitly, *a host*. The class of symbiont-host relationships is called *symbiosis*.

2) A symbiont is called a commensal symbiont, and also a commensal, if it obtains food, protection, or other benefits from its host without damaging and benefiting the latter. The class of commensal-host relationships is called *commensal symbiosis* or *commensalism*.

3) A symbiont is called a mutualistic symbiont, and also a mutualist, if it stands in a mutually beneficial relationship with its host. The class of mutualist-host relationships is called *mutualistic symbiosis* or *mutualism*.

4) A symbiont is called a parasitic symbiont, and also a parasit, if it lives in or on its only host or in association with one or more hosts, from which it obtains both food (by absorbing organic nutrients from the body fluid of a host) and other benefits (as protection or transportation), and at the same time causes overt or covert damage to the host or hosts, but does not directly and immediately kill the latter. The class of parasite-host relationships is called *parasitic symbiosis* or *parasitism*.•

Df 3.13. 1) A plant sensu lato is called a *saprophyte* if it lives on dead or decaying organic mater, such as animal corpses, fallen logs, or the wastes of live organisms, by decomposing the matter and absorbing the necessary nutrients. A saprophyte is called a *perthophyte* if it lives on dead or decaying organic tissue being a part of a living plant sensu stricto.

2) An autotroph or a saprophyte, each taken individually, is indiscriminately called an *autophyte*.•

Df 3.14. 1) A plant-eating animal [sensu stricto] is alternatively (synonymously) called a *herbivorous animal*, and also a *herbivore*.

2) A flash-eating animal [sensu stricto] is alternatively (synonymously) a *carnivorous animal*, and also a *carnivore*.•

Df 3.15. A biont that kills, or overtly damage, and eats other bionts, or their damaged parts (as parts of plants), is called a *predator*, whereas any bionts killed, or overtly damaged, and consumed by the predator for food are called the *pray*. The ecotaxonym "predator" applies, not only to carnivores, which hunt and eat other animals as pray, but also to herbivores, which eat plants as pray.•

Cmt 3.19. All terms as introduced in Dfs 3.12–3.15 are ecotaxonyms, and the categories denoted by these taxonyms are ecocategories. These taxonyms are selective and therefore they are not hierarchical. For instance, the term "symbiont" applies to bionts of all categories, whereas the terms "saprophyte", "perthophyte", and "autophyte", involving the combining form -"phyte" (meaning *plant*) apply only to some plants sensu lato. All possible combined eco-trophotaxonyms and eco-tropho-kellotaxonyms can be formed, and then non-empty ones can be selected out of them, in the same way as described at the beginning of subsection 3.4 for tropho-kellotaxonyms. Here follow most conspicuous eco-trophocategories of bionts.

1°) The absorptive chemomictotrophic (chemoheterotrophic) bionts are divided into four *eco-trophocategories*: *the commensals* (*commensal symbionts*), *the mutualists* (*mutualistic symbionts*), *the parasites* (*parasitic symbionts), the saprophytes*.•

2°) The majority of prokaryotes (bacteria) are the chemomictotrophic (chemoheterotrophic) prokaryotes. The latter category is divided into *the parasitic prokaryotes* and *the saprophytic prokaryotes*.

3°) Fungi sensu stricto, i.e. the tropho-kellokingdom of absorptive euheterotrophic eumetists, is divided into three *eco-tropho-kellocategories*: *the saprophytic fungi sensu stricto, the parasitic fungi sensu stricto*, and *the mutualistic fungi sensu stricto*.•

Essay 7. Basic biochemistry

1. Biochemical (enzymatic chemical) reactions

1.1. Enzymes

Df 1.1. 1) Substance is called a *catalyst of a chemical reaction if it* either (a) initiates the reaction and enables it to run under conditions, under which it is otherwise impossible, or (b) increases the rate the reaction under conditions, under which it runs slower, and if it remains unchanged at the end of the reaction. The effect of catalyst on a chemical reaction is called *catalysis*.

2) "*To catalyze*" a chemical reaction means to bring the reaction to existence or to increase its rate by catalysis.

3) Any of numerous complex *proteins* that are produced by living cells and *catalyze* specific biochemical reactions at body temperatures is called *enzyme.*

4) Names of enzymes are usually formed by attaching the suffix –"*ase*" to the root of a name of the substance, on which a given enzyme acts. For instance, *lactase* is the enzyme that acts on *lactose*.•

Cmt 1.1. In order to act as catalyst, enzyme should not necessarily be in a cell. Many enzymes have been extracted from cells in crystalline form without any loss of their activity. All enzymes that have been extracted so far are *proteins*. •

1.2. Biochemistry of the autotrophic modes of nutrition

Some typical enzymatic chemical reactions, which are driven by photoautotrophs and chemoautotrophs, are made explicit below.

A glucose molecule, $C_6H_{12}O_6$, is synthesized in chloroplasts of a photoautotroph in accordance with the chemical reaction equation

$$6CO_2+6H_2O+E_0\rightarrow C_6H_{12}O_6+6O_2, \tag{1.1}$$

where $E_0>0$ is the photon energy per one synthesized molecule of glucose, which is required for the *endergonic* reaction to run. Particularly, equation (1.1) holds for photosynthesis reactions in photoautotrophic bacteria.

In contrast to photoautotrophic bacteria, which take hydrogen from water and utilize light energy, chemoautotrophic bacteria of some species obtain both hydrogen and energy E_0, required for synthesizing glucose from carbon dioxide, by oxidizing the appropriate inorganic substances, which they absorb from the environment, – such substances, e.g., as hydrogen sulfide, H_2S, or ammonia, NH_3. Chemoautotrophic bacteria of some other species obtain hydrogen from water by means of energy obtained by oxidizing the appropriate hydrogen-free molecules or ions, e.g. ferrous ions, Fe^{2+}. Thus, sulfur and nitrogen bacteria obtain the necessary energy by the respective *exergonic* reactions

$$2H_2S+O_2 \to 2H_2O+2S+E', \tag{1.2}$$

$$4NH_3+3O_2 \to 6H_2O+2N_2+E'', \tag{1.3}$$

where $E'>0$ and $E''>0$ are the energies evolved in the reactions. In this case, the amount of energy E_0 required for synthesizing one molecule of glucose remains the same as in the case of photosynthesis. Addition of equation (1.1) to equation (1.2) multiplied by 6 or to equation (1.3) multiplied by 2 yields:

$$6CO_2+6H_2O+E_0+12H_2S+6O_2 \to 6CO_2+6H_2O+E_0+12H_2O+12S+6E'$$

$$\to C_6H_{12}O_6+6O_2+12H_2O+12S+6E',$$

$$6CO_2+6H_2O+E_0+8NH_3+6O_2 \to 6CO_2+6H_2O+E_0+12H_2O+4N_2+2E''$$

$$\to C_6H_{12}O_6+6O_2+12H_2O+4N_2+2E'',$$

respectively. Upon canceling the similar terms '$6H_2O$' and '$6O_2$' in the first and last terms of each one of the above two trains of equations, and upon adding $-E_0$ to both terms of either equation, one obtains

$$6CO_2+12H_2S \to C_6H_{12}O_6+6H_2O+12S+E_1, \tag{1.4}$$

$$6CO_2+8NH_3 \to C_6H_{12}O_6+6H_2O+4N_2+E_2, \tag{1.5}$$

subject to

$$E_1 \doteqdot 6E' - E_0,\ E_2 \doteqdot 2E'' - E_0, \tag{1.6}$$

where $\doteqdot$ is the sign of equality by definition. Either of equations (1.4) and (1.5) describes the respective enzymatic chemical reaction of synthesizing glucose by a *chemoautotrophic* bacterium. Such a bacterium cannot produce ATP by light energy, and therefore it cannot afford to consume ATP for producing glucose. Therefore, the reactions described by equations (1.4) and (1.5) are *not exergonic*, i.e. $E_1 \geq 0$ and $E_2 \geq 0$. In this case, E_1 or E_2 is the energy evolved in synthesizing one glucose molecule in the respective reaction.

In connection with the enzymatic reactions (1.1), (1.4), and (1.5), the following remarks should be made.

1) Oxygen in reaction (1.1), sulfur and water in reaction (1.4), and nitrogen and water in reaction (1.5) are by-side products of the respective reactions. It is however clear that a sulfur bacterium releases free sulfur atoms because it extracts

hydrogen atoms (and energy) from hydrogen sulfide molecules (H_2S) and bonds them with carbon dioxide molecules (CO_2) into glucose molecules ($C_6H_{12}O_6$). Likewise, a nitrogen bacterium releases free nitrogen molecules (N_2) because it extracts hydrogen atoms (and energy) from ammonia molecules (NH_3) and bonds them with carbon dioxide molecules into glucose molecules. Reaction (1.1) differs somewhat from reactions (1.4) and (1.5). In the course of reaction (1.1), the hydrogen atoms are extracted from the molecules of water (H_2O) and, together with *virtual* carbon oxide radicals (CO), extracted from the carbon dioxide molecules (CO_2), are incorporated into glucose molecules.

An early model of the reaction of photosynthesis of glucose was based on the assumption that a photoautotroph splits carbon dioxide molecules into carbon atoms (C) and oxygen molecules (O_2), bonds free carbon atoms with water molecules into glucose molecules, and releases the free oxygen molecules into the environment. Drawing an analogy between the enzymatic reaction of glucose photosynthesis and the enzymatic reactions of glucose chemosynthesis, C. B. van Neil hypothesized in the 1930s that that photoautotrophic bacterium split molecules of water, and not molecules of carbon dioxide. There is an opinion among biologists (see, e.g., Campbell [1990, pp. 206–208]) that this hypothesis was proved experimentally in the 1950s by using stable heavy isotope atoms ^{18}O as tracers. Still, the very question which one of the two reactants, carbon dioxide or water, is split in photosynthesis of glucose, i.e. the question which one of the two reactant is the *only* source of the free oxygen product, *is incorrect* (*paradoxical*, *unanswerable*). Indeed, six molecules of water, $6H_2O$, occurring on the left-hand side of equation (1.1) contain six oxygen atoms which can form three molecules, $3O_2$. At the same time, there are six free oxygen molecules, $6O_2$, on the right-hand side of equation (1.1). Since twelve hydrogen atoms, which are required for synthesizing one molecule of glucose, are extracted from the reactant $6H_2O$, one may speculate that three of the above-mentioned six free oxygen molecules come from that reactant. Consequently, the three remaining oxygen molecules must come from the reactant $6CO_2$. Incidentally, one of the two by-side products of each of the reactions (1.4) and (1.5) is water. The oxygen which is required for synthesizing the water is certainly extracted from the carbon dioxide, while the hydrogen is, depending on reaction, extracted either from the hydrogen sulfide or from the ammonia. Therefore, if one wishes to visualize any of the three reactions (1.1), (1.4), and (1.5), as a two-step process, then the only possible model of this process is that all reactant molecules of any given reaction, namely, $6CO_2$ and $6H_2O$ in (1.1), $6CO_2$ and $12H_2S$ in (1.4), and $6CO_2$ and $8NH_3$ in (1.5), are, first, enzymatically disassembled into atoms, and then either all the atoms are assembled into the respective product molecules or only some of them are bonded into molecules while the others (as 12S in (1.4)) remain free.

2) In connection with reaction (1.2), the following remark may also be in order. Hydrogen sulfide is colorless gas with the characteristic smell of decomposed protein. In air, the gas burns with a bluish flame in accordance with the reaction equation:

$$2H_2S+3O_2 \rightarrow 2H_2O+2SO_2+E''', \tag{1.7}$$

where $E'''>0$ is the energy evolved in the reaction. Sulfur dioxide, SO_2, is colorless gas with strong smell of burning sulfur. If a cold article, e.g. a china cup, is introduced in the flame accompanying reaction (1.7) then the temperature of the flame essentially decreases so that the burning hydrogen sulfide is oxidized only to free sulfur, which is accumulated on the cup surface in the form of a thin yellow coating. That is to say, in this case, the burning of the hydrogen sulfide goes in accordance with equation (1.2), with the understanding that $E''' > E'' \geq 0$. Addition of equation (1.1) to equation (1.7) multiplied by 2 yields:

$$6CO_2+6H_2O+E_0+4H_2S+6O_2 \rightarrow 6CO_2+6H_2O+E_0+4H_2O+4SO_2+2E'''$$
$$\rightarrow C_6H_{12}O_6+6O_2+4H_2O+4SO_2+2E'''.$$

Upon canceling the similar terms '$4H_2O$' and '$6O_2$' in the first and last terms of this train of equations and upon adding $-E_0$ to both terms, one obtains the following equation of a *hypothetic exegonic* chemical reaction

$$6CO_2+4H_2S+2H_2O \rightarrow C_6H_{12}O_6+4SO_2+2E_3, \tag{1.8}$$

where

$$E_3 \doteqdot 2E''' - E_0, \tag{1.9}$$

so that $E_3>E_2\geq 0$. Using the appropriate enzyme, bacteria of some species drive reaction (1.4), but no bacteria drive reaction (1.8).

2. Polymers (macromolecules) vesus monomers and micromolecules

2.1. Preliminaries

A polymer is a chainlike *macromolecule*, which consists of many (from several to several thousands) identical or similar *monomers* – groups of atoms or ions, *covalently* linked together end to end by *dehydration synthesis*. Polymers of *four* classes are found in cells: *polysaccharides*, *lipids*, *proteins*, and *polynucleotides*, or *nucleic acids.* Most monomers are *dehydrated small organic molecules* subject to the following definition.

Df 2.1. A chemical compound, i.e. a group of bonded atoms, is said to be *dehydrated* if, as compared with the compound to which it is closely related, the compound lacks either hydrogen and oxygen atoms in the proportion of 2 to 1 or bound water molecules. A chemical compound is said to be *dehydrogenated* if it lacks, in the above sense, one or more hydrogen atoms. A chemical compound is said to be *deoxydized* if it lacks, but again in the same sense, one or more oxygen atoms. Both "dehydrated" and "dehydrogenated" are often equivocally abbreviated as either one of the combining forms "*dehydr*" and "*dehydro*", whereas "deoxydized" is abbreviated as either one of the combining forms or adjectives "*deoxy*" and "*desoxy*". Unless stated otherwise, a dehydrated molecule of monosaccharide has two hydrogen atoms and one oxygen atom less than the original non-dehydrated molecule; similarly, a deoxydized, or deoxy, molecule of monosaccharide has one oxygen atom less than the original non-deoxydized molecule.•

Besides polymers, some organic *micromolecules* (*small molecules*), including those of *monosaccharides, disaccharides*, *fatty acids,* and *amino acids* are also found in cells. Monosaccharides, disaccharides, and polysaccharides are collectively called *saccharides.* Saccharides form one of many subfamilies of *carbohydrates*.. It has already been mentioned that glucose is the main immediate nutrient of all cells. Some glucose molecules are oxidized to provide cells with energy, some are incorporated as monomers into disaccharides and polysaccharides by dehydration synthesis, and some others are utilized to drive synthesis of small organic molecules of *fatty acids*, *amino acids*, and *nucleotides*. Fatty acids and amino acids are in turn incorporated as monomers into lipids and proteins by further dehydration syntheses, whereas various nucleotides are immediately joined as monomers into polynucleotides by covalent bonds. Fatty acids and amino acids are in turn incorporated as monomers into lipids and proteins by further dehydration syntheses, whereas various nucleotides are immediately joined as monomers into polynucleotides by covalent bonds.

2.2. Saccharides

In order to function (to live), a living organism requires *organic nutrients* which it utilizes both as a *source of energy* and as a *source of carbon skeletons* for synthesizing *polymers*, some of which are *structural materials* of cells, while the others are *storage materials* found in cells.

The main immediate nutrients, which provide cells both with energy and with molecular carbon skeletons as raw structural material, are *monosaccharides*. Monosacchrides have the general chemical formula $C_nH_{2n}O_n$ with n ranging from 3 to 7, and they are respectively called "*triose*", "*tetrose*", "*pentoze*", "*hexose*", and "*heptose*". Each *n-ose* occurs in various *isomeric forms*. For instance, *ribose* and *ribulose* are two isomeric forms of *pentose*, whereas *glucose*, *galactose*, and *fructose* are three isomeric forms of hexose. Hexose, especially in the form of glucose, is the most common monosaccharide and the major immediate nutrient of cells. Triose and pentose, and especially tetrose and heptose, occur in cells in the form of molecules much rarer than hexose, – mostly as intermediate material in the synthesis of polymers.

An *n*-ose, in which one of the alcoholic hydroxyl groups (OH) is replaced by a hydrogen atom and which has therefore one oxygen atom less, is said to be *deoxy n-ose* or, less explicitly, *deoxy monosaccharide*. In general, if a compound is derivable from another compound, having a certain name, by the removal of one or more oxygen atoms from each one of its molecules then the name of the derivative compound is formed by prefixing the name of the precursor (antiderivative) compound with some one of the words, or combining forms, "*deoxy*" and "*desoxy*". Thus, deoxy monosaccharides have a general chemical formula $C_nH_{2n}O_{n-1}$ with n ranging from 3 to 7, and they are respectively called "*deoxy triose*", "*deoxy tetrose*", "*deoxy pentoze*", "*deoxy hexose*", and "*deoxy heptose*". In this case, the meanings of the similar derivative names such as "*deoxyribose*", "*deoxyglucose*", etc. are self-evident.

Two molecules of monosaccharides are covalently joined together to form one molecule of *disaccharide* in the result of a chemical reaction known as *dehydration* *synthesis* or, briefly, *dehydration*, or else, *condensation*. Disaccharide is disassembled to its constituent monosaccharides by the reverse reaction known as *hydrolysis*. For example, *sucrose*, or *table sugar*, having the formula $C_{12}H_{22}O_{11}$, is a disaccharide formed by linking a molecule of glucose and a molecule of fructose and by extracting one molecule of water (H_2O). Hydrolysis of sucrose with adding of a molecule of water per molecule of sucrose results in the mixture of the two monosaccarides, which is known as *invert sugar*. A higher plant generally transports carbohydrates (saccharides) from one part of it to another in the form of sucrose. *Lactose*, or *milk sugar*, and *maltose*, or *malt sugar*, are also disaccharides which have the same formula as sucrose, with the only difference that the former is formed by linking a molecule of glucose and a molecule of galactose, and the latter is formed by linking of two molecules of glucose; one molecule of water is, as before, extracted. Monomers of polysacchcarides are *dehydrated molecules of monosaccharides*, of which *glucose* is the most common one.

2.3. Lipids

"*Lipids*" is a collective name for various polymers which have the physical property of not mixing with water. *Fats*, *phospholipids*, and *steroids* are three most important categories of lipids. Waxes and some plant or animal pigments are also lipids.

Precursors of the monomers, of which fat and phospholipid polymers are composed, are small molecules of *glycerol* and *fatty acids*. Glycerol is an alcohol, a molecule $H_8C_3O_3$ of which has three carbon atoms, each bearing a hydroxyl group OH. A fatty acid is a carboxyl acid which has a long linear carbon skeleton, most often of 15 or 17 carbon atoms, bonded to a carboxyl group HO_2C. A fatty acid is said to be saturated with hydrogen, or, simply, saturated, if there are no double bonds between its carbon atoms. A fatty acid is said to be unsaturated with hydrogen, or, simply, unsaturated, if there is at least one double bond between its carbon atoms, formed by *dehydrogenation* of, i.e. by removal of certain hydrogen atoms from, the corresponding saturated fatty acid. Thus, for instance, $HO_2C{-}(CH_2)_{15}{-}H$ and $HO_2C{-}(CH_2)_{17}{-}H$ are schematic molecular formulae of the saturated fatty acids called "palmitic" and "stearic", respectively, whereas $HO_2C{-}(CH_2)_7{-}(CH)_2{-}(CH_2)_8{-}H$ is a schematic molecular formula of the unsaturated fatty acid called "oleic". Oleic acid is formed by double (two-fold) *dehydrogenation* of stearic acid. A fat or phospholipid polymer is said to be saturated [with hydrogen] if it contains exclusively saturated fatty acid monomers. The polymer is said to be is said unsaturated [with hydrogen] if it contains at least one unsaturated fatty acid monomer.

A typical fat polymer, called "*triacylglycerol*", is built by dehydration synthesis of one glycerol molecule and three different fatty acid molecules as mentioned above. A typical phospholipid polymer is built by dehydration synthesis of one *choline* molecule $H_{15}C_5NO_2$ (a vitamin of the group B), one *phosphate* (*phosphorus tetraoxide*) *group* PO_4, one glycerol molecule, and *two* fatty acid molecules. Thus, the polymer has a *hydrophilic choline-phosphate-glycerol head* and *two hydrophobic fatty acid tails*. Phospholipids are major building material of cell membranes. In this case, a cell membrane is a *bilayer* of phospholipid polymers such that both surfaces of the bilayer consist of closely packed heads of the polymers, whereas tails of the polymers form the interior of the bilayer. Accordingly, a cell membrane turns out to be hydrophobic in its interior and hydrophilic on its both surfaces, namely on the outer surface of the cell and on the interface between the membrane and the cytoplasm. A steroid polymer is formed from an ensemble of three six-member and one five-member fused carbon rings and an additional group of atoms whose composition depends on the type of steroid. An important animal steroid is *cholesterol*, which is a constituent part of the membranes of animal cells. Most other animal steroids are synthesized from cholesterol.

2.4. Proteins

Proteins are the most common kind of polymers found in cells. More than a half of the entire weight of a living organism falls to the share of proteins. Protein polymers of various kinds serve in a living organism as (i) building, supporting, and storage materials, (ii) antibodies for defense of the organism against foreign substances (as bacteria or viruses); (iii) a vehicle for transport of other substances across a cell's membrane or throughout the organism (as hemoglobin that is the protein serving as the vehicle for transport of oxygen from the lungs to other parts of the organism), hormones (as insulin) that control the concentrations of some other substances (as glucose) in the organism. All enzymes are proteins, but not necessarily vice versa. For instance, insulin is the hormonal protein, which regulates the concentration of glucose in the blood. Chlorophyll, i.e. the enzyme of photoautotrophic biochemical reactions, is also a kind of protein. Precursors of the monomers, of which protein polymers are composed, are small molecules of organic substances, which are collectively called "*amino acids*". There are 20 kinds of amino acids, of which tens of thousands of different kinds of proteins are formed.

2.5. Nucleatides

There are *two* nucleic acids, namely *ribonucleic acid*, or *RNA*, and *deoxyribonucleic acid*, or *DNA*. Particularly, a *ribose* molecule $C_5H_{10}O_5$ and a *phosphate group* PO_4 are covalently linked to a *uracil* molecule $C_4H_4N_2O_2$ to form a nucleotide serving as a monomer for an RNA molecule. Similarly, a *deoxyribose* molecule $C_5H_{10}O_4$ and a *phosphate group* PO_4 are covalently linked to a molecule of *cytosine* $C_4H_5N_3O$, or *thymine* $C_5H_6N_2O_2$, or *adenine* $C_5H_5N_5$, or *guanine* $C_5H_5N_5O$, to form nucleotides of four kinds, which serve as monomers for a molecule of DNA. Cytosine, thymine, and uracil form a family of nucleotides which are collectively called "*pyrimidines*". A molecule of pyrimidine has a six-member ring skeleton made up of four carbon and two nitrogen atoms to which the rest of carbon and nitrogen atoms, along with the hydrogen atoms, are linked in various combinations. Adenine and guanine form another family of nucleotides which are collectively called "*purines*". A molecule of purine has a nine-member closed skeleton made up of five carbon and four nitrogen atoms, to which the fifth nitrogen atom, five hydrogen atoms, and also the only oxygen atom in the case of guanine are linked in various combinations. The purine skeleton is a pyramidine ring, to which a five-member *imidazole* ring, consisting of three carbon and two nitrogen atoms, is fused by one rib of two carbon atoms that the two rings have in common. Nitrogen atoms in a pyramidine or purine tend to take up hydrogen ions from the solution. Therefore, pyramidines and purines are collectively called "*nitrogenous bases*". Thus, a nucleotide that serves as a monomer for a nucleic acid is composed of a nitrogenous base, to which a pentose monosaccharide and a phosphate group are covalently bonded.

Besides nucleotides of the above five kinds, which occur in cells as monomers of nucleic acids, there are nucleotides of some other kinds which occur in cells as individual molecules and which perform important biological functions of their own. *Adenosine triphosphate* ($C_{10}H_{16}N_5P_3O_{12}$), or *ATP*, also called "*adenosinetriphosphate acid*" or "*adenylpyrophosphate*", is one the most important free nucleotides because its function is to transfer energy from one molecule to another in cellular metabolism.

An ATP is a small three-member organic molecule which consists of a ribose molecule ($C_5H_7O_4$) in the form of a pentagon ring, covalently bonded with a phosphate group (P_3O_9) and with an adenine molecule ($C_5H_5N_5$).

3. Basic principles of genetic theory: DNA macromolecules

It will be recalled that a *deoxyribonucleic acid* (*DNA*) macromolecule is a double-helix polynucleotide, i.e. a polymer consisting of two bonded chains of nucleotides as monomers, which spiral around an imaginary axis. A nucleotide is a small three-member organic molecule, which consists of a deoxyribose molecule ($C_5H_{10}O_4$) in the form of a pentagon ring, covalently bonded with a phosphate group (PO_4) and with a nitrogenous base molecule so as to form a stylized blunt angle of about 120° with the deoxyribose molecule at the vortex. The nitrogenous base of each nucleotide monomer found in a DNA polymer is a small asymmetric molecule of one of four kinds: adenine (A, $C_5H_5N_5$), guanine (G, $C_5H_5N_5O$), cytosine (C, $C_4H_5N_3O$), or thymine (T, $C_5H_6N_2O_2$). The phosphate group of a nucleotide molecule can form a covalent bond, called a *phosphodiester linkage*, with the deoxyribose molecule of another nucleotide molecule. In this case, the orientation of the deoxyribose-phosphate quasi-molecule of the first nucleotide turns out to be the same as the orientation of the similar quasi-molecule of the second nucleotide. Therefore, the process of pairwise linkage of nucleotides can be reiterated thus resulting in a *directed* (*oriented*) *polymer* (*chain*, *strand*) of phosphodiesterly bonded deoxyribose-phosphate quasi-molecules as monomers. At the same time, a molecule A or T of one polymer chain and a molecule T or A of the other polymer chain can be linked together by two hydrogen bonds so as to form either a rung A−T or a rung T−A, respectively. Likewise, a molecule C or G of one polymer chain and a molecule G or C of the other polymer chain can be linked together by three hydrogen bonds so as to form either a rung C−G or a rung G−C, respectively. The molecular pairs (A,A), (A,C), (A,G), (C,C), (C,T), (T,T) cannot form bounded states, and therefore A−T, T−A, C−G, and G−C are the only kinds of rungs that occur along a DNA macromolecule. The constituent molecules of any bounded pair A−T, T−A, C−G, or G−C turn out to be oriented relative to each other. The orientation is unique and it has the following general property. Two appropriate nitrogenous bases, as A and T, or C and G, being constituent parts of two paired nucleotides molecules, can form hydrogen bonds if and only if the deoxyribose-phosphate quasi-molecules of the nucleotide molecules are oriented *antiparallel*. Thus, a DNA polynucleotide can be though of as a helical rung-ladder consisting of two antiparallel helical polymers (chains, strands) of deoxyribose-phosphate monomers – the ladder in which every two side-by-side deoxyribose molecules are linked by a rung of two linked molecules of some one of the four kinds: A−T, T−A, C−G, or G−C. The sequence of rungs of the different kinds along the axis of a DNA polynucleotide is not restricted by any topological (geometrical) or physico-chemical constrains, so that the number of such different sequences is astronomically large. Every specific sequence of pairs of the linked nitrogenous bases along a DNA polynucleotide is predetermined exclusively by the genesis of the polynucleotide so as to encode the corresponding genetic information.

Thus, a DNA polynucleotide is a double-stranded helical polymer of nucleotide pairs of four kinds, which differ in their nitrogenous bases; the nucleotide pairs are monomers of the polymer. Four kinds of nitrogenous bases, namely, A (adenine), C (cytosine), G (guanine), and T (thymine)f, and four kinds of bonded pairs of nitrogenous bases, namely A−T, T−A, C−G, and G−C, occur in a DNA polynucleotide. The two constituent strands (chains) of a DNA polynucleotide, which are also polynucleotides in themselves, are *antiparallel* in the sense that any two nucleotides occurring in the DNA polynucleotide are oriented parallel (in the same direction) if they are on the same strand, and antiparallel (in opposite directions) if they are on two different strands. Therefore a DNA polynucleotide has the following important properties.

(1) The structure of either strand a DNA polynucleotide is uniquely determined by the structure of the other strand the polynucleotide.

(2) $N_A=N_T$ and $N_C=N_G$, where N_A, N_T, N_C, and N_G are numbers of adenine, cytosine, guanine, and thymine molecules in the polynucleotide, respectively (*Charhaff's rule*).

The property (1) is of fundamental importance for replication of DNA polynucleotides. Regarding the structure of a DNA polynucleotide, the following remark should also be made. Adenine and guanine belong to a class of nitrogenous bases, which are collectively called "purines", whereas cytosine and thymine belong to another class of nitrogenous bases, which are collectively called "pyramidines". A purine molecule is about twice as large as a pyramidine molecule. However, either bonded molecular pair A−T or G−C is a purine-pyramidine one, whereas either bonded molecular pair T−A or C−G is a pyramidine-purine one. Therefore, the cross-section size of a DNA polynucleatide remains the same along its axis.

A segment of a DNA polynucleotide being a discrete unit of hereditary information is called "*a gene*". Molecular biologists take for granted that a gene is a code of the structure of polypeptides of one specific kind. This axiom is known under the name "*the one gene-one polypeptide hypothesis*". Two preceding successive versions of the axiom had carried the names "*the one gene-one enzyme hypothesis*" and "*the one gene-one protein hypothesis*", which are self-explicative. A polypeptide [molecule] is a linear polymer of dehydrated α amino acid, or, briefly, amino acid, molecules as monomers, which are pairwise linked with each other by covalent bonds, called "peptide bonds". There are 20 [α] amino acids, of the molecules of which any polypeptide is built. Specifically, a peptide bond (CO)−(NH) is formed by dehydration synthesis of the carboxyl group (COOH) of one amino acid molecule and amino group (NH_2) of another amino acid molecule either of the same or of a different kind. Thus, a polypeptide consists of the chain of atoms −N−C−C−N−C−C−, called "the backbone of the polypeptide" or, less explicitly, "a polypeptide backbone", which terminates with an amino group H_2N at one end and with a carboxyl group at the other end, and to which the side hydrogen atoms and side radicals (chains) of the constituent amino acid molecules are attached. A polypeptide has from several to several thousands monomers. One or more polypeptides are twisted, interwoven, and folded to form a three-dimensional macromolecule of substance, which is called "protein". A protein macromolecule has a unique shape, which is determined by the specific sequences of amino acid monomers forming the polypeptide constituents of the macromolecule. The unique shape of a protein macromolecule is said to be *its conformation.* The function of a protein molecule includes its ability to recognize and bind to another molecule of a specific kind. The conformation of a protein molecule is the very property which allows it to recognize and bind to another appropriate molecule by a key-and-lock fit. Therefore, the conformation of a protein molecule corresponds to its function. Except for so-called *restriction enzymes* in bacteria, which are designed to protect a bacterium against intruding foreign DNA molecules from other organisms and which are certain short nucleotide sequences in a bacterium DNA molecule (see Campbell [1990, pp.401, 402] for greater detail), enzyme is protein, but not necessarily vice versa. For instance, karatin and insulin are non-enzymatic proteins. Proteins are the main structural and enzymatic materials, with the help of which the genotype (genetic makeup, genome) of a living organism is expressed as the phenotype (morphology and appearance) of the organism. Molecules of some proteins consist of several molecules of different polypeptides, the structure of a protein molecule being predetermined by the constituent polypeptide molecules. Therefore, the presently common association of a certain polypeptide with a certain gene seems to be natural.

There are solid theoretical and experimental evidences that the smallest units, in which the instructions for a polypeptide structure are encoded within a gene, are sequences of three different nucleotide pairs, collectively called *codons*. Particularly, from a theoretical viewpoint, the number of different sequences of n different objects each (as n-letter words), which are selected from an aggregate of m different objects (as an m-letter alphabet), equals m^n. A DNA polynucleatide consists of paired nucleotides of four different kinds, so that m=4. Therefore, the number of different sequences of n paired nucleotides each, which can be called "n-letter genetic words", equals 4^n. At the same time, a polypeptide consists of amino acid monomers of 20 kinds. To allow encoding 20 monomer kinds by n-letter genetic words, the number n should satisfy the inequality $4^n \geq 20$. The minimal value of 'n', for which this inequality is satisfied, equals 3. In this case, a codon is a 3-letter genetic word encoding the structure of a certain amino acid monomer. Since a polypeptide is a sequence of several hundreds to several thousands monomers, the gene encoding the polypeptide should be a sequence of at least the same number of codons.

A DNA polynucleotide is repeatedly folded together with some protein molecules to form a threadlike construction called a *chromosome*. In this case, the DNA polynucleotide embedded into a chromosome is its genetic material, whereas the protein molecules of the chromosome are its supporting material. If a cell is a prokaryotic one, then its all chromosomes are dispersed in a certain region of the cell, which has no distinct boundary. If a cell is a eukaryotic one then its all chromosomes are located in the nucleus. Since a DNA polynucleotide comprises genes, one may alternatively say that the genome of a cell is the totality of all genes found in its chromosomes. Therefore, the genome of a cell is usually associated with the totality of its chromosomes.

If two cells of the same living organism or of two different living organisms *unite* (*fuse*, *combine*) or *are able* (*have the potential*) *to unite* into one cell, then each one of the two cells is said to be a *gamete*. The cell which is obtained in the result of *unification* (*fusion*, *combination*) of two gametes is said to be a *zygote*. If two gametes that unite or are able to unite to form a zygote are indistinguishable in their morphology, physiology, and behavior then the gametes are said to be *isogametes*; if the two gametes differ in their morphology, physiology, or behavior then they are said to be *heterogametes*. In the latter case, one of the two gametes is typically motile and much smaller than the other, nonmotile gamete. The smaller, motile gamete is said to be a *microgamete*, *male gamete*, *sperm*, *spermatozoid*, or *spermatozoon* ("*spermatozoa*" in the plural). The larger, nonmotile gamete is said to be a *macrogamete*, *female gamete*, *egg*, or *ovum* ("*ova*" in the plural). A large nonmotile female gamete that contains reserve material is also said to be an *oogamete*. Gametes and zygotes are indiscriminately called "*germ cells*". The process of unification of two isogametes into a zygote is said to be *the mutual fertilization of the two gametes*, whereas the zygote is said to be a *fertilized isogamete*. The process of unification of two heterogametes into a zygote is said to be *the fertilization of the female gamete*, whereas the zygote is said to be a *fertilized egg*, or *fertilized ovum*. Reproduction of living organisms by means of isogametes is said to be *isogamic reproduction* or, briefly, *isogamy*. Reproduction of living organisms by means of heterogametes is said to be *heterogamic reproduction* or, briefly, *heterogamy*. Either isogamy or heterogamy is indiscriminately said to be *sexual*, or *gamic*, *reproduction* or, briefly, *gamy*. Reproduction of living organisms, which is not sexual, is said to be *asexual* or *nongamic*. Living organisms that reproduce or are capable of reproducing isogamically are said to be *isogamic*, or *isosexual*, *organisms*. Living organisms that reproduce or are capable of reproducing heterogamically are said to be a *heterogamic*, or *hetrosexual*, *organism*. Both isogamic (isosexual) and heterogamic (hetrosexual) organisms are indiscriminately said to be *gamic*, or *sexual*, *organisms*. Living organisms that reproduce only asexually are said to be *nongamic*, or *asexual*, *organisms*.

Df 3.1. The whole of any organism except its germ cells is called a *soma* ("*somata*" or "*somas*" in the plural). Accordingly, all cells in a multicellular organism except germ cells are called *somatic cells*. In this case, the soma of a unicellular organism is this organism, i.e. this somatic cell. A somatic cell, or a germ cell, is indiscriminately called a *bodily cell*. In other words, a bodily cell is either a somatic cell or a germ cell. Similarly, a *bodily organ*, or, briefly, an *organ*, of a multicellular organism having an organic bodily structure is either a *somatic organ* or a *reproductive organ*.•

In accordance with Df 3.1, all cells of a nongamic living organism are of the same category, namely *somatic*, or *bodily*, *cells*, i.e. the cells of which the organism is made up. By contrast, besides its somatic cells, a gamic organism has normally, within a certain period of its lifetime, gametes and some relevant intermediate cells (as gametes in progress). A living organism that produces isogametes is an *isogamic* (*isosexual*) *organism*, in accordance with the previous definition of the last term. A living organism that has only a male reproductive organ and therefore produces sperms during its lifetime is said to be a *male unisexual organism* or, briefly, a *unisexual male*. A living organism that has only a female reproductive organ and therefore produces eggs during its lifetime is said to be a *unisexual female organism* or, briefly, a unisexual female. Either a male unisexual organism or a female unisexual organism is indiscriminately said to be a *unisexual organism*. A heterogamic organism that has functional male and female reproductive organs and therefore produces sperms and eggs *simultaneously during a certain period of its lifetime* is said to be a *euhermaphroditic*, i.e. *true hermaphroditic*, *organism*, or briefly a *euhermaphrodite*, i.e. *true hermaphrodite*. A heterogamic living organism that has a functional male reproductive organ during some period, or periods, of its lifetime and a functional female reproductive organ during some other period, or periods, of its lifetime is said to be a *sequential hermaphrodite*, or *quasi-hermaphrodite*. A sequential hermaphrodite is said to be a *male quasi-hermaphroditic organism,* or a *quasi-hermaphroditic male*, when it has a male reproductive organ, and a *female quasi-hermaphroditic organism*, or a *quasi-hermaphroditic female*, when it has a female reproductive organ. A quasi-hermaphrodite is said to be a *protogynous* (meaning *female first*) *quasi-hermaphrodite* if it is born as a female and then changes its sex for a male; a quasi-hermaphrodite is said to be a *protandrous* (meaning *male first*) *quasi-hermaphrodite* if it is born as a male and then changes its sex for a female.

The above terms that are relevant to gamic organisms, including the qualifiers “protogynous” and “protandrous”, are derived from the following Greek etymons:

«**ανδρικός** \andrikós\ *a.* of a man, men's; manly; **ανδρικός φύλον** \fílon\ male sex.

Αφρόδιτη \afródite \ *s.f.* Aphrodite.

γυνή \giní \ *s. f.* women; wife.

Έρμης \'hérmes\ *s.m.* Hermes.

ἑρμαφρόδιτος \hérmafróditos\ *s. m.* hermaphrodite (the male god Hermes & the female goddess Aphrodite).

προτού \protú\ *conj. & adv.* before.

πρώτα \próta\ *adv.* (at) first; before.

