Experimental Verification or Refutation of the Electric Charge Conservation using a Cylinder-Capacitor with Rotating Core

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Abstract

The electric force from a uniformly moving point charge onto a resting point charge does not correspond exactly to the Coulomb force. This is a consequence of the Liénard–Wiechert potentials which are derived from Maxwell's equations. If the point charge is moving toward or away from a resting point charge, the electric force seems to be weakened compared to the Coulomb force. In contrary, the electric force appears to be strengthened when the point charge is passing the resting charge sideways. Together, both effects compensate each other so that the total charge is independent of the relative speed.

This article proposes and discusses an experiment with which this claim can be verified. The experiment is of major importance, because besides the field formula of a point charge derived from Maxwell's equations a recently discovered, clearly easier structured alternative exist in which no longer a magnetic part occurs. Although both formulas differ significantly, it is impossible to design experiments with current loops of any form to decide between both alternatives, because theoretical considerations leads always to the same experimental predictions.

The electrical part of both field formulas differs only by a Lorentz factor. This has the consequence that the total charge is in the alternative formula no longer independent from the relative speed between source and destination charge. Thus, the electric charge depends here on the reference frame and we get rest and relativistic charge. The experiment proposed in this article makes it possible to measure this effect so that a decision between both alternatives becomes possible.

Contents

1 Introduction and Starting Point

1.1 Background and Motivation

The electromagnetic field of a uniformly moving ideal point charge can be calculated with Maxwell's equations. A moving point charge q_s with speed $\vec{u}$ which is for the moment at the coordinate origin causes at position $\vec{r}$ the electric field [2, p. 640]

$$\vec{E} = \frac{c\, q_s\, \left(c^2 - u^2\right)\, \vec{r}}{4\,\pi\,\varepsilon_0 \sqrt{r^2\, \left(c^2 - u^2\right) + \left(\vec{r}\,\vec{u}\right)^2}^3}. \tag{1}$$

The magnetic field $\vec{B}$ is

$$\vec{B} = \frac{\vec{u}}{c^2} \times \vec{E}. \tag{2}$$

Actually measurable are only force effects between charges. In order to calculate the force $\vec{F}$ caused from the charge q_s onto another point charge q_d, we also need its speed $\vec{v}$ and the formula of the Lorentz force $\vec{F} = q_d\,\vec{E} + q_d\,\vec{v} \times \vec{B}$. Finally, we obtain the formula

$$\vec{F}_M = \frac{c\, q_s\, q_d\, \left(c^2 - u^2\right)\, \vec{r}}{4\,\pi\,\varepsilon_0 \sqrt{r^2\, \left(c^2 - u^2\right) + \left(\vec{r}\,\vec{u}\right)^2}^3} + \vec{v} \times \left(\frac{\vec{u}}{c^2} \times \frac{c\, q_s\, q_d\, \left(c^2 - u^2\right)\, \vec{r}}{4\,\pi\,\varepsilon_0 \sqrt{r^2\, \left(c^2 - u^2\right) + \left(\vec{r}\,\vec{u}\right)^2}^3} \right) \tag{3}$$

for the force of a uniformly moving ideal point charge q_s onto another uniformly moving ideal point charge q_d at position $\vec{r}$.

It is astonishing that the entire electro- and magnetostatics, including the induction law, can also be derived from the formula [1]

$$\vec{F}_R = \frac{\gamma(w)\, c\, q_s\, q_d\, \left(c^2 - w^2\right)\, \vec{r}}{4\,\pi\,\varepsilon_0 \sqrt{r^2\, \left(c^2 - w^2\right) + \left(\vec{r}\,\vec{w}\right)^2}^3}. \tag{4}$$

In this, γ is the Lorentz factor and $\vec{w}$ the difference speed $\vec{u} - \vec{v}$ between the two point charges. It is obvious that the force (4) has a simpler structure than formula (3) and that the force effect originates here always directly from the source. $\vec{F}_R$ is thus a central force, but $\vec{F}_M$ is not.

