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REPRESENTATION OF GRAPHS USING INTUITIONISTIC NEUTROSOPHIC SOFT SETS

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Abstract. The concept of intuitionistic neutrosophic soft sets can be utilized as a mathematical tool to deal with imprecise and unspecified information. In this paper, we apply the concept of intuitionistic neutrosophic soft sets to graphs. We introduce the concept of intuitionistic neutrosophic soft graphs, and present applications of intuitionistic neutrosophic soft graphs in multiple-attribute decision-making problems. We also present an algorithm of our proposed method.

1. Introduction

Zadeh [39] introduced the concept of fuzzy set, characterized by a membership function in [0, 1], which is very useful in dealing with uncertainty, imprecision and vagueness. Since then, many higher order fuzzy sets have been introduced in literature to solve many real life problems involving ambiguity and uncertainty. Atanassov [5] introduced the concept of intuitionistic fuzzy sets (IFSs) as an extension of Zadeh's fuzzy set [39]. The concept of IFS can be viewed as an alternative approach for when available information is not sufficient to define the impreciseness by the conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but IFS is described by a membership(truth-membership) function and a non-membership(falsity-membership) function, the only requirement is that the sum of both values is less than and equal to one. However, IFSs cannot deal with all types of uncertainty, including indeterminate information and inconsistent information, which exist commonly in different real-world problems. Smarandache [32] introduced the idea of neutrosophic set theory from philosophical point of view. Its prominent characteristic is that a truth-membership degree, an indeterminacy membership degree and a falsity membership degree, in non-standard unit interval $]0^-, 1^+[$, are independently assigned to each element in the set. Moderately, it has been discovered that without a specific description, neutrosophic sets are difficult to apply in the real life applications. After analyzing this difficulty, Wang et al. [34] presented the idea of single-valued neutrosophic set (SVNS) from scientific or engineering point of view, as an instance of the neutrosophic set and an extension of IFS, and provide its various properties. SVNSs represent uncertainty, incomplete,

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imprecise, indeterminate and inconsistent information which exist in real world. On the other hand, Bhowmik and Pal [7] introduced intuitionistic neutrosophic set (INS) and discussed some of its properties.
Molodtsov [26] introduced soft set theory as a new mathematical tool for dealing with imprecision. Soft sets introduced by Molodtsov gave us new technique for dealing with uncertainty after specifying set of parameters. Soft sets has many applications in several fields including operation research, decision-making, probability theory, and smoothness of functions, measurement theory [10, 12, 13]. Maji et al [21, 22, 24] proposed fuzzy soft sets, intuitionistic fuzzy soft sets (IFSSs) and neutrosophic soft sets (NSSs) by combining fuzzy, intuitionistic fuzzy and neutrosophic set theories with soft set theory. Said and Smarandache [30] proposed intuitionistic neutrosophic soft set (INSSs) and its application in decision making-problems. Broumi [11] introduced generalized neutrosophic soft set. Sahin and Kucuk [33] defined similarity and entropy of neutrosophic soft set. Ye [38] proposed correlation coefficients of neutrosophic soft set and its application in decision-making problem. Ye [37] also defined multi criteria decision-making method using aggregation operators.
Akram and Nawaz [1] have introduced the concept of soft graphs and some operation on soft graphs. Certain concepts of fuzzy soft graphs and intuitionistic fuzzy soft graphs are discussed in [2, 3, 29]. Akram and Shahzadi [4] have introduced neutrosophic soft graphs. In this paper, we apply the concept of intuitionistic neutrosophic soft sets to graphs. We introduce the notions of intuitionistic neutrosophic soft graphs and present applications of intuitionistic neutrosophic soft graphs in multiple-attribute decision-making problems.

2. Preliminaries

In this section, we review some basic definitions that will be used in the sequel.