πρωτο \proto\ *comb. form denotes* first. »

The class [of equivalence] of all isogamic organisms is said to be *the isosex* or *the neutral sex*. The class of all euhermophrodotes is said to be *the euhermophrodote sex*. The class of all males, either unisexual or quasi-hermophrodotic, is said to be *the male sex*. The class of all females, either unisexual or quasi-hermophrodotic, is said to be *the female sex*. Any one of four divisions of gamic (sexual) organisms into isogamic (isosexual) organisms, euhermophrodotes, males, and females is indiscriminately said to be a *sex*.

A gamete is said to be a *haploid*, or *monoploid*, *cell*, whereas the number of chromosomes in a gamete is said to be a *haploid*, or *monoploid*, *number*. The number of chromosomes in an isogamete of an isosgamic organism of a certain species or in a female gamete of a heterogamic organism of a certain species is denoted by 'N', with the understanding that N, i.e. the value of 'N', depends on a species. The number N is said to be a *haploid*, or *monoploid*, *number*. Accordingly, a gamete is said to be a *haploid*, or *monoploid*, *cell* and its set of chromosomes is said to be a *haploid*, or *monoploid*, *chromosomal set*. A zygote has two haploid chromosomal sets donated by the two parent gametes, i.e. it has $2N$ chromosomes altogether. The number $2N$ is said to be a *diploid number*. Accordingly, a zygote is said to be a *diploid cell* and its set of chromosomes is said to be a *diploid chromosomal set*. The two sets of chromosomes found in the zygote are said to be *partner sets of chromosomes*, the understanding being that to each chromosome of one of the partner sets there corresponds a unique *partner chromosome* of the other set. Except, perhaps, for one specific pair of partner chromosomes in a warm-blooded animal zygote, which determines the sex of the hybrid to develop from the zygote (see below), two partner chromosomes have the same appearance under the microscope, – particularly, the same shape, length, and stain pattern. Also, two partner chromosomes replicate similarly in the process of subsequent binary fissions of the zygote. At the same time, any chromosome is distinct from any other chromosome, which is not its partner. Each pair of partner chromosomes determines certain traits of the living organism to develop from the zygote.

As the zygote develops into a living organism, any somatic cell of the organism is obtained in the result of the chain of repetitive, so called *mitotic*, binary divisions of the zygote. The division of each cell in the temporal stream (succession, chain) of predecessor-successor (parent-daughter) cells is a two-phase process. The first phase is *mitosis*, the division of the cell's nucleus, in the result of which two new, daughter nuclei, each having a carbon copy of chromosomes of the parent nucleus, arise. The second phase is *cytokinesis*, the binary division of the cytoplasm, in the result of which two new daughter cells, each containing a single nucleus, arise. Therefore, *each somatic cell of any sexual (gamic) organism is a diploid cell*.

As the sexual organism develops from a zygote, successive generations of cells in the temporal stream of cells gradually differentiate to form different organs or systems of organs of the organism, including its reproductive organ. As the organism reaches sexual maturity, its reproductive organ begins to produce gametes. Within the reproductive organ are diploid gamete-producing cells that develop into gametes in the result of a two-stage division process called "meiosis". Each of the two stages, called "meiosis I" and "meiosis II", is in turn a multiphase process. In the result of its meiosis, a diploid gamete-producing cell turns into four gametes.

There are solid experimental evidences that the genomes of *any two single-nucleated somatic cells of a multicellular living organism are carbon copies of each other* up to possible injuries of some genes caused by external agents (as radioactive radiation). The assumption that this is a general property of all living organisms is called "*the axiom of genomic equivalence*". Accordingly, the axiom is briefly expressed by saying that any two single-nucleated somatic cells of a multicellular living organism are genomically equivalent. The axiom of genomic equivalence implies that, as cells of a living organism are differentiated, *they cease to express nonessential genes, but the differentiated cells do not loose the nonessential genes*. The axiom of genomic equivalence allows making the following definition.

The genome of a eukaryote is the genome of its any single-nucleated somatic cell, and vice versa. Putting it more formally, the genome of a living organism is the class [of equivalence] of the genomes of its single-nucleated somatic cell.

Also, owing to the genomic equivalence, one may assert that, all metabolic processes in any cell of a living organism are controlled by *the genome of the cell*, no matter whether the organism is unicellular or multicellular. Equivalently, one may assert that all metabolic processes in any cell of a living organism are controlled by the genome of the living organism. Indeed, if an organism is multicellular, then some metabolic processes in its some specialized cells can be controlled remotely by the genome specimens located in some other cells of the organism, which are specialized differently. However, all genome specimens of the organism are equivalent and hence indistinguishable.

The genome of an organism controls its metabolism through organic molecules of two kinds: *signal molecules* and *transport molecules*. At the same time, in a cell, most of their atoms are bonded into small groups, which are in turn joined together into long linear chainlike organic macromolecules , which are called "*polymers*" and which have been discussed in section 2 of this essay.

Essay 8. Historical remarks on philosophy, logic, and psychology

1. Chronologic qualifiers

Western historiographers divide history of Western civilization into three periods: ancient civilization beginning with the earliest known civilization and extending to the death of Constantine AD 332, the first Christian Emperor of Western Rome Empire, or alternatively to the fall of that empire AD 476; medieval civilization or middle ages, extending from the end of ancient civilization to about *AD 1300, a round year in the life of Dante Alighieri AD 1265–1321, symbolizing* the beginning of Renaissance, which lasted into the 17th century and which was the transitional period between medieval and modern times. Still, some historiographers arbitrarily choose AD 1500 as the boundary between medieval and modern times. The late Renaissance time was the beginning of development of modern philosophy and modern science. In what follows, I shall conventionally qualify various fields of study and discourse either as *traditional* (or *classical*) or as *modern*. In this case, "modern" means: «of or relating to a period extending from a more or less remote point of the modern (post-medieval) time to the present time», whereas "traditional" means: «of or relating either to ancient or to middle (medieval) time». The exact times of activity of some distinguished *dramatis personae* in a given field of study that is characterized either as traditional or as modern will contextually specify more definitely the meaning of the respective qualifier. The most important cultural language of Western Europe until the end of the 17th century was Latin. As compared to Ancient Latin that was the native language of Latins and Romans from the end of the 3rd century BC to the end of the 2nd century AD, the later versions of Latin are chronologized as follows: Late Latin is Latin as used from the 3rd to the 6th centuries inclusive; Medieval Latin is Latin as used from the 7th to the 15th centuries inclusive; New Latin is Latin that as used since the end of the medieval period and especially Latin as used in scientific terminology and classification.

2. Philosophy

2.1. Traditional philosophy versus modern sciences

2.1.1. Traditional philosophy

The word "philosophy" has many meanings in English. Traditionally "*philosophy*" (without any qualifier) meant, and hence "*traditional philosophy*" means, the *general* field of study and discourse that included five *disciplines*, i.e. five *specific* fields of study and discourse: *logic*, *esthetics*, *ethics*, *politics*, and *metaphysics* (cf. Durant [1926, p. 3]), of which logic and metaphysics partly overlapped (to be explicated in due course). Traditional philosophy was based on *ancient Greek philosophy*, primarily on works of *Plato* (428/427–348/347 BC, see Essay 5) and *Aristotle* (384–322 BC, see section 4 of this Essay).

Logic is the study of *ideal forms* of movements of thought in cognitive activities of a sapient subject that involve pairs of complementary methods such as *extrospection* and *introspection*, *practice* (particularly, *observation* or *experiment*) and *theory*, *analysis* and *synthesis*. Consequently, logic is a study that is based on *reason*, and not on belief or intuition, and that utilizes *dialectics* – two complementary methods, one of which is *generalization* (*universalization*, *classification, induction*), including *postulates* (*assumptions*) and *definitions*, and the other one is *specification* (*particularization, deduction*), including *inferences* (*proofs*) and *theorems* as their results, and also including *corollaries* as specifications of postulates, definitions, or theorems; a postulate is called an *axiom* if it is *relatively permanent* and a *hypothesis* if it is an *ad hoc* one.

Esthetics is the study of *ideal forms* of things, which are collectively called *beauty*. Therefore, logic can, by analogy, be called *the esthetic of thought*.

Ethics is the study of *ideal conduct* and of g*ood* and *evil*. The axioms of ethics, which determine ideal conduct, are called *moral principles.* Ideal conduct is of course an abstraction that is supposed to be the standard of personal behavior.

Politics is the study of *idealized social organizations* as monarchy, aristocracy, aristocracy, democracy, anarchism, socialism, communism, conservatism, radicalism, liberalism (as *golden mean* of the previous two), feminism, etc. Politics thus defined is *political philosophy,* which has nothing to do with *real politics*, i.e. with political practice – just as ethics has nothing to do with real personal behavior of individuals.

Metaphysics as an ancient and traditional (medieval) discipline is the inquiry into the «*ultimate Reality*», «*ultimate Mind*», and *their interrelation* – categories that supposedly underlie the «*apparent mind*» of an «*apparent man*», who is conscious (aware) of «*apparent reality*» through his sensations (perceptions). Accordingly, metaphysics is divided (analyzed) into three interrelated specific fields of study and discourse: *ontology, metaphysical psychology*, and *epistemology*. *Ontology* is the inquiry into *the nature of* «*ultimate Reality*» («*ultimate Matter*», «*ultimate Beings*»), supposedly underlying the «*apparent reality*» («*apparent matter*», «*apparent beings*»), of which a sapient subject («apparent man») is conscious (aware) by acquaintance through his sensations. *Metaphysical psychology* is the inquiry into *the nature of* «*ultimate Mind*», the understanding being that this field can be regarded as a part of the field that is presently called *philosophical psychology*. *Epistemology* is the inquiry into *the relation between* «*ultimate Mind*» *and* «*ultimate Reality*», underlying the process of *acquiring knowledge* by a sapient subject through his sensations (perceptions) and conceptions and hence underlying *the relation between* «*ultimate Reality*» *and* «*apparent reality*». Metaphysics is the most baffling branch of philosophy, because it is not, like the other branches of philosophy, a study of certain ideal entities as abstractions of some real entities or, in other words, it is not an attempt to coordinate something real and the corresponding ideal, but it is an attempt to treat *what is real and what is ideal*, and also to treat *what is the mind*, which dares make, not only the above attempts, but which also attempts to treat itself.

Both the whole of logic and the whole of metaphysics are pure mental products of the *brains*, or more precisely *minds* (*cerebral cortices*), of properly schooled, turned, and meditated men (sapient subjects) – products that are produced and studied primarily *introspectively* and that are expressed in terms of the appropriate mutually intelligible graphic symbols. Therefore, logic and metaphysics were formerly regarded as a single area of study or instruction, which was called *mental philosophy*. At the same time, ethics is in fact the study of the aspects of conscious behavior, which are collectively called *moral behavior*. Therefore, ethics is sometimes regarded as a part of modern *philosophical psychology* and is synonymously called *moral philosophy*.

2.1.2. Modern sciences and modern philosophy

According to the contemporary divisions of *art* and *science*, *modern philosophy* and all its branches (subdivisions), which are distinguished by the names "logic", "esthetics", "ethics", and metaphysics, except politics, are qualified as some *humanitarian sciences*, or *humanities*, whereas politics is one of the *social sciences*, i.e. a *branch of social science*. Still, in the following description of the five sciences of the above names in a wide historical prospective, especially logic, I shall associate them with the ancient and traditional sciences of the same names, from which they have stemmed. For avoidance of confusion, I shall, when necessary or desired, distinguish any of those sciences, which are relevant to a certain historical epoch, by means of the corresponding qualifying adjective or phrase, e.g. "ancient", "ancient Greek", "traditional", "modern", "contemporary", etc. A like remark applies to all other *sciences* (*branches of science*), whose embryos existed in the ancient and medieval times, but which were not associated with ancient or traditional philosophy, – such humanitarian sciences, e.g., as mathematics, linguistics, history, etc or such *natural sciences*, e.g., as physics, biology, meteorology, etc. Still, some of ancient mathematics, e.g. Pythagoras theory of numbers, was a part of metaphysics and hence a part of ancient philosophy, in contrast to Euclidean geometry, which were not regarded as a part of ancient philosophy. *Modern philosophy*, which comprises exclusively humanitarian sciences (as mathematics, linguistics, semiotics, semeiotics, history, etc) and no social sciences (as economics, politics, sociology, etc), will be collectively called "*humanitarian philosophy*" – as opposed (antonymous) to "*natural philosophy*", which is a collective name of all modern *natural sciences* (as physics, chemistry, biology, astronomy, geology, geography, oceanography, meteorology, etc).

2.2. Some fundamental traditional and modern philosophical terms

2.2.1. Greek-originated terms

The name "philosophy" and the names of various subdivisions of traditional philosophy, both those used above and some others, have been derived from the Greek etymons as defined in the following vocabulary articles, which are cited, mainly, from the Greek-English-Greek dictionary of Pring [1982] and are supplemented by transcriptions and comments of my own. The transcriptions are made in accordance with the pronunciation remarks of Pring (*ibid.* pp. xiv–xvi) and are given between back-slashed virgules, \ \. The comments are given between angle brackets, { }; "EGP" is an abbreviation of "the English-Greek part of Pring [1982]".

Dictionary 2.1

«**αίσθησις** \éθisis, éthisis\ *s.f.* sense (*bodily faculty*). {from the article **aesthete** of EGP} **αισθητικός** \ethitikós\ *s.m.* **αισθητική** \ethitikí\ *s.f.* aethetics (*or* ethetics)).

γνήσιος \γnésios\ *a.* genuine, true.

γνώσ|ις \γnósis\, **~η** \~i\ *s.f.* knowledge {*knowing*, – by EGP}, cognizance; ~**εις** \~is\ (*pl.*) knowledge {*what a person knows*, – by EGP}, learning.

επιστήμη \epistími\ *s.f.* **:** branch of learning; science.

ηθική \ethikí\ *s,f,* ethics, moral; morality.

λόγια \lójia, lóyia\ *s.n. pl.* words.

λογική \lojikí, loyikí\ *s.f.* logic; way of thinking.

λογικ|ό \lojikó, loyik|ó\ *s.n.* ~ά \~á\ (*pl.*) senses, reason (*sanity*).

λογικός \lojikós, loyikós\ *a.* rational, logical; reasonable.

λόγιος \lóγios\ *a.* learned {*of writing*, – by the article **learn** of EGP}.

λόγ|ος \lóγos\ *s.m.* 1. speech (*faculty*); speech, address; talk; mention, question (*of*); saying; word {*in connected speech*, in contrast to "λε̑ξις" \léksis\ also meaning *a word* but used for mentioning a word *singly*, i.e. actually meaning *a single word*, – see the article **word** in EGP⟩ (*pl.* "τά λόγια" \tá lójia\ {means *the words*}); … 2. reason, ground; account, reckoning; ~ω \~o\ by reason of; …

μετά \metá\ 1. *adv.* afterwards. 2. *prep.* (*with acc.*) after.

μορφή \morfí\ *s.f.* form: face; appearance; aspect; phase.

όν \ón\ *s.n.* being, creature.

οντότης \ontótis\ *s.f.* entity, being; individuality.

πολιτική \politikí\ *s.f.* politics.

πράγμα \práγma\ *s.n.* thing, matter, item.

προτού \protú\ *conj. & adj.* before.

πρώτα \próta\ *adv.* (at) first; before.

ρητορική \ritorikí\ *s.f.* rhetoric; oratory.

σοφίο \sofía\ *s.f.* wisdom; erudition.

ύλη \íli\ *s.f.* matter, substance; material; pus.

φίλος \fílos\ *a. & s.m.* dear, friendly; friend {**φίλη** \fíli\ *s.f.*, – by the article **friend** of EGP}.

φυσικ|ά \fisiká\ *s.n. pl., ~* **ή** \~í\ *s.f.* physics**.**

φυσικός \fisikós\ *a.* natural, physical …

φύσ|ις \físis\ *s.f.* nature; ~ει \~í\ by nature**.**

ψυχή \psixí, psichí\ *s.f.* soul; heart; energy, spirit, courage …»

In this case, and generally in word formations, the i*ntended lexical meaning* (*denotation value*, *denotatum*, pl. "*denotata*") attached to an Anglicized morpheme (as a combining form or word) of Greek or Latin origin should not necessarily exactly coincide with some one of its etymological meanings, although the latter should be akin to the former so as to serve as its logical and mnemonic justification. Consequently, in the general case, in order to turn an Anglicized expression, which has a certain etymological meaning, into a technical term, the expression should be attached with the desired denotatum by the appropriate definition and be used in that *sense* only in the *scope* of that definition for avoidance confusion. Here follow a few important examples.

1) A Merriam-Webster® [1981] (to be referred to as WTNID) defines the meaning (denotatum) of -"*logy*" thus:

«**-logy** … *n comb form* -ES ... **1 :** oral or written expression ⟨phraseo*logy*⟩ **2 :** doctrine, theory, science ⟨socio*logy*⟩ **3 :** discourse, treatise ⟨insecto*logy*⟩»

Particularly, the meaning of the term "epistemology" and the meaning of the cognate term "*gnoseology*" are defined in that dictionary thus:

«**epistemology** … *n* –ES … **:** the study or a theory of the nature and grounds of knowledge esp. with reference to its limits and validity; *broadly* **:** the theory of knowledge – compare GNOSEOLOGY

gnoseology ***or*** **gnosiology** … *n* –ES … **:** the philosophic theory of knowledge **:** inquiry into the basis, nature, validity, and limits of knowledge ⟨~ became coextensive with the whole of metaphysics – C.A.Hart⟩».

Semantically, the definiens of "gnoseology" differs from of the definiens of "epistemology" only by the comment given between angle brackets, so that in the absence of that comment the two nouns would have been synonyms. In accordance with the comment, "gnoseology" should have been explicitly defined as a synonym of "metaphysics" and therefore it comprises ontology, philosophical psychology, and epistemology. To summarize, modern epistemology (gnoseology) deals with the methods of acquiring knowledge, including sensorial experience and abstraction, and with the limits of knowledge.

2) According to the same dictionary, the English combining form "*meta*" means `e.g. in "*meta*physics" or "*meta*mathematics"; *of higher logical type*, e.g. in "metalanguage" or "*meta*system"; *later in time or higher in organization*, *status*, *or in a series*, e.g. in "*Meta*zoa" or "*meta*zoan".

3) The new Anglicized combining form "prota" of my own is the transliteratum of the Greek adverb "πρώτα", which has the same meaning as the combining form "proto" originating from the Greek conjunction and adverb "προτού" \protú\. At the same time, "prota" is consonant with both "meta" and "para". It is therefore convenient to use the morpheme "prota" interchangeably with or instead of the established English (Anglicized) combining form "proto" (or "prot") as a *complimentary antonym of* "meta", which by definition means: *earliest in time or lowest in organization*, *status*, *or in a series*.

4) "Tó óv" \tó ón\ (pl. "τά όντα" \tá ónta\) is an Aristotelian term that is translated into English by the noun "being" and which can be understood as anything that can be described in terms of some instances of the 10 *categories* (*classes of predicates*) of Aristotle [350, *Categories*], namely:

> «substance, quantity, quality, relation, place, time, position, state, action, or affection»,

– according to [ACE, Part 4], or

> «either Substance, or Quantity, or Quality, or Relation, or Where, or When, or Position, or Possession, or Action, or Passion»,

– according to [ACO, Chapter IV]); some scholars use the noun "doing" instead of "action" and "undergoing" instead of either "affection" or "passion". In other words, a being (όν) is *anything that can be treated* (*spoken*) *of* as one that is located in the φύσις \físis\, i.e. in *the nature* or *physical world*. According to Aristotle, every being has a certain *form* (μορφή) and a certain *matter* (ύλη). By contrast, the Aristotelian term "πράγμα" \prágma\ (pl. "πράγματα" \prágmata\), which is translated by the English noun "*thing*", means *anything that can be treated* (*spoken*) *of*, including a being and also including anything *supra-natural* as the Aristotelian God, which is not located in the physical world and which is not, therefore, a being. Thus, "πράγμα" ("thing") in Aristotle's philosophy is a more general and vaguer term than "όν" ("being").

5) The *Peripatetics*[4] divided the entire philosophy (teaching) of Aristotle into *First Philosophy*, or *Metaphysics*, or *Theology*, and *Second Philosophy*, or *Physics*. Physics included everything what was known about the material (natural, physical) world, i.e. about beings (όντα). By contrast, Metaphysics included everything what could be treated of, but what could not be regarded as a part of material world and what could not therefore be described in terms of the 10 Aristotelian categories. This peculiar thing (πράγμα), having *neither matter nor form*, was Aristotelian God, the doctrine of which had come in *Aristotelianism* (*Aristotelian philosophy*) instead of *Platonian ontology* with his transcendental *Universals* (*Forms, Ideas*). Aristotelian God is *not a creator*, but *the prime unmoved mover* (*primum mobile immotum*) of the world, i.e. it is *the prime, sole, and total form-giving cause*, or *motive*, of all changes of *beings*, occurring in the world. God is an incorporeal, spaceless, timeless, sexless, passionless, indivisible, invariable, perfect, and eternal *thing* (*πράγμα*). He or, more correctly, *It* does nothing except giving forms to all beings. That is to say, God is the *pure energy* (*ενέργεια*) or *activity per se* – the Scholastic *Actus Purus*. Like the English Kings and Queens since William III (1650–1702, reigned 1689–1702), Aristotelian God reigns but does not rule. "*Energy*" ("*ενέργεια*") is a fundamental Aristotelian term, being actually a synonym of "God". [5] The etymological sense of the

[4]The school that Aristotle established in the fifty-third year of his age was the walk along the athletic field, on which he strolled up and down together with his scholars when teaching them. The athletic field was a part of the grounds of the temple of Apollo Luceus – the protector of flocks against wolves (from "λύκος" \lúkos\ meaning «wolf»). The walk was called "Peripatos" (from "περίπατος" meaning «walk», «ride», «drive», «trip»). Aristotle's school took the Latinized name "the Luceum" from the name "Apollo Luceus", and the name "Peripatetic School" from "Peripatos". Accordingly, the scholars and later followers of Aristotle are called "Peripatetics".

[5] Although Aristotle is *the founder of logic*, his notion of God (the prime unmoved mover) and his notion of interaction of God with the physical world via energia are alogical (obscure). God is not located in the physical world and therefore it is not a *being* (όν \ón\) that can be treated of in terms of some of the 10 Aristotelian categories. At the same time, energia is the pure activity of God, which should be present in all beings, and therefore it should be a universal, – like form and matter, which are also, not beings, but universals present in beings. I am unable to prescind Aristotelian enérgia from his God, so that I regard, at least in my own use, the names "prime unmoved mover" and "energia" as synonyms.

word “Metaphysics” is explicated below in subsection 3.3. In present usage, “metaphysics” is an equivocal name that has many different senses – like “philosophy” or “religion”.

6) In the Late Ancient Greek philosophy of Neo-Platonism founded by *Plotinus* (Πλωτῖνος \plotínos\, AD ca 204/5–270), the noun “ὀντότης” meant *reality* (*Platonic Universals*). By contrast, in modern English there is a tendency to use the noun “entity” for mentioning anything that can be treated (spoken) of, i.e. to use it in analogy with the Aristotelian term “πράγμα”, translated into English as “thing”, and at the same time to use the noun “thing” in analogy with the Aristotelian term “όν” (“being”) and Plotinus’ term “ὀντότης” (“reality”). In this usage, the noun “thing” is parasynonym of the Latin noun “rēs” (pl. “rēs”), which, according to Simpson [1968], means *a thing, object, matter, circumstance*, and especially *the real thing, fact, truth*. The English words “*reality*” and “*real*” have been derived from that Latin etymon.

2.2.2. Latin-originated terms

Unlike the above terms of Greek origin, the fundamental terms “*art*” and “*science*” have been derived from the *Latin* etymons “*ars*” and “*scientia*” in agreement with the following vocabulary entries, the first two of which are of Latin-English part of Simpson [1968], whereas the latter two are of the English-Latin part of that dictionary.

Dictionary 2.2

«**ars** –tis, f. (root ar-, as in Gr. αραρίσκω) *skill, way, method.* (**1**) *an occupation, profession, art*…(**2**) *skill, knowledge,* as shown in arts…

scĭentĭa –ae, f. (sciens) *a knowing, knowledge of, acquaintance with, skill in*.»

«**art**, *ars* (the most general word; often opp. to *natura* or *ingenium*); *artificium* (the practice of an —, or workmanship); *peritia* (= acquired skill); the fine —s, *artes ingenuae, liberals, humanae, elegantes* (or use compar. of one of these adjectives); a work of —, *artis opus,* (-eries, n.), *opus arte factum.*

science (**1**) = knowledge in gen. *scientia*, (**2**) = systematic knowledge in a particular field, *ars, doctrina, disciplina.* (3.3) in particular: natural — *physica* or *physice, physiologia, rerum naturae scientia, investigatio naturae.*»

Owing to the etymological meanings of the nouns “art” and “science”, it is debatable whether philosophy (especially ancient or traditional one) and some of its subdivisions can be qualified as branches of science, i.e. as branches of systematic knowledge, because logic, e.g., is simultaneously the study, art, and science of reasoning, whereas metaphysics is primarily based on belief and intuition and not on knowledge and reason.

2.2.3. «Organon» and «Metaphysics»

1) Ancient writers credited Aristotle with several hundreds through a thousand volumes, relatively few of which have survived. All extant philosophical and scientific works, which are generally agreed as written by Aristotle himself, and nearly all those, which are attributed to Aristotle but in regard of which his authorship is either disputed or is generally agreed as spurious, – i.e. the complete body of extant works of Aristotle and his school, – have been preserved through their Greek edition in Rome in the first century BC (c60–c40 BC) by Andronicus of Rhodes, a Greek philosopher and the eleventh *scholarch*[6] (the second one in Rome era) of the Lyceum (the Peripatetic school) in Athens, who flourished then in Rome. There are various legends how those works arrived at the hands of Andronicus, which can be found, e.g., in the article **Aristotle** of *Britannica Online Encyclopedia* (*Britannica.com*), to be referred briefly as *BOE*, and in the articles of the same name of *Wikipedia*, *Gale Encyclopedia of Biography* (*GEB*), and *Oxford Companion to Classical Literature* (*OCCL*), on Answers.com. In any case, all later translations of the extant *Aristotelian works*, i.e. *works of Aristotle and his school*, from Greek into Latin and afterwards into various modern languages have been derived and have come down to us mainly from Andronicus' edition. The most fundamental ones of these translations will be discussed in subsection 4.1. The most important works of Aristotle are «*Organon*» and «*Metaphysics*». However, the titles of the works were not given to them by Aristotle himself, so that they are not Aristotle's terms.

2) «*Organon*» comprises six treatises (books): «*Categories*», «*On Interpretation*», «*Prior Analytics*», «*Posterior Analytics*», «*Topics*», and «*Sophistical Refutations*». Most of the Aristotle's biographers and impartial commentators of his works are sure that all works constituting «*Organon*» were written in publishable

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[6]A scholarch (from Greek: "σχολάρχης" \sholarhis, sçolarçis\) is the head of a school, especially the head of a school of philosophy in ancient Athens, such as the Platonic Academy, Aristotelian Lyceum, Zeno's Stoa, and Epicurean Garden, whose first scholarchs were, respectively, Plato (in the years 388–348 BC), Aristotle (in the years 335–322 BC), and Zeno of Citium (in the years c300–c263 BC), and Epicurus (in the years 307–271 BC).

form by Aristotle himself. No such consensus exists among scholars regarding authenticity of many other extant works of Aristotle. The six treatises that are mentioned above form the standard collection of Aristotle's works on logic. However, the conventional order of these works is not the chronological one, which is unknown, but was supposedly (according to Wikipedia, e.g.) chosen by Theophrastus (Θεόφραστος, c371–c287 BC), the successor to Aristotle as the second scholarch of the Lyceum, so as to constitute a well-organized course of lectures on logic. This order was adopted by Andronicus of Rhodes (see below) in his publication of Aristotle's works. The general title «*Organon*» («Ὄργανον» \órganon\), i.e. «*Tool*» or «*Instrument*» in the adequate chaste English translation, which means *the tool*, or *instrument, of correct thinking,* was given to the collection of the six works by the later Peripatetics likely in about the early 1st century BC. By this title, the Peripatetics expressed their view that the study placed under it was not a part of philosophy (in contrast to what the Stoics maintained), and hence not a branch of science (as physics, metaphysics, or mathematics), but rather it was a *tool* of every inquiry. Whether or not this is true regarding logic in the modern sense of the word, is a matter of definition of the words "philosophy", "science", and "logic". However, «*Organon*», especially the formal theory of categorical syllogisms forming the subject matter of his *Prior Analytics*, is surely the most important doctrine and inseparable part of the *entire Aristotle's philosophy*, *humanitarian* and *natural,* called *Aristotelianism* (one of the meanings of the word). Nowadays, either of the two synonymous terms "*organon*" (in the Graecized spelling) and "*organum*" (in the Latinized spelling, pl. -s) is a well-established English noun, which, according to WTNID and in agreement with Peripatetics' interpretation of their Greek etymon, means «an instrument for acquiring knowledge; *specif* **:** a body of methodological doctrine comprising principles for scientific or philosophical procedure or investigation».

3) The word "*Metaphysics*" was reputedly introduced in use by Andronicus who, as the first editor of collected works of Aristotle, gave the name "Τά μετά τά φυσικά" \tá metá tá fisiká\, – i.e., verbatim, "The [writings] after the physics" or, in the modern English terminology, "Metaphysics", – to the works that he placed *after* the works entitled "Τά φυσικά" (from Homer's term "φύσις" \físis\ *of The Odyssey* that was employed by Plato in the sense of "*nature*"), i.e., verbatim, "The natural beings" or, in the modern terminology, "*Physics*". This interpretation of the name "Metaphysics" ("Τά μετά τά φυσικά") is suggested, for instance, in the article

metaphysics of WTNID and in the articles **Aristotle** both in Wikipedia and in OCCL; the latter suggests also some other interpretations of the above name.

4) «*Metaphysics*» comprises 14 books, which are referred to either by Roman numerals from "I" to "XIV" (cf. AMR) or by Arabic numerals from "1"to "14" or else by the Greek letters "A" (the capital alpha), "α" (the small alpha), and from "B" (the capital beta) to "N" (the capital nu) in that order (cf. the article **Metaphysics** in Wikipedia).

5) Besides dealing primarily with the theological doctrine of Aristotle (see the item 5 of sub-subsection 2.2.1.), «*Metaphysics*» has some points of overlap with the works making up «*Organon*». To be specific, the subject matter of Book IV (or Γ) of «*Metaphysics*» [AMR] is primarily a prolix discussion and extensive collection of various formulations of *Law of non-contradiction,* e.g. these ones:

> «But we have now posited that it is impossible for anything at the same time to be and not to be, and by this means have shown that this is the most indisputable of all principles.» (*ibid.* Part 3)

> «Again, if, when the assertion is true, the negation is false, and when this is true, the affirmation is false, it will not be possible to assert and deny the same thing truly at the same time.» (*ibid.* Part 4)

The former quotation is, likely, an alternative translation of the same definition as the following passage from <http://www.non-contradiction.com/>:

> «It is impossible for the same thing at the same time to belong and not belong to the same thing at the same time and in the same respect. Aristotle, Metaphysics, 1005b12-20.»

In this quotation, the reference to the pertinent place in "*Aristotelis Opera*" (The *Aristotelian corpus*) is made by the respective so-called Bekker's numbers to be explicated in subsection 4.1. Law of non-contradiction is one of the most fundamental underlying principles of Aristotle's syllogistics, especially of «*On Interpretations*», where that principle is used but not mentioned. The essay on Law of non-contradiction of «*Metaphysics*» was not included into «*Organon*» by the Peripatetics, and therefore it is not traditionally considered part of «*Organon*», probably because the essay was not known to the Peripatetics or because it was known with a great degree of plausibility not to be an authentic work of Aristotle.

2.2.4. “Hylomorphism”

One of the central and most general doctrines of *Aristotelianism* is *the doctrine* (*principle*) *of opposition and unity of form* (*essence*) *and matter* (*stuff*), which is today called *hylomorphism* or, more specifically, *Aristotelian hylomorphism*. According to hylomorphism, every corporeal entity (being) is a *biune* one that consists of *two inherent principles* (*aspects*), namely a *primordial* (*primary*), *potential* one that is called *matter* and a *secondary, actual* one that is called *form*. That is to say, the matter of a being is its *stuff* or *potency*, whereas the form of the being is its *essence* or *actuality*. In this case, matter and form are two *complementary conceptual* aspects of an entity, which can be distinguished and contrasted, but which cannot be separated from each other. Consequently, the term “hylomorphism” originates from two Greek nouns: “ύλη” \íli\ (pl. “ύλαι” \íle\), meaning *a matter*, and “μορφή” \morfí\ (dual “μορφά” \morfá\, pl. “μορφαί” \morfé\), meaning *a form*. The English nouns “*matter*” and “*form*” are in turn derived respectively from the Latin nouns “mātĕrĭa”, meaning *matter*, *material, stuff of which anything is composed* (besides having some other meanings), and “forma”, meaning *form*, *figure*, *shape* (see Simpson [1968]). Aristotle derived (induced) the doctrine of hylomorphism from his analysis of changes of *particulars*. Namely, when an entity changes (e.g., from being cold to being hot, or from being hard to being soft, or from being solid to being liquid, or from being green to being yellow), its matter remains unaltered throughout the process of change, while its form differentiates any two distinct successive states of the entity. Thus, the matter (stuff) of a real entity (being) is not that entity, because it needs a certain form (essence) to become so. Consequently, it is often convenient to use the term “*protamatter*” for mentioning (denoting) *the matter of a entity (being) as contrasted to the form of the entity* (as in the latter examples) and to use the term “*metamatter*” for mentioning (denoting) *the entity synonymously as a single whole, including its matter* (*protamatter*) *and its form to complete each other.* For instance, in accordance with the doctrine of *physicalistic monism* of *philosophical psychology*, *the mind* (*cerebral cortex*) *of a conscious* (*waking*) *sapient subject* is *the pertinent metamatter*, *the consciousness of the subject* is *the form (essence) of his mind*, and *certain abstract formless* (*unconscious*) *matter* (*material, stuff*), of which the mind is made, is *the protamatter of the conscious* (*waking*) *mind* (*cerebral cortex*).

2.2.5. "Entelechy"

1) Hylomorphism has the following psychological aspect. If a being having a certain form and a certain matter has grown to its modification having a higher form and the same matter then the two hypostases of the same being are habitually and conveniently regarded as two different beings, the former of which is regarded as the *ad hoc* (*immediate*) *matter of the latter*. Hence, a form may in turn be the [ad hoc] matter for growing a higher form. In such ad hoc uses of the generic names "form" and "matter", form and matter, being their denotata, become *epistemologically relativistic notions*, the understanding being that the qualifiers "*epistemologically relativistic*" is in fact (circularly) a synonym of "*ad hoc*". For instance, a man is the form, of which the child was its matter; the child is the form, of which the embryo was its matter; the embryo is the form, of which the ovum was its matter; and so on. In this case, since a form and its ad hoc matter are two epistemologically relativistic hypostases (ways of existence) of each other, one cannot reach the conception of matter without form (in contrast to what is stated in Durant [1926, p. 56]). Any being has a form, so that a formless matter is not a being, i.e. it is not a reality. On the contrary, the form of a being can reach its *highest possible form*, which is called the *first entelechy of the being*. The process of metamorphosis of the being leading to its first entelechy is called the *second entelechy of the being*.

2) "Entelechy" is the Anglicized Aristotelian term "*εντελέχεια*" /enteléçia, entélehia/ – the noun that is composed of the following three etymons: "*εντός*" /entós/, adv. & prep., meaning *inside* or *within,* "*τέλος*" /telós/, noun, meaning *an end*, and "*έχω*" /éxo, écho/**,** v.t. & i., meaning *to have*, *keep*, or *hold*. That is to say, etymologically, *an entelechy is an entity havin*g (echo) *its end* (telos) *within* (entos) *itself*. By definition, the Aristotelian entelechy of a being is a *biune* entity that has two distinct *hypostases* (*ways of existence*, *aspects*): the first entelechy and the second entelechy. In other words, in Aristotelianism, the term "entelechy" assumes two distinct senses. This is why "entelechy" is often interpreted differently by different translators and interpreters. The first Aristotelian entelechy of a being is the *full realization of its form-giving cause*, which is called the *energy* ("*ενέργεια*") or *God*. For instance, an egg of a tortoise that has become another tortoise and not a crocodile is the first entelechy of the egg, and some ice that has become water and not oil is the first entelechy of the ice. By contrast, the process of metamorphosis of the egg of a

tortoise into another tortoise is the second entelechy of the egg, and the process of metamorphosis of some ice into water is the second entelechy of the ice.

3) “Entelechy’ is a magnificent Aristotelian term that, from the viewpoint of modern science, gathered together such notions as *phenotype* and *genotype* in application to any thing, and not just to a *biont* (*living organism*), In presently common usage in English, the noun “entelechy” means *the final end* or *purpose*, thus being an analogue of “phenotype”. Accordingly, the postpositive qualifier “*in entelechy*” and the kindred prepositive qualifier “*entelechial*” are *synonyms* of the postpositive qualifiers “*in full realization*”, “*in actuality*”, and “*in extension*”, and also of the prepositive qualifiers “*actual*” and “*extensional*”. Hence, the qualifiers “*in entelechy*” and “*entelechial*” are, at the same time, *antonyms* of the postpositive qualifiers “*in potency*” and “*in intension*” and of the kindred prepositive qualifiers “*potential*” and “*intensional*”. In Modern Greek, “ενέργεια” means *activity*, *action*, *operation*, *effort*, and also *energy* in the conventional physical interpretation of the word. Therefore, in contrast to “*potential energy*”, “*entelechial energy*” can be understood as a synonym of “*kinetic energy*”. It is also noteworthy that in Modern Greek, the noun “*εντέλεια*” \entélia\, meaning *perfection*, and the adjective “*εντελής*” \entelís\, meaning *perfect* or *complete*, are cognate with Aristotelian “*εντελέχεια*”.

3. Logic in historical prospective

3.1. Formal logic versus material logic

“Logic” is a generic name, which is used by different writers for mentioning various topics and doctrines. The most extensive meaning (denotatum, denotation value) of the noun “*logic*” in a wide historical prospective can briefly be defined in the following two equivalent ways.

Df 3.1. i) *Logic* (in Greek: “λογική” \lojikí, loyikí\ from the etymon “λόγος” \lóγos\ in the sense of “reason”) is the graphic (written) theory (theoretical study) of thought and its movements in any *cognitive activities* of a sapient subject, i.e. mental activities of the sapient subject that are aimed at *acquiring knowledge*.

ii) *Logic* is the study, art, science, and discourse of *correct reasoning*, the understanding being that an instance of reasoning is either a right judgment or a right inference of the right judgment from other right judgments. Hence, most briefly, *logic* is the study of ways of making right judgments.•

Cmt 3.1. 1) In accordance with Df 3.1, "logic" means *cultivated logic*, i.e. logic that is cultivated as a branch of science and art as opposed to *naïve logic* – an inborn or acquired power of a man (sapient subject) to behave and judge adequately and reason validly about states of affairs (events) of his everyday life.

