Key point of the proof (Proof of Global in Time Solvability of Incompressive NSIVP in the Whole Space Using Time Transformation Analysis)

Global in Time Solvability is confirmed, based on nonincreasing of $\boldsymbol{L}^2$-norm of the solution $\|\boldsymbol{u}\|_2 \leq \|\boldsymbol{a}\|_2$ gotton as a priori estimation, by means of upper limit estimation of solution based on integral equation with regard to the solution.

However, estimation by means of simple combination of following integral equation (1.3) and Hölder inequality and Young's inequarity has limitation and can't give result to be proved.

$$(1.3)\quad \boldsymbol{u}_t = K_t * \boldsymbol{a} - \int_0^t d\tau\, \mathcal{P}(\boldsymbol{\partial} K_{t-\tau} * \cdot \boldsymbol{u}_\tau \boldsymbol{u}_\tau)$$

For examble, simply using $\|\boldsymbol{u}\|_2 \leq \|\boldsymbol{a}\|_2$ and equation above, following pro forma inequality is confirmed.

$$\|\boldsymbol{u}_t\|_q \leq \|\boldsymbol{a}\|_q + C\|\boldsymbol{a}\|_2^2 \int_0^t d\tau\, \|\boldsymbol{\partial} K_{t-\tau}\|_q$$

However, considering $\|\boldsymbol{\partial} K_{t-\tau}\|_q = c(t-\tau)^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$, for $n \geq 3, q \in (n, \infty]$, this factor diverges in case of integrating with regard to τ.

In the paper, avoiding divergence like the abobe by means of time transfered integral equation (1.4), usuful estimation is confirmed and global in time solvability is proived.

$$(1.4)\quad \boldsymbol{u}_t^{\Phi} = K_t^{\Phi} * \boldsymbol{a} - \int_0^t d\tau\, \mathcal{P}(\boldsymbol{\partial} K_{t-\tau}^{\Phi} * \varphi_\tau \cdot \boldsymbol{u}_\tau^{\Phi} \boldsymbol{u}_\tau^{\Phi})$$

By means of this relation and $\|\boldsymbol{u}\|_2 \leq \|\boldsymbol{a}\|_2$, following relation is confirmed.

$$\|\boldsymbol{u}_t^{\Phi}\|_q \leq \|\boldsymbol{a}\|_q + C\|\boldsymbol{a}\|_2^2 \int_0^t d\tau\, \varphi_\tau \|\boldsymbol{\partial} K_{t-\tau}^{\Phi}\|_q$$

Based on $\|\boldsymbol{\partial} K_{t-\tau}^{\Phi}\|_q = c(\Phi(t-\tau))^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$ and using appropriate time transformation funstion $\Phi = \Phi(t)$, even though for $n \geq 3, q \in (n, \infty]$, this factor doesn't diverge for integrating with regard to τ and with integral factor φ integral keeps finete, useful estimation can be confirmed.

$$\|\boldsymbol{u}_t^{\Phi}\|_q \leq \|\boldsymbol{a}\|_q + Cc\|\boldsymbol{a}\|_2^2 \int_0^t d\tau\, \varphi_\tau {\Phi_{t-\tau}}^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$$

note

Global in time solvability of nonlinear problems were proved depending typically on local in time solvability and a priori estimations.

As for NSIVP in the whole space, local in time solvability for initial value which belongs to $\boldsymbol{L}^q{}_{,\, q\in(n,\infty]}$ and a priori estimation with regard to $\boldsymbol{L}^2$-norm of solution are proved simplly by means of well known relations like integral equation and Young's inequarity.

Therefore, if appropriate a priori estimation with regard to $\boldsymbol{L}^q$-norm of solution is confirmed, global in time solvability is proved.

However, analysis based on well known relations like integral equation and Young's inequality has never provided appropriate a priori estimation with regard to $\boldsymbol{L}^q$-norm of solution.

On the other hand, analysis based on time transformed integral equation and Young's inequality provides appropriate a priori estimation with regard to $\boldsymbol{L}^q$-norm of solution.