Definition 2.1. [31] Let U be a universe of discourse. A neutrosophic set $\mathcal{N}$ in U is characterized by a truth membership function $\sigma_{\mathcal{N}}$, an indeterminacy membership function $\phi_{\mathcal{N}}$ and a falsity membership function $\psi_{\mathcal{N}}$, where $\sigma_{\mathcal{N}}$, $\phi_{\mathcal{N}}$, $\psi_{\mathcal{N}}$: $U \to]0^-, 1^+[$ are real standard or nonstandard subsets of $]0^-, 1^+[$.
It is written as

$$\mathcal{N} \quad = \quad \{< r, (\sigma_{\mathcal{N}}(r), \phi_{\mathcal{N}}(r), \psi_{\mathcal{N}}(r)) >: r \in U\},$$

where the sum of $\sigma_{\mathcal{N}}(r)$, $\phi_{\mathcal{N}}(r)$ and $\psi_{\mathcal{N}}(r)$ has no restriction, so $0^- \leq \sigma_{\mathcal{N}}(r) + \phi_{\mathcal{N}}(r) + \psi_{\mathcal{N}}(r) \leq 3^+$.
The neutrosophic set from philosophical point of view, takes the value from the real standard or non-standard subsets of $]0^-, 1^+[$. Since $]0^-, 1^+[$ will be difficult to handle in real life applications such as in engineering and scientific problems. So, for technical applications, we have to take the standard unit interval [0, 1] instead of $]0^-, 1^+[$.

Definition 2.2. [7] An element x of X is called significant with respect to neutrsophic set A of X if the degree of truth-membership or falsity-membership or indeterminancy-membership value, i.e., $T_A(x)$or $I_A(x)$ or $F_A(x) \geq 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminancy-membership and falsity-membership all can not be significant.

We define an intuitionistic neutrosophic set by $A^* =< x, T_{A^*}(x), I_{A^*}(x), F_{A^*}(x) >$, where $\min\{T_{A^*}(x), F_{A^*}(x)\} \leq 0.5$, $\min\{T_{A^*}(x), I_{A^*}(x)\} \leq 0.5$, and $\min\{F_{A^*}(x), I_{A^*}(x)\} \leq 0.5$, for all $x \in X$, with condition $0 \leq T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.3. [8] Let X, Y and Z be three ordinary nonempty sets. An INS relation (INSR) is defined as an intuitionistic neutrosophic subset of $X \times Y$, having the form $R = \{< (x, y), T_R(x, y), I_R(x, y), F_R(x, y) >: x \in X, y \in Y\}$, where $T_R : X \times Y \to [0, 1], I_R : X \times Y \to [0, 1], F_R : X \times Y \to [0, 1]$ satisfy the condition $0 \leq T_R(x, y) + I_R(x, y) + F_R(x, y) \leq 2$.
The collection of all INSR on $X \times Y$ is denoted as $GR(X \times Y)$.

3. Intuitionistic neutrosophic soft graphs

Definition 3.1. [30] Let U be an initial universe, and let P be the set of all parameters. $\mathcal{N}(U)$ denotes the set of all INSSs of U. Let N be a subset of P. A pair (F, N) is called an *intuitionistic neutrosophic soft set* INSS over U.

Let $\mathcal{N}(V)$ denotes the set of all INSSs of V and $\mathcal{N}(E)$ denotes the set of all INSSs of E.

Definition 3.2. An intuitionistic neutrosophic soft graph on a nonempty V is an ordered 3-tuple $G = (F, K, N)$ such that

(1) N is a non-empty set of parameters,
(2) (F, N) is an INSS over V,
(3) (K, N) is an intuitionistic neutrosophic soft relation on V, i.e., $K : N \to \mathcal{N}(V \times V)$, where $\mathcal{N}(V \times V)$ is an intuitionistic neutrosophic power set,
(4) $(F(e), K(e))$ is an ING for all $e \in N$.

That is,

$$T_{K(e)}(xy) \leq \min\{T_{F(e)}(x), T_{F(e)}(y)\},$$

$$I_{K(e)}(xy) \leq \min\{I_{F(e)}(x), I_{F(e)}(y)\},$$

$$F_{K(e)}(xy) \leq \max\{F_{F(e)}(x), F_{F(e)}(y)\},$$

such that $0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 2 \ \forall\ e \in N,\ x, y \in V$.
The intuitionistic neutrosophic graph (ING) $(F(e), K(e))$ is denoted by $H(e)$. Note that $T_{K(e)}(xy) = I_{K(e)}(xy) = 0$ and $F_{K(e)}(xy) = 1$ for all $xy \in V \times V - E, e \notin N$. (F, N) is called an intuitionistic neutrosophic soft vertex and (K, N) is called an intuitionistic neutrosophic soft edge.
Thus, $((F, N), (K, N))$ is called an INSG if