2) Letting aside hypothetical Kantian *a priori* (inherent, inborn) knowledge, it is at present postulated that there are two kinds of knowledge: *knowledge by acquaintance* (through percepts) and *knowledge by description* (through concepts). The latter includes *knowledge by reason* (*by induction or deduction*). Therefore, knowledge and reason are two distinct but inseparable aspects of *relationships between the mind of a sapient subject and entities* (*beings*), *of which the subject is conscious*, *including the subject himself*. I call such an entity "*a coentity of the sapient subject*". For description and study, cognitive activities of a sapient subject are most generally divided into two complementary methods in three independent ways: (a) *extrospection* (particularly, *observation* or *experiment*) and *introspection*, (b) *analysis* and *synthesis*, and (c) *practice* and *theory*.

3) Unfortunately, the generic name "logic" has been labeled to many different theories of reasoning, which were developed under other names, – such theories, e.g., as Aristotelian «*Organon*» (meaning *Tool*, or *Instrument*, *for acquiring knowledge*), *epistemology*, and *semiotic* or any of its three branches: *syntactics*, *semantics*, and *pragmatics*. In fact, the only way to define the contemporary denotatum of the term "logic" is to give the complete long list of the names of all specific fields of study and discourse, which are regarded as branches of logic, – no matter whether the name of a branch contains the generic name "logic" or not.

4) During his lifetime, any reasonable sapient subject makes an indefinite number of judgments; each judgment has its own matter (content, sense), but many *materially different judgments may have the same form*. Therefore, in analyzing judgments and reasoning, the nouns "*form*" and "*matter*" (or "*content*") are traditionally used as complementary antonyms and consequently their kindred (derivational) adjectives "*formal*" and "*material*" are used likewise. Consequently, the entire logic is traditionally *divided* (*bifurcated*) into two supposedly complementary fields: *formal logic* and *material logic*. This traditional division of logic seems to be fundamental and therefore it is maintained by likely all contemporary philosophers, logicians, and lexicographers, so that the generic names "formal logic" and "material logic" are used as antonyms.

5) In the literature on logic and philosophy and in explanatory dictionaries of the English language, there are many adequate and basically equivalent concise descriptive (conceptual) definitions of the term "formal logic", although some of them differ from one another in some covert mutually contradictory subtleties. By contrast, various concise descriptive definitions of the name "material logic", which can be found in the pertinent literature, are overtly inconsistent and the definientia of all such definitions that I know are not substantiated and illustrated sufficiently for grasping their meanings, so that their definiendum, "material logic", turns out to be a *nomen nudum*. Therefore, it is not clear what fields of study and discourse should be included under the name "material logic" in accordance with its descriptive definitions, – in the exclusion of some concrete logical systems, which are just labeled with this generic name as an *ad hoc* proper name. Aristotelian «*Organon*», especially in the modified form taught in the Middle Ages in the framework of the *Trivium* (see Cmt 3.2 below), is one of such concrete systems. Since the *sense* (subject matter), which is assigned to the name "material logic" by its various descriptive definitions, either is blurred or is ambiguous for one reason or another, therefore the sense of the name "formal logic", being the would-be antonym of "material logic", turns out to be blurred or ambiguous as well.

6) As I have pointed out previously, since the two fields of study and discourse, which are called "formal logic" and "material logic", are supposedly *mutually complementary*, therefore uncertainty in meaning of the latter term leads to uncertainty in meaning of the former term; hence, the generic term "logic" becomes ill-defined as well. For instance, WTNID defines the meanings of the above two names thus:

> «**formal logic** *n* **:** a system of logic (as Aristotelian logic or symbolic logic) that abstracts the forms of thought from its content to establish abstract criteria of consistency – contrasted with *material logic*
>
> **material logic** *n* **:** logic that is valid within a certain universe of discourse or field of application because of certain peculiar property of that universe or field – contrasted with *formal logic*»

Except for the phrase «contrasted with *material logic*», the above Webster's definition of "formal logic" is adequate. However, the Webster's definition of "material logic" describes this name expressly vaguely: it does not give any criteria of belonging a separate judgment or a system of interrelated judgments to material logic and it is not illustrated in any way so that, given a discourse, that definition does not allow deciding to which field of study the given discourse belongs, to material logic or to formal logic or to both, or to no logic at all. That is to say, “material logic” thus defined is a nomen nudum (naked name), while “formal logic” becomes a nomen nudum as well owing to the phrase «contrasted with *material logic*» in its definition. Concise conceptual definitions of the term “material logic”, which can be found in other authoritative publications, are not any better. To compare, Wiktionary, e.g., defines the denotatum of that term in this manner:

> «**Material logic** is «the branch of logic that focuses on the content of reasoning».

This definition is circular and therefore the expression “material logic”, being its definiendum, is a nomen nudum again.

7) The above “formal logic”/“material logic” puzzle or apparent paradox, whatever you call the above confusion and controversy in using the terms “formal logic” and “material logic”, arises from forgetting the following fact. In accordance with the general philosophical doctrine of hylomorphism, given a system of reasoning, that is called a *logical system* or a *logic*, its *form* (*essence*), called a *formal logical system* or a *formal logic*, and its *matter* (*stuff, raw material*), called *the pertinent formal logical system* or *the pertinent material logic*, are two complementary and inseparable *hypostases* (*ways of existence*, *aspects*), of each other. That is to say, if *formal logic* (*FL*) is thought of as a *certain totality of formal logical systems* then *material logic* (*ML*) should be thought of as *the* ***respective*** *material logic*, i.e. as *the totality of the respective material logical systems*, and not as an *incoherent totality of various independent material logical systems.* For avoidance of confusion ans controversy, he phrase «contrasted with *material logic*» in the Webster’s definition of *formal logic* should be understood in that very sense.

8) In accordance with the item 1 of sub-subsection 2.2.5, given two increasingly high formal systems of logic (formal logics, forms of logic), e.g. a first-order predicate calculus and a sentential calculus or Aristotelian syllogistics, which have the same counterpart material system of logic (material logic, matter of logic), e.g. the same system of interrelated declarative sentences as the interpretand of either one of the two formal system, the lower formal system can *ad hoc* be regarded as the

epistemologically relativistic matter of the higher formal system, while the latter can *ad hoc* be regarded as the epistemologically relativistic form of the former. For instance the *organon* A_1 is the form, of which its *conservative conformal catlogographic* (*CCFCL*) *interpretand* is its matter; the CCFCL interpretand is the form, of which its *progressive conformal catlogographic* (*PCFCL*) *interpretand* is its matter; the PCFCL interpretand is the form, of which the class of appropriate *English declarative sentences* (*EDS's*) is its matter; the class of EDS's is the form, of which the totality of facts (states of affairs), to which separate members (EDS's) of that class are conformable, is the matter of the class of EDS's. It is understand that any *fact* (*state of affairs*), to which a certain EDS conforms, has its own form, so that it is a being, which is not however an object of formal logic.•

Cmt 3.2. 1) Monastic schools ("scholae monasticae" in Latin) and cathedral schools were most important institutions of higher education in medieval Western Europe from early Middle Ages until 12th century. Many of these schools evolved into medieval and modern universities. Along with religious studies, the standard curriculum of a medieval higher school comprised *seven secular disciplines* (*liberal arts*), *divided into two groups*.

2) *Grammar*, *rhetoric*, and *logic* constituted the elementary division of the curriculum, called the *Trivium*, successful study of which was required of all who would obtain the B.A. degree. In this case, as has been mentioned above, logic of the Trivium was the pertinent version of Aristotelian «*Organon*», which is called by contemporary writers "*material logic*" and which was called by various medieval writers at different times "*critical logic*" or briefly "*criticism*", and also "*major logic*" or "*greater logic*" as opposed to Aristotelian syllogistics, being a part of «*Organon*», which was called "*minor logic*" or "*lesser logic*" (cf., e.g., Turner [1910, **Logic**] and the Memoria Press program *Material Logic* on the Internet, <http://www.memoriapress.com/descriptions/logic/material_ logic1.html>), and also "*term logic*".

3) *Arithmetic, geometry, astronomy,* and *music* constituted the higher division of the curriculum, called the *Quadrivium*, successful three-year study of which was required of all who would obtain the M.A. degree. It seems to be not accidental that music was included into the Quadrivium together with the three exact sciences: arithmetic, geometry, and astronomy, especially together with the last one. Such a

composition of the Quadrivium was likely due to the fact that Pythagoras and other ancient mathematicians proclaimed the existence of the so-called *music of the spheres* – a theoretic harmony or music, which was created by the movements of the planets and heavenly bodies and which was inaudible to human beings.•

3.2. Applied logic

1) Besides formal logic and material logic, some contemporary writers attempt to define another branch of logic, which they style "*applied logic*". Unfortunately, all definitions of this would-be self-subsistent field of logic that I have encountered in the literature are persuasive ones being, if I may put it this way, even less convincing than the above quoted definitions of "material logic". It is, of course, possible to declare that epistemology, e.g., is one of the branches of applied logic. However, a substantiated conceptual (and not ostensive) definition of the term "applied logic" should, particularly, unambiguously describe the differentia that is supposed to be denoted by the qualifier "applied" as compared with the differentia that is denoted by the qualifier "material". Until this is done, the meaning of the name "applied logic" is indistinguishable from the meaning of the name "material logic". There are two similar general philosophical principles of «saving thoughts», namely the principle of *Ockham's razor*, saying that *entities should not be multiplied unless necessary*, and *Leibniz's principle of identity of indiscernibles* (*principium identitatis indiscernibilium*), saying that *no two objects have exactly the same properties.* Therefore, in accordance with either of the two principles, the name "applied logic" should be disregarded as the definiendum of many definitions.

2) Nevertheless, *applied logic* other than *material logic* certainly exists. I shall not however attempt to give any concise definition of the general term "applied logic" for the fear that it will be another unsubstantiated one. I shall only remark that there is rigorous reasoning of at least one kind that should be relegated to applied logic, and not to material logic, because it is not adjoint of any *formal logistic system* (*FLS*), but rather it is based on a completely different rigorous principle. I mean formation and use of a *rigorous univocal system of taxography* (*taxographonymy*), i.e. *of graphic* (*written*) *taxonomy*, *of a specialized field*, each *element* (*member*) of which, called a *taxograph* (*taxographonym*), i.e. graphic (written) *taxonym* (*name of a taxon*, i.e. *of a taxonomic class*), is a *description,* or more explicitly *description of the species* (*specific class*, *subclass)*, *through a genus and the difference*, or *differences*, – briefly

DcTrG&D, *DcSTrG&D*, *DcTrG&Ds*, or *DcSTrG&Ds* in that order, in Latin *descriptio*, or *descriptio species*, *per genus et differentiam*; or *differentias*, respectively. Such a system can be called an *onomatological*, or *onomastic*, *system*. At the same time, a *definition* whose definiens is a DcTrG&D or DcTrG&Ds is a traditional *definition through the genus and difference* (*differentia*), or *differences* (*differentiae*), – briefly a *DfTrG&D* or *DfTrG&Ds*, in Latin *definitio per genus et differentiam*, or *differentias*, which was introduced by Aristotle [350 BCE, «*Posterior Analytics*»] and which is often called a *real*, or *explicative*, *definition*. Therefore, I relegate every onomastic system to *applied logic* and I call applied logic of such systems *onomastic logic* (*OL*).•

3.3. Aristotle is the founder of logic

1) Aristotle is commonly and undisputedly called *the founder of logic*, most often without specifying the kind or kinds of logic, which he inaugurated. Here, for instance, follows an interpretation of Aristotle's title of founder of logic from article **syllogistics** of BOE:

> «Aristotle's claim to be the founder of logic rests primarily on the *Categories*, the *De interpretatione*, and the *Prior Analytics*, which deal respectively with words, propositions, and syllogisms. These works, along with the *Topics*, the *Sophistical Refutations*, and a treatise on scientific method, the *Posterior Analytics*, were grouped together in a collection known as the *Organon*, or "tool" of thought.»

In agreement with the above quotation, Aristotle's right to the title of founder of logic rests on his «*Organon*», which is often synonymously called "*Aristotelian logic*". Therefore, the occurrence of the word "logic" in Aristotle's title «founder of logic» should be understood as «*deductive formal, material, and applied logic, and also inductive logic*», although Aristotle's influence on the appearance and development of inductive logic is as a rule unjustifiably ignored. Consequently, the name "Aristotelian logic" is equivocally used in various senses from its broad sense as a synonym of Aristotle's «*Organon*» to its narrow sense as a synonym of the term "*Aristotelian syllogistics*". The latter denotes a system of *14 syntactic* (*formal*) *deductive three-judgment rules of inference*, called *categorical syllogisms*, which Aristotle developed in «*Prior Analytics*» on the basis of «*Categories*» and «*On Interpretations*», and which was later supplemented with *another 5 categorical* *syllogisms* reputably by Galen of Pergamum (AD c130–c200), a prominent Roman (of Greek ethnicity) physician, surgeon, and philosopher, who gathered up and systematized ancient knowledge of medicine and anatomy and remained the supreme authority in these fields for more than a thousand years. Galen also wrote about logic and philosophy. The *fourth syllogistic figure*, comprising the additional 5 categorical syllogisms, is sometimes called the *Galenian figure*, whereas the version that Galen was the first scholar to use and possibly to discover it is explicitly supported, e.g., in the article **1figure** of WTNID. Still, according to the article **Prior Analytics** of Wikipedia, the fourth figure was added after Aristotle's death by Theophrastus (c372–c287 BC), a student and close associate of Aristotle, who succeeded him on his retirement as scholarch (head) of the Lyceum in Athens and who led the school for more than three decades.

2) *Syntax of reasoning*, i.e. correct use of words for expressing form of reasoning, and also some *moods of correct reasoning*, e.g. the rule of *reductio ad absurdum* (or *reductio ad imposibile*) and the principle of illegitimacy of *vicious infinite regress*, were devised by Greek philosophers before Aristotle. Particularly, Aristotle was aware of works of *Zeno of Elea* (c495–c430 BC), the Greek philosopher and mathematician, whom Aristotle called the inventor of *dialectics*. *Zeno's paradoxes*, especially the noblest one of the race of Achilles and a tortoise, strikingly illustrated the paradoxicality of *vicious infinite regresses* and contributed to the development of rigorous methods of logical reasoning and of precise concepts of continuity and infinity.[7] However, the entire field of study and discourse, which was scientifically investigated, systematized, and inaugurated by Aristotle in «*Organon*», especially in its first three treatises, had not existed earlier. In [ASR, part 34], Aristotle himself says:

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[7]At present, the Zeno's paradox is solved elementarily. In accordance with the modern concepts of motion of a material point in the tree-dimensional Euclidean space, due to Galileo Galilei (1564–1642) and Isaac Newton (1642–1727), the trajectory of such a motion relative to an inertial frame of reference is uniquely determined by the initial position and initial velocity of the point and by its acceleration at every later moment. If Achilles and the Tortoise move steadily in the same direction along the same straight line with [constant linear] velocities v_A and v_T and if the initial position of the Tortoise is at a distance s_0 in front of Achilles then the current distance $s(t)$ between the two at a moment t is determined as: $s(t)=s_0+v_Tt-v_At$. Hence, at the moment $t_*=s_0/(v_A-v_T)$, satisfying the equation $s(t_*)=0$, Achilles overtakes the Tortoise.

«Of this inquiry, on the other hand, it was not the case that part of the work had been thoroughly done before, while part had not. Nothing existed at all.»

3) The following definitions illustrate the ambiguity of the term "Aristotelian logic". WTNID defines this and the related terms "traditional logic" and "symbolic logic" thus:

«**aristotelian logic** *n, usu cap A* **:** the logic of Aristotle: **a :** the total organon of Aristotle including his theory of predicables and categories, of definitions and syllogistic **b :** the traditional formal logic inaugurated by Aristotle – compare TRADITIONAL LOGIC

traditional logic *n* **1 :** a system of formal logic mainly concerned with syllogistic form of deduction that is based on Aristotle and includes some of the changes and elaborations made by Stoics and Scholastics **:** ARISTOTELIAN LOGIC – compare IMMEDIATE INFERENCE, OPPOSITION SUBJECT-PREDICATE, SYLLOGISM, SYMBOLIC LOGIC **2 :** inductive logic esp. as developed by Francis Bacon and J. S. Mill

Here follow two relevant definitions of the Internet:

«In philosophy, **term logic**, also known as **traditional logic**, is a loose name for the way of doing logic that began with Aristotle and that was dominant until the advent of modern predicate logic in the late nineteenth century… The fundamental assumption behind the theory is that propositions are composed of two terms – hence the name "two-term theory" or "term logic" – and that the reasoning process is in turn built from propositions…» (*Wikipedia* on *Answers.com*)

«**Aristotelian logic**: 1. the logic of Aristotle, especially in the modified form taught in the Middle Ages; 2. traditional logic: the logic of the Late Middle Ages, derived from Aristotelian logic, and concerned esp with the study of syllogism.» (*Dictionary.com*)

According to the above definitions, both names "Aristotelian logic" and "traditional logic" are ambiguous. Therefore, I shall, as far as possible, distinguish among different meanings of these names by supplementing them with appropriate qualifiers that will be defined formally in Df 3.3 below in this section in the framework of a certain general partition of traditional logic. Meanwhile, a few following terminological remarks.

4) Aristotle's «*Organon*» together with all later contributions and comments is called *Aristotelian major,* or *greater*, or *metamaterial*, *logic*. The collection of the first three treatises of Aristotle's «*Organon*», namely «*Categories*», «*On Interpretations*», and «*Prior Analytics*» together with all later contributions and comments is called *Aristotelian minor,* or *lesser*, *logic* and also *Aristotelian deductive logic* (*ADdL*), *Aristotelian syllogistics* (*AS*), *categorical syllogistics* (*CS*), *predicate syllogistics* (*PS*), *traditional Aristotelian deductive logic* (*TrADdL*), and *traditional predicate logic* (*TrPL*), and *term*, or *two-term, logic*. The collection of the last three treatises of Aristotle's «*Organon*», namely «*Posterior Analytics*», «*Topics*», and «*Sophistical Refutations*» but again together with all later contributions and comments are called *Aristotelian epistemology*, meaning *Aristotelian theory of knowledge.* Accordingly, the denotatum of the name "Aristotelian logic" as defined in the item **a** of the former above Webster's definition or in the item 1 of the above definition of *Dictionary.com* is *Aristotelian major logic*, whereas the denotatum of the name "Aristotelian logic" as defined in the item **b** of the same Webster's definition or in the item 2 or of the same definition of *Dictionary.com*, and also the denotatum of the name "traditional logic" as defined in the item 1 of latter above Webster's definition or in the above definition of *Wikipedia* is *TrADdS.* By contrast, the denotatum of the name "traditional logic" as defined in the item 2 of the last Webster's definition is distinguished by either of the self-explanatory names "*traditional inductive logic*" ("*TrIdL*") and "*Bacon-Mill's inductive logic*".

5) It is understood that Aristotelian epistemology includes *scientific method* of acquiring knowledge, which was developed in his *Posterior Analytics* (cf. the quotation of BOE at the beginning of this subsection). Thus, Aristotle's title of founder of logic implies that Aristotle is also the founder of *scientific epistemology*. In this connection, the following remark will be in order. The categorical syllogisms, which have been developed in *Prior Analytics*, are *deductive moods of reasoning*, i.e. rigorous moods of inference of a particular (concrete) instance from general principles and, perhaps, from other particular instances. At the same time, in accordance with Aristotle's *theory of knowledge*, all *primary* knowledge of a man, i.e. knowledge that does not requires any demonstration (proof), comes from experience of the man (cf. Aristotle [APsAM, Book 1, Part 3]). Therefore, Aristotle *recognized* the importance

of *inductive moods of reasoning*, i.e. rigorous moods of inference of general principles from concrete instances. Consequently, Aristotle is not only the founder of deductive logic literally, but he can also be regarded the inspirer (spiritual precursor) of inductive logic. However, he *developed* in closed form only deductive logic of categorical syllogisms, which was adopted and polished by Scholastics after a millennium. Consequently, the later modern critics and opponents of Scholastics associated them, along with Aristotle, as advocates of exclusively deductive logic. For instance, approximately two millennia after Aristotle, Francis Bacon (1561–1626) published his most important treatise under the head "*Novum Organum*" ("*The New Organon*"), in which he flung a challenge to all medieval metaphysics, based on Aristotelian logic, by developing *inductive logic*. He says: «To go beyond Aristotle by the light of Aristotle is to think that a borrowed light can increase the original light from which it is taken» (see Durant [1926, p. 100]). Inductive logic was developed further by Mill [1843].

3.4. "Logic"

Aristotle himself did not give any indications that he considered the six treatises compiled in «*Organon»* as a single whole study and he did not employ either of the terms "Organon" and "logic" either as the title or in the title of any of his treatises or their parts. At the same time, Aristotle used the generic name "analytics" in the titles of his two treatises, namely «*Prior Analytics*» and «*Posterior Analytics*», which he thus regarded as one work. The term "logic" he employed as a close synonym of "*dialectics*" ("διαλεκτική" \ðialectikí\ *s.f.*, dual διαλεκτικά \ðialectiká\, pl. διαλεκτικαί \ðialectiké\) meaning, collectively, *induction and deduction* or, distributively and indiscriminately, *either induction or deduction*, i.e., in general, *induction or deduction* (with inclusive "or"). It is suggested in [WA] that: «What we today call Aristotelian logic, Aristotle himself would have labeled "analytics".

Regarding conjectural etymology of the Aristotelian term "αναλυτός" \analutós\ and its English parasynonym "analytics", it is said in WPA:

> «Analytics comes from the Greek word "analutos" meaning "solvable" and the Greek verb "analuein" meaning "to solve". However, in Aristotle's corpus, there are distinguishable differences in the meaning of "analuein" and its cognates. There is also the possibility that Aristotle may have borrowed his use of the word "analysis" from his teacher Plato. On the other hand, the meaning that best fits the Analytics is one derived from the study of Geometry

and this meaning is very close to what Aristotle calls ἐπιστήμη “episteme”, knowing the reasoned facts. Therefore, Analysis is the process of finding the reasoned facts…»

Still, according to Pring [1982] (see Dict A1.1), Aristotle’s term ““επιστήμη” (dual επιστήμα \epistíma\, pl. επιστήμαι \epistíme\)” has acquired a completely different sense in Modern Greek, whereas in Modern English it has been used as an etymon in forming the term “epistemology” whose meaning as explicated in the item 2 of subsection 2.3 is distinct from that of both “analytics” and “logic”. At the same time, the etymological sense of “analytics” seems not to be as straightforward as suggested in the above quotation from [WPA].

According to various Greek-English-Greek dictionaries, including Pring [1982], *analytical* (*detailed*) *etymology* of the Greek words “αναλυτός” and “ανάλυσις” \análusis\ and of their English parasynonyms “analytics” and “analysis” is based on the following *basic* Greek etymons:

“α”- \a\ or “αν”- \an\ (*privative prefix*) un-, in-, -less.

“λύω” \lúo\ *v.* solve.

“λύσις” \lúsis\ s.f. termination, dissolution (of partnership, etc); solving, settling; solution, answer; dénouement, ending.

“λυτός” \lutós\ *a.* solvable.

Consequently, the following morphological constructions are derivational:

“ανα”- \ana\ (*double privative prefix* –Ya. I.) *denotes* up, back, again, intensification.

“άλυσις” \álusis\ *s.f.* chain, sequence, succession.

“άλυτος,” \álutos\ *a.* not united; unsolved; unsolvable; (*fig.*) indissoluable

“ανάλυσις” \análusis\ *s.f.* analysis.

“αναλυτικός” \analutikós\ *a.* analytic(al); detailed.

The peculiarity of the prefix “ανα”- is that it is the sequence of the privative prefix “α”- and its allomorph “αν”, so that it is an *intensifying positive* one. Therefore, just as “λυτός”, the adjective “αναλυτός” means *solvable* but apparently in the more specified sense of the expression “*solvable in detail*”. The Greek noun, or absolute adjective, “αναλυτός” and hence the derived English noun “analytics” should be understood respectively. At the same time, the term “logic” originates from the following etymons (see e.g. Pring [1982]), the first two of which have already been mentioned in Dict A2.1:

“λόγος” \lóγos\ *s.m.* 1. speech (*faculty*); speech (*communication or expression of thoughts in spoken words*), address; talk, mention, question (*of*); saying; word (see the next entry). 2. reason, ground; account, reckoning.

“λόγια” \lójia, lóyia\ *s.n. pl.* words (*in connected speech*, – in accordance with the article **word** of EG; opposed to “λε̃ξις” \léksis\ *s.f.* meaning a separate word).

“λογική” \lojikí, loyikí\ *s.f.* way of thinking; logic.

“λογικό” \lojikó, loyikí\ *s.n*, “λογικά” \~á\ (*pl.*) senses, reason (*sanity*).

“λογικός” \lojikos\ *a.* rational; reasonable; logical.

In the light of the above etymological senses of the word “analytics” and “logic”, Aristotle’s conjectural view on what he did in the treatises «*Prior Analytics*» and «*Posterior Analytics*» and also in the related treatises can briefly be described thus: *detailed solution of problems of dialectic* (*inductive or deductive*) *reasoning with the help of words connected so as to form interrelated judgments*. By definition, a *judgment* is a *true proposition*, whereas a *proposition* is a *truth-functional declarative sentence*, i.e. a *declarative sentence that can be either true or antitrue* (*false*). Therefore, either of the two words “analytics” and “logic” could be used as a synonym of “Organon”. However, the former has not been used in this collective sense. The word “λογική” as a name of the science founded by «*Organon»* was reputedly used in writings of the Stoics. However, the Latin version of this name, “*logika*”, from which the English noun “logic” is derived, was coined by Marcus Tullius Cicero (106–43 BC), a Roman statesman, orator, and author. Here follows the article of Simpson [1968] confirming this fact:

«**lŏgĭcus**, -a, -um (λογικός), *logical*; n. pl. as subst. **lŏgĭca**, - ōrum, *logic*: Cic.».

3.5. Aristotelian logic

1) *Aristotelian logic* (*AL*) or *Aristotelian syllogistics* (*AS*), called also *predicate traditional logic* (*PTrL*), has two physically inseparable *psychical aspects*, one of which is its *form*, called *Aristotelian formal logic* (*AFL*) or *Aristotelian formal syllogistics* (*AFS*), while the other one is its *matter*, called *Aristotelian material logic* (*AML*) or *Aristotelian material syllogistics* (*AMS*).

a) *AFL* (*AFS*) is *a system of 19 formal deductive three-judgment three-term rules*, or *moods, of inference*, called *formal*, or *schematic*, *categorical syllogisms* (*FCS's* or *SCS's*) and also, more precisely, *categorical syllogism-schemata* or *categorical syllogism-forms* (*CSS'ta* or *CSF's*) *of categorical syllogism instances* (*CSI's*), called also *material categorical syllogisms* (*MCS's*). The entire set of 19 FCS's divided into *4 ordered syllogistic figures* containing 4, 4, 6, and 5 FCS's respectively. The 14 FCS's comprised in the first three figures were laid down by Aristotle himself in his «*Prior Analytics*» [APrAJ] on the basis of his «*Categories*», [ACE] or [ACO]), and «*On Interpretations*», [AIE]), whereas the 5 remaining FCS's comprised in the fourth figure was reputedly added by Galen of Pergamum (AD c130–c200), a prominent Roman (of Greek ethnicity) physician, surgeon, and philosopher, who gathered up and systematized ancient knowledge of medicine and anatomy and remained the supreme authority in these fields for more than a thousand years. Galen also wrote about logic and philosophy. The fourth syllogistic figure is sometimes called the *Galenian figure*, whereas the version that Galen was the first scholar to use and possibly to discover it is explicitly supported, e.g., in the article ^{1}**figure** of WTNID. Still, according to the article **Prior Analytics** of Wikipedia, the fourth figure was added after Aristotle's death by Theophrastus (c372–c287 BC), a student and close associate of Aristotle, who succeeded him on his retirement as scholarch (head) of the Lyceum in Athens and who led the school for more than three decades.

b) *AML* (*AMS*) comprises concerete MCS's, bing *material instances* (*matters*) of FCS's, which are expressed in a certain native language (as English), into which AFL is incorporated.

2) An FCS (SCS, CSS, CSF) is a formal (schematic) rule (mood) of deductive inference of a *categorical judgment*, called the *conclusion*, *from two known categorical judgments*, called the *premises*. A *syllogistic judgment* (*SJ*) is a *veracious* (*accidentally true*) *simple extended declarative sentence of a standard form* as specified below. The two *premise schemata* (*forms*) and the *conclusion schema* (*form*) of an FCS are certain *instances* of the appropriate three *syllogistic judgment-schema* (*SJS*), or *syllogistic judgment-form* (*SJF*), *placeholders* (*PH's*), i.e. three *SJSPH's* or *SJFPH's*, selected out of the following four:

"All ***u*** are ***v***", "All ***u*** are not ***v***", "Some ***u*** are ***v***", "Some ***u*** are not ***v***", (3.1) which will be called (in that order) *universal affirmative* (*UA*), *universal negative* (*UN*), *particular affirmative* (*PA*), and *particular negative* (*PN*) *SJSPH's* in the sense that they are *placeholders* (*PH's*) *of schemata* (*S*) *of universal affirmative*, *universal negative*, *particular affirmative*, and *particular negative syllogistic judgments* (*SJ*) respectively. Consequently, either bold-faced italic letter '***u***' or '***v***' is a *placeholder*, whose range is the set of any three light-faced italic letters as specified, e.g., '*u*', '*v*', and '*w*', or '*A*', '*B*', and '*C*', or '*P*' ("*P*redicate"), '*S*' ("*S*ubject"), and '*M*' ("*M*iddle term"), the understanding being that each of the light-faced letters is, in turn, *a placeholder, whose range is the class of English count names in the plural number form*. Therefore, given a set of such three light-faced placeholders, say {'*u*','*v*','*w*'}, any one of the above four wordy SJSPH's (3.1), has six instances that corresponds to six possible instances of the ordered pair (***u***, ***v***), namely

$$(\boldsymbol{u},\boldsymbol{v}) \in \{(u,v),(u,w),(v,w),(v,u),(w,u),(w,v)\}. \tag{3.2}$$

3) Either one of the synonymous generic terms "*syllogistic judgment-schema*" ("*SJS*", pl. "*SJS'ta*") and "*syllogistic judgment-form*" ("*SJF*", pl. "*SJF's*") is an antonym of either one of the synonymous generic terms "*syllogistic judgment-instance*" ("*SJI*", pl. "*SJI's*") and "*syllogistic judgment*" ("*SJ*", pl. "*SJ's*"). The generic term "*formal syllogistic judgment*" ("*FSJ*", pl. "*FSJ's*") will be used as another synonym of the former two, whereas its antonym "*material syllogistic judgment*" ("*MSJ*", pl. "*MSJ's*") will be used as another synonym of the later two synonyms. Also, employing the appropriate qualifier "*existential*" of modern symbolic logic, the combined qualifiers "*existential affirmative*" ("*EA*") and "*existential negative*" ("*EN*") can be used interchangeably with the respective qualifiers "*particular affirmative*" (*"PA*") and "*particular negative*" ("*PN*") that have been introduced in the previous item. Consequently, an SJS of the range of the UA, UN, PA (EA), or PN (EN) SJSPH of the list (3.1) is briefly called an UA, UN, PA (EA), or PN (EN) SJS, and similarly with "SJF" or "FSJ" in place of "SJS".

4) The conventional form of an FCS, in which the premises and conclusion are stated as three separate FSJ's, each of which ends with a full stop, can be called *the staccato form of the FCS* (*SFFCS*). Alternatively, an FCS can be asserted in the form of a *hypothetical statement schema* in which the antecedent is the conjunction of two premises and the consequent is the conclusion. This form of an FCS can be called *the legato form of the FCS* (*LFFCS*) and also a *formal hypothetico-categorical syllogism*

(*FHCS*) or a *formal quantified transitive law* (*FQTL*) or a *formal syllogistic implication* (*FSI)*. It is essential that in passage from the staccato form of an FCS to its legato form, the premises and conclusion do not alter and hence they remain categorical (unconditional), i.e. neither hypothetical nor disjunctive. Therefore both forms are equivalent, while the legato form is preferable because it is naturally incorporated into logistic systems.

5) The SJSPH's (3.1) will be denoted logographically by '$\mathrm{A}(\boldsymbol{u},\boldsymbol{v})$', '$\mathrm{E}(\boldsymbol{u},\boldsymbol{v})$', '$\mathrm{I}(\boldsymbol{u},\boldsymbol{v})$', and '$\mathrm{O}(\boldsymbol{u},\boldsymbol{v})$' in that order, i.e.

$$\begin{gathered}\mathrm{A}(\boldsymbol{u},\boldsymbol{v}) \to [\text{All } \boldsymbol{u} \text{ are } \boldsymbol{v}],\, \mathrm{E}(\boldsymbol{u},\boldsymbol{v}) \to [\text{All } \boldsymbol{u} \text{ are not } \boldsymbol{v}],\\ \mathrm{I}(\boldsymbol{u},\boldsymbol{v}) \to [\text{Some } \boldsymbol{u} \text{ are } \boldsymbol{v}],\, \mathrm{O}(\boldsymbol{u},\boldsymbol{v}) \to [\text{Some } \boldsymbol{u} \text{ are not } \boldsymbol{v}],\end{gathered} \tag{3.3}$$

where '$\to$' is a *rightward synonymic definition sign* meaning "*stands for*", the round brackets are mentioned, while the square brackets are used but not mentioned. The four capital letters 'A', 'E', 'I', 'O' and the corresponding small ones 'a', 'e', 'i', 'o' are *conventional code* (*catch*) *letters* for any SJS'ta (SJF's), called also *syllogistic propositional schemata* (*SPS'ta*) or *syllogistic propositional forms* (*SPF's*), that are associated with the four SJSPH's of the list (3.1). The code letters are derived as the vowels of the two Latin words "***a****ff****i****rmo*" and "*n****e****g****o***". However, in contrast to the conventional use of these code letters, I employ the *capital code letters* as *logical predicates of the SJSPH's* and as *logical predicates of the SJS'ta* in the ranges of the SJSPH's. For instance, one of the 19 FCS's, which is *mnemonically* denoted as 'Barbara(u, w, v)' has the *semi-verbal staccato form*:

$$\text{«}\,\mathrm{A}(w,v).\;\; \mathrm{A}(u,w).\, \text{Therefore, } \mathrm{A}(u,v).\text{»} \tag{3.4}$$

or the tantamount *logographic legato form*:

$$\text{«}[\mathrm{A}(w,v) \wedge \mathrm{A}(u,w)] \Rightarrow \mathrm{A}(u,v)\text{»}, \tag{3.5}$$

where, and generally in what follows, '$\wedge$' stands for 'and', and '…$\Rightarrow$—' stands for either of the two synonyms "… only if —" and "if … then —".

6) A *judgment instantiating* a certain SJF (SJS) is called a *material instance*, or *material interpretand* (i.e. the result of interpreting), or *matter* (*content*) *of the SJF* (*SJS*). Accordingly, an instance of the FCS is called a *material interpretand*, or *matter* (*content*), of the FCS, and also, less explicitly, a *material categorical syllogism* (*MCS*). There exists an indefinite number of material interpretands of any FCS. For instance, the FCS 'Barbara(u, w, v)' can be materialized (instantiated) thus:

Barbara(mammals, vertebrates, animals), i.e. «All vertebrates are animals. All mammals are vertebrates. Therefore, all mammals are animals.»

7) The words occurring in (3.1) have been derived as translations into English of the corresponding Greek words employed by Aristotle in the pertinent treatises of «Organon», primarily in «On Interpretation» and «Prior Analytics», or of the Latin words employed in translations of «Organon» into Latin. The *plural number form of the quantifiers*, occurring in the placeholders of the list (3.1), is predetermined by the fact that names of *nonempty individuals*, i.e. of *primary substances* in Aristotelian coinage, are rejected in AL (see, e.g., Łukasiewicz [1951] and Lamontagne and Woo [2008]). Therefore, there are no singular judgments in AL at all. AL is often introduced by stating the following argument as a typical example of categorical syllogisms:

«All men are mortal. Socrates is a man. Therefore, Socrates is mortal.»

This argument is not, however, an Aristotelian syllogism. An appropriate example of categorical syllogisms would be the following:

«All men are mortal [beings]. All Greeks are men. Therefore, all Greeks are mortal [beings].»

The reason for excluding primary substances, called also *individual subjects* or *singular terms*, in Aristotelian syllogistics is that subjects and predicatives ("predicates" in the Aristotelian terminology) must be exchangeable in the sense that the subject of one proposition can be the predicative of another proposition and vice versa. But a primary substance cannot be predicated of ("said-of") any other substance, and therefore it is not admissible in AL.

8) Still, the basic property of AL not to deal with nonempty individual (primary substances) remains unaltered if the SJSPH's (3.1) are represented in *the equivalent single number form* as:

"Every ***u*** is a ***v***", "Every ***u*** is not a ***v***", "Some ***u*** is a ***v***", "Some ***u*** is not a ***v***". (3.1a)

However, unlike English, both Ancient and Modern Greek *have no indefinite article*, whereas Latin *has no articles at all*, either definite or indefinite. In this respect, Hebrew, e.g., is similar to Greek, whereas Russian, e.g., is similar to Latin. Therefore, the classification of the English word occurring in the proposition-schema placeholders (3.1a) should differ somewhat from the classification of the words occurring in the Greek or Latin counterparts of those placeholders.

9) The range of either term placeholder ‘***u***’ or ‘***v***’ occurring in the SJSPH’s (3.1a) is the same as that of the homographic term placeholder occurring in the SJSPH’s (3.1), e.g. the set {‘*u*’,‘*v*’,‘*w*’}. However, the term placeholders ‘*u*’, ‘*v*’, and ‘*w*’ are now replaceable with *count names in the singular number form*. Therefore, the logical predicates, which are associated with the SJSPH’s (3.1a), are distinct from the logical predicates, which are associated with the SJSPH’s (3.1), and which have been denoted by ‘A’, ‘E’, ‘I’, and ‘O’. In order to maintain the distinction between the plural and logical predicates symbolically, the former can be redenoted as ‘A_p’, ‘E_p’, ‘I_p’, and ‘O_p’, while the latter are denoted as ‘A_s’, ‘E_s’, ‘I_s’, and ‘O_s’, the understanding being that the subscript ‘$_p$’ stands for “plural” and ‘$_s$’ for “singular”. At the same time, in the general discussion of categorical syllogisms, I may use ‘A(***u***, ***v***)’, e.g., for equivocally mentioning both ‘A_p(***u***, ***v***)’ and ‘A_s(***u***, ***v***)’, and I may likewise use ‘A(*u, v*)’ for equivocally mentioning, e.g., both ‘A_p(*u, v*)’ and ‘A_s(*u, v*)’. In this case, once I substitute concrete count names for ‘*u*’ and ‘*v*’, either in the plural or in the singular, the subscript to ‘A’ can immediately be restored from the *number form* of the names. For instance, “A(squares, polygons)” stands for “A_p(squares, polygons)” and hence for “All squares are polygons”, whereas “A(square, polygon)” stands for “A_s(square, polygon)” and hence for “Every square is a polygon”. Likewise, the FCS ‘Barbara(*u*, *w*, *v*)’ can be materialized (instantiated), either thus: Barbara(squares, rectangles, polygons), i.e. «All rectangles are polygons. All squares are rectangles. Therefore, all squares are polygons.», or thus: Barbara(square, rectangle, polygon), i.e. «Every rectangle is a polygon. Every square is a rectangle. Therefore, every square is a polygons.»

