

The suggestion that 2-probable primes satisfying Even Goldbach conjecture are possible

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Abstract: The even Goldbach conjecture suggests that every even integer greater than four may be written as the sum of two odd primes. This conjecture remains unproven. We explore whether two probable primes satisfying the Fermat’s little theorem can potentially exist for every even integer greater than four. Our results suggest that there are no obvious constraints on this possibility.

Results:

Let every even integer $2n$ greater than four be expressible as the sum of two probable odd primes p and q .

Then $2n=p+q$

And by definition of 2-probable primes using Fermat’s little theorem

$$2^{p-1} \cong 1 \pmod{p}$$

$$2^{q-1} \cong 1 \pmod{q}$$

$$2^{p-1} . 2^{q-1} \cong 1 \pmod{pq}$$

$$2^{p+q-2} \cong 1 \pmod{pq}$$

$$2^{2n-2} \cong 1 \pmod{pq}$$

Therefore if for every even integer $2n$ there exists 2-probable primes p and q then the above conditions must be satisfied.

$$\text{So } 2^{2n-2} = pqr+1$$

Where r is a suitable odd integer.

$$4^{n-1} - pqr = 1 \dots\dots\dots(I)$$

We are interested in all cases where n is a positive integer greater than 2. Since p and q are two probable odd primes, therefore $\text{gcd}(4^{n-1}, pq) = 1$

and by Bezout’s identity it should be possible to find integers a and b such that

$$a.4^{n-1} + b.pq = 1$$

Expression (I)

$4^{n-1} - pqr = 1$ is one such form.

Alternative Bezout’s coefficients may be identified as follows

$$4^{n-1} - 4^{n-1}xpq + 4^{n-1}xpq - pqr = 1$$

$$(1 - pqx)4^{n-1} + (4^{n-1}x - r)pq = 1$$

So Bezout coefficients, "a" is of the form $(1 - pqx)$ and "b" is of the form $(4^{n-1}x - r)$ where x is any integer and "r" is a value that is specific for every integer 2n.