$$T_{K(e)}(xy) \leq \min\{T_{F(e)}(x), T_{F(e)}(y)\},$$
$$I_{K(e)}(xy) \leq \min\{I_{F(e)}(x), I_{F(e)}(y)\},$$
$$F_{K(e)}(xy) \leq \max\{F_{F(e)}(x), F_{F(e)}(y)\},$$

such that $0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 2 \ \forall\ e \in N,\ x, y \in V$. In other words, an INSG is a parameterized family of INGs. The class of all INSGs is denoted by $\mathcal{INS}(G^*)$. The *order* of an INSG is

$$O(G) = \Big(\sum_{e_i \in N} (\sum_{w \in V} T_{F(e_i)}(w)), \sum_{e_i \in N} (\sum_{w \in V} I_{F(e_i)}(w)), \sum_{e_i \in N} (\sum_{w \in V} F_{F(e_i)}(w)) \Big).$$

The *size* of an INSG is

$$S(G)=\Big(\sum_{e_i\in N}(\sum_{wv\in E}T_{K(e_i)}(wv)),\sum_{e_i\in N}(\sum_{wv\in E}I_{K(e_i)}(wv)),\sum_{e_i\in N}(\sum_{wv\in E}F_{K(e_i)}(wv))\Big).$$

Example 3.1. Consider a simple graph $G^*=(V,E)$ such that $V=\{w_1,w_2,w_3,w_4,w_5\}$ and $E=\{w_1w_2,w_2w_3,w_1w_3,w_1w_5,\}$. Let $N=\{e_1,e_2,e_3\}$ be a set of parameters and let (F,N) be an INSS over V with intuitionistic neutrosophic approximation function $F:N\to\mathcal{N}(V)$ defined by
$F(e_1)=\{(w_1,0.4,0.5,0.3),(w_2,0.5,0.4,0.6),(w_3,0.6,0.5,0.4),\}$,
$F(e_2)=\{(w_1,0.6,0.2,0.3),(w_3,0.6,0.5,0.3),(w_5,0.7,0.5,0.4)\}$,
$F(e_3)=\{(w_1,0.8,0.5,0.4),(w_2,0.5,0.5,0.3),(w_3,0.6,0.5,0.4)\}$. Let (K,N) be an INSS over E with intuitionistic neutrosophic approximation function $K:N\to\mathcal{N}(E)$ defined by
$K(e_1)=\{(w_1w_2,0.3,0.3,0.6),(w_2w_3,0.5,0.4,0.6)\}$,
$K(e_2)=\{(w_1w_3,0.6,0.2,0.2),(w_1w_5,0.6,0.1,0.4)\}$,
$K(e_3)=\{(w_1w_2,0.4,0.5,0.4),(w_1w_3,0.6,0.5,0.3)\}$.
Clearly, $H(e_1)=(F(e_1),K(e_1))$, $H(e_2)=(F(e_2),K(e_2))$ and $H(e_3)=(F(e_3),K(e_3))$ are INGs corresponding to the parameters e_1, e_2 and e_3, respectively as shown in Figure 3.1.

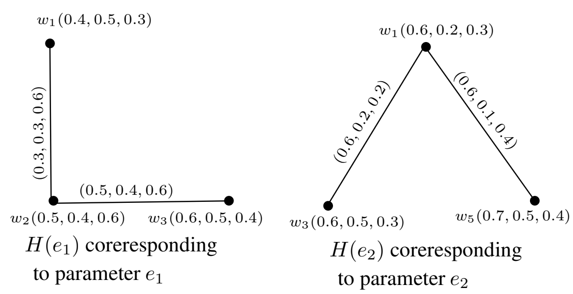

FIGURE 3.1. Intuitionistic neutrosophic soft graph $G=\{H(e_1),H(e_2),H(e_3)\}$.

Hence $G=\{H(e_1),H(e_2),H(e_3)\}$ is an INSG of G^*. Tabular representation of an INSG is given in Table 1.

TABLE 1. Tabular representation of an intuitionistic neutrosophic soft graph.