Despite the different form, both force formulas lead to the same predictions when we analyze, instead of single charges, currents and interpreting them as closed chains of uniformly moving point charges. This means that it is impossible to decide with the laws of classical electrical engineering which force formula is actually correct. On the other hand, however, it is not so that both formulas would be equivalent, because there are tiny differences for moving point charges. At the same time, the field

$$\vec{\mathcal{E}} := \vec{F}_R / q_d = \gamma\, \vec{E} \tag{5}$$

does not correspond to the electric field $\vec{E}$ because of

$$\varepsilon_0 \oiint_{\partial V} \vec{\mathcal{E}} \cdot \mathrm{d}\vec{A} = \gamma\, q_s, \tag{6}$$

what does not fit, due to the additional Lorentz factor, to the first of Maxwell's equations.

The fact that the complex relations of classical electrical engineering can also be derived from equation (4) leads inevitably to the question whether the usually used partitioning of electromagnetic force into an electrical and magnetic part is appropriate and if electrical charge possibly raises with an increasing speed. For mass, this is taken for granted. The same might be true for electric charge.

1.2 The Field of Uniformly Moving Point Charges

When comparing the equations (3) and (4), it is noticeable that both formulas differ only by a Lorentz factor $\gamma(u)$ if the target charge q_d does not move. For $\vec{v}=0$ is therefore

$$\vec{F}_R = \gamma(u)\,\vec{F}_M. \tag{7}$$

For the further it is helpful to understand how the shape of the electric field $\vec{F}_M/q_d$ depends on the velocity $\vec{u}$ of the source charge from the viewpoint of a resting target charge. For this purpose, in equation (3), the speed of the target charge $\vec{v}$ is set to zero. After a rearrangement of the terms we get

$$\vec{F}_M = \zeta_M \cdot \frac{q_s\,q_d}{4\,\pi\,\varepsilon_0}\,\frac{\vec{r}}{r^3}. \tag{8}$$

with

$$\zeta_M = \frac{\left(1-\frac{u^2}{c^2}\right)}{\sqrt{\left(1-\frac{u^2}{c^2}\right)+\left(\frac{\vec{r}}{r}\,\frac{\vec{u}}{c}\right)^2}^{\,3}}. \tag{9}$$

The posterior part of expression (8) corresponds exactly to the Coulomb law and is therefore of no interest. The direction-dependent, unitless and scalar factor ζ_M is however important. For $\vec{r}\parallel\vec{u}$, ζ_M becomes $1/\gamma(u)^2$, as can easily be verified by insertion. But, if $\vec{r}$ is oriented perpendicular to the velocity $\vec{u}$, ζ_M is exactly $\gamma(u)$. Eventually we have

$$\zeta_M^{\parallel} = \frac{1}{\gamma(u)^2} \quad \text{and} \quad \zeta_M^{\perp} = \gamma(u). \tag{10}$$

Because the Lorentz factor $\gamma(u)$ is principally greater or equal one, the electric force seems to be amplified transversely to the moving point charge but weakened longitudinally.

These statements apply not only to solution (3) resulting from the Maxwell equations, but also to the competing force formula (4). But here is because of relation (7)

$$\zeta_R^{\parallel} = \frac{1}{\gamma(u)} \quad \text{and} \quad \zeta_R^{\perp} = \gamma(u)^2. \tag{11}$$

The object of the experiment discussed in the following is to determine the extremely small factor $\zeta^{\perp}$ as precisely as possible with minimal implicit assumptions[1] and to determine whether $\zeta^{\perp}=\gamma(u)$ or $\zeta^{\perp}=\gamma(u)^2$ holds true.

2 The Experiment

2.1 Principle Design and Description of the Measuring Process

The experiment uses a cylinder capacitor with an internal cylinder, which can rotate with a given rotational speed. The entire capacitor have to be in a vacuum chamber. In the first step of the experiment, the outer hollow cylinder is short-circuited to ground and the inner cylinder is subjected with a positive high voltage (figure 1). After the capacitor is fully charged, the voltage source is removed, the connection of the outer cylinder is disconnected from ground, and a voltage measuring device with a very high internal resistance is interposed. Because the outer cylinder is still at the same reference potential as ground, the voltage measuring device will display exactly zero. Subsequently, the inner cylinder is slowly brought to a definite final rotational speed. After the disappearance of the induction current in the outer hollow cylinder, a voltage measurement is carried out.

