Engineering Approach to Explanation of Special Relativity Theory

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Abstract: In this short note it is shown that the Special Relativity Theory (SRT) is a theory describing the reality correctly providing that certain conditions, which are encapsulated in the theory’s assumptions, are satisfied. This is demonstrated by using a simple capacitor, which avoids many difficulties of a relatively complex mathematics that leads many physics hobby enthusiasts to a wrong conclusion claiming that the theory cannot be correct and needs to be changed.

Introduction: The electronic gadgets that have been developed during the past decades and are being successfully used by many people; such as cell phones, TV sets, GPS navigation devices in cars, Laptop computers, etc., which are all based on Maxwell’s Equations that are used in describing some of the details of workings of all these devices and in their designs. Electronic engineers are very happy with these equations and believe in their correctness as is confirmed by the successes of all these applications. It is therefore puzzling that there are still some engineers that do not believe that SRT is correct, because SRT clearly follows from these equations. In order to avoid a complexity of mathematics, this will be demonstrated using a simple capacitor.

Simple background: The capacitor model that will be used for the simple derivations is shown in Fig.1. It is not possible to avoid the mathematics completely, so the reader must be patient and be familiar with only a simple algebra:

Fig.1 Parallel plate capacitor with the dielectric plates spaced at a distance \( d \) and charged with a uniformly embedded charge \( +Q = +mq \) and \( -Q = -mq \) respectively. The plates’ area: \( A = W\cdot L \) is large in comparison to \( d \) and the plates’ thickness \( z_p \) is small and can be neglected.

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It is well known in the electronic industry that the energy $E_n$ stored in the capacitor is equal to:

$$E_n = \frac{1}{2} CV^2 \quad (1)$$

where $C$ is the capacitance and $V$ the voltage across the capacitor. This formula is used for calculating the power consumption of all electrical gadgets and is well tested and experimentally verified. For our purposes, however, it will be expressed in terms of charge, because charge is an invariant quantity that does not depend on motion. Since it holds that: $Q = CV$ the result is:

$$E_n = \frac{1}{2} \frac{Q^2}{C} \quad (2)$$

As is also well known the parallel plate capacitor capacitance $C$ is calculated from the formula:

$$C = \frac{\varepsilon_o A}{d} \quad (3)$$

Where $\varepsilon_o$ is the vacuum dielectric constant and where it was considered that there is only a vacuum between the plates. The formula in Eq.2 can thus be rewritten as follows:

$$E_n = \frac{1}{2} \frac{Q^2 d}{\varepsilon_o A} \quad (4)$$

In the next step let’s consider further that this capacitor is moving in the direction parallel to the plates’ plane with a constant velocity $v$ relative to the laboratory coordinate system. The moving plates’ charge will, of course, appear as a current that will be observed from the laboratory point of view. The current will cause a magnetic field to appear between the plates with the magnetic field intensity $H$ according to the formula:

$$H = \frac{vQ}{2A} \quad (5)$$

It is now possible to calculate the energy stored in the capacitor moving in the direction of plates’ plane starting with the Lorentz force formula for the force acting between the plates:

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}) \quad (6)$$

By substituting into Eq.6 for the electric field intensity: $E = Q/Cd$, the magnetic field: $B = \mu_o H$, and using the relation that: $\varepsilon_o \mu_o = c^{-2}$ where $c$ is the speed of light, the energy is obtained by integrating the force over the distance $d$ with the result as follows:

$$E_n = \frac{1}{2} \frac{Q^2 d}{\varepsilon_o A} \left(1 - \frac{v^2}{c^2}\right) \quad (7)$$
This is a nice and simple result. The derivation seems correct, but this result cannot be right for a simple reason. It is not possible for the moving capacitor to have less energy than the stationary capacitor \( v = 0 \). Something is missing, so let’s call on SRT to see if this problem can be solved by using some of the formulas from that theory. In the next derivations the stationary parameters will have the subscript zero in order to distinguish them from the moving parameters. In SRT it is derived that the moving rods appear shorter, this is called the length contraction effect, which can now be applied to the capacitor area \( A: \ A = A_o \sqrt{1 - v^2 / c^2} \). This will result in the following modification of Eq.7:

\[
E_{n\parallel} = \frac{1}{2} \frac{Q^2 d_o}{\varepsilon_o A_o} \sqrt{1 - \frac{v^2}{c^2}}
\]  

(8)

This result is encouraging, but still we have a reduction in the energy when the capacitor is moving. Let’s examine the capacitor motion in the direction that is perpendicular to the plates. This will change the apparent plate distance: \( d = d_o \sqrt{1 - v^2 / c^2} \), while the area \( A \) will not be reduced. The result will be as follows:

\[
E_{n\perp} = \frac{1}{2} \frac{Q^2 d_o}{\varepsilon_o A_o} \sqrt{1 - \frac{v^2}{c^2}}
\]  

(9)

This is nice; at least we have the same result in both directions because the energy of the moving capacitor cannot depend on the direction of motion. However, the problem of reduced energy of a moving capacitor still has not been solved. Something is missing.