F	w_1	w_2	w_3	w_4	w_5
e_1	$(0.4,0.5,0.3)$	$(0.5,0.4,0.6)$	$(0.6,0.5,0.4)$	$(0.0,0.0,0.0)$	$(0.0,0.0,0.0)$
e_2	$(0.6,0.2,0.3)$	$(0.0,0.0,0.0)$	$(0.6,0.5,0.3)$	$(0.0,0.0,0.0)$	$(0.7,0.5,0.4)$
e_3	$(0.8,0.5,0.4)$	$(0.5,0.5,0.3)$	$(0.6,0.5,0.4)$	$(0.0,0.0,0.0)$	$(0.0,0.0,0.0)$

K	w_1w_2	w_2w_3	w_1w_3	w_1w_5
e_1	$(0.3,0.3,0.6)$	$(0.5,0.4,0.6)$	$(0.0,0.0,0.0)$	$(0.0,0.0,0.0)$
e_2	$(0.0,0.0,0.0)$	$(0.0,0.0,0.0)$	$(0.6,0.2,0.2)$	$(0.6,0.1,0.4)$
e_3	$(0.4,0.5,0.4)$	$(0.0,0.0,0.0)$	$(0.6,0.5,0.3)$	$(0.0,0.0,0.0)$

The order of INSG is G is $O(G)=\big((0.4+0.5+0.6)+(0.6+0.6+0.7)+(0.8+0.5+0.6),(0.5+0.4+0.5)+(0.2+0.5+0.5)+(0.5+0.5+0.5),(0.3+0.6+0.4)+(0.3+$

$0.3+0.4)+(0.4+0.3+0.4)\big) = (5.3, 4.1, 3.4)$. The size of intuitionistic neutrosophic soft graph G is $S(G) = \big((0.3+0.5)+(0.6+0.6)+(0.4+0.6), (0.3+0.4)+(0.2+0.1)+(0.5+0.5), (0.6+0.6)+(0.2+0.4)+(0.4+0.3)\big) = (3.0, 2.0, 2.5)$.

Definition 3.3. Let $G_1 = (F_1, K_1, N_1)$ and $G_2 = (F_2, K_2, N_2)$ be two INSGs of G_1^* and G_2^*, respectively. The *Cartesian product* of G_1 and G_2 is an INSG $G = G_1 \times G_2 = (F, K, N_1 \times N_2)$, where $(F = F_1 \times F_2, N_1 \times N_2)$ is an intuitionistic neutrosophic soft set over $V = V_1 \times V_2$, $(K = K_1 \times K_2, N_1 \times N_2)$ is an INSS over $E = \{((w, v_1), (w, v_2)) : w \in V_1, (v_1, v_2) \in E_2\} \cup \{((w_1, v), (w_2, v)) : v \in V_2, (w_1, w_2) \in E_1\}$ defined as

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1, v_2)\ \forall\ w \in V_1, (v_1, v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = T_{F_2(e_2)}(v) \wedge T_{K_1(e_1)}(w_1, w_2)$,
$I_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = I_{F_2(e_2)}(v) \wedge I_{K_1(e_1)}(w_1, w_2)$,
$F_{K(e_1,e_2)}\big((w_1, v), (w_2, v)\big) = F_{F_2(e_2)}(v) \vee F_{K_1(e_1)}(w_1, w_2)\ \forall\ v \in V_2, (w_1, w_2) \in E_1$.

$H(e_1, e_2) = H_1(e_1) \times H_2(e_2)$ for all $(e_1, e_2) \in N_1 \times N_2$ are intuitionistic neutrosophic graphs.

Definition 3.4. The *cross product* of G_1 and G_2 is an INSG $G = G_1 \circledcirc G_2 = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, N_1 \times N_2)$ is an INSS over $E = \{((w_1, v_1), (w_2, v_2)) : (w_1, w_2) \in E_1, (v_1, v_2) \in E_2\}$ defined as

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$

(ii) $T_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = T_{K_1(e_1)}(w_1, w_2) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = I_{K_1(e_1)}(w_1, w_2) \wedge I_{K_2(e_2)}(v_1, v_2)$,
$F_{K(e_1,e_2)}\big((w_1, v_1), (w_2, v_2)\big) = F_{K_1(e_1)}(w_1, w_2) \vee F_{K_2(e_2)}(v_1, v_2)\ \forall\ (w_1, w_2) \in E_1, (v_1, v_2) \in E_2$.

