Examination of sufficient conditions for inflation, e folds, density fluctuations, and a modified Heisenberg Uncertainty Principle

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Abstract
First, referring to an article by K. Freeze on “Natural Inflation”, we review e folds data, and also the role of a modified Heisenberg Uncertainty principle as to what to expect from initial conditions as to satisfying the onset of inflation.

Key words, Modified Heisenberg Uncertainty Principle, Sufficient condition for inflation, E folds, density fluctuations.
1. What is important about the modified HUP and applying conditions brought up by Freeze, as to natural inflation in [1]?

We will first of all refer to two necessary and sufficient conditions for the onset of inflation, given in [1], and the modified HUP, given in [2], combined with Padmanabhan’s reference [3].

I.e. what we will be doing is to re do the reference calculations given in [1] with an eye toward initially using the potential \( V \sim \text{Modified HUP Energy} E \), as the input into [1]’s criteria into inflation generation. The given innovation in our procedure will be to have the following phase change, in terms of initial conditions

\[
V_{\text{Pre-Planckian}} \sim \left( \Delta E \sim \frac{h}{\delta t \cdot a_{\text{min}}^2 \phi_{\text{inf}}} \right)_{\text{Planckian-space-time}} \rightarrow V \approx V_0 \cdot \exp \left\{ -\left( \frac{16\pi G}{\gamma} \cdot \phi(t) \right) \right\}.
\]

Here, we will be using in the Pre Planckian potential the inputs from the data usually associated with [3]

\[
a \approx a_{\text{min}} t^\gamma
\]

\[
\Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot t \right\}
\]

\[
\Leftrightarrow V \approx V_0 \cdot \exp \left\{ -\left( \frac{16\pi G}{\gamma} \cdot \phi(t) \right) \right\}
\]

In other words, we will be using the inflation given by

\[
\phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot t \right\}
\]

As far as applications to:[1]

a) Sufficient inflation: the 65 E fold limit

\[
N_{\text{-foldings}} = -\frac{8\pi}{m_{\text{Planck}}^2} \int_{\phi_1}^{\phi_2} V(\phi) \left( \frac{\partial V(\phi)}{\partial \phi} \right) d\phi \geq 65
\]

b) Amplitude to density fluctuations, given by

\[
\frac{\delta \rho}{\rho} \approx \left. \frac{H^2}{\dot{\phi}} \right|_{\text{Horizon}} \geq \frac{\delta T}{T} \leq O \left( 10^{-5} \right)
\]

Eq. (5) and Eq. (6) will use Eq.(3) as the inflaton, while also using Eq. (4) as the Pre Planck Potential.
2. Filling in the details, and what it indicates as far as physics

What we are doing is a way to give more substance to the calculations given in [4], and we find that for Eq. (5) we have

\[
N_{\text{e-foldings}} = \frac{8\pi}{m_{\text{Planck}}^2} \left( \ln \frac{t_2}{t_1} \right)^2 \sim \frac{8\pi}{m_{\text{Planck}}^2} \left( M_{\text{max-number}} \right)
\]  

(7)

If we utilize the Planck Units, the above then changes to:

\[
N_{\text{e-foldings}} \bigg|_{\text{Planck-units}} = 8\pi \cdot \left( \ln \frac{t_2}{t_1} \right)^2 \sim 8\pi \cdot \left( M_{\text{max-number}} \right)
\]  

(8)

For the Eq. (6) we would have

\[
\frac{\delta \rho}{\rho} \simeq \frac{H^2}{\dot{\phi}} \bigg|_{\text{Horizon}} \sim \frac{8\pi GV_0}{\gamma \cdot (3\gamma - 1)} \cdot \frac{\dot{t} \cdot H^2}{\sqrt{4\pi G}} \bigg|_{\text{Horizon}} \sim \frac{t \cdot H^2}{\sqrt{\gamma} \cdot \sqrt{4\pi G}} \bigg|_{\text{Horizon-Planck-units}}
\]  

\[
\sim \frac{t \cdot H^2}{\sqrt{\gamma} \cdot \sqrt{4\pi G}} \bigg|_{\text{Horizon-Planck-units}} \leq O\left(10^{-5}\right)
\]  

(9)

This, especially, Eq. (9) puts severe restraints upon \( H \)

Here we will be referencing a model for \( a \approx a_{\text{min}}^t \) meaning that the constraint Eq. (9), bottom part, becomes affected by

\[
H \sim (\dot{a} / a) \sim \gamma / t
\]  

(10)

Then

\[
\frac{\delta \rho}{\rho} \sim \frac{t \cdot H^2}{\sqrt{\gamma} \cdot \sqrt{4\pi G}} \bigg|_{\text{Horizon-Planck-units}} \sim \frac{\gamma^{3/2} \sqrt{4\pi}}{(t / t_{\text{Planck}})} \bigg|_{\text{Horizon-Planck-units}} \leq O\left(10^{-5}\right)
\]  

(11)
3. How to verify the above. What it is saying?

There is nothing too surprising about Eq. (11) above. However, Eq. (8) states that we really DO need to, if we assume the HUP modified, as given in our derivations, that the fine details of the E folding depend upon our choice of time \( t_1 \) and time \( t_2 \), in crucial ways. If \( t_1 \) is say in the Pre Planckian era, before Planck time, and \( t_2 \) is say in the electro weak, the implications are profound.

In addition this may also affect [5] and [6] in terms of phenomenology, all of which needs to be vetted in careful data set analysis.

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Reference