10) In accordance with the common practice, I shall often use the short SJFPH’s “No ***u*** are ***v***” and “No ***u*** is a ***v***” synonymously (interchangeably) with “All ***u*** are not ***v***” and “Every ***u*** is not a ***v***” respectively, although the former introduce undesirable asymmetry in classification of the similarly positioned parts of the four SJFPH’s (3.1).

11) In AL, a *proposition* is by definition a *declarative sentence*, *affirmative or negative*, of one of the four standard forms (3.1) that can either be *materially* (*m*-) *veracious*, i.e. *accidentally m-true*, or be *m-antiveracious*, i.e. *accidentally m-antitrue* (*accidentally m-false*). In this case, “*accidentally*” (or “*circumstantially*”) means *not* *universally*, i.e. *not tautologously*, whereas "*m-veracious*" ("*materially veracious*") means *conformable to a certain state of affairs*, i.e. *a certain fact, case, event,* etc. Accordingly, a proposition is said to be *m-antiveracious* if its negation is *m-veracious.* An m-veracious proposition (and not just an undecided one) is by definition called here a *judgment*, although the Greek parasynonym of the English noun "*judgment*", namely "κρίσις" \krísis\ (from the kindred verb "κρίνω" \kríno\ meaning [*I*] *judge*, *deem*, *consider*, or *decide*) is not an Aristotelian term.

12) In accordance with Aristotle's terminology, the SJFPH's (3.1) and (3.1a) can be analyzesd as follows: ***u*** is *the subject* and ***v*** is *the predicate* of a SJF, "all", "some", or "every" is the *quantity* of the subject, "are" or "is" is *an affirmative quality*, and "are not" or "is not" is *a negative quality,* of the predicate. An affirmative quality or a negative quality is indiscriminately called a *quality*. Thus, any *simple proposition* has one subject, one predicate, one quality, and at most one quantity. The subject or the predicate of a proposition is indiscriminately called a *term* of the proposition, so that, conversely, a term of a proposition is either the subject or the predicate of the proposition. The quantity (if present) or the quality of a proposition is indiscriminately called a *property* of the proposition, so that, conversely, a property of a proposition is either its quantity or its quality. The subject together with its quantity (if present) forms *the grammatical subject* of the proposition. The quantities "all" and "every" are called the *universal* quantities. The quantity "some" is called the *particular* quantity. The presently common substitute for "particular" in this context is "*existential*". The [Aristotelian] predicate together with its quality forms *the grammatical predicate* of the proposition, whereas the predicate alone is known in English as *the predicative*. A subject or a predicate is called a *universal term* or simply a *universal* if it is a quantifiable (count) name and a *singular*, *single*, or *individual*, *term* if it is a proper name.

13) A proposition is said to be a *universal one* if it has a universal quantity, a *particular* (*existential,* – to use the more appropriate modern qualifier) *one* if it has the particular (existential) quantity, and a *singular* (*single*, *individual*) if it has no quantity. A proposition is said to be an *affirmative one* if it has an affirmative quality and a *negative one* if it has a negative quantity. Aristotle excluded from his syllogistics singular propositions such as «Socrates is a man»:

> «We call those propositions single which indicate a single fact, or the conjunction of the parts of which results in unity: those propositions, on the

other hand, are separate and many in number, which indicate many facts, or whose parts have no conjunction.» [AIE, Chap 5]

14) Chapter 6 and a part of chapter 7 of [AIE] are concerned with relationships among four types of *non-singular propositions with the same terms*: *universal affirmative*, A(*u, v*); *universal negative*, E(*u, v*); *particular* (*existential*) *affirmative*, I(*u, v*); and *particular* (*existential*) *negative*, O(*u, v*). Any two of such propositions are said to be *opposite propositions* or *opposites*. Specifically, two non-singular propositions with the same terms are said to be: (a) *contradictory ones* or *contradictories* if they differ both in quantity and in quality, (b) *contrary ones* or *contraries* if they differ in quality, (c) *alternate ones* or *alterns* if they differ in quantity. Consequently, A(*u*, *v*) and O(*u*, *v*) or E(*u, v*) and I(*u, v*) are contradictories, A(*u, v*) and E(*u, v*) or I(*u, v*) and O(*u, v*) are contraries, and A(*u, v*) and I(*u, v*) or E(*u, v*) and O(*u, v*) are alterns. More specifically, A(*u, v*) and E(*u, v*) are called *supercontraries*, and I(*u, v*) and O(*u, v*) *subcontraries*.

15) The above relations among the four opposites are conventionally illustrated in the form of a so-called *square of oppositions*, whose vertices are labeled either with the SJFPH's 'A(***u***, ***v***)', 'E(***u***, ***v***)', 'I(***u***, ***v***)', and 'O(***u***, ***v***)' or with the SJF's 'A(*u*, *v*)', 'E(*u*, *v*)', 'I(*u*, *v*)', and 'O(*u*, *v*)' (e.g.) in that order in the clockwise direction starting from the upper left vertex. In this case, each diagonal joins two contradictories, each vertical side joins two alterns, and each horizontal side joins two contraries, the upper one joining two supercontraries and the lower one two subcontraries. The square of oppositions is implied by Aristotle's fourfold taxonomic schema of simple non-singular propositions, but that square was not drawn by Aristotle himself.

16) I have rigorously proved in the TTL that

$$\mathrm{O}(\boldsymbol{u},\boldsymbol{v}) \Leftrightarrow \neg\mathrm{A}(\boldsymbol{u},\boldsymbol{v}), \mathrm{I}(\boldsymbol{u},\boldsymbol{v}) \Leftrightarrow \neg\mathrm{E}(\boldsymbol{u},\boldsymbol{v}) \tag{3.6}$$

and that hence conversely

$$\mathrm{A}(\boldsymbol{u},\boldsymbol{v}) \Leftrightarrow \neg\mathrm{O}(\boldsymbol{u},\boldsymbol{v}), \mathrm{E}(\boldsymbol{u},\boldsymbol{v}) \Leftrightarrow \neg\mathrm{I}(\boldsymbol{u},\boldsymbol{v}), \tag{3.6a}$$

where, and generally in what follows, '⇔' stands for "if and only if"; and I have also proved there that

$$\mathrm{A}(\boldsymbol{u},\boldsymbol{v}) \Rightarrow \mathrm{I}(\boldsymbol{u},\boldsymbol{v}), \mathrm{E}(\boldsymbol{u},\boldsymbol{v}) \Rightarrow \mathrm{O}(\boldsymbol{u},\boldsymbol{v}). \tag{3.7}$$

It is understood that the relations (3.6)–(3.7) are tautologies. In agreement with (3.6), two contradictories cannot both be either true or antitrue (false) simultaneously.

Unlike either pair of contradictories, the supercontraries A($\boldsymbol{u}$, $\boldsymbol{v}$) and E($\boldsymbol{u}$, $\boldsymbol{v}$) can be both antitrue but they cannot be both true, whereas the subcontraries I($\boldsymbol{u}$, $\boldsymbol{v}$) and O($\boldsymbol{u}$, $\boldsymbol{v}$) can be both true but they cannot be both antitrue. At the same time, in agreement with (3.7), if A($\boldsymbol{u}$, $\boldsymbol{v}$) is true then I($\boldsymbol{u}$, $\boldsymbol{v}$), being its altern, is also true, and likewise if E($\boldsymbol{u}$, $\boldsymbol{v}$) is true then O($\boldsymbol{u}$, $\boldsymbol{v}$), being its altern, is also true.

17) By definition, the particular logical quantifier "*strictly some*" means *some but not all.* From this definition, it immediately follows that

$$[\text{Strictly some } \boldsymbol{u} \text{ are } \boldsymbol{v}] \Rightarrow [\text{Strictly some } \boldsymbol{u} \text{ are not } \boldsymbol{v}],$$

and that conversely

$$[\text{Strictly some } \boldsymbol{u} \text{ are not } \boldsymbol{v}] \Rightarrow [\text{Strictly some } \boldsymbol{u} \text{ are } \boldsymbol{v}].$$

Hence,

$$[\text{Strictly some } \boldsymbol{u} \text{ are } \boldsymbol{v}] \Leftrightarrow [\text{Strictly some } \boldsymbol{u} \text{ are not } \boldsymbol{v}]. \tag{3.8}$$

At the same time, according to Aristotle, propositions I($\boldsymbol{u}$, $\boldsymbol{v}$) and O($\boldsymbol{u}$, $\boldsymbol{v}$), i.e. $\text{Some } \boldsymbol{u} \text{ are } \boldsymbol{v}$ and $\text{Some } \boldsymbol{u} \text{ are not } \boldsymbol{v}$, are not equivalent. This means that the logical quantifier (quantity) "*some*" is used in AL in *inclusive sense*, i.e. as an abbreviation of the expression "*strictly some or all*" or of the sense-concurrent expression "*less than all or all*".

18) An FCS is a *latent quantified predicate* (*functional*) *rule of inference* that is composed of three SJSPH's of standard forms selected out of (3.1) or (3.1a), two of which are for premises and one for a conclusion. In this case, given an MCS, being an instance of the FCS, in agreement with the above item 11, the premises the MCS are *m-veracious*, i.e. *accidentally m-true*, either *by assumption* or *by the previous knowledge*, while the conclusion of the MCS is *m-veracious by inference* (*deduction*) *of the MCS itself.*

19) Each one of the three terms and each one of the two premises of an FCS were provided by Aristotle with a proper name as follows. The predicate of the conclusion is called the *major term of the FCS*, while the subject of the conclusion is called the *minor term of the FCS*. The two premises have one term in common, which is called the *middle term of the FCS*, while the two other terms of the premises *extreme terms of the FCS*, the understanding being that one of the extreme terms is major and the other one is minor. The premise that contains the major term is called the *major premise of the FCS* and the premise that contains the minor term is called the *minor premise of the FCS*. The order of the premises is not important. However, in stating an FCS, the major premise is always *conventionally* stated first, the minor premise is stated second, and the conclusion comes last.

20) The order, in which the middle term of an FCS is arranged relative the extreme terms of the major and minor premises *determines* or *is* just *called* the *figure of the FCS,* which is less explicitly called a *syllogistic figure*. There are *four syllogistic figures* altogether, which are represented in the *legato style* by the *definientia* of the following four definition schemata:

$$\begin{aligned}
&(1\mathbf{UVW})(u,w,v) \to [\mathbf{U}(w,v) \wedge \mathbf{V}(u,w) \Rightarrow \mathbf{W}(u,v)],\\
&(2\mathbf{UVW})(u,w,v) \to [\mathbf{U}(v,w) \wedge \mathbf{V}(u,w) \Rightarrow \mathbf{W}(u,v)],\\
&(3\mathbf{UVW})(u,w,v) \to [\mathbf{U}(w,v) \wedge \mathbf{V}(w,u) \Rightarrow \mathbf{W}(u,v)],\\
&(4\mathbf{UVW})(u,w,v) \to [\mathbf{U}(v,w) \wedge \mathbf{V}(w,u) \Rightarrow \mathbf{W}(u,v)],
\end{aligned} \tag{3.9}$$

where each of the letters ‘**U**’, ‘**V**’, and ‘**W**’ is a placeholder whose range is the set of four letters ‘A’, ‘E’, ‘I’, and ‘O’. It is understood that each the 19 FCS’s fits in exactly one of the above four definition schemata. Namely, the strings ‘(1**UVW**)’, ‘(2**UVW**)’, ‘(3**UVW**)’, and ‘(4**UVW**)’ are placeholders having the following ranges:

$$\begin{aligned}
&(1\mathbf{UVW}) \in \{(1\mathrm{AAA}),(1\mathrm{EAE}),(1\mathrm{AII}),(1\mathrm{EIO})\},\\
&(2\mathbf{UVW}) \in \{(2\mathrm{AOO}),(2\mathrm{AEE}),(2\mathrm{EAE}),(2\mathrm{EIO})\},\\
&(3\mathbf{UVW}) \in \{(3\mathrm{AAI}),(3\mathrm{OAO}),(3\mathrm{AII}),(3\mathrm{IAI}),(3\mathrm{EAO}),(3\mathrm{EIO})\},\\
&(4\mathbf{UVW}) \in \{(4\mathrm{AAI}),(4\mathrm{AEE}),(4\mathrm{IAI}),(4\mathrm{EAO}),(4\mathrm{EIO})\}.
\end{aligned} \tag{3.10}$$

Any concrete string in the range of each of the above four placeholders is a *logographic logical predicate-sign* (*LLPS*) of the corresponding FCS. In the three SJF’s of a FCS, there are three variables: ‘*u*’, ‘*v*’, and ‘*w*’ altogether, which are indiscriminately called *terms* of the FCS. Discriminately, ‘*u*’ and ‘*v*’ are respectively called the *subject* and the *predicative* both of the conclusion and of the FCS, while ‘*w*’ is called the *middle term* of the FCS. In principle, the premises of an FCS are commutable. However, in each of the four syllogistic figures, the order of premises is fixed. In this case; the first premise is said to be *major* and the second one *minor*, while each figure is determined by the order of tokens of the variables ‘*u*’, ‘*v*’, and ‘*w*’ in the premises.

21) Thus, the definitions (3.9) subject to (3.10) determine the *four syllogistic figures* that I have preliminarily described in the item 1a of this subsection. As I have indicated there, the 14 FCS’s comprised in the first three figures were laid down by Aristotle himself, whereas the 5 FCS’s comprised in the fourth figure, in which *the middle term is the predicate of the major premise and the subject of the minor* *premise*, were reputedly added by Galen; this figure is therefore called the *Galenian one*. In accordance with (3.9) subject to (3.10), each one of the 19 FCS's is identified by its LLPS. Alternatively, each FCS is identified by the respective *conventional three-syllable catchword*, whose vowels are selected out of the four small letters 'a', 'e', 'i', and 'o' in a certain order so as to indicate the mood of the FCS.. In this case, the sequence of consonants occurring in a catchword and hence the catchword as a whole are uniquely associated with the figure of the FCS. Hence, the catchword of an FCS is concurrent to its LLPS, and it will therefore be called the *phonographic,* or *verbal*, *logical predicate-sign* (*PhLPS* or *VLPS*) *of the FCS*. The latter predicate-signs are defined as follows:

List 1.1: *The catchwords of the 19 FCS's.*

Figure 1:

Barbara→1AAA, Celarent→1EAE, Darii→1AII, Ferio→1EIO.

Figure 2:

Baroco→2AOO, Camestres→2AEE, Cesare→2EAE, Festimo→2EIO.

Figure 3:

Barapti→3AAI, Bocardo→3OAO, Datisi→3AII, Disamis→3IAI,
Felapton→3EAO, Feriso→3EIO.

Figure 4:

Bamalip→4AAI, Calemes→4AEE, Dimatis→4IAI, Fesapo→4EAO,
Fresison→4EIO.

22) In investigation of the *validity properties* of separate FCS's, it is convenient to divide the 19 kinds of FCS into 10 *types* in accordance with the ordered triples of the vowels, which occur in their catchwords, namely, AAA, AAI, AII, IAI, AEE, EAE, EAO, EIO, AOO, OAO. In this case, for the purpose of description and study, two FCS's with the same first two letters in either order are included into the same *group*. Thus, there are six different groups of FCS's, namely, AAA&AAI, AII&IAI, AEE&EAE, EAO, EIO, AOO&OAO. Although the consonant letters occurring in the catchword of a syllogism are irrelevant to the types of the FSJ's forming a syllogism, it turns out that *all catchwords of the syllogisms of each of the six groups begin with the same consonant letter*, provided that the conventional catchword "*Darapti*" is replaced with "*Barapti*". Consequently, the catchword "Barapti" of my own is used in the TTL and in this essay instead of the conventional catchword "Darapti". In the following definition, all 19 FCS's are defined in accordance with both the four syllofistic figures and the above-mentioned six syllogistic groups.

Df 3.2.

1°) Group AAA&AAI

1) $\text{Barbara}(u,w,v) \rightarrow (1\text{AAA})(u,w,v) \leftrightarrow [\text{A}(w,v) \wedge \text{A}(u,w) \Rightarrow \text{A}(u,v)]$.

2) $\text{Barapti}(u,w,v) \rightarrow (3\text{AAI})(u,w,v) \rightarrow [\text{A}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

3) $\text{Bamalip}(u,w,v) \rightarrow (4\text{AAI})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

2°) Group AII&IAI

4) $\text{Darii}(u,w,v) \rightarrow (1\text{AII})(u,w,v) \leftrightarrow [\text{A}(w,v) \wedge \text{I}(u,w) \Rightarrow \text{I}(u,v)]$.

5) $\text{Datisi}(u,w,v) \rightarrow (3\text{AII})(u,w,v) \leftrightarrow [\text{A}(w,v) \wedge \text{I}(w,u) \Rightarrow \text{I}(u,v)]$.

6) $\text{Disamis}(u,w,v) \rightarrow (3\text{IAI})(u,w,v) \leftrightarrow [\text{I}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

7) $\text{Dimatis}(u,w,v) \rightarrow (4\text{IAI})(u,w,v) \leftrightarrow [\text{I}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{I}(u,v)]$.

3°) Group EAE&AEE

8) $\text{Celarent}(u,w,v) \rightarrow (1\text{EAE})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{A}(u,w) \Rightarrow \text{E}(u,v)]$.

9) $\text{Camestres}(u,w,v) \rightarrow (2\text{AEE})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{E}(u,w) \Rightarrow \text{E}(u,v)]$.

10) $\text{Cesare}(u,w,v) \rightarrow (2\text{EAE})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{A}(u,w) \Rightarrow \text{E}(u,v)]$.

11) $\text{Calemes}(u,w,v) \rightarrow (4\text{AEE})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{E}(w,u) \Rightarrow \text{E}(u,v)]$.

4°) Group EAO

12) $\text{Felapton}(u,w,v) \rightarrow (3\text{EAO})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)]$.

13) $\text{Fesapo}(u,w,v) \rightarrow (4\text{EAO})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)]$.

5°) Group EIO

14) $\text{Ferio}(u,w,v) \rightarrow (1\text{EIO})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{I}(u,w) \Rightarrow \text{O}(u,v)]$.

15) $\text{Festino}(u,w,v) \rightarrow (2\text{EIO})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{I}(u,w) \Rightarrow \text{O}(u,v)]$.

16) $\text{Feriso}(u,w,v) \rightarrow (3\text{EIO})(u,w,v) \leftrightarrow [\text{E}(w,v) \wedge \text{I}(w,v) \Rightarrow \text{O}(u,v)]$.

17) $\text{Fresison}(u,w,v) \rightarrow (4\text{EIO})(u,w,v) \leftrightarrow [\text{E}(v,w) \wedge \text{I}(w,u) \Rightarrow \text{O}(u,v)]$.

6°) Group AOO&OAO

18) $\text{Baroco}(u,w,v) \rightarrow (2\text{AOO})(u,w,v) \leftrightarrow [\text{A}(v,w) \wedge \text{O}(u,w) \Rightarrow \text{O}(u,v)]$.

19) $\text{Bocardo}(u,w,v) \rightarrow (3\text{OAO})(u,w,v) \leftrightarrow [\text{O}(w,v) \wedge \text{A}(w,u) \Rightarrow \text{O}(u,v)]$.$\bullet$

Cmt 3.3. In principle, an MCS may contain three or more premises, but such an MCS is always a combined one that can be reduced to a sequence of Aristotelian MCS's. For instance, the following three-premise syllogism, due to Lewis Carroll (*Symbolic Logic*, Part I, 1896), is given and illustrated by the pertinent *Venn diagrams* in Lipschutz [1964, pp. 225–226]:

«Babies are illogical. Nobody is despised who can manage a crocodile. Illogical people are despised. Hence, babies cannot manage crocodiles.»

This unconventional MCS is however reduced to the following sequence of two conventional MCS's:

a) Barbara(illogical people, babies, despised people): All illogical people are despised ones. All babies are illogical people. Hence, all babies are despised people.

b) Celarent(despised people, people who can manage crocodiles, babies): All despised people are not people who can manage crocodiles. All babies are despised people. Hence, all babies are not people who can manage crocodiles.

Incidentally, the above two conventional MCS's can be restated as the following equivalent singular MCS's:

a') Barbara(illogical man, baby, despised man): Every illogical man is a despised one. Every baby is an illogical man. Hence, every baby is a despised man.

b') Celarent(despised man, man who can manage crocodile, baby): Every despised man is not a man who can manage a crocodile. Every baby is a despised man. Hence, every baby is not a man who can manage a crocodile.•

Cmt 3.4. When a taxonym of a BTB is used without any of the qualifiers "sensu lato" and "sensu stricto", it is necessary to indicate, to which taxonomy the taxonym belongs. Otherwise, confusion is unavoidable. For instance, the categorical syllogism Barbara(alga, bacterium, plant), i.e.

«A blue alga (cyanobacterium) is a bacterium. A bacterium is a plant. Hence, a blue alga is a plant.»

is effective (applicable) in the LT and is ineffective (inapplicable) in LWT, whereas the categorical syllogism Celarent(alga, bacterium, plant), i.e.

«A blue alga (cyanobacterium) is a bacterium. No bacterium is a plant. Hence, a blue alga is not a plant.»

is effective (applicable) in the LWT and is ineffective (inapplicable) in LT. By contrast, both syllogisms Barbara(alga, bacterium, plant sensu lato) and Celarent(alga, bacterium, plant sensu stricto) are valid.•

23) In his «*Prior Analytics*», [APrAJ], Aristotle divides syllogisms into two groups: *perfect syllogisms* and *imperfect syllogisms*. The perfect syllogisms are those four, which have the first figure. Aristotle takes them for granted as *valid axioms*. Then he uses these axioms to prove the imperfect syllogisms from some intuitive considerations based on *conversions of judgments* (*true propositions*). A *conversion* is the act of altering a proposition by exchanging its subject and its predicate, while preserving its quality. There are two types of conversions: a *simple conversion* when the quantity of the converted proposition is kept unaltered and a *conversion per accidens* when the universal quantity of the converted proposition is changed to the particular one. Some conversions are antiveracious (do not exist). Also, in accordance with the previous item, only the first three syllogistic figures of FCS's are original Aristotelian ones, while the fourth figure is Galenian or somebody else's one, which comprises five more improper syllogisms. Aristotle's conversions of standard judgments are trivial and they can be summarized as follows.

Table 1: *Conversions of standard judgments*

Original judgment	*Simple conversion*	*Per accidens conversion*
A($\boldsymbol{u}$, $\boldsymbol{v}$)	"A($\boldsymbol{v}$, $\boldsymbol{u}$)", antiveracious	I($\boldsymbol{v}$, $\boldsymbol{u}$)
E($\boldsymbol{u}$, $\boldsymbol{v}$)	E($\boldsymbol{v}$, $\boldsymbol{u}$)	O($\boldsymbol{v}$, $\boldsymbol{u}$)
I($\boldsymbol{u}$, $\boldsymbol{v}$)	I($\boldsymbol{v}$, $\boldsymbol{u}$)	"A($\boldsymbol{v}$, $\boldsymbol{u}$)" , antiveracious
O($\boldsymbol{u}$, $\boldsymbol{v}$)	"O($\boldsymbol{v}$, $\boldsymbol{u}$)", antiveracious	"E($\boldsymbol{v}$, $\boldsymbol{u}$)", antiveracious

For instance, the judgments: (a) A(*men, mammals*), i.e. *All men are mammals*; (b) E(*fishes, mammals*), i.e. *No fishes are mammals*; (c) I(*chordates, fishes*), i.e. *Some chordates are fishes*; (d) O(*vertebrates, mammals*), i.e. *Some vertebrates are not mammals* have the following *simple conversions*: (a') "A(*mammals*, *men*)", i.e. *"All mammals are men*", which is antiveracious; (b') E(*mammals, fishes*), i.e. *No mammals are fishes*; (c') I(*fishes, chordates*), i.e. *Some fishes are chordates*; (d') "O(*mammals*, *vertebrates*)", i.e. "*Some mammals are not vertebrates*", which is antiveracious; and the former also have the following *conversions per accidens*: (a") I(*mammals*, *men*), i.e. *Some mammals are men*; (b") O(*mammals*, *men*), i.e. *Some mammalss are not men*; (c") "A(*fishes*, *chordates*)", i.e. *"All fishes are chordates*", which is antiveracious; (d") "E(*mammals*, *vertebrates*)", i.e. *"No mammals are vertebrates*", which is antiveracious. Conversions of proper syllogisms are discussed in detail in Lamontagne and Woo [2008].

24) The subject matter of the original treatises of «*Organon*», especially Aristotelian syllogistics constituting the subject matter of «*Prior Analytics*», has been developed further by many generations of later contributors. They have sifted the original theory of Aristotelian syllogisms from prolix abstruse discourses and unnecessary subtleties and have essentially improved it by supplementing it with the convenient mnemonic and graphic technique. Therefore, for the purposes of this essay and of the entire Psychologistics, it would be counterproductive to discuss Aristotelian syllogistics in its original form. When I wish to emphasize that I refer to or discuss an original work of Aristotle without its later modifications or interpretations, I indicate this with the help of the appropriate added words or I employ the qualifier "Aristotle's" and not "Aristotelian".

3.6. Partition of conventional dual formal logic (DFL)

Df 3.3. 1) In a broad historical prospective, *dual formal logic* (*DFL*) as a single whole field of study and discourse is divided into *traditional* (*classical*) *formal logic* (*TrFL*), *algebraic logic*, called also *old mathematical,* or *old symbolic, logic* (*OMhL* or *OSbL*), i.e. *symbolic logic of the middle of 19th century*, and *new* (*contemporary, modern*) *mathematical,* or *symbolic, logic* (*NMhL* or *NSbL*) *that arouse at the joint of 19th and 20th centuries and have been developing through 20th century*. TrFL is divided into *traditional deductive formal logic* (*TrDdFL*) and *traditional inductive*, or *Bacon-Mill's*, *formal logic* (*TrIdFL*). TrDdFL is, in turn, divided into *traditional sentential formal logic* (*TrSFL*) and *traditional predicate formal logic* (*TrPFL*), called also *predicate formal syllogistics* (*PFS*), *Aristotelian deductive formal logic* (*ADdFL*), *Aristotelian formal syllogistics* (*AFS*), *categorical formal syllogistics* (*CFS*), or *traditional Aristotelian deductive formal logic* (*TrADdFL*). Of the last five synonymous names, the first two, "traditional *predicate* formal logic" ("Tr*P*FL") and "*predicate* formal syllogistics" ("*P*FS"), are descriptive of the fact that, from the standpoint of NSbL and especially from the standpoint of $\mathcal{A}_1$, any one of 19 *categorical syllogism-schemata* (*syllogism-forms*, *syllogism-rules*) comprised in TrPFL (PFS) is a *latent quantified predicate* (*functional*) *rule of deductive inference.* By contrast, from the same standpoint, TrSFL comprises *tautologous sentential forms, overwhelming most* of which can be regarded (used) as *syllogism-forms*, i.e. as *syllogistic rules of deductive inference*, according to which, from a certain number 1 to 3 of *judgment-forms*, i.e. *formally veracious* (*f-veracious*) *sentential forms*, as *premises*, another judgment-form is *immediately inferred* as *conclusion* – just as in the case of the categorical syllogism-forms. Accordingly, TrSFL can be divided into *sentential formal syllogistics* (*SFS*), comprising *sentential syllogism-forms* (*SSF's*), and *supplementary sentential formal logic* (*SSFL*), comprising few *non-syllogistic sentential forms* (*NSSF's*) that are not comprised in SFS. It is understood that all tautologous sentential forms, comprised in TrSFL, are expressed in terms of modern symbolic logic (NSbL). Therefore, any one of the tautologous sentential forms, comprised in SSFL, might have been resulted by inadequate incorporation of its original verbal or semi-verbal laws into NSbL (to be illustrated). In any case, for convenience in description and study in terms of NSbL, I regard SSFL as a part of *miscellaneous sentential formal logic* (*MscSFL*) that comprises, by definition, all NSSF's of SSFL and, in addition, *Law of double negation* ($\neg\neg \boldsymbol{P} \Leftrightarrow \boldsymbol{P}$), which MscSFL shares (has in common) with SFS (cf. subsection III.1.2 of the TTL, being the panlogographic precursor of MscSFL). TrPFL (AFL) can be called a *semi-verbal formal logic* because the *primitive copulas* (*link-verbs*) (e.g. "*is*" or "*is not*") and the *quantifiers of universality and particularity* (e.g. "all" and "some" respectively), which are employed in this logic, are verbal expressions of a certain native language (as Greek, Latin, or English), into which the logic is incorporated.

2) The term "*traditional logic*" ("*TrL*") alone, without any additional qualifier, denotes a totality of *traditional logical theories* (*TrLT's*), each of which determines a *traditional FLS* (*TrFLS*) along with the respective *traditional MLS* (*TrMLS*), such as a certain system of *declarative sentences* (*DS's*) of a given *written native language* (*WNL*), e.g. written English. Consequently, in accordance with the previous item, TrL can be divided into three parts: *traditional deductive logic* (*TrDdL*), *traditional inductive*, or *Bacon-Mill's*, *logic* (*TrIL*), and *traditional*, or *Aristotelian, epistemology* (*TrE* or *AE*). TrDdL is divided into *traditional sentential logic* (*TrSL*) and *traditional predicate logic* (*TrPL*); the latter is called also *predicate syllogistics* (*PS*); *Aristotelian*

deductive logic (*ADdL*); *Aristotelian syllogistics* (*AS*); *Aristotelian minor,* or *lesser*, *logic*: *categorical syllogistics* (*CS*); *term*, or *two-term, logic*; and *traditional Aristotelian deductive logic* (*TrADdL*). The union of TrDdL (Aristotelian minor logic) ant TrE (AE) is, by definition, *Aristotelian major*, or *greater*, *logic*, i.e. the *cumulative theory of* «*Organon*». TrSL is divided into *sentential syllogistics* (*SS*) and *supplementary sentential logic* (*SSL*).

3) Algebraic logic is divided into *Boolean algebra*, called also *Algebra of classes*, and *Algebra of relations*, called also *Calculus of relations*. The former was developed in 1840–1850 by English mathematician and logician George Boole (1815–64). The latter was developed somewhat later by another English mathematician and logician Augustus De Morgan (1806–71) in his *Syllabus of a Proposed System of Logic* (1860) for dealing with relations in the same manner, in which classes are dealt in Boolean algebra. With reference to their formulations in 19^{th} century, *algebra of classes* and *algebra of relations* are collectively called *algebraic logic*.

4) In order to list conveniently FLS's included in NSbFL, I shall use the abbreviations: "*ALC*" for "*axiomatic logical calculus*", "*ASC*" for "*axiomatic sentential calculus*", "*APC*" for "*axiomatic predicate calculus*", and "*AFC*" for "*axiomatic functional calculus*", the understanding being that "*APC*" and "*AFC*" are synonyms and that an ALC is either an ASC or an APC. I shall also abbreviate "*plain*" as "*P*" and "*modal*" as "*M*", so that the abbreviations "*PALC*", "*PASC*", "*PAPC*" ("*PAFC*"), "*MALC*", "*MASC*", and "*MAPC*" ("*MAFC*") are self-explanatory, the understanding being that the occurrences of the prepositive qualifier "plain" ("P") in the pertinent full (correspondingly, abbreviated) terms can be used interchangeably with occurrences of the qualifier "*non-modal*" ("*NM*"). The plural number forms of any one of the above abbreviations will be formed by suffixing it with "'i", where the apostrophe should be understood as an operator of substitution of the ending "i" for the ending "us" in the word "calculus", for which the last letter "C" of the abbreviation stands.

5) An ALC, i.e. an ASC or an APC, is a *P*ALC (*NM*ALC), i.e. a *P*ASC (*NM*ASC) or a *P*APC (*NM*APC), respectively, if *the set of its logical* (*sentential*, *relational*) *connectives* comprises *one* or *two primary atomic plain* (*non-modal*) *logical* (*sentential*, *relational*) *connectives*, e.g. *Sheffer's stroke* |, /, or **|** (see, e.g.,

Hilbert and Ackermann [1950, pp. 11, 29]) in the former case, or ¬ (*not*) and ∨ (*or*, *vel*) or ¬ and ∧ (*and*) in the latter case, and also any *secondary plain* (*non-modal*) *logical connectives,* which are definable in terms of the pertinent primary one or ones.

6) An ALC, i.e. an ASC or an APC, is an *M*ALC, i.e. an *M*ASC or an *M*APC respectively, if among its *sentential connectives* are two singulary (one-place) prepositive *modal operators* (*modal sentential connectives*): (i) ‘□’ (sometimes ‘L’ or ‘N’) for “*Necessarily*” or “*It is necessary that*” and (ii) ‘◊’ (or ‘M’) for “*Possibly*” or “*It is possible that*”, which qualify the truth of a postpositive relation. The operators ‘□’ and ‘◊’ are interrelated as: ◊↔¬□¬ and □↔¬◊¬, where ‘↔’ is the metalinguistic sign of synonymity and ‘¬’ is the object sign of negation. In logic, the property of a statement to express the *possibility*, *impossibility*, *necessity*, *contingency*, or *probability* is called a *modality* of the statement. Consequently, an ALC is a PALC if it does not have any modality operators and that hence does not incorporate any modalities.

7) A PAPC is called a *first-order*, or *lowest-order, PAPC* (*FOPAPC* or *LOPAPC*) if the notation for *quantification* that it has applies only to atomic term-variables (see, e.g., Hilbert and Ackermann [1950, Chapter III] or Church [1956, Chapters III and IV]); and it is said to be a *second-order*, or *higher-order, PAPC* (*SOPAPC* or *HOPAPC*) if the notation for quantification that it has applies, not only to atomic term-variables, but also to atomic predicate-variables or to atomic relation-variables or to both (see, e.g., Hilbert and Ackermann [1950, Chapter IV] and Church [1956, Chapter V]).

8) A PASC or an FOPAPC (LOPAPC) that is used or referred to in the literature most frequently is called a *conventional axiomatic sentential* (*propositional*) *calculus* (briefly *CASC*) or a *conventional axiomatic predicate* (*functional*) *calculus* (briefly *CAPC* or *CAFC*) respectively. A CASC or a CAPC (CAFC) will indiscriminately be called a *CALC*, i.e. a *conventional axiomatic logical calculus*. In this case, the qualifier “conventional” can be used interchangeably with “*classical*”; the latter also abbreviated as “*C*” so that all above abbreviations remain unaltered.

9) Various systems of nomenclature of CALC’i are discussed, e.g., in Hilbert and Ackermann [1950, Editor’s Notes, pp. 165, 166]. For the sake of being specific, by a CASC, I shall, in agreement with Df I.2.1 of the TTL, mean either the *Russell logistic system*, denoted by ‘$\mathrm{P_R}$’, or the *equivalent Russell-Bernays logistic system*,

denoted by ‘P_{RB}’, whereas by a CAPC I shall mean either the system F^1 of Church [1956, Chapters III and IV] or the first-order predicate calculus that is developed in Hilbert and Ackermann [1950, Chapter III] under the heading “*The restricted predicate calculus*” (cf. the pertinent definition of the item 1 of subsection 3.5 and that of the item 6 of subsection 3.6). P_R is based on five axioms, which were for the first time published in Russell [1908] and which were afterwards used in Whitehead and Russell [1910; 1962, pp. 96, 97]) as items **∗1·2–∗1·6.** Bernays [1926] discovered the non-independence of Russell’s axiom **∗1·5**, so that P_{RB} is based on the remaining four Russell’s axioms. The calculi P_R and P_{RB} are discussed, e.g., in Hilbert and Ackermann [1950, §10, pp. 27–30] and in Church [1956, §25, pp. 136–138; §29, p. 157]. The axioms of P_{RB} are also used in Bourbaki [1960, Chapter I, §3, S1–S4]. By definition, all axiomatic propositional and functional calculi that are developed or mentioned in Church [1956] are *P*ALC’i, but only a part of them are *C*ALC’i.

10) To recapitulate, NSbL (NMhL) includes TrDFL, all PASC’i and all MASC’i, and it also includes all first- and second-order PAPC’i (PAFC’i) and MAPC’i (PAFC’i), Hence, NSbL includes particularly all CASC’i and all CAPC’i (CAFC’i). In this case, TrDFL, all CASC’i, and all CAPC’i (CAFC’i) are *slaves* of the PLTFL $\mathcal{A}_1$ in the sense that every axiom of the former is a theorem of the latter.•

Cmt 3.5. 1) Formal logic is based on the following pairs of mutually complementary mental operations or their results: *generalizations* and *specifications*, *assumptions* and *inferences* (*proofs*, *arguments*), and *inductions* and *deductions*, collectively called *dialectics.* An *assumption* is a generalization that results in a *relation* (particularly in a *statement*), which is taken for granted to be either *valid* or *veracious*, depending on the pertinent FLS, and which I shall most generally call a *postulate*. I say that a postulate is an *axiom* if I regard it as a *permanent* one in a given situation or universally and a *hypothesis* if I regard it as a *temporary* (*ad hoc*) one. An axiom, belonging to the IML of an FLS, is called a *rule of inference of the FLS* if, according to the axiom, from appropriate valid, or veracious, relations as *premises*, another valid, or correspondingly veracious, relation is *immediately inferred* as *conclusion* that is generally called a *theorem.*

2) In general, the inference of a valid, or veracious, relation is qualified *immediate* if it requires one application of a single rule of inference. Therefore, *any syllogism*, e.g. *is an immediate inference of its conclusion, no matter how many*

premises it has. This implicit definition of any grammatical form of the infinitive verb equivalent "*to immediately infer*" and of its kindred substantive "*immediate inference*" is in agreement with the first part of following definition by Church [1956, p. 49, n. 115] of the latter substantive in modern symbolic logic:

> «…We term the inferences *immediate* in the sense of requiring only one application of a rule of inference – not the traditional sense of (among other things) having only one premiss.»