**Missing energy:** It seems that we have not included a kinetic energy of the field into calculations when the capacitor is moving. If the field between the plates had a mass associated with it, even if there is only a vacuum there, then moving capacitor would certainly have a kinetic energy associated with that mass. This is suggesting that the vacuum with the field in it is obviously not empty; it must be filled with something massive, at least when it is deformed by the field, such as for example, the dark matter or the old æther. The field mass can be determined by dividing the energy by \( c^2 \). It therefore follows that the rest mass of the field in the capacitor can be defined as:

\[
m_o = \frac{1}{2} \frac{Q^2 d_o}{\varepsilon_o A_o c^2}
\]  

(10)

Here it was considered that the plates’ mass itself can be neglected (very thin charged plastic foils) and that only the field in the vacuum has a mass. Let’s consider the field kinetic energy to be equal to:

\[
E_{n,k} = \frac{m_o v^2}{\sqrt{1 - v^2 / c^2}}
\]  

(11)

The total energy for the capacitor in motion in either direction thus will be:
\[
E_{\text{tot}} = m_o c^2 \sqrt{1 - \frac{v^2}{c^2}} + \frac{m_o v^2}{\sqrt{1 - v^2 / c^2}} = \frac{m_o c^2}{\sqrt{1 - v^2 / c^2}} \approx m_o c^2 + \frac{1}{2} m_o v^2 + \ldots
\]  

(12)

This result agrees with our expectations. SRT therefore resolved the problem, which would otherwise not be possible to solve. This simple calculation thus confirms that SRT is a correct theory and that there might be an æther or a dark matter in the vacuum that mediates the action of a field.

**Time dilation:** Because we have now verified the correctness of SRT length contraction effect and the inertial mass increase with velocity, the attention will turn to time dilation to complete the analysis. It will now be assumed that the plates have a small rest inertial mass \(m_{\text{ipo}}\) and will be released from a mutual distance \(d\) to collide with each other. This will be analyzed for the plates that are moving in a direction parallel to the plates with a velocity \(v\). The collision time will thus represent the ticking of clocks, because it can be periodically repeated. From the Newton force equation (slow plate motion in the perpendicular direction to the plates’ plane compare to \(c\)) and the force equation in Eq.6 assuming again that the velocity \(u\) of plates travelling towards each other is arbitrarily small, the resulting formula that will determine the time of collision will be as follows:

\[
m_{\text{ipo}} \frac{du}{dt} = \frac{Q^2}{2e_o A} \left(1 - \frac{v^2}{c^2}\right)
\]

(13)

After two times integrating this equation and considering that the distance to collision is \(d_o/2\) the result for the time to collision is:

\[
t_c^2 = \frac{2e_o d_o m_{\text{ipo}} A}{Q^2 \left(1 - v^2 / c^2\right)} = \frac{2e_o d_o m_{\text{ipo}} A_o}{Q^2 \left(1 - v^2 / c^2\right)}
\]

(14)

This is the famous SRT time dilation formula. It is worth noting that the plates’ inertial mass increase due to the velocity \(v\) is exactly compensated by the plates’ area reduction. It is thus clear that for the apparent ticking time of moving clocks (colliding plates) holds the following:

\[
t_A = \frac{t_{\text{col}}}{\sqrt{1 - v^2 / c^2}}
\]

(15)

The case for the motion perpendicular to the plane of plates is not easy to analyze. Analysis is difficult due to the non-uniform plates’ motion in that direction. However, this case is already superfluous for the proof of time dilation effect as shown in Eq.15.

Having thus derived the length contraction and time dilation formulas it is now easy to understand the Lorentz coordinate transformation equations:

\[
x_o = \frac{x - vt}{\sqrt{1 - v^2 / c^2}} \quad t_o = \frac{t - vx / c^2}{\sqrt{1 - v^2 / c^2}}
\]

(16)
and from them derive the well-known and important coordinate transformation invariant:

\[ s^2 = (ct)^2 - x^2 = (ct_o)^2 - x_o^2 \] (17)

**Discussion:** Several important observations can be made in the above derivations as follows:

1. It is possible to assign a gravitating mass to the first term in Eq.12: \( m_g = m_o \sqrt{1 - v^2 / c^2} \), and the inertial mass to the second term: \( m_i = m_o / \sqrt{1 - v^2 / c^2} \), as has already been found in previous publications \([1,2]\). These relations can be generalized and applied to all bodies. In particular the photons do not have any gravitational mass they have only inertial mass.

2. The second important observation is that the kinetic energy when the capacitor motion is in the parallel direction to the plates plane is carried only by the magnetic field, while the energy of the capacitor when moving in the direction perpendicular to the plates plane is carried only by the electric field. This is because there is no magnetic field in the capacitor in that case.

3. The third observation is that while the gravitating mass is reduced by the motion this deficit is more than compensated for by the increase of the inertial mass due to the motion to obtain the well-known formula for the kinetic energy valid for small velocities.

4. The fourth observation is that the field kinetic energy does not have a factor \( \frac{1}{2} \) in front of the formula. The interplay between the rest energy and the kinetic energy is, therefore, interesting and may lead to a deeper understanding of what is going on in the masses of atomic nuclei.

5. The last observation is that the total rest energy of the capacitor is equal to:

\[ E_{\text{rest}} = m_o c^2 \] (18)

**Conclusions:** Finally, it is clear that SRT is a correct theory that is valid for any uniform inertial motion and that it is closely describing the reality. It is thus perplexing that there are still some scientists and engineers claiming that it is wrong. When the paradoxes are encountered in SRT it is necessary to always examine assumptions and the details how the theory is applied. The author hopes that this short note may be helpful to such disbelieving engineers.

**References:**