[1] relativistic dynamics

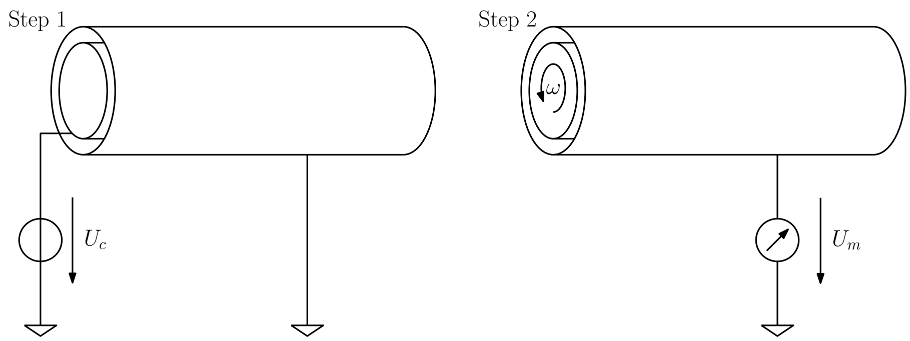

Figure 1: A cylinder capacitor is charged and then decoupled from voltage source and ground. In the second step, the inner cylinder is brought into rotation.

If the electric charge is independent of the relative speed, no voltage should be measured even when the inner cylinder is rotating. In order to avoid errors, any charge balancing between the cylinders and the environment has to be prevented very carefully. This is only possible if the distance between the capacitor plates is sufficiently high and when the experiment is operated in a vacuum.

2.2 Theoretical Analysis of the Measurement Setup

In Maxwell's electrodynamics, a *resting* charge is only influenced from the electric field $\vec{E}$. This even applies when the field-generating charges are moving. Looking at the side view of the cylinder capacitor (figure 2), it becomes clear that the positively charged electron holes of the inner cylinder, which are moved with the velocity $\omega\, R_i$, cannot magnetically affect the electrons in the outer cylinder because these are in rest. From the viewpoint of Maxwell's electrodynamics, it is therefore sufficient to consider only the electric field. This can be determined with Maxwell's first equation

$$\varepsilon_0 \oiint_{\partial V} \vec{E} \cdot \mathrm{d}\vec{A} = Q, \tag{12}$$

where Q is the total charge stored on the inner cylinder.

The central statement of Maxwell's first equation is that the electrical flux through any closed surface does exactly correspond to the enclosed total charge Q. It can be calculated that this is always true for the electric field of a moving point charge given by equation (1). Considering that the electric field of each charge distribution, including that of the inner cylinder, is a sum of the fields of all single charges, it becomes clear that Maxwell's first equation is generally valid, and that the velocities of the charges in the inner cylinder does not influence the electric field $\vec{E}$.

The electric field of a hollow cylinder with the radius R rotating with the angular velocity ω can be easily determined with the first Maxwell equation. First of all, it is obvious that the field inside has to be zero, since any closed surface that is inside of the cylinder does not include any charge. The electric field to the outside can be calculated by enclosing the hollow cylinder into a cylinder with a larger radius. The total charge Q of the hollow cylinder is therefore completely within.

It can now be concluded that, for symmetry reasons, the electric field can have only a radial component, and therefore $\vec{E} = E\, \vec{e}_r$ has to be valid. The same is true for the surface element $\mathrm{d}\vec{A}$ on the lateral area of the cylinder. If we consider that a cylinder having the length L and the radius r has a lateral surface of $2\,\pi\, r\, L$, equation (12) gives the relation $\varepsilon_0\, E\, 2\,\pi\, r\, L = Q$. That means, at the distance r from the symmetry axis the field strength is