At the same time, Church's reservation regarding the special *traditional sense of* "*immediate reference*" as «*having only one premiss*» contradicts the fact that any *traditional categorical* (*unconditional*) or *conditional* (*disjunctive* or *hypothetical*) *syllogism* ***immediately infers*** *its conclusion from its* **two** *premises* (*premisses*), whereas any *traditional dilemma* (*dilemmatic syllogism*) ***immediately infers*** *its conclusion from its* **three** *premises.* In this case, *modus ponendo ponens*, being a *traditional hypothetical syllogism,* is conventionally used as *the main primary rule of inference in all axiomatic systems of modern sentential* (*propositional*) *and predicate* (*functional*) *calculi*, so that its applications are *immediate inferences*, in accordance with the first part of Church's definition. Still, Church's contradictory definition of the traditional sense of "immediate reference" is supported, e.g., by the following definition of WTNID:

> «**immediate inference** *n* **1 :** an inference drown from a single premise **2 :** the operation of drawing an inference from a single premise»

This semantic paradox can be solved by admitting that a syllogistic inference from two or three premises is, by that definition, *mediate* (*not immediate*). However, if one admits Church's thesis that an inference by *modus ponendo ponens* is immediate in symbolic logic then he must admit that a like inference in traditional logic is immediate as well. Incidentally, *modus ponendo ponens* is a *slave theorem*, and not a primary rule of inference, of either organon A_1 or $\mathbf{A}_1$ of the TTL.

3) A finite sequence of valid, or veracious, relations is said to be a *proof* of the last relation in the sequence if each relation in the sequence is either an axiom or is a theorem that is immediately inferred from preceding valid, or veracious, relations of the sequence. A relation that has a proof is called a *theorem*.

4) In the TTL, a theorem of the trial formal logic (cumulative organon $\mathcal{A}_1$) is called a *master*, or *decision*, *theorem* (*MT* or *DT*) if its form allows classifying another

relation, called *the slave relation of the MT* (*DT*), either as *valid*, *antivalid*, or *vav-neutral* (*vav-indeterminate, neither valid nor antivalid*) or as *veracious*, *antiveracious,* or *vravr-neutral* (*vravr-indeterminate, neither veracious nor antiveracious*). The slave relation of an MT is also *the slave relation of the proof of the MT*, which is alternatively called an *algebraic decision procedure* (*ADT*) *for the slave relation*.

5) A linguistic device, with the help of which a new, more expressive or shorter, name is introduced instead of existing ones, is called a *definition*. A straightforward specification of an axiom, theorem, or definition that does not requires any proof is conventionally called a *corollary*.•

Cmt 3.6. 1) Since atomic predicate-variables and atomic relation-variables can, in principle, be quantified, therefore a CAPC (CAFC) is not a closed system of logical thinking. Still, owing to its relative simplicity and wide scope, a CAPC is commonly used as underlying logical discipline of *set theory*, which is in turn used as underlying discipline of mathematics.

2) After creation of *set theory*, – at first of the naive one by Georg Cantor (1845–1918) during the years 1878-84, and then of the axiomatic one by Ernst Zermelo [1908], – certain special classes were called *sets*, although it took some time with mathematicians and logicians to realize that a set was a class, but not necessarily vice versa. It will be recalled that, in outline, a set is a class, which has permanent member population and which can be linearly ordered in the sense that it can serve as a domain of definition of the linear order relation $\leq$. Therefore, I alternatively call a set "a regular class" and a class that is not a set "an irregular class" (see subsection I.9.3 in the TTL). In the literature, the classes of the two kinds are distinguished by the names "small class" and "proper class" in that order (see, e.g., Fraenkel et al [1973, pp. 128, 134–135, 167] for the former term or the article «**class**» in Wikipedia for both terms). Thus, a set is a class, but not necessarily vice versa. The difference between a set and a class not being a set is discussed in detail in subsection I.9.3 of the TTL.•

The way, by which I have developed $\mathcal{A}_1$, can be generalized as the following definition.

Df 3.4: ***Definition of the term*** "***formal logic***". A) *Formal logic* (*FL*) is a *study of the* ***form of reasoning*** *in abstraction from its matter*, along with or without a *study of the* ***form*** *of relationship between the former form and its matter*.

B) The matter of reasoning comprises *complex objects* of a logician (thinker, interpreter, sapient subject), which are called *states of affairs* and also *facts*, *events*, etc, and which *are not objects of FL*. However, in accordance with the above definition, a study of relationship between form and matter of reasoning is a part of FL. Therefore, within the *trial FL* (*TFL*), has been done that study has been done as *a study of the relationship of conformity of valid and vav-neutral* (*vav-indeterminate*, *neither valid nor antivalid*) *euautographic* (*genuinely autographic*, *pure syntactic*) *relations* (*ER's*) *of the organon* A_1 *to the* ***formal matters*** (***f-matters***) *of those ER's in the form of the respective conformal catlogographic* (*CFCL*) *f-tautologous* (*f-tautological, universally f-true*) *and f-veracious* (*accidentally f-true*) *f-ttatt-neutral* (*f-ttatt-indeteminate*, *neither f-tautologous nor f-antitautologous*) *relations* (briefly *CFCLR's*), being *the respective CFCL interpretands of the ER's*. In other words, the relationship of conformity under study is *the relationship of conformity of the validity-values* (*validity-classes*) *validity and vav-neutrality* (*vav-indeterminacy*), i.e. *neutrality* (*indeterminacy*) *with respect to the validity-values validity and antivalidity*, *of ER's of* A_1 *to the* ***f-matters*** *of the CFCL interpretands of the ER's in the form of the f-tautologousness-value* (*f-tautologousness-class*) *f-tautologousness and in the form of the f-veracity-value f-veracity of the respective CFCL interpretands.*

C) In the case of the TFL, the negation of a valid relation is an antivalid relation and vice versa, whereas the negation of a vav-neutral relation is another vav-neutral relation; the negation of an f-tautologous relation is an f-antitautologous relation and vice versa, whereas the negation of an f-ttatt-neutral relation is another f-ttatt-neutral relation; the negation of an f-veracious relation is an f-antiveracious relation and vice versa, whereas the negation of an f-vravr-neutral (f-vravr-indeterminate, neither an f-veracious nor an f-antiveracious) relation is another an f-vravr-neutral relation. Also, relations of the TFL satisfy the following secondary taxonomies.

i) A relation of the TFL is said to be:

a) *f-true* if it is either *f-tautologous* (*universally f-true*) or *f-veracious* (*accidentally f-true*);

b) *f-antitrue* (*f-false*) if it is either *f-antitautologous* (*universally f-antitrue*, *f-contradictory*) or *f-antiveracious* (*universally f-antitrue*);

c) *neutral* (*indeterminate*) *with respect to the f-truth-values f-truth and f-antitruth*, i.e. *neither f-true nor f-antitrue,* – briefly *f-tat-neutral* (*f-tat-indeterminate*), if it is *f-vravr-neutral* (*f-vravr-indeterminate*).

In this case, the negation of an f-true relation is an f-antitrue relation and vice versa, whereas the negation of an f-tat-neutral relation is another an f-tat-neutral relation. The qualifiers "tat-neutral" ("tat-indeterminate") and "vravr-neutral" ("vravr-indeterminate") are synonyms.

ii) A relation of the TFL is said to be:

a) *invalid* if it is antivalid or vav-neutral, *non-antivalid* if it is valid or vav-neutral, and *vav-unneutral* (or *vav-determinate*) if it is valid or antivalid;

b) *f-atautilogous* if it is f-antitautologous or f-ttatt-neutral, *f-non-antitautologous* if it is f-tautologous or f-ttatt-neutral, and *f-ttatt-unneutral* (or *f-ttatt-determinate*) if it is f-tautologous or f-antitautologous;

c) *f-unveracious* if it is f-veracious or f-vravr-neutral, *f-non-antiveracious* if it is f-veracious or f-vravr-neutral, and *f-vravr-unneutral* (or *f-vravr-determinate*) if it is f-veracious or f-antiveracious;

d) *f-untrue* if it is f-true or f-tat-neutral, *f-non-antitrue* if it is f-true or f-tat-neutral, and *f-tat-unneutral* (or *f-tat-determinate*) if it is f-true or f-antitrue.

D) In the case of *dual FL* (*DFL*), vav-neutral, f-ttatt-neutral, f-vravr-neutral, and f-tat-neutral relations are disregarded. Since f-veracious (accidentally f-true) and f-antiveracious (accidentally f-antitrue, accidentally f-false) relations are f-ttatt-relations, therefore the former relations are disregarded along with the latter. Consequently, the qualifiers to relations of DFL of each one of the following four lists i–iv are synonyms:

i) "f-valid" and "f-non-antivalid";

ii) "f-antivalid" and "f-invalid";

iii) "f-tautologous" ("universally f-true"), "f-true", "f-non-antitautologous" ("f-uncontradictory"); and "f-non-antitrue";

iv) "f-antitautologous" ("f-contradictory", "universally f-antitrue", "universally f-false"), "f-antitrue" ("f-false"), "f-atautologous".

Also, the *validity-value validity* or *invalidity* of a relation of DFL can be qualified as an *f-truth-functional* one and likewise the *f-truth-value f-truth* or *f-untruth* of a relation of DFL can be qualified as a *validity-functional* one in the sense that a relation of DFL is said to be *valid* if and only if it is *f-true* and *invalid* if and only if it is *f-untrue* (*f-false*). That is to say, the qualifiers to relations of DFL of either one of the following two pairs are also synonyms:

a) "valid" and "f-true",

b) "antivalid" and "f-antitrue".

Therefore, the six qualifiers of the lists i and iii or those of the lists ii and iv are synonyms.

E) From the previous item, it follows that the relationship of the TFL, which models *the relationship between the form of reasoning, abstracted from its matter, and that same matter itself,* i.e. *between the TFL and the pertinent TML, becomes an identity, and hence it disappears, in DFL.* Hence, the definition of *formal logic* (*FL*), which was made in the item A of this definition, applies only to *the TFL*, i.e. with "*Trial formal logic* (*TFL*)" in place of "*Formal logic* (*FL*)". In the case of *DFL*, that item should be restated as follows.

F) *Dual formal logic* (*DFL*) is a *study of the form of reasoning in abstraction from its matter*. The relationship between the DFL and the *material logic* (*ML*), being *its matter*, is beyond the scope of the DFL.

G) In accordance with the items A and F, TFL or DFL is a *single trial* or *dual formal logical system* (*TFLS* or *DFLS*) or a *totality of TFLS's* or *DFLS's*, respectively. In this case, a *formal logical system* (*FLS*) is either a *totality of separate axiomatic rules of inference of one type* or an *axiomatic logical calculus*, *basic* (*canonic, plain*, *non-modal*) or *modal*, the understanding being that some of the former axiomatic rules of inference can be theorems of a certain axiomatic logical calculus. Any *FLS*, *dual* or *trial*, is or is supposed to be developed with the help and within its *inclusive metalanguage* (*IML*), which is called *the theory of the FLS* or generally a *logical theory*. A logical theory is said to be a *dual one* (*DLT*), if it determines a DFLS, and a *trial one* (*TLT*), if it determines a TFLS. Several logical theories may determine recognizably the same FLS. Besides the FLS that is prescinded from its theory, that theory may determine the pertinent *material logical system* (*MLS*) (e.g., a certain system of English declarative sentences) and also determine the relationship between the FLS and the MLS.

H) The noun "logic" alone, without qualifiers, is an equivocal (multisemantic) generic term. By default, this noun most generally means (denotes, is used for mentioning) a *certain totality of logical theories*. At the same time, "logic" can be used synecdochically for mentioning, e.g. either FL, i.e. a single FLS or a totality of

FLS's, or a single logical theory. Therefore, it is convenient to make the following stipulatory definition. If a logical theory determines an FLS along with the pertinent MLS as an interpretand of the FLS then that logical theory is alternatively (synonymously) a *metamaterial logical theory* (*MMLT*) or *metamaterial logic* (*MML*). In this case, by contrast, the MLS can alternatively (synonymously) be called a *protamaterial logical theory* (*PMLT*) or *protamaterial logic* (*PML*).•

Cmt 3.7. 1) The term "tautology" has arisen in the conventional (dual) truth-functional FL after Wittgenstein [1921], who applied that term to any quantifier-free or quantified statement, being *universally true* by virtue solely of the *abstract truth-functional validity of its syntactic form.* Such use of the term "tautology" has been adopted by all modern logician and mathematicians. At the same time, Wittgenstein suggested as a thesis the doctrine that *all logic* and *all mathematics* is tautological. This thesis has commonly been regarded as one that is difficult to defend and therefore it has never been adopted by logical and mathematical society (cf. Quine [1951, p. 55]). Df 3.4 allows reaching complete clarity regarding Wittgenstein's thesis.

2) In accordance with the item D of that definition, the only *truth-values truth* and *antitruth* (*falsehood*) that exist in DFL are *formal truth-values* (*f-truth-values*) *universal formal truth* (*universal f-truth*) and *universal formal antitruth* (*universal f-antitruth*), i.e. *f-tautologousness-values f-tautologousness* and *f-antitautologousness* (*f-contradictoriness*), respectively. Hence, if «*all logic*» is understood either as *all DFL* or as *all dual logic* (*DL*), i.e. as *all DFL together with the pertinent dual material logic* (*DML*), *adjoint of the DFL,* in the hypostasis of a certain rich WNL (e.g. rich written English), *whose materially tautologous* (*m-tautologous, universally m-true*) *DS's interpret f-tautologous relations of DFL*, – which *was obviously meant* by Wittgenstein, – then the part of Wittgenstein's thesis concerning «*all logic*» is *accidentally* (*not universally, not tautologously*) *true*.

3) By contrast, in TFL, besides the above *universal f-truth-values*, there are the *f-veracity-values f-veracity* and *f-antiveracity*, i.e. the *accidental f-truth-values accidental f-truth* and *accidental f-antitruth*, and there is also the *f-veracity-value f-vravr-neutrality* (*f-vravr-indeterminacy*), i.e. *neutrality* (*indeterminacy*) *with respect to the f-veracity-values f-veracity and f-antiveracity*. In this case, an *f-veracious*, *f-aniveracious*, or *f-vravr-neutral* (*f-vravr-indeterminate*) *relation* of TFL is an *f-ttatt-*

neutral (*f-ttatt-indeterminate*) *relation*, i.e. *neutral* (*indeterminate*) *with respect to the f-tautologousness-values f-tautologousness and f-antitautologousness*. An f-veracious relation of TFL is *interpretable* (*replaceable*) by any appropriate *m-veracious* (*accidentally m-true*, *fact-conformable*) *DS*, which belongs to a *certain rich WNL* (e.g. *rich written English*), being the *pertinent trial material logic* (*TML*) *adjoint of the TFL*. Here follow some examples of m-veracious sentences.

a) Each one of the following DS's is a *proper m-veracious DS*, because it conforms to the respective *permanent* historical, geographical, or geopolitical fact:

"Sir Walter Scott is the author of *Waverley*", "Abraham Lincoln was the 16th president of the USA", "London is in Europe", "Chicago is North of New York", "Moscow is the capital of Russia".

b) Each one of the following DS's is also a *proper m-veracious DS*, because it conforms to the respective *permanent* mathematical fact:

"To each natural number there is a strictly larger natural number", "The sum of angles of a triangle equals π", "Two infinite straight parallel lines in a 3-dimensional affine real Euclidean space do not intersect", "$2>1$", "$3^2+4^2=5^2$".

c) "It is raining" is a *common m-veracious DS* and "It is not raining" is a *common m-antiveracious DS there and then, where and when it is raining*, while on the contrary, "It is raining" is a *common m-antiveracious DS* and "It is not raining" is a *common m-veracious DS there and then, where and when it is not raining.*

d) In the TTL, I have proved that, in the exclusion of Bamalip, Barapti (former Darapti), Felapton, and Fesapo, the remaining 15 of 19 categorical syllogisms are *f-tautologous*, i.e. *universally f-true*. The former four categorical syllogisms are *f-ttatt-neutral* (*f-ttatt-indeterminate*) ones, which turn into *f-veracious*, i.e. *accidentally f-true*, ones owing to certain *additional f-veracious so-called catlogographic axioms*. This result is in agreement with the finding of Hilbert and Ackermann [1950, pp. 48–54, 53ff] that all categorical syllogisms in the exclusion of the above four are deducible from Boolean algebra. Incidentally, in view of the additional veracious catlogographic axioms, the four peculiar syllogisms cannot, strictly speaking be qualified *categorical*, i.e. *unconditional.*

The above examples have the following general implications.

i) If «*all logic*» is understood either as *all TFL* or as *all trial logic* (*TL*), i.e. *all TFL together with the pertinent trial material logic* (*TML*) *of a certain rich WNL* (e.g.

rich written English), *whose m-veracious* (*fact-conformable*) *DS's interpret f-veracious relations of TFL*, – which *was obviously not meant* by Wittgenstein, – then the part of Wittgenstein's thesis concerning «*all logic*» is *accidentally* (*not universally, not tautologonusly*) *m-antitrue* (*m-false*).

ii) The class of ttatt-neutral DS's is an inexhaustible source of *mathematical postulates*, both *permanent* ones called *axioms* and *ad hoc* ones called *hypotheses*, and also of *mathematical theorems*, which are therefore m-veracious (accidentally m-true) and not m-tautological (not universally m-true). Hence, the part of Wittgenstein's thesis concerning *supposed tautologousness of all mathematics* is also *accidentally m-antitrue* (*m-false*).

4) I cannot report on my studies of various f-true, i.e. f-tautologous (universally f-true) and relations of TFL in detail within the scope of this general outline. I shall only remark that all unquantified f-tautologous relations of the TFL and a few quantified ones (as the 15 tautologous categorical syllogisms) are effective mainly as *rules of inference*, whereas all quantified f-tautologous relations of the TFL are substantiated relations in themselves like most f-veracious ones.•

3.7. "Syllogism"

Preliminary Remark 3.1. TrSFL comprises *tautologous sentential forms* (*schemata*), some of which, including De Morgan's laws, were invented by ancient Greek philosophers, pre-Aristotelian ones and post-Aristotelian ones (particularly, by Stoics), whereas the other ones were invented by medieval Scholastics, and all of which were later deduced in the *conventional axiomatic sentential*, or *propositional, calculi* (*CASC'i*) constituting a part of NMhL (NSbL). Most of these tautologous sentential forms, namely those comprised in SFS, are *tautologous sentential forms* (*schemata*, *rules*) *of deductive inference*, which can therefore be alternatively called *sentential syllogism-forms* (*syllogism-schemata*, *syllogism-rules*) or *formal*, or *schematic*, *sentential syllogisms* (briefly, *FSS's* or *SSS's*). At the same time, in authoritative explanatory dictionaries and in encyclopedias, the term "syllogism" is as rule defined in the narrow sense of "categorical syllogism". Therefore, for avoidance or confusion, I shall stick to the following definition of term "syllogism".•

Df 3.5. By Df 3.3(1), TrDdFL is a part of *dual* FL (*D*FL). However, in the following classification of individual inference rules of TrDdFL some metaterms of the *trial* FL (*T*FL) $\mathcal{A}_1$, belonging to the IML of the latter, i.e belonging to the TTL, are used because these metaterms are absent in the IML of any *dual formal logistic system* (*DFLS*).

1) A *judgment* is an *m-veracious* (*accidentally m-true*) and hence *m-ttatt-neutral* (*m-ttatt-indeterminate*) *declarative sentence* (*DS*), i.e. a *one-sentence statement*, in any basic or rich written native language (WNL) as English, and vice versa. Consequently, an *m-tautologous statement is not a judgment.* Accordingly, a *judgment-form* or *judgment-schema* is an *f-veracious* (*accidentally f-true*) and hence *f-ttatt-neutral* (*f-ttatt-indeterminate*) *sentential form* and vice versa.

2) A *syllogism-form* is an *f-true form of deductive inference* (*proof*) *of an f-veracious form*, i.e. *of a judgment-form*, called the *conclusion-form*, from one or more *f-veracious forms of known judgments*, called the *premise-forms*; "form" can be used interchangeably with "*schema*", "*pattern*", or "*rule*". Consequently, a *syllogism-instance* is an *m-true instance of the pertinent f-true syllogism-form, of deductive inference* (*proof*) *of the judgment,* being the pertinent *m-veracious conclusion-instances of the conclusion-form*, from *known judgments*, being the pertinent *m-veracious premise-instances of the premise-forms*.

3) A syllogism-form can either be f-tautologous (universally f-true), as any one of the sentential syllogism-forms or as any one of the 15 *tautologous* categorical syllogisms, or be f-veracious (accidentally f-true), as any one of the 4 *veracious* categorical syllogisms Bamalip, Barapti (former Darapti), Felapton, and Fesapo, indicated in Cmt 3.7(3d), Consequently, all syllogism-instances of an f-tautologous syllogism-form are m-tautologous, whereas all syllogism-instances of an f-veracious

4) A *syllogism-form* or a *syllogism-instance* is indiscriminately called *a syllogism.* Consequently, the *premise-forms* or the *premise-instances* are indiscriminately called the *premises* and the *conclusion-form* or the *conclusion-instance* is indiscriminately called the *conclusion.*

5) A syllogism that has *n* judgments, subject to $n \geq 2$, i.e. *n*–1 *premises* and one *conclusion*, is said to be an *n-judgment syllogism* or an (*n*–1)-*premise syllogism*.

6) The generic term "*syllogism*" is derived from the Greek etymons "συλλογή" \silloγí\ *s.f.*, meaning *a collection, thought*, or *reflection,* and "συλλογισμός" \silloγismós\ *s.m.*, meaning *a reflection* or, tautologically, *a syllogism*.•

Cmt 3.8. 1) Each rule of TrSFL had been regarded as valid in its own right until all rules of TrSFL *were incorporated as tautologies* into every modern CASC and hence into every modern CAPC. I have elementarily proved all these rules in the framework of $\mathcal{A}_0$, and hence in the framework of $\mathcal{A}_1$ containing $\mathcal{A}_0$ as its self-subsistent part, by the pertinent *basic algebraic decision procedures* (*BADP's*). In the result, any law (tautologous sentential form) of TrSFL exists now in various equivalent variants of two *syntactic forms*: *logographic form* and s*emi-verbal* (*logophonograpic*, *phonologographic*) *form*, while some rules of TrSFL are also expressible in pure *verbal* (*phonographic*) *form*. In this case, a tautologous sentential form is a *sentential syllogism-form* if it is either (a) an implication such as $\boldsymbol{P}\Rightarrow\boldsymbol{Q}$, where the antecedent '$\boldsymbol{P}$' is a placeholder of a single concrete premise or of the conjoined concrete premises and the consequent '$\boldsymbol{Q}$' is a placeholder of the pertinent concrete conclusion; or (b) a biimplication (bihypothetical, equivalence) such as $\boldsymbol{P}\Leftrightarrow\boldsymbol{Q}$, where '$\boldsymbol{P}$' is a placeholder of the premise or of the conjoined premises and '$\boldsymbol{Q}$' is a placeholder of the conclusion, or vice versa. A tautologous sentential form that is neither an implication nor a biimplication is not a syllogism-schema.

2) At the same time, any syllogism-schema of TrSFL, i.e. of SFS, can be represented either in a *staccato form* (*style*) or in a *legato form* (*style*).

a) A *staccato form* of a sentential syllogism-schema is a *form*, in the framework of which all premise-schemata and the conclusion-schema are asserted separately from one another after the manner of simple declarative sentences. Hence, *a staccato form of a sentential syllogism-schema is always either a semi-verbal one or a pure verbal one, but not necessarily vice versa*.

b) A *legato form* of a sentential syllogism-schema is a form, in the framework of which the syllogism-schema is represented a *single whole,* after the manner of a complex sentence. Hence, *a logographic form of a sentential syllogism-schema is necessarily a legato one, but not necessarily vice versa*.•

4. The Aristotelian corpus and the literature on the extant Aristotelian works

Aristotle (Αριστοτέλης \Aristotélís\, Αριστοτέλους \Aristotélus\, Latin: Aristotelēs), 384–322 BC, was a student at the Academy of Plato in Athens, who became the greatest «Greek philosopher and scientist whose thought determined the course of Western intellectual history for two millennia», – as said in the article of the same name of BOE (Britannica Online Encyclopedia, Britannica.com).

I have already said in the item 1 of sub-subsection 2.2.3 that the complete body of extant works of Aristotle and his school was published in Greek by Andronicus of Rhodes in Rome in the middle of the first century BC (c60–c40 BC). I have also mentioned that all later translations of the extant Aristotelian works into Latin and afterwards into various modern languages were derived and have come down to us mainly from Andronicus' edition. The *Aristotelian corpus* (in Latin: *Corpus Aristotelicum*), i.e. the complete body of works of Aristotle and his school, which survived from antiquity through the Middle Ages to the modern times, has been published in Latin under the heading "*Aristotelis Opera*" in the Prussian Academy of Sciences edition (*edidit Academia Regia Borussica*, Berlin, 1831–1870) by the editor August Immanuel Bekker (1785-1871), a German philologist. The interested reader will find the full information about *Corpus Aristotelicum* and about its English translations, e.g., in the article of the same name of Wikipedia. That article also contains information about authentic, debatably spurious, and agreeably spurious Aristotle's works and about Latin and English translations of the fragments of supposedly spurious Aristotle's works, which were located after publication of Bekker's edition of Corpus Aristotelicum. The Wikipedia's article has links to the online English translations both of the treatises of Corpus Aristotelicum and of the above-mentioned fragments. The following brief remarks about the extant Aristotelian works, which are based on the Wikipedia's article and on some other sources, are made for convenience in discussion of some pertinent topics in the TTL and also as a guide to more detailed information about those works, which can be found in the Internet.

Most works of *Aristotelis Opera* (Corpus Aristotelicum) and their classification originate with Andronicus' edition. At the same time, the form of organization and reference to the works of Aristotle, which was adopted in *Aristotelis Opera* and which is called "Bekker numbers", has become standard and it is also widely used by contemporary writers on Aristotle's works. A Bekker number comprises up to four Arabic digits for a page number of Bekker's edition, a letter "a" or "b" for a column number, and then the Arabic numeral for a line number, when necessary or desired. In English, separate Aristotelian treatises are informally mentioned either by their English titles as given in *The Complete Works of Aristotle*, edited by Jonathan Barnes, 2 vols. Princeton University Press, 1984, or by their Latin titles of Bekker's edition. Accordingly, all those names are said to be *conventional*.

The complete list of Aristotelian works is given in the above-mentioned Wikipedia's article by their conventional English and Latin names and also by their Bekker numbers, if exist. The contents of the *original* Bekker's Aristotelian corpus and the pertinent classification of the extant Aristotelian works are represented by the following modification of the Wikipedia's list, in which all Bekker numbers are omitted, but the marks * and **, indicating respectively disputed works and works generally agreed to be spurious, are preserved.

1) Logic (*Organon*):

Categories (*Categoriae*),
On Interpretation (*De Interpretatione*),
Prior Analytics (*Analytica Priora*),
Posterior Analytics (*Analytica Posteriora*),
Topics (*Topica*),
Sophistical Refutations (*De Sophisticis Elenchis*).

2) Physics (The study of nature):

Physics (*Physica*),
On the Heavens (*De Caelo*),
On Generation and Corruption (*De Generatione et Corruptione*),
Meteorology (*Meteorologica*),
*On the Universe*** (*De Mundo*),
On the Soul (*De Anima*),
Little Physical Treatises (*Parva Naturalia*):
- *Sense and Sensibilia* (*De Sensu et Sensibilibus*),
- *On Memory* (*De Memoria et Reminiscentia*),
- *On Sleep* (*De Somno et Vigilia*),
- *On Dreams* (*De Insomniis*),
- *On Divination in Sleep* (*De Divinatione per Somnum*),
- *On Length and Shortness of Life* (*De Longitudine et Brevitate Vitae*),
- *On Youth, Old Age, Life and Death, and Respiration* (*De Juventute et Senectute*, *De Vita et Morte*, *De Respiratione*),

*On Breath*** (*De Spiritu*),
History of Animals (*Historia Animalium*),
Parts of Animals (*De Partibus Animalium*),
Movement of Animals (*De Motu Animalium*),

Progression of Animals (*De Incessu Animalium*),

Generation of Animals (*De Generatione Animalium*),

Minor works:

*On Colors*** (*De Coloribus*),

*On Things Heard*** (*De Audibilibus*),

<u>*Physiognomonics*</u>** (*Physiognomonica*),

<u>*On Plants*</u>** (*De Plantis*),

*On Marvellous Things Heard*** (*De Mirabilibus Auscultationibus*),

*Mechanics*** (*Mechanica*),

<u>*Problems*</u>* (*Problemata*),

Minor works:

*On Indivisible Lines*** (*De Lineis Insecabilibus*),

*The Situations and Names of Winds*** (*Ventorum Situs*),

*On Melissus, Xenophanes, and Gorgias***

3) Metaphysics:

Metaphysics (*Metaphysica*).

4) Ethics and politics:

Nicomachean Ethics (*Ethica Nicomachea*),

*Great Ethics** (*Magna Moralia*),

<u>*Eudemian Ethics*</u> (*Ethica Eudemia*),

*On Virtues and Vices*** (*De Virtutibus et Vitiis Libellus*),

<u>*Politics*</u> (*Politica*),

*Economics** (*Oeconomica*).

5) Rhetoric and poetics:

Rhetoric (*Ars Rhetorica*),

*Rhetoric to Alexander*** (*Rhetorica ad Alexandrum*),

Poetics (*Ars Poetica*).

Cmt 4.1. 1) As was mentioned in the item 2 of sub-subsection 2.2.3, the six treatises that are mentioned in the item 1 of the above list form the standard collection of Aristotle's works on logic (see also the item 5 of sub-subsection 2.2.3 and the subsection 3.5).

2) The order, in which Aristotle's treatises are arranged in the Andronicus' and Bekker's editions, and particularly the order of the six treaties of «*Organon*» (cf. Cmt

4.1), is regarded as logically justified, but it is not necessarily the chronological one, which is unknown. At the same time, regarding the debatable Aristotle's authorship of many treatises credited to him, Durant [1926, p. 47] says:

> «It does not appear that Aristotle published in his life-time any technical writings except those on logic and rhetoric; and the present form of the logical treatises is due to later editing. In the case of the *Metaphysics* and the *Politics* the notes left by Aristotle seem to have been put together by his executors without revision and alteration. Even the unity of style which marks Aristotle's writings, and offers an argument to those who defend his direct authorship, may be, after all, merely a unity given them through common editing by the Peripatetic School. About this matter there rages a sort of Homeric question, of almost epic scope, into which the busy reader will not care to go, and on which a modest student will not undertake to judge.[10] We may at all events be sure that Aristotle is the spiritual author of all those books that bear his name: that the hand may be in some cases another's hand, but that the head and the heart are his.»

Similar remarks can be found, e.g., in the articles **Aristotle** of Wikipedia, GEB, and OCCL.•

Cmt 4.2: ***The literature on the extant Aristotelian works.*** An ordinary interested reader, not being a schooled expert in Aristotle's works, should have no difficulties in locating additional information about separate Aristotle's works elsewhere, particularly in the Internet. The following critical remarks are designed for a reader of this kind.

I) Any comment on works of Aristotle that can be found in the literature is an interpretation that was made from the viewpoint of the author or authors of the comment in another language and in another time of what Aristotle wrote in Ancient Greek more than two millennia ago and more than one millennium before invention of most physical instruments that are necessary for precise measurements or observations, – such instruments, e.g., as the chronometer, thermometer, barometer, telescope, microscope, and precision balance, – and even before invention of *null*, which was made in the ninth century of the Christian era, to say nothing of invention of modern mathematics and numerical analysis and their applications to physics and logic. Aristotle has only the ruler, pair of compasses, primitive substitutes for some other modern measuring instruments, and «*Organon»*, which he invented, but did not mint any general name to this embryo of modern logic.

II) Plato was a talented author-philosopher, who embedded his philosophical teaching into literary productions. At the same time, he knew his contemporary mathematics due to Pythagoras (570–500 BC) and considered it as a bridge created by the God between nature and philosophy, particularly between nature and his philosophical theory of transcendental Forms. By contrast, Aristotle was naturalist-philosopher, who embedded his philosophical view on nature in concentrated abstract technical terminology and phraseology, although his Weltanschauung, like that of Plato, rested on religious mythology. It would, however, be incorrect to think, as it is suggested by some commentators, that in contrast to Plato Aristotle disliked or perhaps did not know mathematics. More likely, he just rejected the Pythagoras teaching, according to which nature was a world of mathematics whose objects were «eternal and immovable and separable» [from matter] numbers – the world that was ruled by mathematical regularities and constancies. By contrast, natural (physical) objects were, according to Aristotle's philosophy, movable ones, whose form was distinct but inseparable from their matters (cf. Aristotle [AMR, Book VI, Part 1]). In its spirit, Pythagorean teaching was close to that of Platonic Forms, but it was unacceptable to Aristotle. In connection with Plato's philosophy, Durant [1926, pp. 26, 27] says:

> «Plutarch tells us that according to Plato "God always geometrizes"; or, as Spinoza puts the same thought, God and the universal laws of structure and organization are one and the same reality. To Plato, as to Bertrand Russell, mathematics is therefore the indispensable prelude to philosophy, and its highest form; over the doors of his Academy, Plato placed, Dantesquely, these words, "Let no man ignorant of geometry enter here."»[8];

[8] In the first part «*Inferno*» (Italian for "Hell") of his epic poem «*Divine Comedy*» («*La Divina Commedia*») of the 14th century, Dante Alighieri (1265–1321) passes through the gate of Hell, which bears an inscription, the ninth (and final) line of which is the famous phrase "Lasciate ogne speranza, voi ch'intrate", or "Abandon all hope, ye who enter here"[4]. There are many English translations of this famous line, some of which are cited in the article «**Inferno (Dante)**» of Wikipedia. As far as the above-cited inscription above the doors of Platonic Academy is concerned, it is noteworthy that by "geometry" Plato meant pre-Euclidean geometry because Euclid (325–265 BC) flourished after both

and he continues in regard to Aristotle's study of nature (*ibidem*, p. 53):

> «If we begin here chronologically, with his *Physics*, we shall be disappointed; for we find that this treatise is really *metaphysics*, an abstruse analysis of matter, motion, space, time, infinity, cause, and other such "ultimate concepts." One of the more lively passages is an attack on Democritus' "void"; there can be no void or vacuum in nature, says Aristotle, for in a vacuum all bodies would fall with equal velocity; this being impossible, "the supposed void turns out to have nothing in it" – an instance at once of Aristotle's very occasional humor, his addiction to unproved assumptions, and his tendency to disparage his predecessors in philosophy. It was the habit of our philosopher to preface his works with historical sketches of previous contributions to the subject in hand, and to add to every contribution an annihilating refutation. "Aristotle, after the Ottoman manner," says Bacon, "thought he could not reign secure without putting all his brethren to death"… But to this fratricidal mania we owe much of our knowledge of pre-Socratic thought.
>
> For reasons already given, Aristotle's astronomy represents very little advance upon his predecessors. He rejects the view of Pythagoras that the sun is the center of our system; he prefers to give that honor to the earth.»

III) In spite of the above criticism, Aristotle invented an extensive terminology of science and philosophy, the elements of which translated or transliterated into Latin in the middle ages and into major European languages in the modern times, – such English words, e.g., as "*category*", "*energy*", "*kinetic*", "*dynamic*", "*substance*", "*quantity*", "*quality*", "*actuality*", "*potentiality*", "*matter*", "*form*", "*syllogism*", "*axiom*", "*principle*", "*maxim*" (meant, in Aristotelianism, the major premise of a syllogism), etc, – have become indispensable coins in studying and treating physical (natural) and psychical (mental) phenomena. Still, just as the entire philosophy of Plato, the entire philosophy of Aristotle as treated in his Metaphysics and other works has the character of a religious legend. Therefore, while the theological understanding of deity or the scientific understanding of nature and mind (consciousness) have been changing, the meanings of transliterations of the original Aristotle's etymons in Latin

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Plato and Aristotle. Also, «*La Divina Commedia*» was written more than a millennium after Plato's time. Therefore, it would have been correct to say that the Dante's famous inscription is a Platonic one.

and other European languages have been changing accordingly, so that they have at present nothing or almost nothing to do with their etymons.

IV) As far as «*Organon*» is concerned, it is quite rare that a modern commentator on these treatises is simultaneously a logician and a schooled philologist, having command in Ancient Greek and Latin. Therefore, most comments on «*Organon*» that have been written by contemporary busy logicians for other busy logicians have unavoidably been derived from its translation into the language of a commentator or from some existing comments, most (if not all) of which have recursively the same character. In this case, the medieval Latin or modern English terms or expressions that are evoked as counterparts of certain original Aristotle's terms or expressions, which are not often demonstrated and not translated literally, but rather are transliterated and translated freely, may subtly differ from the original ones in meaning. The only thing that a contemporary English-writing non-philologist commentator of an Aristotle's work can do is to compare several existing comments on that same work with one another and perhaps with its translations from Ancient Greek or Latin into English, in order to decide what in these comments expresses Aristotle's thoughts and what is the result of free translation or of interpretation of the commentators, and then to recapitulate his findings in his own words for his purpose at hand.

V) To complete the above remarks, they should be supplemented with subsection 2.2 of Essay 5 and especially with the items I–VII of that subsection.

VI) I am not a schooled expert either in Aristotelian works or in ancient languages. Therefore, in this essay and also in Essay 5, I have made general comments of my own on some most conspicuous doctrines and peculiarities of Aristotelian works, especially of his «*Organon*» and his *Metaphysics*, from the standpoint of my TTL by the method described above in the above item IV. In this case, I have mentioned some sources or literally quoted wordings of other writers only if these are appropriate (cf. my quotations of Durant). I have not, however, made any scrupulous references to concrete places of Aristotelian works by their Bekker numbers, except for the cases when such references occur in the quotations that I have made. When necessary, I have referred to online English translations of Aristotelian works, which occur among other sources in the list of references at the end of the TTL or in the list of references at the end of this treatise. With the help of various Greek-English-Greek dictionaries (particularly with the help of Pring [1982]), and also with the help of various comments found in the Internet, I have tried, when necessary or desirable, to analyze the etymology of some original Aristotelian terms or even to restore some others.

Essay 9. A commutative semigroup calculus:

Generalized associative and commutative laws for an abstract binary composition operator

Abstract

The generalized associative and commutative laws for an abstract binary composition operator relative to an abstract binary equivalence operator are formulated and proved rigorously in the most general form in the framework of the appropriate logistic commutative semigroup calculus, by the method of mathematical induction along with the pertinent non-trivial combinatorics.

1. Introduction

In this essay, I formulate in the most general form and rigorously prove the generalized associative and commutative laws for an abstract binary composition operator (placeholder) '◊' relative to an abstract binary equivalence operator (placeholder) '∼'. In order to do so, I develop a subsidiary calculus, which is denoted by 'C_0' and is called the *commutative semigroup calculus* (*CSGC*). The latter taxonomic name is descriptive of the following properties of C_0. All well-formed formulas of C_0 are of *two* kinds: *terms* and *binary equivalence relations* between terms. A term of C_0 is either atomic (primitive) or combined (composite). Composite terms are formed of primitive ones or of some other composite terms with the help of the abstract binary substantial composition operator '◊'. Composite terms of C_0 are assumed to satisfy *the basic associative and commutative laws with respect to the equivalence relational operator* '∼'. Consequently, C_0 is an *abstract commutative semigroup*, whose terms are related by the equivalence operator '∼' instead of an equivalence sign such as '↔' or '⇔', which is used in symbolic logic (cf. Suppes [1957, p. 10]), and instead of the equality sign such as '=' or '$\hat{=}$', which is used in class (or particularly set) theory and in algebra (cf. Mac Lane and Birkhoff [1967, p. 61]) The basic associative and commutative laws for the binary logical connectives '∨' ("inclusive or", "vel") and '∧' ('&'), for the binary class-theoretic (particularly set-theoretic) operators '∪' and '∩', or '$\bigcup$' and '$\bigcap$', and also those for the binary algebraic operators '+' and '·', or '$\hat{+}$' and '$\hat{\cdot}$', are stated respectively in symbolic logic, in class (or set) theory, and in algebra as the pertinent instances of '◊' relative to the respective equivalence signs '$\leftrightarrow$' or '$\Leftrightarrow$', and '=' or '$\triangleq$' as the pertinent instances of '~'.

Both generalized laws are rigorously proved by induction with respect to a number of atomic terms occurring in a general term of C_0 – induction that is augmented by some pertinent non-trivial combinatorics. Although the generalized laws are always intuitively understood, their rigorous formal statements and proofs cannot, to the best of my knowledge, be found in the mathematical literature.

2. Underlying meta-definitions

Definition 2.1. 1) In order to state a *binary asymmetric synonymic definition* (*BASD*) conveniently and formally, I shall make use of either one of the horizontal arrows $\rightarrow$ and $\leftarrow$, which belong to the *inclusive language* (*IML*) of C_0 and which are indiscriminately called a *universal asymmetric*, or *one-sided*, *synonymic definition signs* or, discriminately, the *universal rightward synonymic definition sign* and the *universal leftward one* respectively. At the head of an arrow I shall write the *material definiens* – the graphonym, which is already known either from a previous definition or from another source. At the base of the arrow I shall write the *material definiendum* – the new graphonym, which is being introduced by the definition and which is designed to be used instead of or interchangeably with the definiens. Accordingly, the arrow $\rightarrow$ is rendered into ordinary language thus: "*is to stand as a synonym for*" or straightforwardly "*is the synonymous definiendum of*", and $\leftarrow$ thus: "*can be used instead of or interchangeably with*" or straightforwardly "*is the synonymous definiens of*". The [material] definiendum and [material] definiens of a BASD are indiscriminately called the *terms* of the definition. A BASD, which is made with the help of $\rightarrow$ or $\leftarrow$, is said to be a *formal BASD* or briefly an *FBASD*. Neither the definiendum nor the definiens of an FBASD should involve any *function symbols*, particularly any outermost (enclosing) quotation marks, that are not their constituent parts and that are therefore used but not mentioned. If an arrow stands between a definiendum schema and a definiens schema then the arrow is supposed to apply simultaneously to the schemata and to every pair of interrelated instances (denotata, interpretands) of the schemata, unless stated otherwise

2) The *scope* of a FBASD is the part of the text following but not including the FBASD. Accordingly, though the definiendum and definiens of a FBASD are not exchangeable in the FBASD, after the FBASD is stated a token of the definiendum can be used *instead of* or *interchangeably with* a token of the definiens.

3) An FBASD is said to be an *abbreviative* FBASD or simply a *formal abbreviative definition* (*FAD*) if it prescribes that the definiendum is to stand as an abbreviation for the definiens. In this case, the arrow → can more specifically be rendered into ordinary language thus: "*is to stand as an abbreviation for*" and ← thus: "*is to be abbreviated as*". In the *scope* of a FAD, the definiendum is usually used instead of its definiens.

4) In order to state formally that two old or two new graphonyms are or are to be used interchangeably (synonymously), I shall write the graphonyms, without any quotation marks that are not their constituent parts, in either order on both sides of the two-sided arrow ↔ belonging to the IML. Such a relation is called a *formal binary symmetric synonymity*, or *concurrency*, *relation* (FBSSR), whereas ↔ is accordingly called a *synonymity*, or *concurrency*, *sign*. The two graphonyms standing on both sides of ↔ are called the *terms* of the FBSSR. If an FBSSR is a *corollary* from the pertinent FBASD stated previously then ↔ is read as "*is concurrent to*" or, alternatively, **"**— ↔ …**"** is read as **"**— *and … are concurrent***"** or as **"**— *and … are synonyms***"**, where alike ellipses should be replaced alike and then the bold-faced double quotation marks should be replaced with the light-faced ones. If an FBSSR is stated in no connection with any previous FBASD then the FBSSR is said to be a *formal binary symmetric synonymic definition* (*FBSSD*), whereas ↔ is called the *symmetric*, or *two-sided*, *synonymic definition sign*. In this case ↔ is read as "*is to be concurrent to*" or, alternatively, **"**— ↔ …**"** is read as **"**— *and … are to be concurrent***"** or as **"**— *and … are to be synonyms***"**, where alike ellipses should, as before, be replaced alike, while the bold-faced double quotation marks are placeholders for the light-faced ones. Just as in the case of → or ←, if ↔ stands between schemata then the arrow is supposed to apply simultaneously to the schemata and to every pair of interrelated instances (denotata, interpretands) of the schemata, unless stated otherwise.

5) When the signs $\rightarrow$, $\leftarrow$, and $\leftrightarrow$ apply to number-valued constants or variables, they can be replaced with the respective signs $\doteq$, $\doteq$, and $\doteq$, which are called *the ordinary rightward, leftward, and two-sided signs of equality by definition*.•

Definition 2.2. $\{x|x \in A \text{ and } \boldsymbol{P}(x)\}$ is the class (or particularly set) of elements of the class (set) A having the property (predicate) $\boldsymbol{P}$.•

Definition 2.3. 1) 'ω_0' *denotes*, i.e. ω_0 *is*, the set of all *natural numbers* from 0 to infinity.

2) Given $n \in \omega_0$,

$$\omega_n \doteq \{i|i \in \omega_0 \text{ and } i \geq n\}, \tag{2.1}$$

i.e. 'ω_1', 'ω_2', etc *denote* the sets of natural numbers from 1, 2, etc respectively to infinity.

3) Given $m \in \omega_0$, given $n \in \omega_m$,

$$\omega_{m,n} \doteq \{i|i \in \omega_0 \text{ and } n \geq i \geq m\}, \tag{2.2}$$

i.e. '$\omega_{m,n}$' *denotes* the set of natural numbers from a given number m to another given number n subject to $n \geq m$. It is understood that

$$\omega_{m,m} = \{m\},\ \omega_{m,\infty} = \omega_m,\ \omega_{m,n} = \varnothing \text{ if } m \geq n. \tag{2.3}$$ •

Comment 2.1. Definition 2.3(1) is an *explicative* one. A theory of natural integers in particular, and a theory of any numbers (as rational, real, or complex ones) in general can consistently be deduced from the five Peano axioms, which are, in turn, theorems of an axiomatic set theory (see, e.g., Halmos [1960, pp. 46–53], Burrill [1967], Feferman [1964]).•

3. The setup of C_0

Definition 3.1: *The primitive basis of* C_0. The *atomic,* or *primitive, symbols of* C_0 are three *atomic operators* (*improper symbols*)

$$\Diamond, \sim, [\ ] \tag{3.1}$$

and an infinite list of *atomic operata* (*proper symbols*)

$$x_1,\ x_2,\ \ldots, \tag{3.2}$$

i.e., in general, x_i for each $i \in \omega_1$. In this case, $\Diamond$ is *the composition operator*, ~ is *the relation operator*, [is a *left square bracket*, or *square bra*,] is a *right square bracket,* or *square ket*. The atomic (primitive) proper symbols are alternatively called the *atomic* (*primitive*) *terms*. The infinite list (3.2) is called the *alphabet of terms of* C_0, whereas the linearly ordered set ω_1 is called the *alphabetic order of the terms of* C_0 •

Comment 3.1. 1) It is understood that the commas and ellipses occurring on the lists (3.1) and (3.2) are ordinary punctuation marks of the *exclusive meta-language* (*XML*), which have nothing to do with the atomic terms.•

Definition 3.2. A single atomic symbol or a finite linear sequence of atomic symbols of C_0 is called an *assemblage of* C_0.•

Definition 3.3. 1) Each one of the bold roman English minuscule letters '**x**', '**y**', '**z**', alone or together with any number of primes is an *atomic* (*primitive*) *syntactic place-holding variable* whose range is the set of atomic terms of C_0.

2) Either of the bold roman English capitals '**U**' and '**V**' is an *atomic syntactic variable placeholder* (*place-holding variable*) *whose range is the [class of] assemblages of* C_0.•

Axiom 3.1: *The formation rules of* C_0.

1) An *atomic term* **x** of C_0 standing alone is a *term* of C_0.

2) If **U** and **V** are terms of C_0 then $[\mathbf{U}\Diamond\mathbf{V}]$ is a term of C_0.

3) If **U** and **V** are terms of C_0 then $[\mathbf{U}\sim\mathbf{V}]$ is a *relation [of the terms] of* C_0.

4) **U** is a term of C_0 if and only if its being so follows from the rules 1 and 2. **U** is a relation of C_0 if and only if its being so follows from the rule 3.

5) **U** is a *formula* of C_0 if and only if it either is a term or a relation of C_0.•

Definition 3.4. Each one of the bold roman English majuscule letters '**X**', '**Y**', '**Z**', alone or together with any number of primes is a primitive syntactic variable whose range is the terms of C_0.•

Definition 3.5: *Composite terms of* C_0. 1) Given $i\in\omega_1$, given $j\in\omega_2$, an arrangement of $j-1$ pairs of square brackets in the string

$$x_i\Diamond x_{i+1}\Diamond\ldots\Diamond x_{i+j-2}\Diamond x_{i+j-1} \tag{3.3}$$

is said to be *congruous* if and only if the arrangement is made by repeated applications of the formation rule Definition 3.4(2), according to which *a pair of square brackets is an integral part of the symbol* [◊]. The string, which is obtained by a congruous arrangement of $j-1$ pairs of square brackets in the string (3.3), is called *a j-ary term over the string (3.3) or a j-ary term with the basis*

$$x_i,\ x_{i+1},\ \ldots,\ x_{i+j-2},\ x_{i+j-1}. \tag{3.4}$$

A primitive term of C_0 is called a *singulary term*, and vise versa. A *j*-ary term of C_0 with $j \in \omega_2$ is indiscriminately called a *composite term.*

2) Given $i \in \omega_1$, given $j \in \omega_2$, let k_i, k_{i+1}, ..., k_{i+j-2}, k_{i+j-1} be *j* given natural numbers of the set ω_1 in a given order, alphabetic or not. The above item applies with 'x'' in place of 'x' subject to the definitions:

$$x'_i \to x_{k_i}, x'_{i+1} \to x_{k_{i+1}}, \ldots, x'_{i+j-2} \to x_{k_{i+j-2}}, x'_{i+j-1} \to x_{k_{i+j-1}}, \tag{3.5}$$

where '*k*' is a *virtual ad hoc label* that can be replaced by any other appropriate one such as '*l*', '*m*', or '*n*'.•

Comment 3.2. In accordance with Definition 3.5, a composite *j*-ary term over the string (3.3) is a formula, in which occurrences of the primitive improper and proper symbols satisfy the following rules:

1) A square bra immediately precedes either another square bra or one of the symbols (3.4) of the list (but not ◊).

2) An square ket is immediately preceded either by another square ket or by one of the symbols of the list (3.4) (but not by ◊).

3) The primitive terms x_i and x_{i+j-1} occur in the *j*-ary term through the strings $\left[x_i\lozenge\right.$ and $\left.\lozenge x_{i+j-1}\right]$, respectively.

4) Each primitive term x_k at $k \in \omega_{i+1,i+j-2}$ occurs in the *j*-ary term either through the string $\left[x_k\lozenge\right.$ or through the string $\left.x_k\right]$.

It is understood that $\left[x_k\right] \to x_k$ for each $k \in \omega_{1,i}$, i.e. $\left[x_k\right]$ is by definition the same term as x_k. Still, if I use at least one pair of the total number $j-1$ pairs of square brackets for enclosing, e.g., any one of the primitive terms of the list (3.4) individually then $j-1$ pairs of brackets will not suffice to form a *j*-ary term from the string (3.3). For forming such a term, it is necessary to insert all $j-1$ pairs of brackets into the string (3.3) in accordance with Definition 3.5, i.e. in accordance with the above rules 1–4.•

Definition 3.6. A composite term has the form $[\mathbf{X}\lozenge\mathbf{Y}]$ in only one way. The occurrence of ◊ between **X** and **Y** is called *the principal occurrence of* ◊ *in the term* $[\mathbf{X}\lozenge\mathbf{Y}]$ or *the principal composition operator of the term* $[\mathbf{X}\lozenge\mathbf{Y}]$. The terms **X** and **Y**, each of which can be either primitive or composite, are called *the first and second* *principal constituent terms* (or, briefly, *constituents*) *of the composite term* $[\mathbf{X}\lozenge\mathbf{Y}]$ respectively.•

Comment 3.3. If the number *j* is not too large, so that a *j*-ary term is not too long, then the principal occurrence of ◊ in the term may be recognized as such at a glance. Otherwise the principal occurrence of ◊ can be detected by the following counting procedure which is a modification of the similar method suggested by Church [1956, pp. 70 and 71] in a somewhat different context.

Given a *j*-ary composite term with $j \in \omega_3$, let us label all square brackets of the term by the integer numbers from 1 to 2*j* in the direction from left to right. Let us, also, assign the number +1 to each bra, and -1 to each ket. Let, accordingly, 'I_l' be an integer-valued variable such that for each $l \in \omega_{1,2j}$: $I_l = 1$ if the *l*th bracket is a bra, and $I_l = -1$ if the *l*th bracket is a ket. Let S_l be the sum of all such numbers for the first *l* brackets; that is, recursively, for each $l \in \omega_{1,2j}$: $S_l = S_{l-1} + I_l$, subject to $S_0 = 0$ and $S_1 = I_1 = 1$. The *j*-ary term necessarily begins with a bra [and ends with a ket]. The second symbol must be either a primitive term or another bra. If the second symbol is a primitive term then the third symbol must be ◊, and this ◊ is necessarily the principal composition operator. If the second symbol is another bra then for that bra $l = 2$ and $S_2 = 2$. In this case, there must exist the minimal integer $l_* \in \omega_{3,2j}$ such that $S_{l_*} = 1$, with the understanding that the bracket of the number l_* is necessarily a ket and that this ket is necessarily followed by ◊. This operator ◊ is necessarily the principal one. It is evident that the number l_* is necessarily odd.•

Definition 3.7. For each $i \in \omega_1$: for each $j \in \omega_2$:

$$
\begin{aligned}
\bigDiamond_{k=1}^{j} x_{i+k-1} &\to \left[\bigDiamond_{k=1}^{j} x_{i+k-1}\right] \to \left[\left[\bigDiamond_{k=1}^{j-1} x_{i+k-1}\right]\lozenge x_{i+j-1}\right] \\
&\to \left[\left[\left[\ldots\left[\left[x_i \lozenge x_{i+1}\right]\lozenge x_{i+2}\right]\ldots\lozenge x_{i+j-3}\right]\lozenge x_{i+j-2}\right]\lozenge x_{i+j-1}\right] \\
&\leftarrow \bigDiamond\left(x_i, x_{i+1}, \ldots, x_{i+j-2}, x_{i+j-1}\right).
\end{aligned}
\tag{3.6}
$$

The symbol '$\bigDiamond_{k=1}^{k=j}$' can be used interchangeably with '$\bigDiamond\limits_{k=1}^{j}$'.•

Comment 3.4. Definition (3.6) can, e.g., be syntactically interpreted (specified) by replacing the ordered triple ⟨◊, $\bigDiamond$, *x*⟩ with any one of the following:

(1) $\langle\vee, \bigvee, \mathbf{P}\rangle$, (2) $\langle\wedge, \bigwedge, \mathbf{P}\rangle$, (3) $\langle\cup, \bigcup, x\rangle$ or $\langle\bigcup, \bigcup, x\rangle$, (4) $\langle\cap, \bigcap, x\rangle$ or $\langle\bigcap, \bigcap, x\rangle$, (5) $\langle+, +, x\rangle$ or $\langle+, \Sigma, x\rangle$, (6) $\langle\cdot, \cdot, x\rangle$ or $\langle\cdot, \Pi, x\rangle$, (7) $\langle\hat{+}, \hat{+}, \mathbf{I}\rangle$ or $\langle\hat{+}, \hat{\Sigma}, \mathbf{I}\rangle$, (8) $\langle\hat{\cdot}, \hat{\cdot}, \mathbf{I}\rangle$ or $\langle\hat{\cdot}, \hat{\Pi}, \mathbf{I}\rangle$,

subject to certain subsequent syntactic or semantic (mental) interpretations of the symbols '**P**', 'x', and '**I**' involved. The variants of definition (3.6) subject to the above substitutions (1)–(8) will in the sequel be referred to as (3.6_1)–(3.6_8) respectively. In this case, the square brackets can be replaced with brackets of any other appropriate shape with some other brackets, when necessary or desirable. In accordance with definitions (3.6_5) and (3.6_6), the symbols '$\underset{k=1}{\overset{j}{+}}$' and $\underset{k=1}{\overset{j}{\cdot}}$ are introduced instead of the conventional symbols '$\sum_{k=1}^{j}$' and '$\prod_{k=1}^{j}$' respectively. In the sequel, the multiplication sign '$\cdot$' between numerical variables or constants will conventionally be omitted.

2) At $i \doteqdot 1$ and $j \doteqdot n$, definition (3.6) reduces to

$$\underset{k=1}{\overset{n}{\Diamond}} x_k \to \left[\underset{k=1}{\overset{n}{\Diamond}} x_k\right] \to \left[\left[\underset{k=1}{\overset{n-1}{\Diamond}} x_k\right] \Diamond x_n\right] \to [[[...[[x_1 \Diamond x_2] \Diamond x_3]...\Diamond x_{n-2}] \Diamond x_{n-1}] \Diamond x_n]$$
$$\leftarrow \Diamond(x_1, x_2, ..., x_{n-1}, x_n) \text{ for each } n \in \omega_2, \tag{3.7}$$

which can be particularized in analogy with (3.6_1)–(3.6_8).

3) Definition (3.6) can also be particularized thus:

$$\underset{k=1}{\overset{j}{,}} x_{i+k-1} \to \left(\underset{k=1}{\overset{j}{,}} x_{i+k-1}\right) \to \left(\left(\underset{k=1}{\overset{j-1}{,}} x_{i+k-2}\right), x_{i+j-1}\right)$$
$$\to (((...((x_i, x_{i+1}), x_{i+2})..., x_{i+j-3}), x_{i+j-2}), x_{i+j-1}) \tag{3.8}$$
$$\leftarrow (x_i, x_{i+1}, ..., x_{i+j-2}, x_{i+j-1}) \leftrightarrow \langle x_i, x_{i+1}, ..., x_{i+j-2}, x_{i+j-1}\rangle$$
$$\leftarrow \text{a leftt-associated ordered } j\text{-tuple}$$

and alternatively with $\langle\ \rangle$ in place of ( ).•

Axiom 3.2: ***The properties of*** $\sim$ ***relative to terms of*** C_0**.**

1) **X**~**X**. (*Reflexivity law*)

2) *If* $[\mathbf{X} \sim \mathbf{Y}]$ *then* $[\mathbf{Y} \sim \mathbf{X}]$. (*Symmetry law, Commutative law*)

3) *If* $[\mathbf{X} \sim \mathbf{Y}]$ *and* $[\mathbf{Y} \sim \mathbf{Z}]$ *then* $[\mathbf{X} \sim \mathbf{Z}]$.

(*Rule of categorical syllogism, Transitive law.*)

4) *If* $[\mathbf{X}' \to \mathbf{X}]$ *then* $\vdash [\mathbf{X}' \sim \mathbf{X}]$. (*Rule of a definition,*

Rule of an implied equivalence.)

Comment 3.5. In accordance with the names of laws 1–3 of Axiom 3.2, the equivalence operator ~ is conventionally said to be *reflexive*, *symmetric*, and *transitive*.•

Axiom 3.3: ***The properties of ◊ relative to ~.***

1) $[[\mathbf{X}\lozenge\mathbf{Y}]\lozenge\mathbf{Z}]\sim[\mathbf{X}\lozenge[\mathbf{Y}\lozenge\mathbf{Z}]]$. (*Associative law*)

2) $[\mathbf{X}\lozenge\mathbf{Y}]\sim[\mathbf{Y}\lozenge\mathbf{X}]$. (*Commutative law)*.

3) $[\mathbf{X}\sim\mathbf{Y}]$ *if and only if* $[\mathbf{X}\lozenge\mathbf{Z}]\sim[\mathbf{Y}\lozenge\mathbf{Z}]$. (*Imprinting versus erasing law*)

4. The generalized associative law for an associative binary composition operator

4.1. The total number of composite terms with a given basis

Lemma 4.1. Given $i\in\omega_1$: given $j\in\omega_2$, the number τ_j of *j*-ary terms that can be constructed by all possible congruous arrangements of *j*–1 pairs of square brackets in the string (3.3) satisfies the equation

$$\tau_j = \mathop{+}_{k=1}^{j-1}\tau_k\tau_{j-k}\,, \tag{4.1}$$

subject to

$$\tau_1 \doteqdot 1\,. \tag{4.2}$$

Proof: It follows from (4.1) subject to (4.2) that

$$\tau_2 = (\tau_1)^2 = 1,\ \ \tau_3 = 2\tau_1\tau_2 = 2\,, \tag{4.3}$$

so that (4.1) subject to (4.2) is true both for $j\doteqdot 2$ and for $j\doteqdot 3$. At the same time, given $j\in\omega_4$: given $l\in\omega_{1,j-1}$: the number of *j*-ary terms that can be obtained by congruous arrangements of missing *j*–3 pairs of square brackets in each one of the following strings:

$$\left[x_i\lozenge\left[x_{i+1}\lozenge\ldots\lozenge x_{i+j-2}\lozenge x_{i+j-1}\right]\right], \tag{4.4}$$

$$\left[\left[x_i\lozenge x_{i+1}\lozenge\ldots\lozenge x_{i+l-2}\lozenge x_{i+l-1}\right]\lozenge\left[x_{i+l}\lozenge x_{i+l+1}\lozenge\ldots\lozenge x_{i+j-2}\lozenge x_{i+j-1}\right]\right] \tag{4.5}$$
$$\textit{for each } l\in\omega_{2,j-2},$$

$$\left[\left[x_i\lozenge x_{i+1}\lozenge\ldots\lozenge x_{i+j-2}\right]\lozenge x_{i+j-1}\right] \tag{4.6}$$

can evidently be written as $\tau_l \tau_{j-l}$, the understanding being that $l \doteq 1$ in (4.4) and that $l \doteq j{-}1$ in (4.6). Hence, the total number τ_j of j-ary terms with all possible $l \in \omega_{1,j-1}$ is given by (4.1) subject to (4.2). QED.•

Comment 4.1. Given $m \in \omega_1$: equation (4.2) can be written as

$$\tau_{2m} = 2 \sum_{k=1}^{2m-1} \tau_l \tau_{2m-l} + \tau_m^2 \text{ if } j \doteq 2m, \tag{4.7}$$

$$\tau_{2m+1} = 2 \sum_{k=1}^{2m} \tau_l \tau_{2m-l+1} \text{ if } j \doteq 2m+1. \tag{4.8}$$

For $m \doteq 1$, equations (4.7) and (4.8), subject to (4.2), yield (4.3). Here follow some further instances of (4.7) and (4.8:

$$\begin{aligned} \tau_4 &= 2\tau_1\tau_3 + (\tau_2)^2 = 5,\ \tau_5 = 2(\tau_1\tau_4 + \tau_2\tau_3) = 14, \\ \tau_6 &= 2(\tau_1\tau_5 + \tau_2\tau_4) + (\tau_3)^2 = 42, \\ \tau_7 &= 2(\tau_1\tau_6 + \tau_2\tau_5 + \tau_3\tau_4) = 132. \end{aligned} \tag{4.9}$$

In the following theorem, the problem of computation of τ_j is solved in a closed analytical form.•

Theorem 4.1. For each $j \in \omega_2$:

$$\tau_j = \frac{1}{j} C_{2(j-1)}^{j-1} = \frac{(2j-2)(2j-3)...(j+1)}{(j-1)!}, \tag{4.10}$$

where '$C_{2(j-1)}^{j-1}$' is a binomial coefficient.

Proof: Let

$$g(x) \doteq \sum_{j=1}^{\omega_0} \tau_j x^j, \tag{4.11}$$

where 'x' is a real-valued variable, whose range will *a posteriori* be chosen so as to guarantee convergence of the power series in (4.11). It follows from (4.11) that

$$[g(x)]^2 = \sum_{k=1}^{\omega_0} \sum_{l=1}^{\omega_0} \tau_k \tau_l x^{k+l} = \sum_{j=2}^{\omega_0} x^j \sum_{k=1}^{j-1} \tau_k \tau_{j-k} = \sum_{j=2}^{\omega_0} \tau_j x^j. \tag{4.12}$$

In developing the final expression in (4.12), a new variable of summation 'j' has been introduced instead of 'l' by the equation $j = k + l$, and then use of (4.1) has been made. By (4.2) and (4.11), equation (4.12) can be rewritten as

$$[g(x)]^2 - g(x) + x = 0, \tag{4.13}$$

subject to

$$g(0) = 0, \tag{4.14}$$

which follows from (4.12). The solution of the quadratic equation (4.13), which satisfies (4.14), is given by

$$g(x) = \frac{1}{2}\left(1 - \sqrt{1 - 4x}\right). \tag{4.15}$$

If $|x| < \frac{1}{4}$ then the expression on the right-hand side of equation (4.15) can be expanded in a power series with respect to '$4x$' thus giving

$$g(x) = x + \mathop{+}_{j=2}^{\omega_0} \frac{1}{j} C_{2(j-1)}^{j-1} x^j. \tag{4.16}$$

Comparison of (4.11), subject to (4.2), and (4.16) yields (4.10), the understanding being that

$$C_{2(j-1)}^{j-1} = \frac{(2j-2)!}{\left[(j-1)!\right]^2} = \frac{(2j-2)(2j-3)\ldots(j+1)j}{(j-1)!}. \tag{4.17}$$

QED.•

Comment 4.2. For successive values of j from 2 through 7, equation (4.10) recovers the values of 'τ_j' as given in (4.3) and (4.9).•

Comment 4.3. The proof of Lemma 4.1 applies, *mutatis mutandis*, with ',' '⟨', and '⟩' in place of '◊', '[', and ']', respectively. Hence, in accordance with Theorem 4.1, *the number of iterative* (j-1)*-fold ordered pairs with a given basis equals* τ_j *as defined by (4.10)*. •

4.2. The principal occurrence of ◊ in a composite term

Definition 4.1. 1) For each $j \in \omega_1$, denoted by 'j': each one of the symbols 'κ_j', 'λ_j', 'μ_j', 'ν_j', 'ξ_j', 'ρ_j' is a variable whose range of values is the set ω_{1,τ_j}. The base letter of any one of the above variables can be provided with any number of primes or with an alpha-numeric superscript, thus giving another variable with the same range.

2) Item 1 applies with any natural-number-valued variable, as 'i', 'k', 'l', 'm', 'n', in place of 'j'.•

Corollary 4.1. With '1' or '2' in place of 'j', each one of the symbols as introduced in item 1 of Definition 4.1 is a constant denoting the natural number 1; that is, e.g.,

$$\kappa_1 = \kappa_2 = 1. \tag{4.18}$$

Proof: The corollary follows from Definition 4.1, because $\tau_1 = \tau_2 = 1$, by (4.2) and (4.3).•

Definition 4.2. For each $j \in \omega_3$, all composite terms with any given *j*-term basis are said to be *numbered in the universal order*, or briefly *universally ordered*, with natural integers of the set $\omega_{1,j}$ if and only if the ordinal number of each composite term uniquely identifies the congruous arrangement of *j*–1 pairs of square brackets in that term independent of the primitive terms forming the basis. In this case, $\left[x_i \Diamond x_{i+1} \Diamond \ldots \Diamond x_{i+j-2} \Diamond x_{i+j-1}\right]_{\lambda_j}$, e.g., is a variable, which takes on, as its value, the *j*-ary term over the string (3.3) of a number λ_j; and similarly with 'x'' in place of 'x' subject to the definitions (3.5).•

Corollary 4.2. For each $i \in \omega_1$,

$$[x_i]_1 \to [x_i] \to x_i . \; [x_i \Diamond x_{i+1}]_1 \to [x_i \Diamond x_{i+1}]. \tag{4.19}$$

Proof: The corollary follows from Definition 4.2 by Corollary 4.1.•

Definition 4.3. For each $i \in \omega_1$,

1) $y_{i,i} \to x_i$.

2) $y_{i,i+j-1} \to x_i \Diamond x_{i+1} \Diamond \ldots \Diamond x_{i+j-2} \Diamond x_{i+j-1}$ for each $j \in \omega_2$.•

Corollary 4.3. For each $i \in \omega_1$,

1) $[y_{i,i}]_1 \leftrightarrow [x_i]_1 \leftrightarrow x_i \leftrightarrow y_{i,i}$,

2) $\lfloor y_{i,i+1} \rfloor_1 \leftrightarrow [x_i \Diamond x_{i+1}]_1 \leftrightarrow [x_i \Diamond x_{i+1}]$,

3) $\lfloor y_{i,i+j-1} \rfloor_{\kappa_j} \leftrightarrow \lfloor x_i \Diamond x_{i+1} \Diamond \ldots \Diamond x_{i+j-2} \Diamond x_{i+j-1} \rfloor_{\kappa_j}$ for each $j \in \omega_2$ and each $\kappa_j \in \omega_{1,\tau_j}$,

the understanding being that the items 2 and 3 are instances of the item at $j \stackrel{.}{=} 1$ and $j \stackrel{.}{=} 2$ respectively. In accordance with Definition 2.1(4), the sign $\leftrightarrow$ is a bilateral definition sign such that the symbols standing on both sides of it can be used interchangeably.

Proof: The corollary follows from Definition 4.2 by Definitions 2.1(4,5) and 4.1 and by Corollary 4.2.•

Theorem 4.2. For each $i \in \omega_1$, for each $j \in \omega_2$, for each $\kappa_j \in \omega_{1,\tau_j}$, there are exactly one $k \in \omega_{1,j-1}$, exactly one $\lambda_k \in \omega_{1,\tau_k}$, and exactly one $\mu_{j-k} \in \omega_{1,\tau_{j-k}}$ such that the composite term $\left[y_{i,i+j-1}\right]_{\kappa_j}$ is defined as

$$\left[y_{i,i+j-1}\right]_{\kappa_j} \leftrightarrow \left[\left[y_{i,i+k-1}\right]_{\lambda_k} \Diamond \left[y_{i+k,i+j-1}\right]_{\mu_{j-k}}\right]. \tag{4.20}$$

The occurrence of ◊ between $\left[y_{i,i+k-1}\right]_{\lambda_k}$ and $\left[y_{i+k,i+j-1}\right]_{\mu_{j-k}}$ is evidently the principal occurrence of ◊ in the term $\left[y_{i,i+j-1}\right]_{\kappa_j}$.

Proof: The corollary follows from Definition 4.1 by Definition 4.2 and Corollaries 4.2 and 4.3.•

4.3. Two recursive methods of numbering of the composite terms with a given basis

There are $\tau_j!$ different ways to order all *j*-ary terms over the string (3.3). For proving the generalized associative and commutative laws for composite terms, it is sufficient just to assume that for each $j \in \omega_3$ there exists a certain universal order of the *j*-ary terms with a given basis, i.e. an order which allows identifying each term by its ordinal number. Any one of $\tau_j!$ orders of the *j*-ary terms can in principle be selected as the universal one, while all other orders should be disregarded. Still, in accordance with (4.10), the number τ_j rapidly increases with *j* (see, e.g., (4.3) and (4.9)), so that the corresponding number $\tau_j!$ becomes enormous. Therefore, starting from the value 5 of '*j*', the problem of numbering the *j*-ary terms *in extension* (*in entelechy, in actuality*) is practically unsolvable. Two algorithms of recursively numbering the *j*-ary terms *in intension* (*in potency*) are suggested below. One algorithm will be called *the method of numbering the composite terms by association of the square brackets to the left* or *the method of numbering the composite terms by association of the principal composition operator ◊ to the right.* Briefly, this algorithm will be called *the first algorithm* or *the first method*. The other algorithm will be called *the method of numbering the composite terms by association of the square brackets to the right* or *the method of numbering the composite terms by association of the principal composition operator ◊ to the left.* Briefly, this algorithm will be called *the second algorithm* or *the second method*. Every variable whose range is the j-ary terms numbered is accordance with the first method will carry a roman English capital subscript 'L'. Every variable whose range is the j-ary terms numbered in accordance with the first method will carry a roman English capital subscript 'R'. In what follows, the two algorithms are introduced heuristically with the help of simple examples.

By Corollary 4.2 and Definition 4.3, there is a unique way to number both a singular term ($j \doteqdot 1$) and a binary term ($j \doteqdot 2$) independent of any numbering method; namely, for each $i \in \omega_1$:

$$\lfloor y_{i,i} \rfloor_1 \to \lfloor y_{i,i} \rfloor \to y_{i,i} \leftrightarrow x_i \leftarrow [x_i] \leftarrow [x_i]_1, \tag{4.21}$$

$$\lfloor y_{i,i+1} \rfloor_1 \to \lfloor y_{i,i+1} \rfloor \leftrightarrow [x_i \Diamond x_{i+1}] \leftarrow [x_i \Diamond x_{i+1}]_1. \tag{4.22}$$

At the same time, in accordance with the two methods of numbering, the ternary ($j \doteqdot 3$) and quaternary ($j \doteqdot 4$) terms are ordered as follows:

$$\begin{gathered}
\lfloor y_{i.i+2} \rfloor_{\mathrm{L1}} \to [[x_i \Diamond x_{i+1}] \Diamond x_{i+2}] \leftarrow \lfloor y_{i.i+2} \rfloor_{\mathrm{R2}}, \\
\lfloor y_{i.i+2} \rfloor_{\mathrm{L1}} \to [x_i \Diamond x_{i+1} \Diamond x_{i+2}]_{\mathrm{L2}} \to [x_i \Diamond [x_{i+1} \Diamond x_{i+2}]] \\
\leftarrow [x_i \Diamond x_{i+1} \Diamond x_{i+2}]_{\mathrm{R1}} \leftarrow \lfloor y_{i.i+2} \rfloor_{\mathrm{R1}},
\end{gathered} \tag{4.23}$$

$$\begin{gathered}
\lfloor y_{i.i+3} \rfloor_{\mathrm{L1}} \to [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{L1}} \to [[[x_i \Diamond x_{i+1}] \Diamond x_{i+2}] \Diamond x_{i+3}] \\
\leftarrow [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{R5}} \leftarrow \lfloor y_{i.i+3} \rfloor_{\mathrm{R5}}, \\
\lfloor y_{i.i+3} \rfloor_{\mathrm{L2}} \to [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{L2}} \to [[x_i \Diamond [x_{i+1} \Diamond x_{i+2}]] \Diamond x_{i+3}] \\
\leftarrow [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{R4}} \leftarrow \lfloor y_{i.i+3} \rfloor_{\mathrm{R4}}, \\
\lfloor y_{i.i+3} \rfloor_{\mathrm{L3}} \to [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{L3}} \to [[x_i \Diamond x_{i+1}] \Diamond [x_{i+2} \Diamond x_{i+3}]] \\
\leftarrow [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{R3}} \leftarrow \lfloor y_{i.i+3} \rfloor_{\mathrm{R3}}, \\
\lfloor y_{i.i+3} \rfloor_{\mathrm{L4}} \to [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{L4}} \to [x_i \Diamond [[x_{i+1} \Diamond x_{i+2}] \Diamond x_{i+3}]] \\
\leftarrow [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{R2}} \leftarrow \lfloor y_{i.i+3} \rfloor_{\mathrm{R2}}, \\
\lfloor y_{i.i+3} \rfloor_{\mathrm{L5}} \to [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{L5}} \to [x_i \Diamond [x_{i+1} \Diamond [x_{i+2} \Diamond x_{i+3}]]] \\
\leftarrow [x_i \Diamond x_{i+1} \Diamond x_{i+2} \Diamond x_{i+3}]_{\mathrm{R1}} \leftarrow \lfloor y_{i.i+3} \rfloor_{\mathrm{R1}},
\end{gathered} \tag{4.24}$$

in compliance with Definition 4.3 and Corollary 4.3. These two methods of ordering (successively numbering) ternary and quaternary terms over the respective strings (3.3) with j=3 and j=4 can be generalized for any given $i \in \omega_1$ and any given $j \in \omega_4$ as follows.