$$\vec{E}(r) = \frac{Q}{2\,\pi\,\varepsilon_0\, r\, L}\,\vec{e}_r. \tag{13}$$

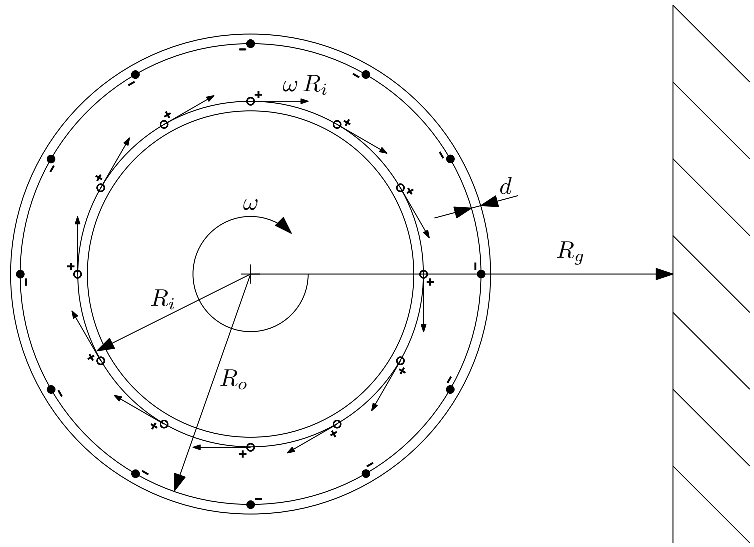

Figure 2: Side view

The field $\mathcal{E}$ can be calculated because of the relation (5) in an equivalent manner. We get

$$\vec{\mathcal{E}}(r,\omega) = \frac{\gamma(\omega\,R)\,Q}{2\,\pi\,\varepsilon_0\,r\,L}\,\vec{e}_r. \tag{14}$$

It was exploited that the Lorentz factor γ is in this particular case the same for all charges of the rotating cylinder. Thus it is a constant which does not have to be considered for the integration over the charge density.

For the field of the cylinder capacitor shown as a side view in figure 2, the relation is

$$\vec{E}(\vec{r}) = \frac{Q}{2\,\pi\,\varepsilon_0\,r\,L}\,\vec{e}_r \cdot \begin{cases} 0 & \text{for } r < R_i \\ 1 & \text{for } R_i \le r < R_o \\ 0 & \text{for } R_o \le r \end{cases} \tag{15}$$

when the electric charge is relativistic invariant.

If, however, the electric charge depends, contrary to the statement of Maxwell's first equation, on the differential velocity the field should have the form

$$\vec{\mathcal{E}}(\vec{r}) = \frac{Q}{2\,\pi\,\varepsilon_0\,r\,L}\,\vec{e}_r \cdot \begin{cases} 0 & \text{for } r < R_i \\ \gamma(\omega\,R_i) & \text{for } R_i \le r < R_o \\ \gamma(\omega\,R_i) - 1 & \text{for } R_o \le r. \end{cases} \tag{16}$$

Thus, a force $\vec{F} = q_d\,\vec{\mathcal{E}}$ on any resting charge q_d outside from the cylinder capacitor would occur. This means furthermore that, if electric charge is not relativistic invariant, a very small, angular velocity dependent voltage

$$U_m = \int\limits_{R_o+d}^{R_g} \vec{\mathcal{E}} \cdot \vec{e}_r\,\mathrm{d}r = (\gamma(\omega\,R_i) - 1)\;Q\,\frac{\ln(R_g) - \ln(R_o + d)}{2\,\pi\,\varepsilon_0\,L} \tag{17}$$

from the outer surface of the capacitor against ground at distance R_g from the symmetry axis of the capacitor should be measurable (figure 2).

Because the capacitor is charged only when the internal cylinder is in rest, the stored charge quantity on the internal cylinder is

$$Q = C\,U_c \tag{18}$$

for a charge voltage U_c. The capacitance C of the capacitor is a pure geometric parameter and can be accurately calculated and measured. A calculation in this case results in

$$C = \frac{2\,\pi\,\varepsilon_0\,L}{\ln(R_o) - \ln(R_i)}. \tag{19}$$

Substituting the formulas (18) and (19) into the equation (17) yields for the measured voltage the equation

$$U_m = (\gamma(\omega\,R_i) - 1)\;U_c\,\frac{\ln(R_g) - \ln(R_o + d)}{\ln(R_o) - \ln(R_i)}, \tag{20}$$

which contains only parameters which can be easily controlled and measured in the experiment.