Given $m \in \omega_2$, given $j \in \omega_{m+1}$, for each $\kappa_{\mathrm{L}m} \in \omega_{1,\tau_m}$,

$$\begin{gathered}
\lfloor y_{i,i+j-1} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor y_{i,i+m} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor\lfloor y_{i,i+m-1} \rfloor_{\mathrm{L}\kappa_m} \Diamond x_{i+m} \rfloor \\
\leftarrow \lfloor y_{i,i+m} \rfloor_{\mathrm{R}(\tau_{m+1}-\kappa_{\mathrm{L}m}+1)} \leftarrow \lfloor y_{i,i+j-1} \rfloor_{\mathrm{R}(\tau_j-\kappa_{\mathrm{L}m}+1)} \text{ if } j \doteqdot m+1, \\
\lfloor y_{i,i+j-1} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor y_{i,i+m+1} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor\lfloor\lfloor y_{i,i+m-1} \rfloor_{\mathrm{L}\kappa_m} \Diamond x_{i+m} \rfloor \Diamond x_{i+m+1} \rfloor \\
\leftarrow \lfloor y_{i,i+m+1} \rfloor_{\mathrm{R}(\tau_{m+2}-\kappa_{\mathrm{L}m}+1)} \leftarrow \lfloor y_{i,i+j-1} \rfloor_{\mathrm{R}(\tau_j-\kappa_{\mathrm{L}m}+1)} \text{ if } j \doteqdot m+2, \\
\lfloor y_{i,i+j-1} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor\lfloor \ldots \lfloor y_{i,i+m-1} \rfloor_{\mathrm{L}\kappa_m} \Diamond x_{i+m} \rfloor \Diamond \ldots x_{i+j-2} \rfloor \Diamond x_{i+j-1} \rfloor \\
\leftarrow \lfloor y_{i,i+j-1} \rfloor_{\mathrm{R}(\tau_j-\kappa_{\mathrm{L}m}+1)} \text{ if } j \in \omega_{m+3}.
\end{gathered} \tag{4.25}$$

Particularly, at $i \doteqdot 1$ and $j \doteqdot n$, for each $n \in \omega_{m+1}$, definitions (4.25) reduce to

$$\begin{gathered}
\lfloor y_{1,n} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor y_{1,m+1} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor\lfloor y_{1,m} \rfloor_{\mathrm{L}\kappa_m} \Diamond x_{m+1} \rfloor \\
\leftarrow \lfloor y_{1,im+1} \rfloor_{\mathrm{R}(\tau_{m+1}-\kappa_{\mathrm{L}m}+1)} \leftarrow \lfloor y_{1,n} \rfloor_{\mathrm{R}(\tau_n-\kappa_{\mathrm{L}m}+1)} \text{ if } n \doteqdot m+1, \\
\lfloor y_{1,n} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor y_{1,m+2} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor\lfloor\lfloor y_{1,m} \rfloor_{\mathrm{L}\kappa_m} \Diamond x_{m+1} \rfloor \Diamond x_{m+2} \rfloor \\
\leftarrow \lfloor y_{1,m+2} \rfloor_{\mathrm{R}(\tau_{m+2}-\kappa_{\mathrm{L}m}+1)} \leftarrow \lfloor y_{i,i+j-1} \rfloor_{\mathrm{R}(\tau_n-\kappa_{\mathrm{L}m}+1)} \text{ if } n \doteqdot m+2, \\
\lfloor y_{1,n} \rfloor_{\mathrm{L}\kappa_{\mathrm{L}m}} \to \lfloor\lfloor \ldots \lfloor y_{1,m} \rfloor_{\mathrm{L}\kappa_m} \Diamond x_{m+1} \rfloor \Diamond \ldots x_{n-1} \rfloor \Diamond x_n \rfloor \\
\leftarrow \lfloor y_{1,n} \rfloor_{\mathrm{R}(\tau_j-\kappa_{\mathrm{L}m}+1)} \text{ if } n \in \omega_{m+3}.
\end{gathered} \tag{4.26}$$

In this case, given $j \in \omega_3$, denoted by '*j*', '$\kappa_{\mathrm{L}j}$' is a variable that takes on the ordinal numbers of the *j*-ary terms from 1 to τ_j in accordance with the first algorithm, while '$\kappa_{\mathrm{R}j}$' is the like variable for the second algorithm, the understanding being that

$$\kappa_{\mathrm{R}j} \doteqdot \tau_j - \kappa_{\mathrm{L}j} + 1, \tag{4.27}$$

in compliance with (4.23)–(4.25); and similarly with 'λ', 'μ', 'ν', 'ξ', or 'ρ' in place of 'κ', in compliance with Definition 4.1. Thus, in the framework of the second method, the same *j*-ary terms are numbered in the opposite direction. Since the same numbers from 1 to τ_j are employed in both ordering methods, therefore the two methods are incompatible in the sense that they cannot be applied to the same *j*-ary terms simultaneously.•

Convention 4.1. In the sequel, I shall adopt the first algorithm, so that for each $j \in \omega_3$,

$$\kappa_j \doteqdot \kappa_{\mathrm{L}j} \in \tau_j, \tag{4.28}$$

and similarly with 'λ', 'μ', 'ν', 'ξ', or 'ρ' in place of 'κ', in compliance with Definition 4.1.•

By the pertinent instances of Axiom 3.3(1), it follows from (4.23) and (4.24) that

$$[[x_i \Diamond x_{i+1}] \Diamond x_{i+2}] \sim [x_i \Diamond [x_{i+1} \Diamond x_{i+2}]] \tag{4.29}$$

$$\begin{gathered}
[[[x_i \Diamond x_{i+1}] \Diamond x_{i+2}] \Diamond x_{i+3}] \sim [[x_i \Diamond [x_{i+1} \Diamond x_{i+2}]] \Diamond x_{i+3}], \\
[[[x_i \Diamond x_{i+1}] \Diamond x_{i+2}] \Diamond x_{i+3}] \sim [[[x_i \Diamond x_{i+1}] \Diamond [x_{i+2} \Diamond x_{i+3}]]] \sim [x_i \Diamond [x_{i+1} \Diamond [x_{i+2} \Diamond x_{i+3}]]], \\
[x_i \Diamond [[x_{i+1} \Diamond x_{i+2}] \Diamond x_{i+3}]] \sim [x_i \Diamond [x_{i+1} \Diamond [x_{i+2} \Diamond x_{i+3}]]].
\end{gathered} \tag{4.30}$$

Consequently, by the pertinent instances of Axiom 3.2(3), it follows from (4.30) that all five composite terms occurring in (4.24) are mutually (pairwise) equivalent. Proceeding from this fact and from (4.29) at $i \doteq 1$, the equivalence of all *j*-ary terms over the string (3.3) is proved in the next subsection.

4.4. The generalized associative law

Theorem 4.3: ***The generalized associative law for* $\Diamond$ *with respect to* $\sim$.** Under Axiom 3.3, for *each* $n \in \omega_3$, for each $\kappa_n \in \omega_{1,\tau_n}$, for each $\kappa'_n \in \omega_{1,\tau_n}$:

$$\lfloor y_{1,n} \rfloor_{\kappa_n} \sim \lfloor y_{1,n} \rfloor_{\kappa'_n}, \tag{4.31}$$

the understanding being that (4.30) is not trivial if and only if $\kappa_n \neq \kappa'_n$.

Proof: The theorem will be proved by induction with respect to values of the variable 'n'. In accordance with (4.29) and (4.30), the *induction hypothesis* can most generally be stated thus: Let for *some* $n \in \omega_3$, for each $\kappa_n \in \omega_{1,\tau_n}$, for each $\kappa'_n \in \omega_{1,\tau_n}$, (4.31) holds. Therefore, by Axiom 3.3(3), under the same quantifiers

$$[\lfloor y_{1,n} \rfloor_{\kappa_n} \Diamond x_{n+1}] \sim [\lfloor y_{1,n} \rfloor_{\kappa'_n} \Diamond x_{n+1}], \tag{4.32}$$

Hence, by Definition 4.3 and Axiom 3.3(1), above hypothesis implies for *some* $n \in \omega_3$, , for each $\kappa_n \in \omega_{1,\tau_n}$, for each $\kappa'_n \in \omega_{1,\tau_n}$, for each $j \in \omega_{1,n}$, for each $\kappa_j \in \omega_{1,\tau_j}$, for each $\lambda_{n-j+1} \in \omega_{1,\tau_{n-j+1}}$, for each $\kappa'_j \in \omega_{1,\tau_j}$, for each $\lambda'_{n-j+1} \in \omega_{1,\tau_{n-j+1}}$:

$$\begin{gathered}
[\lfloor y_{1,n} \rfloor_{\kappa_n} \Diamond x_{n+1}] \sim [\lfloor y_{1,j} \rfloor_{\kappa_j} \Diamond \lfloor y_{j+1,n} \Diamond x_{n+1} \rfloor_{\lambda_{n-j+1}}] \sim [\lfloor y_{1,j} \rfloor_{\kappa_j} \Diamond \lfloor y_{j+1,n+1} \rfloor_{\lambda_{n-j+1}}], \\
[\lfloor y_{1,n} \rfloor_{\kappa'_n} \Diamond x_{n+1}] \sim [\lfloor y_{1,j} \rfloor_{\kappa'_j} \Diamond \lfloor y_{j+1,n} \Diamond x_{n+1} \rfloor_{\lambda'_{n-j+1}}] \sim [\lfloor y_{1,j} \rfloor_{\kappa'_j} \Diamond \lfloor y_{j+1,n+1} \rfloor_{\lambda'_{n-j+1}}],
\end{gathered} \tag{4.33}$$

while by Theorem 4.2 there are exactly one $\kappa_{n+1} \in \omega_{1,\tau_{n+1}}$ and exactly one $\kappa'_{n+1} \in \omega_{1,\tau_{n+1}}$ such that

$$[\lfloor y_{1,j} \rfloor_{\kappa_j} \Diamond \lfloor y_{j+1,n+1} \rfloor_{\lambda_{n-j+1}}] \sim \lfloor y_{1,n+1} \rfloor_{\kappa_{n+1}}, [\lfloor y_{1,j} \rfloor_{\kappa'_j} \Diamond \lfloor y_{j+1,n} \Diamond x_{n+1} \rfloor_{\lambda'_{n-j+1}}] \sim \lfloor y_{1,n+1} \rfloor_{\kappa'_{n+1}}. \tag{4.34}$$

Thus, by Axiom 3.2(3), it follows from (4.32)–(4.34), that for the given $n \in \omega_3$, for each $\kappa_{n+1} \in \omega_{1,\tau_{n+1}}$, for each $\kappa'_{n+1} \in \omega_{1,\tau_{n+1}}$:

$$\lfloor y_{1,n+1} \rfloor_{\kappa_{n+1}} \sim \lfloor y_{1,n+1} \rfloor_{\kappa'_{n+1}}, \tag{4.35}$$

the understanding being that (4.31) is not trivial if and only if $\kappa_{n+1} \neq \kappa'_{n+1}$. QED.•

Corollary 4.4. Under Axiom 3.3, for each $n \in \omega_3$: for each $\kappa_n \in \omega_{2,\tau_n}$:

$$\lfloor y_{1,n} \rfloor_{\kappa_n} \sim \lfloor y_{1,n} \rfloor_1, \tag{4.36}$$

the understanding being that

$$\begin{gathered} \lfloor y_{1,n} \rfloor_1 \leftrightarrow [x_1 \Diamond x_2 \Diamond \ldots \Diamond x_{n-1} \Diamond x_n]_1 \leftrightarrow \Diamond(x_1, x_2, \ldots, x_{n-1}, x_n) \\ \leftrightarrow \left[\mathop{\Diamond}_{k=1}^{n} x_k \right] \rightarrow \left[\left[\mathop{\Diamond}_{k=1}^{n-1} x_k \right] \Diamond x_n \right] \rightarrow [[[\ldots[[x_1 \Diamond x_2] \Diamond x_3] \ldots \Diamond x_{n-2}] \Diamond x_{n-1}] \Diamond x_n], \end{gathered} \tag{4.37}$$

by Corollary 4.3(3) at $i \doteqdot 1$ and $j \doteqdot n$, and also by (3.7), (4.26), and (4.28).

Proof: (4.36) is the instance of (4.31) at $\kappa'_n \doteqdot 1$.•

5. The generalized commutative law for an associative and commutative binary composition operator

Definition 5.1. For each $n \in \omega_2$, the sequence i_1, i_2, ..., i_{n-1}, i_n is any one of the n! permutations of the n successive numbers 1, 2, ..., n–1, n, and, hence, the sequence x_{i_1}, x_{i_2}, ..., $x_{i_{n-1}}$, x_{i_n} is the corresponding permutation of the n successive primitive terms x_1, x_2, ..., x_{n-1}, x_n.•

Corollary 5.1. Under Definition 5.1,

1) $y_{i_k,i_k} \rightarrow x_{i_k}$ for each $k \in \omega_{1,n}$.

2) $y_{i_1,i_n} \rightarrow x_{i_1} \Diamond x_{i_2} \Diamond \ldots \Diamond x_{i_{n-1}} \Diamond x_{i_n}$.

Proof: At $i \doteqdot 1$ and $j \doteqdot n$, items 1 and 2 of Definition 4.3 reduce to

$$y_{1,1} \rightarrow x_1,$$

$$y_{1,n} \rightarrow x_1 \Diamond x_2 \Diamond \ldots \Diamond x_{n-1} \Diamond x_n \text{ for each } n \in \omega_2.$$

Items 1 and 2 of the corollary are the variants of the last two definitions with 'i_1', 'i_2', ..., 'i_{n-1}', 'i_n' in place of '1', '2', ..., 'n–1', 'n', respectively. Hence, the corollary follows from Definition 4.3 by Definition 5.1.•

Corollary 5.2. Under Definition 5.1,

1) $\left[y_{i_k,i_k} \right]_1 \leftrightarrow \left[x_{i_k} \right]_1 \leftrightarrow x_{i_k} \leftrightarrow y_{i_k,i_k}$ for each $k \in \omega_{1,n}$.

2) $\lfloor y_{i_k,i_{k+1}} \rfloor_1 \leftrightarrow \lfloor x_{i_k} \Diamond x_{i_{k+1}} \rfloor_1 \leftrightarrow \lfloor x_{i_k} \Diamond x_{i_{k+1}} \rfloor$ for each $k \in \omega_{1,n-1}$.

3) $\left[y_{i_1,i_n} \right]_{\kappa_n} \leftrightarrow \left[x_{i_1} \Diamond x_{i_2} \Diamond \ldots \Diamond x_{i_{n-1}} \Diamond x_{i_n} \right]_{\kappa_n}$ for each $\kappa_n \in \omega_{1,\tau_n}$.

Proof: At $i \equiv 1$ and $j \equiv n$, items 1–3 of Corollary 4.3 reduce to

$$\left[y_{1,1}\right]_1 \leftrightarrow \left[x_1\right]_1 \leftrightarrow x_1 \leftrightarrow y_{1,1},$$

$$\left[y_{i,i+1}\right]_1 \leftrightarrow \left[x_i \Diamond x_{i+1}\right]_1 \leftrightarrow \left[x_i \Diamond x_{i+1}\right],$$

$$\left[y_{1,n}\right]_{\kappa_n} \leftrightarrow \left[x_1 \Diamond x_2 \Diamond \ldots \Diamond x_{n-2} \Diamond x_n\right]_{\kappa_n} \text{ for each } \kappa_n \in \omega_{1,\tau_n}.$$

Items 1–3 of the corollary are the variants of the last three definitions with 'i_1', 'i_2', ..., 'i_{n-1}', 'i_n' in place of '1', '2', ..., 'n–1', 'n', respectively. Hence, the corollary follows from Corollary 4.3 by Definition 5.1.•

Corollary 5.3. For each $n \in \omega_2$,

$$\begin{aligned}&\mathop{\Diamond}_{k=1}^{n} x_{i_k} \to \left[\mathop{\Diamond}_{k=1}^{n} x_{i_k}\right] \to \left[\left[\mathop{\Diamond}_{k=1}^{n-1} x_{i_k}\right] \Diamond x_{i_n}\right] \\ &\to \left[\left[\left[\ldots\left[\left[x_{i_1} \Diamond x_{i_2}\right] \Diamond x_{i_3}\right] \ldots \Diamond x_{i_{n-2}}\right] \Diamond x_{i_{n-1}}\right] \Diamond x_{i_n}\right].\end{aligned} \tag{5.1}$$

Proof: (5.1) is the variant of (3.7) with 'i_1', 'i_2', ..., 'i_{n-1}', 'i_n' in place of '1', '2', ..., 'n–1', 'n' respectively.•

Definition 5.2. For each $n \in \omega_2$,

$$\left[y_{i_1,i_n}\right]_1 \to \left[x_{i_1} \Diamond x_{i_2} \Diamond \ldots \Diamond x_{i_{n-1}} \Diamond x_{i_n}\right]_1 \to \mathop{\Diamond}_{k=1}^{n} x_{i_k} \tag{5.2}$$

subject to (5.1). Particularly,

$$\left[y_{1,n}\right]_1 \to \left[x_1 \Diamond x_2 \Diamond \ldots \Diamond x_{n-1} \Diamond x_n\right]_1 \to \mathop{\Diamond}_{k=1}^{n} x_k, \tag{5.3}$$

subject to (3.6).•

Comment 5.1. By (4.35), Definition 5.2 holds automatically if the n-ary terms with a given basis are numbered in accordance with the first recursive method. However, the method, by which the n-ary terms are numbered is irrelevant both to the proof of Theorem 4.3 and to the proof of the following Theorem 5.1. Therefore, Definition 5.2 can be regarded as an isolated definition of the first n-ary term, which has been made for convenience in making further statements. The remaining $\tau_n - 1$ n-ary terms are supposed to be numbered in an arbitrary, but fixed way.•

Theorem 5.1: ***The generalized commutative law for $\Diamond$ with respect to $\sim$.*** Under Axioms 3.2 and 3.3, for each $n \in \omega_2$, for each $\kappa_n \in \omega_{1,\tau_n}$,

$$\left[y_{i_1,i_n}\right]_{\kappa_n} \sim \left[y_{1,n}\right]_1. \tag{5.4}$$

the understanding being that (5.4) is not trivial if and only if the sequence x_{i_1}, x_{i_2}, ..., $x_{i_{n-1}}$, x_{i_n} differs from the sequence x_1, x_2, ..., x_{n-1}, x_n and $\kappa_n \neq 1$.

Proof: At $n \doteqdot 2$, (5.4) becomes

$$\left[y_{i_1,i_2}\right]_1 \sim \left[y_{1,2}\right]_1, \tag{5.5}$$

which reduces to

$$\left[x_2 \Diamond x_1\right] \sim \left[x_1 \Diamond x_2\right], \tag{5.6}$$

by Corollary 4.3(2) at $i \doteqdot 1$ and also by Definition 5.1 and Corollary 5.2(2). Relation (5.6) is true because it is an instance of Axiom 3.3(2). The theorem will therefore be proved by induction on 'n' from the following *induction hypothesis*.

For *some* $n \in \omega_2$, for each $m \in \omega_{2,n}$, for each $\kappa_m \in \omega_{1,\tau_m}$,

$$\left[y_{i_1,i_m}\right]_{\kappa_m} \sim \left[y_{1,m}\right]_1, \tag{5.7}$$

the understanding being that (5.7) is not trivial if and only if the sequence x_{i_1}, x_{i_2}, ..., $x_{i_{n-1}}$, x_{i_m} differs from the sequence x_1, x_2, ..., x_{m-1}, x_m and $\kappa_m \neq 1$.

Given *n* of the induction hypothesis, let the sequence

$$x_{j_1},\ x_{j_2},\ ...,\ x_{j_n},\ x_{j_{n+1}} \tag{5.8}$$

be, in accordance with Definition 5.1, any one of the (*n*+1)! permutations of the *n*+1 successive primitive terms x_1, x_2, ..., x_n, x_{n+1}. Let for each $\lambda_{n+1} \in \omega_{1,\tau_{n+1}}$, $\left[y_{j_1,j_{n+1}}\right]_{\lambda_{n+1}}$ be an (*n*+1)-ary term with the basis (5.8), i.e.

$$\left[y_{j_1,j_{n+1}}\right]_{\lambda_{n+1}} \to \left[x_{j_1} \Diamond x_{j_2} \Diamond ... \Diamond x_{j_n} \Diamond x_{j_{n+1}}\right]_{\lambda_{n+1}}. \tag{5.9}$$

Let '*s*' be an integer-valued variable such that $s \in \omega_{1,n+1}$. Given $s \in \omega_{1,n+1}$, let $j_s \doteqdot n+1$ and hence

$$x_{j_s} \to x_{n+1}. \tag{5.10}$$

Depending on the value of '*s*', any given (*n*+1)-ary term $\left[y_{j_1,j_{n+1}}\right]_{\lambda_{n+1}}$ subject to (5.9) can be developed as follows.

1) $s \doteqdot 1$. In this case, $j_1 \doteqdot n+1$ so that $x_{j_1} \to x_{n+1}$ by (5.10). Hence,

$$y_{j_1,j_{n+1}} \to x_{n+1} \Diamond y_{j_2,j_{n+1}}, \tag{5.11}$$

where

$$y_{j_2,j_{n+1}} \to x_{j_2} \Diamond x_{j_3} \Diamond ... \Diamond x_{j_n} \Diamond x_{j_{n+1}}. \tag{5.12}$$

By (5.11), it follows from Theorem 4.3 that for each $\lambda_{n+1} \in \omega_{1,\tau_{n+1}}$: for each $\mu_n \in \omega_{1,\tau_n}$:

$$\left[x_{n+1} \Diamond y_{j_2,j_{n+1}}\right]_{\lambda_{n+1}} \sim \left[x_{n+1} \Diamond \left[y_{j_2,j_{n+1}}\right]_{\mu_n}\right], \tag{5.13}$$

while, by Axiom 3.3(2), for each $\mu_n \in \omega_{1,\tau_n}$:

$$\left[x_{n+1} \Diamond \left[y_{j_2,j_{n+1}}\right]_{\mu_n}\right] \sim \left[\left[y_{j_2,j_{n+1}}\right]_{\mu_n} \Diamond x_{n+1}\right]. \tag{5.14}$$

The sequence x_{j_2}, x_{j_3},..., x_{j_n}, $x_{j_{n+1}}$ is evidently a permutation of the sequence x_1, x_2, ..., x_{n-1}, x_n. Therefore, by the induction hypothesis,

$$\left[y_{j_2,j_{n+1}}\right]_{\mu_n} \sim \left[y_{1,n}\right]_1 \text{ for each } \mu_n \in \omega_{1,\tau_n}. \tag{5.15}$$

By Definition 5.2, it follows that

$$\left[\left[y_{1,n}\right]_1 \Diamond x_{n+1}\right] \leftrightarrow \left[\left[\mathop{\Diamond}_{k=1}^{n} x_k\right] \Diamond x_{n+1}\right] \leftrightarrow \mathop{\Diamond}_{k=1}^{n+1} x_k \leftrightarrow \left[y_{1,n+1}\right]_1. \tag{5.16}$$

Since $x_{n+1} \leftrightarrow x_{j_1}$, therefore by (5.15) and (5.16), it follows from (5.14) that

$$\left[y_{j_1,j_{n+1}}\right]_{\lambda_{n+1}} \sim \left[y_{1,n+1}\right]_1 \text{ for each } \lambda_{n+1} \in \omega_{1,\tau_{n+1}}. \tag{5.17}$$

2) $s \in \omega_{2,n}$. In this case, by (5.10),

$$y_{j_1,j_{n+1}} \to y_{j_1,j_{s-1}} \Diamond x_{n+1} \Diamond y_{j_{s+1},j_{n+1}}, \tag{5.18}$$

where

$$y_{j_1,j_{s-1}} \to x_{j_1} \Diamond x_{j_2} \Diamond \ldots \Diamond x_{j_{s-2}} \Diamond x_{j_{s-1}}, \tag{5.19}$$

$$y_{j_{s+1},j_{n+1}} \to x_{j_{s+1}} \Diamond x_{j_{s+2}} \Diamond \ldots \Diamond x_{j_n} \Diamond x_{j_{n+1}}. \tag{5.20}$$

By (5.18), it follows from Theorem 4.3 that for each $\lambda_{n+1} \in \omega_{1,\tau_{n+1}}$, for each $\mu_n \in \omega_{1,\tau_n}$, for each $\nu_{n-s+1} \in \omega_{1,\tau_{n-s+1}}$:

$$\left[y_{j_1,j_{n+1}}\right]_{\lambda_{n+1}} \sim \left[\left[y_{j_1,j_{s-1}}\right]_{\mu_{s-1}} \Diamond \left[x_{n+1} \Diamond \left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}}\right]\right], \tag{5.21}$$

while by Axiom 3.3(2) for each $\nu_{n-s+1} \in \omega_{1,\tau_{n-s+1}}$:

$$\left[x_{n+1} \Diamond \left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}}\right] \sim \left[\left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}} \Diamond x_{n+1}\right]. \tag{5.22}$$

Hence, (5.21) becomes

$$\left[y_{j_1,j_{n+1}}\right]_{\lambda_{n+1}} \sim \left[\left[y_{j_1,j_{s-1}}\right]_{\mu_{s-1}} \Diamond \left[\left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}} \Diamond x_{n+1}\right]\right]. \tag{5.23}$$

while

$$\begin{aligned}&\left[\left[y_{j_1,j_{s-1}}\right]_{\mu_{s-1}}\Diamond\left[\left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}}\Diamond x_{n+1}\right]\right]\\&\sim\left[\left[\left[y_{j_1,j_{s-1}}\right]_{\mu_{s-1}}\Diamond\left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}}\right]\Diamond x_{n+1}\right].\end{aligned}\tag{5.24}$$

by Axiom 3.3(1). At the same time, from the pertinent variant of Corollary 4.4, it follows that

$$\begin{aligned}&\left[\left[y_{j_1,j_{s-1}}\right]_{\mu_{s-1}}\Diamond\left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}}\right]\\&\sim\left[x_{j_1}\Diamond x_{j_2,j_{s-1}}\ldots\Diamond x_{j_{s-2}}\Diamond x_{j_{s-1}}\Diamond x_{j_{s+1}}\Diamond x_{j_{s+2}}\Diamond\ldots\Diamond x_{j_n}\Diamond x_{j_{n+1}}\right]_1.\end{aligned}\tag{5.25}$$

The sequence

$$x_{j_1},\ x_{j_2},\ldots,\ x_{j_{s-2}},\ x_{j_{s-1}},\ x_{j_{s+1}},\ x_{j_{s+2}},\ \ldots,\ x_{j_n},\ x_{j_{n+1}}$$

is evidently a permutation of the sequence x_1, x_2, ..., x_{n-1}, x_n. Therefore, by the induction hypothesis,

$$\left[x_{j_1}\Diamond x_{j_2}\Diamond\ldots\Diamond x_{j_{s-2}}\Diamond x_{j_{s-1}}\Diamond x_{j_{s+1}}\Diamond x_{j_{s+2}}\Diamond\ldots\Diamond x_{j_n}\Diamond x_{j_{n+1}}\right]_1\sim\left[y_{1,n}\right]_1.\tag{5.26}$$

Hence, (5.25) becomes

$$\left[\left[y_{j_1,j_{s-1}}\right]_{\mu_{s-1}}\Diamond\left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}}\right]\sim\left[y_{1,n}\right]_1.\tag{5.27}$$

By (5.24) and (5.27), it follows from (5.23) that

$$\left[y_{j_1,j_{n+1}}\right]_{\lambda_{n+1}}\sim\left[\left[y_{1,n}\right]_1\Diamond x_{n+1}\right],\tag{5.28}$$

which reduces to (5.17).

In this connection, the following remark should be made. If $s \stackrel{=}{=} n$ then $j_n \stackrel{=}{=} n+1$, so that $x_{j_n} \to x_{n+1}$. In this case, by Corollary 5.2(1),

$$\left[y_{j_{s+1},j_{n+1}}\right]_{\nu_{n-s+1}}\leftrightarrow\left[y_{j_{n+1},j_{n+1}}\right]_{\nu_1}\leftrightarrow\left[y_{j_{n+1},j_{n+1}}\right]_1\leftrightarrow x_{j_{n+1}},\tag{5.29}$$

the understanding being that necessarily $\nu_{n-s+1} \stackrel{=}{=} \nu_1 \stackrel{=}{=} 1$. Hence, owing to the notation used, the above proof of (5.17) from the induction hypothesis, subject to $s\in\omega_{2,n}$, has no peculiarities in the particular case where $s \stackrel{=}{=} n$.

3) $s \stackrel{=}{=} n+1$. In this case, $j_{n+1} \stackrel{=}{=} n+1$, so that $x_{j_{n+1}} \to x_{n+1}$. Hence,

$$y_{j_1,j_{n+1}}\to y_{j_1,j_n}\Diamond x_{n+1},\tag{5.30}$$

where

$$y_{j_1,j_n}\to x_{j_1}\Diamond x_{j_2}\Diamond\ldots\Diamond x_{j_n},\tag{5.31}$$

in accordance with Corollary 5.2(3) with ‘j’ in place of ‘i’. By (5.30), it follows from Theorem 4.3 that for each $\lambda_{n+1} \in \omega_{1,\tau_{n+1}}$: for each $\mu_n \in \omega_{1,\tau_n}$:

$$\left[y_{j_1,j_n} \Diamond x_{n+1}\right]_{\lambda_{n+1}} \sim \left[\left[y_{j_1,j_n}\right]_{\mu_n} \Diamond x_{n+1}\right]. \tag{5.32}$$

The sequence x_{j_1}, x_{j_2}, ... , $x_{j_{n-1}}$, x_{j_n} is evidently a permutation of the sequence x_1, x_2, ..., x_{n-1}, x_n. Therefore, by the induction hypothesis,

$$\left[y_{j_1,j_n}\right]_{\mu_n} \sim \left[y_{1,n}\right]_1 \text{ for each } \mu_n \in \omega_{1,\tau_n}. \tag{5.33}$$

Hence, (5.32) reduces to (5.17).

Thus, (5.4) implies (5.17) in all the three cases. QED.•

Comment 5.2. Given $n \in \omega_2$: let p_n be a *permutation function on the set* $\omega_{1,n}$, such that for each $k \in \omega_{1,n}$: there is exactly one $i_k \in \omega_{1,n}$ such that $i_k = p_n(k)$. In this case, the sequence i_1, i_2, ..., i_{n-1}, i_n can be rewritten as $p_n(1)$, $p_n(2)$, ..., $p_n(n-1)$, $p_n(n)$. Accordingly, the corresponding sequence x_{i_1}, x_{i_2}, ..., $x_{i_{n-1}}$, x_{i_n} becomes $x_{p_n(1)}$, $x_{p_n(2)}$, ..., $x_{p_n(n-1)}$, $x_{p_n(n)}$.

There are n! different permutation functions on the set $\omega_{1,n}$. Therefore, a separate permutation function should more explicitly be denoted, for instance, by the symbol ‘$p_n^{(m)}$’, so that for each $k \in \omega_{1,n}$, for each $m \in \omega_{1,n!}$, there is exactly one $i_k^{(m)} \in \omega_{1,n}$ such that $i_k^{(m)} = p_n^{(m)}(k)$. In this case, application of the function $p_n^{(m)}$ to each number of the sequence 1, 2, ..., n–1, n results in the sequence $p_n^{(m)}(1)$, $p_n^{(m)}(2)$, ..., $p_n^{(m)}(n-1)$, $p_n^{(m)}(n)$, which can briefly be written as $i_1^{(m)}$, $i_2^{(m)}$, ..., $i_{n-1}^{(m)}$, $i_n^{(m)}$. The corresponding permutation of the of the n successive primitive terms x_1, x_2, ..., x_{n-1}, x_n results in the sequence $x_{p_n^{(m)}(1)}$, $x_{p_n^{(m)}(2)}$, ..., $x_{p_n^{(m)}(n-1)}$, $x_{p_n^{(m)}(n)}$ or, briefly, $x_{i_1^{(m)}}$, $x_{i_2^{(m)}}$, ..., $x_{i_{n-1}^{(m)}}$, $x_{i_n^{(m)}}$. Accordingly, Corollary 5.3 can, more explicitly, be restated as follows.•

Corollary 5.3a. For each $n \in \omega_2$, for each $m \in \omega_{1,n!}$:

$$\begin{aligned} &\overset{n}{\underset{k=1}{\Diamond}} x_{p_n^{(m)}(k)} \to \left[\overset{n}{\underset{k=1}{\Diamond}} x_{p_n^{(m)}(k)}\right] \to \left[\left[\overset{n-1}{\underset{k=1}{\Diamond}} x_{p_n^{(m)}(k)}\right] \Diamond x_{p_n^{(m)}(n)}\right] \\ &\to \left[\left[\left[\ldots\left[\left[x_{p_n^{(m)}(1)} \Diamond x_{p_n^{(m)}(2)}\right] \Diamond x_{p_n^{(m)}(3)}\right] \ldots \Diamond x_{p_n^{(m)}(n-2)}\right] \Diamond x_{p_n^{(m)}(n-1)}\right] \Diamond x_{p_n^{(m)}(n)}\right] \end{aligned} \tag{5.34}$$

or briefly

$$\mathop{\Diamond}_{k=1}^{n} x_{i_k^{(m)}} \to \left[\mathop{\Diamond}_{k=1}^{n} x_{i_k^{(m)}}\right] \to \left[\left[\mathop{\Diamond}_{k=1}^{n-1} x_{i_k^{(m)}}\right] \Diamond x_{i_n^{(m)}}\right]$$
$$\to \left[\left[\left[\ldots\left[\left[x_{i_1^{(m)}} \Diamond x_{i_2^{(m)}}\right] \Diamond x_{i_3^{(m)}}\right] \ldots \Diamond x_{i_{n-2}^{(m)}}\right] \Diamond x_{i_{n-1}^{(m)}}\right] \Diamond x_{i_n^{(m)}}\right]. \qquad (5.35)\bullet$$

Essay 10. The theory of trial logic and Psychologistics

1. The background of the treatise

1) Based on two papers by Kurt Gödel [1930, 1931], Alonzo Church [1936a, 1936b] proved that the *decision problem* for a *conventional axiomatic first-order predicate calculus* is unsolvable. Regarding his unsolvable decision problem, Church [1936b, p. 41, footnote 6] says:

> «By the Entscheidungproblem of a system of symbolic logic is here understood the problem to find an effective method by which, given any expression Q in the notation of the system, it can be determined whether or not Q is provable in the system.»

At the same time, Hilbert and Ackermann [1950, p. 124] comment on the papers of Church thus:

> «Results by A. Church based on papers by K. Gödel show that the quest for a general solution of the decision problem must be regarded as hopeless. We cannot report on these researches in detail within the limits of this book. We shall only remark that a general method of decision would consist of a certain recursive procedure for the individual formulas which would finally yield for each formula the value truth or the value falsehood. Church's work proves, however, the non-existence of such a recursive procedure; at least, the necessary restrictions would not fall under the general type of recursion set up by Church, who has given to the somewhat vague intuitive concept of recursion a certain precise formalization.»

Thus, the decision problem, which was dealt with by Church, should have been explicitly called a *dual* (*two-valued*, *two-fold*) *decision problem* in the sense that, if existed, its solution for a given relation would have discriminated between the pertinent *positive value* of the relation as its *provability* or *truth* (*validity*) and the respective *negative value* as its *improvability* or *untruth* (*falsehood, invalidity*). However, modern *formal logic* is *dual* (*two-valued*) and therefore it has not dealt with any decision problems other than dual ones. Consequently, the *generic name* "decision problem" was unfortunately used in the literature on logic, particularly by Church himself and by the commentators on his works, *synecdochically* instead of the more correct *specific name* "dual decision problem" – just as the generic name "formal logic" is as a rule used *synecdochically* instead of the more correct specific name "dual formal logic". Since the dual character of Church's decision problem was blurred, therefore by the fact of proving its insolvability Church actually eliminated the entire *subject category* called "*decision problem"* from the *subject taxonomy* (*partition*) *of symbolic logic*. The logicians of the generation, succeeded that of Church and his contemporaries, have *in fact* abandoned the very concept of decision problem – just as long ago the physicists abandoned their concept of *ether* and just as long ago the mathematicians abandoned their concept of *infinitesimals* as being supposedly infinitely small but nonzero real numbers. In the modern mathematics the latter notion is replaced by the so-called *ε&δ-language* (*epsilon-and-delta-language*). Thus, the theorem of Church, which was of course a distinguished achievement of symbolic logic, paradoxically became at the same time detrimental to symbolic logic from the standpoint of prospective trends of its further development. Particularly, it was discouraging logicians to attempt formulating and solving a *trial* (*three-fold*) *decision problem* of some kind so as to contradict neither to the results of Gödel nor to the results of Church.

2) After Whitehead and Russell [1910; 1962, p. 6ff], relations of any *conventional axiomatic logical calculus* (briefly *CALC*), a *sentential* one (briefly *CASC*) or a *first-order predicate* one (briefly *CAFOPC* or synecdochically *CAPC*), is supposed to be *propositional* or *dualistic truth-functional* in the sense that every relation of any CALC that is not paradoxical can be either *true* or *untrue* (*false*), the understanding being that the *negation* of a true relation is an untrue (false) relation and vice versa. In general, the *validity* or *invalidity* of a relation of *dual formal logic* can be qualified as a *truth-functional* one, and likewise the *truth* or *untruth* (*falsehood*) of a relation of dual formal logic can be qualified as a *validity-functional* one, in the sense that a relation of dual formal logic is said to be *valid* if and only if it is *true* and *invalid* if and only if it is *untrue* (*false*). Consequently, the *negation* of a valid (true) relation is an invalid (untrue, false) relation and vice versa.

3) Based on the results of Church, the skepticism of Hilbert and Ackermann regarding possibility to solve the dual decision problem for first-order predicate calculus has been shared by some other authoritative logicians, who have not, however, explicitly mentioned that the problem in question is dual – just as Hilbert and Ackermann and Church have not explicated this fact. Here follows one of the

most categorical statements, if not the most categorical one, regarding such a decision problem by Suppes [1957, pp. 69–70]:

> «In chapter 2 we saw that there was a mechanical method (by use of truth tables) for testing the truth-functional validity or invalidity of an argument. Such a mechanical method is often called a *decision procedure*. In one sense the existence of a decision procedure for truth-functional arguments trivializes the subject. Fortunately or unfortunately, no such trivialization of the logic of quantification is possible. It was rigorously proved in 1936 by the contemporary American logician Alonzo Church that there is no decision procedure, that is, no mechanical test, for the validity of arbitrary formulas in first-order predicate logic.* Since all of mathematics may be formalized within first-order predicate logic,† the existence of such a decision procedure would have startling consequences: a machine could be built to answer any mathematical problem or to decide on the validity or invalidity of any mathematical argument. But Church's theorem ruins at a stroke all such daydreams of students of logic and mathematics. Not only there is no known decision procedure: his theorem establishes that there never be any.
>
> ———————
>
> *First-order predicate logic is the logic of sentential connectives and quantifiers for individual variables, that is, the logic of the formulas defined in Chapter 3. "First-order" refers to the fact that no quantification of predicates is permitted.
>
> †The standard developments of axiomatic set theory has one of their aims to establish this fact in substantive details.»

Unlike Hilbert and Ackermann, who associate solution of the decision problem for a given relation with the possibility to decide whether the relation is *true* or *false*, Suppes associates solution of the decision problem for a given relation with the possibility to decide whether the relation is *valid* or *invalid*. However, in accordance with the above item 2 the values *truth* and *falsehood* of a propositional (dualistic truth-functional) relation are tantamount to its values *validity* and *invalidity* respectively. Therefore, Suppes speaks about the same dual decision problem as Hilbert and Ackermann.

4) Should the dual decision problem be solvable, Suppes misinterprets implications of its expected solution in mathematics simply because a system of *class*, or particularly *set*, *theory* is a *semantic theory* that cannot be equivalent to any system of first-order predicate calculus. Particularly, a class theory should necessarily contain a *class-builder* such as '$\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle\}$' or a concurrent *contextual axiomatic definition mode*, which is designed to convert a given *relation* (*condition*) $\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle$ into the respective additional *constant* or *variable class-valued* (or *set-valued*) *ordinary term* $\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle\}$ ('$\boldsymbol{P}$', '$\boldsymbol{x}$', '$\boldsymbol{x}_1$', '$\boldsymbol{x}_2$', …, '$\boldsymbol{x}_n$' are atomic placeholders having the appropriate ranges). Such a term is automatically included into the range of the variable $\boldsymbol{x}$ (not '$\boldsymbol{x}$'), although it is not introduced by formation rules of any first-order predicate calculus. Particularly, any axiomatic system of set theory has a certain axiom, which makes that system *self-consistent* (*non-paradoxical*) and which necessarily involves, explicitly or implicitly, a certain *set-builder*. This axiom was originally called "*Axiom of Ausonssonderung*" by Zermello [1908], i.e. "*Axiom of sifting*", and it is most often called in English "*Axiom of specification*" or "*Axiom of separation*". For instance, Halmos [1960, p. 6] states such an axiom thus:[9]

> «**Axiom of specification**. *To every set A and to every condition S*(*x*) *there corresponds a set B whose elements are exactly those elements x of A for which S*(*x*) *holds*.»

Formally, this axiom can be restated thus:

$$(\forall A)(\forall S)(\exists B)(\forall x)[[x \in B] \leftrightarrow [x \in A] \& S(x)]$$

or thus:

[9]The same axiom is semi-formally stated under the same name in Suppes [1960, p. 21] and informally (in the intuitive manner of Halmos' formulation) it is stated in Bernays [1958, p. 11] under the name *Axiom of Subsets*. Supposedly the same axiom is semi-formally stated in Fraenkel *et al* [1973, p. 31] under the name *Axiom of comprehension*. However, in the latter formulation of the axiom, the condition analogous to the condition '$x \in A$' of the formalized axiom schema of Halmos is missing. Therefore, Axiom of comprehension of Fraenkel *et al* is contradictory (paradoxical). In the set-theoretic system by Bourbaki [1960], the axiom separation schema is stated under the logographic name S8 and verbal name "La schéma de sélection et réunion", i.e. "The schema of selection and reunion" [*ibid*. Chap. II, §1, n°6]. There occurs in Bourbaki's schema the syntactic variable $\boldsymbol{R}$, whose range is [the set of] the so-called *relations* of the theory, i.e. the well-formed sentence-valued formulas of the theory.

$$(\forall S)(\exists B)[B = \{x|[x \in A] \,\&\, S(x)\}],$$

where the quantifier '$(\forall S)$' should be understood as: «for every predicate S that is defined in terms of $\in$ and perhaps of some sentential connectives present in the given set theory». The occurrence of that quantifier in Axiom of specification evidences that a system of set theory is not a *first-order predicate* one at all. Also, if a class (or set) theory involves *nonempty individuals* then the latter can be introduced only by *verbal* axioms (cf., e.g., Fraenkel *et al* [1973, pp. 24–25]) and hence *informally.*

5) Somewhat more than 30 years ago, I became obsessed with a *trial* (*three-valued*) *decision problem* for a *properly designed axiomatic first-order algebraico-predicate calculus*. In contrast to A. Church [1936a, 1936b], who proved that a dual decision problem for the conventional axiomatic first-order predicate calculus is unsolvable, I have solved the trial decision problem algebraically (and hence analytically, not tabularily) and have successfully applied the pertinent *algebraic decision procedures* to all conceivable logical relations of interest, including the 19 categorical syllogisms.

6) The solution of the trial decision problem and all my most conspicuous relevant findings are collected in a treatise, which I informally call it "*The Theory of Trial Logic*" or "*The Trial Logic Theory*" – briefly "*The TLT*"; the formal title of my treatise will be discussed before long. I employ the first sentence of the quotation of Hilbert and Ackermann [1950, p. 124], given above in the item 1, as an epigraph to my treatise in order to emphasize the fact that the generic name "decision problem" without either additional qualifier "dual" or "trial" is a *misnomer* that results in confusion, while the fact that the trial decision problem has turned out to be solvable does not contradict the results of Church and agrees with the results of Gödel.

7) In order to solve the trial decision problem, I have developed the *entire system of new notions*, to which the conventional dualistic terminology that is *from the very beginning* based on using semantic terms such as "proposition", "truth", and "falsehood" is inapplicable, except for the case of restricted dualistic interpretation of the *final* results. Voltaire said, «If you wish to converse with me, define your terms». Accordingly, in order to make my treatise communicable, I have developed the appropriate new comprehensive (all-embracing) system of *pasigraphic*, i.e. *logographic* and *euautographc, notation* and *phonographic*, i.e. *wordy*, *terminology*. Etymologically, I have derived the adjective "pasigraphic" from the Greek adjective

"πάς" \pás\ meaning *all* or *every*, so that it lexically means «*commonly intelligible*, *i.e. capable of being shared by all people independent of the languages they use*», – like «logographic» and «pictographic» («iconographic»). Owing to its comprehensive *nomenclature* (*pasigraphic notation and phonographic terminology*), it is impossible to break the treatise into intelligible separate journal articles, to say nothing of reducing it to a single journal article of a reasonable size. The only possible format for the subject matter of the treatise is a single whole full-scaled book, which spread over more than a thousand 1.5-line-spaced Word-set pages and which occupies about 7 MB in the PDF.

8) Going ahead the story, here follow some simplest examples (to be discussed in the item 16 of this letter in greater detail) of phonographic (wordy) trichotomies and dichotomies of relations of the TLT – in contrast to the dichotomy *truth* vs. *untruth* (*falsehood*), i.e. *truth-functional validity* vs. *truth-functional invalidity*, of relations of dual logic. Either *declarative sentence* (*DS*) "Abraham Lincoln was or was not the 16th president of the USA" or "Abraham Lincoln was or was not the 28th president of the USA" is, *syntactically*, *valid* and hence, *semantically, m-tautologous* (*universally m-true*), where "*m-*" is an abbreviation of "*materially-*", meaning *conformable to facts.* Consequently, either DS "It is not the case that Abraham Lincoln was or was not the 16th president of the USA" or "It is not the case that Abraham Lincoln was or was not the 28th president of the USA" is, *syntactically*, *antivalid* and hence, *semantically, m-antitautologous* (*universally m-antitrue*, *universally m-false*, *universally m-contradictory*). In this case, either *proper simple declarative affirmative sentence* (*PSDAS*) "Abraham Lincoln was the 16th president of the USA" or "Abraham Lincoln was the 28th president of the USA" is, syntactically, *neutral*, or *indeterminate¸ with respect to validity and antivalidity* – briefly *vav-neutral* or *vav-indeterminate*, and hence it is, semantically, *neutral*, or *indeterminate¸ with respect to tautologousness and antitautologousness* – briefly *ttatt-neutral* or *ttatt-indeterminate.* Consequently, either one of the two sentences (relations) is, syntactically, *invalid* in the sense that it belongs to the union of the class of antivalid and of the class of vav-neutral sentences (relations), and it is, semantically, *m-atautologous* in the sense that it belongs to the union of the class of m-antitautologous and of the class of ttatt-neutral sentences (relations). At the same time, in accordance with the pertinent *historical facts*, the former sentence is, semantically, *m-veracious* (*accidentally m-true*), whereas the latter sentence is *m-antiveracious* (*accidentally m-*

antitrue, *accidentally m-false*). If, however, no pertinent historical facts are known to a given interpreter of the above two PSDAS's then either one of the sentences is *neutral,* or *indeterminate, with respect to m-veracity and m-antiveracity* – briefly *m-vravr-neutral* or *m-vravr-indeterminate, relative* (*in relation*) *to that interpreter.* Likewise, here and now, given an interpreter, *the vav-neutral common simple declarative affirmative sentence* (*CSDAS*) "It is raining" is, relative to that interpreter, m-veracious (accidentally m-true) if it is raining, m-antiveracious (accidentally m-antitrue) if it is not raining, and m-vravr-neutral (m-vravr-indeterminate) if the weather condition outdoors is not known to the interpreter. An m-tautologous (universally m-true) or m-veracious (accidentally m-true) sentence is indiscriminately called an *m-true sentence*, whereas an m-antitautologous (universally m-antitrue) or m-antiveracious (accidentally m-antitrue) sentence is indiscriminately called an *m-antitrue* (*m-false*) *sentence*. At the same time, an m-vravr-neutral sentence is *alternatively* (*synonymously*) said to be *neutral,* or *indeterminate, with respect to m-truth and m-antitruth* – briefly *m-tat-neutral* or *m-tat-indeterminate.* It is understood that the *negation* of a valid relation is an antivalid relation and vice versa, whereas the *negation* of a vav-neutral relation is another vav-neutral relation; and similarly with "*tautologous*" and "*ttatt*', "*veracious*" and "*vravr*", or "*truth*" and "*tat*" in place of "*valid*" and "*vav*" respectively in all occurrences. In order to express various trichotomies and dichotomies of relations in the TLT phonographically (wordily), I establish the appropriate hierarchy of English privative prefixes. The adverbial qualifier opposite to "*materially*" ("*m*") is "*formally*" ("*f*").

9) In spite of the large length of the TLT and also in spite of a wide variety of its aspects, its subject matter can be condensed in the following *Abstract.*

«In contrast to Church, who proved in 1936, based on papers by Gödel, that a *dual decision problem* for the conventional axiomatic first-order predicate calculus is unsolvable, I have solved a *trial decision problem algebraically* (and hence analytically, not tabularily) for a properly designed *axiomatic first-order algebraico-predicate calculus*, called briefly the *trial logic* (*TL*), and have successfully applied the pertinent *algebraic decision procedures* to all conceivable logical relations of academic or practical interest, including the 19 categorical syllogisms. The structure of the TL is a *synthesis* of the structure of a *conventional axiomatic first-order predicate calculus* (briefly *CAPC*) and of the structure of an *abstract integral domain*. Accordingly, the TL contains as its autonomous parts the so-called *Predicate-Free*

Relational Trial Logic (*PFRTL*), which is parallel to a *conventional axiomatic sentential calculus* (CASC*)*, and the so-called *Binder-Free Predicate Trial Logic* (*BFPTL*), which is parallel to the predicate-free part of a *pure* CAPC. This treatise, presenting some of my findings, is alternatively called "*the Theory of Trial Logic*" ("*the TTL*") or "*the Trial Logic Theory*" ("*the TLT*"). The treatise *reopens* the entire topic of symbolic logic that is called "*decision problem*" and that Church actually *closed by the fact of synecdochically calling* the specific *dual* decision problem, the insolvability of which he had proved, by the *generic* name "decision problem", without the qualifier "*dual*". Any additional axiom that is incompatible with the *algebraic decision method* of the trial logic and that is therefore detrimental for that method is regarded as one belonging to either to another logistic system or to mathematics.»

The previous portion of this essay was designed so as to be in principle accessible a wide scientific community, especially that of logicians, mathematicians, and linguists. The rest of the essay, which is somewhat more demanding, can be seen as an extended abstract of the treatise, which are designed to give the reader an impression of various trichotomies and dichotomies of relations, which are established in the treatise, and also an impression of their implications in mathematics and in ordinary languages.

10) Using the appropriate elements of the new terminology, introduced in my treatise, I formally entitle it as:

> «*A theory of the Comprehensive Endosemasiopasigraphic Algebraico-Predicate Organon and its conformal catlogographic interpretations*: *A general analytical solution of trial decision problems for first-order predicate calculus*».

This title should be understood as follows. I denote the entire algebraico-predicate calculus addressed in the treatise by '$\mathcal{A}_1$' and briefly called it the *Trial Logic* or more precisely the *Trial Formal Logic* (*TFL*). Accordingly, I informally and loosely (less informatively) call the treatise «*The Trial Logic Theory*» («*The TLT*»). $\mathcal{A}_1$ is *the sequence of the four interrelated logistic systems* $\mathbf{A}_1$, A_1, I_1, *and* A_1 *in that order*, which are interrelated as follows. The principal *insignificant* (*semantically uninterpreted*, *chess-like*) *calculus of* $\mathcal{A}_1$ is denoted by 'A_1' and called the *Comprehensive Euautographic Algebraico-Predicate Organon*, whereas the *calculus* *of placeholders of euautographic relations of* A_1 is denoted by '$\mathbf{A}_1$' and called the *Comprehensive Panlogographic Algebraico-Predicate Organon*. I use the term "Organon" in analogy with Aristotelian «*Organon*» and also in analogy with Francis Bacon's «*Novum Organum*»; the qualifier "*comprehensive*" means «*having an infinite number of branches that share the same algebraic decision method*»; "*euautographic*" means «*graphic* (*written*) *and genuinely self-referential*»; and "*panlogographic*" means «*logographic over* (*above, assuming, taking on*, *interpretable by*) *euautographs*». The union and superposition of A_1 and $\mathbf{A}_1$ is denoted by 'A_1' and called the *Comprehensive Biune Euautographic and Panlogographic Algebraico-Predicate Organon* or concisely the *Comprehensive Endosemasiopasigraphic Algebraico-Predicate Organon* – the name that occurs in the title of the treatise. The adjective "pasigraphic", being a combining form of the complex *monomial* qualifier "*endosemasiopasigraphic*", have been defined in the item 4 so that it means «*either euautographic or panlogographic* (in general, *logographic*)». Consequently, the qualifier "endosemasiopasigraphic" to A_1 means that all relations and all terms of A_1 are *pasigraphic,* i.e. either those of A_1 or those of $\mathbf{A}_1$, while the complex prepositive prefix "*endosemasio*" (in contrast to "*exosemasio*") emphasizes the fact that *any pasigraph of* A_1 *neither has nor assumes* (*takes on*) *any signification* (*import value*) *beyond* A_1, i.e. that A_1 is *semantically close.* In contrast to "panlogographic", the qualifier "*catlogographic*" means «*logographic under* (*below, interpreting*) *euautographs beyond the scope of* A_1». Etymology of all unconventional terms that I use is explained in the treatise.

11) Since the calculus A_1 is semantically close (endosemasiopasigraphic), therefore in order to solve the trial decision problem for any *relation of academic or practical interest of* A_1, i.e. either of A_1 or of $\mathbf{A}_1$, which is qualified as a *slave relation* (*SR*), a *euautographic one* (*ESR*) or a *panlogographi*c *one* (*PLSR*) respectively, I algebraically *prove* (*deduce*) for it the pertinent *master*, or *decision*, *theorem* (*MT* or *DT*) – the *euautographic one* (*EMT* or *EDT*) or the *panlogographic one* (*PLMT* or *PLDT*) respectively. From the [syntactic] form of the MT, I unambiguously classify its SR as a [syntactically] *valid* one or as an *antivalid* one, or else as a *vav-neutral* (*vav-indeterminate*, *neither valid nor antivalid*) one. In this case, *the negation of a valid relation is an antivalid relation, and vice versa*, whereas *the negation of a vav-*

neutral relation is *another vav-neutral relation*. An MT is by definition a *valid master*, or *decision, relation* (*MR* or *DR*). The totality of rules of inference and decision of A_1 or that of $\mathbf{A}_1$, which allows classifying any *slave euautographic relation* (*ESR*) of interest of A_1 or any *slave panlogographic relation* (*PLSR*) of interest of $\mathbf{A}_1$ in the above way, is denoted by 'D_1' or '$\mathbf{D}_1$', respectively; D_1 is called the *Euautographic*, and $\mathbf{D}_1$ the *Panlogographic*, *Advanced Algebraic Decision Method* – briefly *EAADM* and the *PLAADM* respectively. The union of D_1 and $\mathbf{D}_1$ is denoted by 'D_1' and is called the *Endosemasiopasigraphic AADM*.

12) A *euautographic relation* (*ER*) of A_1 or a *panlogographic relation* (*PLR*) of $\mathbf{A}_1$ of a certain one of the above three classes: *validity*, *antivalidity*, and *vav-neutrality* (*vav-indeterminacy*) is indiscriminately called a *vavn-decided relation* or specifically a *decided ER* (*DdER*) of A_1 or a *decided PLR* (*DdPLR*) of $\mathbf{A}_1$ respectively. Although I use the adjectival qualifier "indeterminate" as a synonym of qualifier "neutral', *there is no indeterminacy* (*uncertainty*) *in relegating a ESR of* A_1 (e.g.) *to the class of vav-neutral* (*vav-indeterminate*) *ER's if it is so*. A vav-neutral ER of A_1 is *not an improvable relation of the Gödelian type*, because it *is proved to be vav-neutral* – just as a valid ER, other than a euautographic axiom of A_1, is proved to be valid and just as an antivalid ER is proved to be antivalid. The notion of a proposition is not applicable to the *dramatis personae* of such an *algebraic decision procedure* – either in the Aristotelian sense of "proposition" as a *truth-functional declarative sentence* or in the Frege-Church sense of "proposition" as *the [Platonic] sense a truth-functional declarative sentence.* The division of the *decided ER's* (*DdER's*) into the three classes: the valid, antivalid, and vav-neutral ER's is called the *primary*, or *basic*, *decisional trichotomy* (*trisection*, *trifurcation*) *of the DdER's*. A DdER of A_1 is said to be: *invalid* if it is antivalid or vav-neutral, *non-antivalid* if it is valid or vav-neutral, and *vav-unneutral* or *vav-determinate* if it is valid or antivalid. In accordance with this definition, the DdER's of A_1 are divided into *two complementary classes in three ways*, namely: (a) the *valid* ER's and the *invalid* ER's, (b) the *antivalid* ER's and the *non-antivalid* ER's, (c) the *vav-neutral*, or *vav-indeterminate*, ER's and the *vav-unneutral*, or *vav-determinate*, ER's. These three divisions the DdER's are called the *secondary,* or *subsidiary*, *decisional dichotomies* (*bisections*, *bifurcations*) *of the DdER's*. Like remarks apply to *panlogographic relations* (*PLR's*) *of* $\mathbf{A}_1$ with the following provisos. A PLR of $\mathbf{A}_1$ is valid, or antivalid, if and only if

every ER of A_1 in *its range* is valid, or antivalid, respectively, whereas the range of a vav-neutral PLR of $\mathbf{A}_1$ can in the general case contain ER's of A_1 of all the three classes: valid, antivalid, and vav-neutral, and also some *suspended* ER's.

13) The organon A_1 is succeeded in $\mathcal{A}_1$ with a logistic system that is denoted by 'I_1' and called the *conservative conformal catlogographic* (*CCFCL*) *interpretation of* A_1. I_1 is the set of *CCFCL interpretations* of ER's of A_1 of three kinds: (a) some selective *valid ESR's*, (b) some selective *vav-neutral ESR's,* (c) the *EMT's* (*EDT'*s) of the selective vav-neutral ESR's. The totality of rules of I_1, denoted by 'I_1', comprises replacements of the occurrences of *atomic euautographic ordinary terms* (*AEOT's*), as u to z, u_1 to z_1, u_2 to z_2, etc, and $\emptyset$, and of *atomic euautographic relations* (*AER's*), as p to s, p_1 to s_1, p_2 to s_2, etc, throughout the above *euautographic interptetantia* (*interpreted euautographic relations*) with occurrences of the respective *atomic conformal catlogographic terms* (*ACFCLT's*) u to z, u_1 to z_1, u_2 to z_2, etc, and $\varnothing$ and *atomic conformal catlogographic relations* (*ACFCLR's*) p to s, p_1 to s_1, p_2 to s_2, etc, without any quotation marks. The CCFCL interpretand of a DdER of A_1, which is less explicitly (more generally) called a *conservative catlogographic relation* (*CCLR*)*,* is semantically interpreted, but it preserves the validity-value of the DdER; the latter is the *euautographic interpretans* (pl. "*interpretantia*") of the former. Consequently, in the result of the above *conformal catlogographic replacements*, a valid ESR is transduced into the respective so-called *formally tautologous* (*f-tautologous*, *universally f-true*) *CCLR*, a vav-neutral ESR is transduced into the respective so-called *f-ttatt-neutral* (*f-ttatt-indeterminate, neither f-tautologous nor f-antitautologous*) *CCLR*, and the EMT (EDT) of a vav-neutral ESR is transduced into the *CCFCLMT* (*CCFCLDT*) of the respective f-ttatt-neutral *consercative catlogographic slave relation* (*CCLSR)*.

14) Some f-ttatt-neutral CCLSR's can be postulated to be *formally veracious* (*f-veracious*, *accidentally f-true*), thus turning into a *catlogographic postulate.* A catlogographic postulate is called a *catlogographic hypothesis* if it is a *temporary* (*ad hoc*) one and a *catlogographic axiom* if it is a *permanent* one. The CCFCLMT of an slave f-ttatt-neutral CCLSR that involves some one or some more of the catlogographic postulates can be developed further with the help of the totality of the inference and decision rul*es*, D_1, being CCFCL interpretands of those forming D_1; i.e. formally D_1=I_1(D_1). In the result, the CCFCLMT reduces to a certain ultimate

progressive conformal catlogographic master, or *decision*, *theorem* (*PCFCLMT* or *PCFCLDT*), from the form of which I unambiguously classify its f-ttatt-neutral CCLSR as a *progressive catlogographic slave relation* (*PCLSR*) of exactly one of the following three kinds: an *f-veracious* (*accidentally f-true*) *PCLSR*, an *f-antiveracious* (*accidentally f-antitrue*, *accidentally f-false*) *PCLSR*, or an *f-vravr-neutral* (*f-vravr-indeterminate*, *neither f-veracious nor f-antiveracious*) *PCLSR*. In this case, *the negation of an f-veracious relation is an f-antiveracious relation and vice versa*, whereas *the negation of an f-vravr-neutral* (*vravr-indeterminate*) *relation* is *another f-vravr-neutral* (*vravr-indeterminate*) *relation*. The calculus, in the framework of which catlogographic postulates are laid down and PCFCLMT's (PCFCLDT's) are established, is an *interpreted logistic system*, i.e. a *formalized language*, which is denoted by '$\mathcal{A}_1$' and called the *Comprehensive Catlogographic Algebraico-Predicate Organon* or *Comprehensive Catlogographic Advanced Algebraico-Logical Organon*. $\mathcal{A}_1$ has no formation, no transformation (inference), and no decision rules of his own: all input CCLR's are supplied to $\mathcal{A}_1$ by I_1, whereas $\mathcal{D}_1$ is the *Catlogographic AADM of* $\mathcal{A}_1$. Accordingly, I_1 plays two interrelated roles: first, it is the *most immediate interpretational supplement to* A_1 and, second, it is the *interpretational interface between* A_1 *and* $\mathcal{A}_1$. In fact, $\mathcal{A}_1$ is just a *simplest illustration* (*model*) *of mathematics*.

15) The term "tautology" has arisen in the conventional truth-functional dual formal logic FL after Wittgenstein [1921], who applied that term to any quantifier-free or quantified statement, being *universally true* by virtue solely of the *abstract truth-functional validity of its syntactic form.* Such use of the term "tautology" has been adopted by all modern logicians and mathematicians. At the same time, Wittgenstein suggested as a *thesis* the doctrine that *all logic* and *all mathematics*, i.e. in fact *all postulates and all theorems of dual logic and those of mathematics based on dual logic*, are tautological. I call a postulate "a *hypothesis*" if it is a *temporary* (*ad hoc*) one and "an *axiom*" if it is a permanent one. The above Wittgenstein thesis has commonly been regarded as one that is difficult to defend and therefore it has never been adopted by logical and mathematical community (cf. Quine [1951, p. 55]). In the trial logic, all euautographic and panlogographic axioms and all like master and slave theorems are valid relations. Hence, the CCFCL interpretands of all above ER's f-tautologous *catlogographic relations* (*CLR's*). By contrast, the PCFCL interpretands of all vav-neutral ESR's and of their EMT's are f-veracious and hence f-ttatt-neutral CLR's.

16) In accordance with the item 4, in order to develop a full-scale class, set, or mass theory, A_1 should be augmented by an *additional formation rule*, according to which to any given condition-relation $\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle$ ('$\boldsymbol{P}$' is an atomic placeholder for a ttatt-neutral CLR, while each one of the atomic logographs '$\boldsymbol{x}$', '$\boldsymbol{x}_1$', '$\boldsymbol{x}_2$', …, '$\boldsymbol{x}_n$' is a placeholder for any ACFCLT mentioned in the item 13), there corresponds a class (particularly, set) or mass, which is denoted by '$\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle\}$'. This formation rule is in fact a *contextual definition* of the *operator of abstraction* $\{\ |\ \}$, which is called a *general builder of an ordinary term* and which allows prescinding a class, set, or mass $\{\boldsymbol{x}|\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle\}$ (depending on the given theory) from the given condition-relation $\boldsymbol{P}\langle\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots,\boldsymbol{x}_n\rangle$. All other operators that are used in a class or mass theory, – such operators, e.g., as the *binary operators* $\cup$, $\cap$, and **-** *of union*, *intersection*, and *difference of classes* or the *operator of aggregation* $\{\ ,\ ,\ldots,\ \}$ *of elements* (*classes* or *sets*), called also a *concrete set-builder,* – can contextually be defined in terms of the operator $\{\ |\ \}$. Also, sets (but not irregular classes and not masses) should be allowed to be *domains of definitions* of various order relations and thus to become *ordered*. It is understood that all the above operators and all order relations should be subjected to or be introduced by the appropriate *semantic axioms along with the appropriate definitions*. Therefore, a full-scale class, set, or mass theory cannot have any decision method after the manner of D_1 and D_1. Thus, my solution of the trial decision problem do not fulfill all enthusiastic expectations regarding would-be decisional proofs of mathematical theorems, which were unjustifiably associated with a hypothetical solution of the dual decision problem before the latter was proved to be unsolvable. However, the trial algebraic decision method is a powerful and simple tool for various three-fold classifications of all logical relations of practical or academic interest and of some mathematical relations and it is also an indispensable source of logical, mathematical, and linguistic wisdom.

17) A *catlogographic relation* (*CLR*)*, conservative* (*CCLR*) or *progressive* (*PCLR*), is indiscriminately said to be: (a) *f-true* if it is *f-tautologous* (*universally f-true*) or *f-veracious* (*accidentally f-true*); (b) *f-antitrue* or *f-false* if it is *f-antitautologous* (*f-contradictory, universally f-antitrue*) or *f-antiveracious* (*accidentally f-antitrue*, *accidentally f-false*); (c) *f-tat-neutral* (*f-tat-indeterminate, neither f-true nor f-antitrue*) if it is *f-vravr-neutral*. In this case, *the negation of an f-true relation is an f-antitrue relation and vice versa*, whereas *the negation of an f-tat-*

neutral (*f-tat-indeterminate*) *relation* is *another f-tat-neutral* (*f-tat-indeterminate*) *relation*. Consequently, a CLR is said to be (cf. the item 8): *f-untrue* if it is f-antitrue or f-tat-neutral, *f-non-antitrue* if it is f-true or f-tat-neutral, and *f-tat-unneutral* or *f-ta - determinate* if it is f-true or f-antitrue. In accordance with this definition, the CLR's are divided into *two complementary classes in three ways*, namely: (i) the *f-true* CLR's and the *f-untrue* CLR's, (ii) the *f-antitrue* CLR's and the *f-non-antivalid* CLR's, (iii) the *f-tat-neutral*, or *f-tat-indeterminate*, CLR's and the *f-tat-unneutral*, or *f-tat-determinate*, CLR's.

18) By contrast, in the framework of *dual formal logic* (*DFL*), relations of any *conventional axiomatic logical calculus* (briefly *CALC*), a *sentential* one (briefly *CASC*) or a *first-order predicate* one (briefly *CAFOPC* or synecdochically *CAPC*), are supposed, likely after Whitehead and Russell [1910; 1962, p. 6ff] (cf. the item 2), to be *propositional* or *dualistically f-truth-functional* in the sense that every relation of any CALC that is not paradoxical can either be *f-true* or *f-untrue* (*f-false*), the understanding being that the *negation* of an f-true relation is an f-untrue (f-false) relation and vice versa. In general, the *validity* or *invalidity* of a relation of DFL can be qualified as an *f-truth-functional* one, and likewise the *f-truth* or *f-untruth* (*f-falsehood*) of a relation of DFL can be qualified as a *validity-functional* one, in the sense that a relation of DFL is said to be *valid* if and only if it is *f-true* and *invalid* if and only if it is *f-untrue* (*f-false*). Consequently, the *negation* of a valid (f-true) relation is an invalid (f-untrue, f-false) relation and vice versa. In accordance with the above-said, my treatise has the following fundamental implication. The occurrence of the noun "*Principia*" in the title "*Principia Mathematica*" of the known 3-volume monograph by Whitehead and Russell [1910–13] means *dual-logic principles of.* By contrast, I regard my treatise as *Principia Nova Mathematica*, the understanding being that the occurrence of the substantive "*Principia Nova*", i.e. "*new principles of*", in the above name means *trial-logic principles of.* The latter, higher principles allow answering epistemological questions and solving logical problems (the decision problem is among them), being beyond the scope of *Principia Mathematica* and also beyond the scope of any other *dual* logical theory that has stemmed from or been inspired by the above monograph.

19) I include my treatise under the banner "*Psychologistics*", which is an abbreviation of the descriptive name "*Psychological foundations of logic and logical foundations of psychology*" ("*PFL & LFP*"). Accordingly, I develop the *PTFL*

(*predicate trial formal logic*) along with its inseparable (built-in) *AADM* (*algebraic decision method*) as an inseparable part of *Psychologistics* – a new *biune* field of study and discourse of my own that includes its two *complementary conceptual hypostases* (*ways of existence*, *aspects*), namely *Psychological foundations of logic* (*PFL*) and *Logical foundations of psychology* (*LFP*), which can be distinguished and contrasted, but cannot be separated from each other, like *matter* and *form* of a thing. PFL, called also the *psychologistic logic* (*PLL*), include *cognitive* and *conative* aspects (as opposed to *affective* ones) of traditional *introspective psychology of my own* (as opposed to various trends of modern *extrospective psychology*), along with the doctrine of *physicalistic monism*, according to which *my mind is my cerebral cortex and vice versa.* LFP include the pertinent material logic also some modes of the PTFL, used reflexively in the meta-language without mentioning them.

20) Conservative and progressive catlogographic relations (CCLR's and PCLR's) can be interpreted further *materially* by appropriate *declarative sentences* (*DS's*) of a certain native language (as English). Although such a material interpretation of CLR's is beyond the scope of the TFL, I shall briefly illustrate it below by a few examples. Beyond the scope of both I_1 and $\mathcal{A}_1$, every ACFCLR '*p*' to '*s*', 'p_1' to 's_1', 'p_2' to 's_2', etc (see the item 13) can be regarded as a placeholder, whose range is a certain class of *m-ttatt-neutral* (*m-ttatt-indeterminate*), i.e. *neither m-tautologous nor m-contradictory*, *rational* (to be explained) *simple declarative affirmative sentences* (*RSDAS*) of written English or of another WNL; "*m-*" is an abbreviation of "*materially-*". For instance, the DS's "Abraham Lincoln was the 16th president of the USA" and "Abraham Lincoln was the 28th president of the USA" are *catphonographi*c (*wordy*) *relations* (*CPR's*), and *not catlogographic* (*not logographic*) *ones*. Consequently, *like* and at the same time *unlike f-ttatt-neutral* (*f-ttatt-indeterminate, neither f-tautologous nor f-antitautologous*) *CLR's*, they are *m-ttatt-neutral* (*m-ttatt-indeterminate, neither m-tautologous nor m-antitautologous*) *CPR's*. In this case, the former DS is *m-veracious* (*accidentally m-true*) because it, or more precisely *the proposition being its sense*, is conformable to the pertinent historical fact (state of affairs), whereas the latter DS is *m-antiveracious* (*accidentally m-antitrue*, *accidentally m-false*), because it is not conformable to any pertinent historical fact, while its negation is. Incidentally, the versions of the above two sentences with the phrases "the 16th president of the USA in the years 1861–65" and "the 28th president of the USA in the years 1913–21" instead of the respective shorter

phrases “the 16^{th} president of the USA” and “the 28^{th} president of the USA” have the same semantic properties. However, the DS’s “Abraham Lincoln was the 16^{th} president of the USA in the years 1913–21” and “Abraham Lincoln was the 28^{th} president of the USA in the years 1861–65” are both *paralogous* (from the Greek adjective “παράλογος” \paráloγos\ meaning *unreasonable* or *absurd*), and hence neither m-veracious nor m-antiveracious, because they involve as their constituent parts the respective phrases “the 16^{th} president of the USA in the years 1913–21” and “the 28^{th} president of the USA in the years 1861–65”, which are *contradictiones in adjecto* and which have therefore *no denotata* (denotation values). For the like reason, paralogous are, e.g., also the following DS’s: “A centaur is a mammal”, “A centaur is not a mammal”, “A centaur is a mammal or a centaur is not a mammal”, “The present king of Israel is as wise as the king Solomon”, “The present king of Israel is not as wise as the king Solomon”, the disjunction of the above two, “The capital of the USSR in AD2000 was in Europe”, “The capital of the USSR in AD2000 was not in Europe”, the disjunction of the above two, etc. Thus, the qualifier “rational” has been used above as an antonym of “paralogous”. Besides ttatt-neutral RSDAS’s as those mentioned above, which can be qualified *proper*, the range of an ACFCLR, e.g. the range of ‘*p*’, contains ttatt-neutral RSDAS’s such as: “It is raining”, “The night is light”, “The night is dark”, “This water is cold”, “This water is hot”, “This meal is testy”, “This meal is not testy”, “I am hungry”, “I am full up”, etc, which can be qualified *common* in the following sense. In any given place and at any given time, the DS “It is raining”, e.g., is (a) m-veracious, or (b) m-antiveracious (accidentally m-true), or (c) m-vravr-neutral (m-vravr-indeterminate) and hence m-tat-neutral (m-tat-indeterminate) if and only if in the given place and at the given time (a') it is raining, or (b') it is not raining, or (c') the pertinent weather conditions are not known, respectively.

21) I solved the trial decision problem for the principal euautographic algebraico-predicate organon A_1 and for its panlogographic *interpretans* (*anti-interpretand*) $\mathbf{A}_1$, i.e. for A_1, naïvely, *from intuitive considerations*, a long time ago, at the joint of eighties and nineties years of the last century. Accordingly, once the single whole organon $\mathcal{A}_1$ is set up and learned, it can be executed without mentioning its theory – just as a native language is used in everyday communication without mentioning its grammar. Particularly, the algebraic decision procedures for all

relations of $\mathcal{A}_1$ of academic or practical interest turn out to be almost as simple as computational procedures of primary school arithmetic with integers. $\mathcal{A}_1$ contains as its self-contained *autonomous* parts a *predicate-free organon* $\mathcal{A}_0$ and a *binder-free* (*contractor-free*, *quantifier-free*) *predicate organon* $\mathcal{A}_1{}^0$, which are briefly called the *Predicate-Free Relational Trial Logic* (*PFRTL*) and the *Binder-Free Predicate Trial Logic* (*BFPTL*) respectively, or more precisely by the variants of the above nameswith "*Formal Logic*" ("*FL*") in place of "*Logic*" ("*L*"), and for which all algebraic decision procedures are especially simple. As compared to the combined algebraic decision methods $\mathcal{D}_0$ and $\mathcal{D}_1{}^0$ of the latter two organons, the combined algebraic decision method $\mathcal{D}_1$ of $\mathcal{A}_1$ contains some additional, more sophisticated rules for handling the binders (contractors, quantifiers). However, all executions of $\mathcal{D}_1$ are after all as straightforward and intelligible as executions of $\mathcal{D}_0$ and $\mathcal{D}_1{}^0$. The most difficult aspects of the organon $\mathcal{A}_1$ and of its combined algebraic decision method $\mathcal{D}_1$, which lie far beyond the scope of the primary school arithmetic, are the rigorous setup of all logistic systems comprised in of $\mathcal{A}_1$ (including all pertinent logical and algebraic formation rules and all pertinent rules of inference and decision) and also various *epistemological aspects* of those systems, including significant (semantic) interpretations of *decided relations of* A_1 that should be done beyond the scope of A_1. I had been trying to reach complete rigor and perfectness and had forgotten the fact that I am mortal before some day when my health suddenly began rapidly deteriorate.

22) In the previous item, it goes without saying that $\mathcal{A}_0$ is *the sequence of the four logistic systems* $\mathbf{A}_0$, A_0, I_0, *and* A_0, and similarly $\mathcal{A}_1{}^0$ is *the sequence of the four logistic systems* $\mathbf{A}_1{}^0$, $\mathsf{A}_1{}^0$, $\mathsf{I}_1{}^0$, *and* $A_1{}^0$, the understanding being that the logistic systems of either quadruple are interrelated in the same way as $\mathbf{A}_1$, A_1, I_1, and A_1 with the index '$_0$' or '$_1{}^0$' respectively in place of the index '$_1$'. Accordingly, the totalities of rules of inference and decision of the organons A_0, $\mathbf{A}_0$, A_0, $\mathsf{A}_1{}^0$, $\mathbf{A}_1{}^0$, and $A_1{}^0$ are denoted by 'D_0', '$\mathbf{D}_0$', 'D_0', '$\mathsf{D}_1{}^0$', '$\mathbf{D}_1{}^0$, and '$D_1{}^0$', whereas the cumulative rules of the interpretations I_0 and $\mathsf{I}_1{}^0$ are denoted by 'I_0' and 'I_1'(the same as that of I_1), respectively. The wordy names of D_0, $\mathbf{D}_0$, and D_0 or those of $\mathsf{D}_1{}^0$, $\mathbf{D}_1{}^0$, and $D_1{}^0$ are variants of the wordy names of D_1, $\mathbf{D}_1$, and D_1 with respectively "*Basic*" ("*B*") or "*Rich Basic*" ("*RB*") in place of "*Advanced*" ("*A*"). Thus, D_0, $\mathbf{D}_0$, or D_0 is commonly (indiscriminately) called a *basic algebraic decision method* (*BADM*) and $\mathsf{D}_1{}^0$, $\mathbf{D}_1{}^0$, or $D_1{}^0$ is commonly called a *rich basic algebraic decision method* (*RBADM*) – in

contrast to D_1, $\mathbf{D}_1$, or D_1, which s commonly called an *advanced algebraic decision method* (*AADM*). The rules constituting D_1^0, $\mathbf{D}_1^0$, or D_1^0 are essentially the same as the rules constituting D_0, $\mathbf{D}_0$, or D_0 respectively, so that the former rules differ from the latter rules only in their domains of applications. In contrast to any one of any one of the AADM's D_1, $\mathbf{D}_1$, and D_1, which is incompatible with introducing a class, set, or mass builder (see the item 16), the latter affects neither any one of the BADM's D_0, $\mathbf{D}_0$, and D_0 nor any one of the RBADM's D_1^0, $\mathbf{D}_1^0$, and D_1^0. Therefore, while the TL (Trial Logic) as a single whole is destroyed by adding the class, set, or mass builder to it, both the PFRTL (Predicate-Free Relational Trial Logic) and the BFPTL (Binder-Free Predicate Trial Logic), being its *autonomous* parts, are preserved.

23) Besides the TLT, being the principal part of Psychologistics, the latter is supposed to include an indefinite number of relatively independent essays, which are called *Psychologistic Essays*, or briefly *Essays*, and which form supplementary material to the subject matter of the TLT that is included und`er the common heading "*Essays on Psychologistic Themes*" (abbreviated as "*EPT*"). Since each Essay is relatively autonomous, therefore the subject matter of the EPT is a cumulative one that comprises the subject matters of the separate constituent Essays. By this moment, EPT comprises ten Essays, whose subject matters have been recapitulated in Preface of the EPT under the heading "*Abstract of the current EPT*".

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BOE — Britanica Online Encyclopedia (Britanica.com).

GWST — *Greek Word Study Tool*. Online at <http://www.perseus.tufts.edu/hopper/morph?l=logion&la=greek>.

OCCL — *The Oxford Companion to Classical Literature*. Online at <http://en.wikipedia.org/wiki/The_Oxford_Companion_to_Classical_Literature>