3 Summary

Figure 3 shows the expected measured voltage under realistic assumptions. As can be seen, the expected voltage is in the microvolt range and is thus measurable. At the same time, it becomes apparent that the effect is very small and can only be noticed if absolutely every charge balancing is prevented. For this reason, it is not possible to observe this effect in the case of current-carrying conductor loops, because the voltage source, which causes the current flow, provides charge carriers with which the conductor neutralizes itself towards the outside.

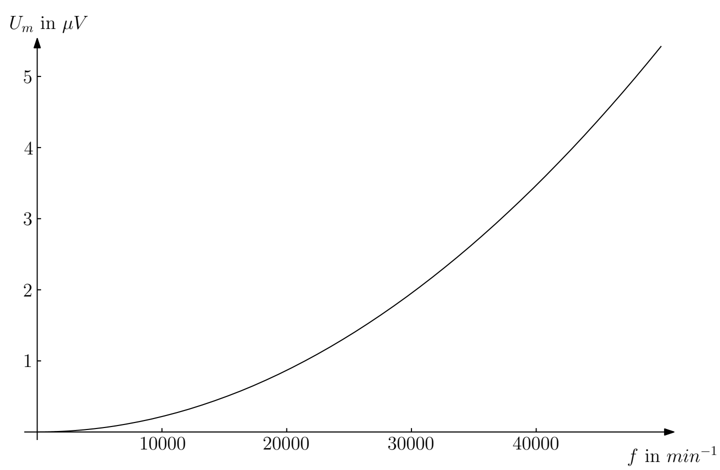

Figure 3: Expected voltage U_m for $R_i = 10cm$, $R_o = R_i + 3mm$, $R_g = 30cm$, $d = 3mm$ and $U_c = 10kV$ in dependency of the rotational speed $f = \frac{\omega}{2\,\pi}$.

A further possibility to detect this effect would be to generate a current flow in a superconductor and then to heat it so that the current comes to a standstill when the critical temperature is exceeded. If in the experiment charge balancing would be suppressed, the object would no longer be electrically neutral after warm-up. But here, too, the measured voltages would be low, and it would seem to be plausible to explain the effect with electrostatic charging.

In this context, it might be understandable that the author of this article considers the possibility that the electrical charge could be relative and that this has been overlooked so far. After all, Maxwell's electrodynamics is one of the best-tested theories at all and forms the basis of many theories and technologies. At this point, however, the reader should recognize that the subject studied here is not concerned with the propagation of electromagnetic waves. The temporal behavior of the propagation of electromagnetic waves is very probably correctly described by Maxwell's equations. Only their amplitudes are discussed, and the deviations are so small that they are of no importance in electrical engineering, where charges are practically always slow in comparison to the speed of light.

Nevertheless, it is extremely important to determine whether the Maxwell equations are correct with respect to fast-moving charge carriers. Strangely enough, it has rarely been questioned that the electrical charge is relativistically invariant. Calculations which, for example, use the formula $q\,\vec{v} \times \vec{B}$ for very fast moving electrical particles would be incorrect and it might be that this error is up to now unconsciously compensated with an additional Lorentz factor in the relativistic mass of the particle. If, therefore, this experiment did not yield a null result, this would have a serious impact on relativistic dynamics. The fact that the classical electro- and magnetostatics can be significantly simplified by the additional Lorentz factor should prepare for a possible non-null result.

References

[1] Steffen Kühn. Magnetism, interpreted as a multi-particle effect. *vixra:1611.0287*, 2016.

[2] Günter Lehner. *Elektromagnetische Feldtheorie*. Number ISBN 3-540-00998-1. Springer-Verlag Berlin Heidelberg New York, 2004.