$H(e_1, e_2) = H_1(e_1) \circledcirc H_2(e_2)$ for all $(e_1, e_2) \in N_1 \times N_2$ are intuitionistic neutrosophic graphs.

Definition 3.5. The *lexicographic product* of G_1 and G_2 is an INSG $G = G_1 \odot G_2 = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, N_1 \times N_2)$ is an INSS over $E = \{((w, v_1), (w, v_2)) : w \in V_1, (v_1, v_2) \in E_2\} \cup \{((w_1, v_1), (w_2, v_2)) : (w_1, w_2) \in E_1, (v_1, v_2) \in E_2\}$ defined as

(i) $T_{F(e_1,e_2)}(w, v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w, v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w, v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w, v) \in V, (e_1, e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1, v_2)$,
$I_{K(e_1,e_2)}\big((w, v_1), (w, v_2)\big) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1, v_2)$,

$F_{K(e_1,e_2)}\big((w,v_1),(w,v_2)\big) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1,v_2)\ \forall\ w \in V_1, (v_1,v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = T_{K_1(e_1)}(w_1,w_2) \wedge T_{K_2(e_2)}(v_1,v_2)$,
$I_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = I_{K_1(e_1)}(w_1,w_2) \wedge I_{K_2(e_2)}(v_1,v_2)$,
$F_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = F_{K_1(e_1)}(w_1,w_2) \vee F_{K_2(e_2)}(v_1,v_2)\ \forall\ (w_1,w_2) \in E_1, (v_1,v_2) \in E_2$.

$H(e_1,e_2) = H_1(e_1) \odot H_2(e_2)$ for all $(e_1,e_2) \in N_1 \times N_2$ are INGs.

Definition 3.6. The *strong product* of G_1 and G_2 is an INSG $G = G_1 \otimes G_2 = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, A \times N_2)$ is an INSS over $E = \{((w,v_1),(w,v_2)) : w \in V_1, (v_1,v_2) \in E_2\} \cup \{((w_1,v),(w_2,v)) : v \in V_2, (w_1,w_2) \in E_1\} \cup \{((w_1,v_1),(w_2,v_2)) : (w_1,w_2) \in E_1, (v_1,v_2) \in E_2\}$ such that

(i) $T_{F(e_1,e_2)}(w,v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w,v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w,v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w,v) \in V, (e_1,e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}\big((w,v_1),(w,v_2)\big) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1,v_2)$,
$I_{K(e_1,e_2)}\big((w,v_1),(w,v_2)\big) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1,v_2)$,
$F_{K(e_1,e_2)}\big((w,v_1),(w,v_2)\big) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1,v_2)\ \forall\ w \in V_1, (v_1,v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = T_{F_2(e_2)}(v) \wedge T_{K_1(e_1)}(w_1,w_2)$,
$I_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = I_{F_2(e_2)}(v) \wedge I_{K_1(e_1)}(w_1,w_2)$,
$F_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = F_{F_2(e_2)}(v) \vee F_{K_1(e_1)}(w_1,w_2)\ \forall\ v \in V_2, (w_1,w_2) \in E_1$,

(iv) $T_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = T_{K_1(e_1)}(w_1,w_2) \wedge T_{K_2(e_2)}(v_1,v_2)$,
$I_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = I_{K_1(e_1)}(w_1,w_2) \wedge I_{K_2(e_2)}(v_1,v_2)$,
$F_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = F_{K_1(e_1)}(w_1,w_2) \vee F_{K_2(e_2)}(v_1,v_2)\ \forall\ (w_1,w_2) \in E_1, (v_1,v_2) \in E_2$.

$H(e_1,e_2) = H_1(e_1) \otimes H_2(e_2)$ for all $(e_1,e_2) \in N_1 \times N_2$ are INGs.

Definition 3.7. The *composition* of G_1 and G_2 is an INSG $G = G_1[G_2] = (F, K, N_1 \times N_2)$, where $(F, N_1 \times N_2)$ is an INSS over $V = V_1 \times V_2$, $(K, N_1 \times N_2)$ is an INSS over $E = \{((w,v_1),(w,v_2)) : w \in V_1, (v_1,v_2) \in E_2\} \cup \{((w_1,v),(w_2,v)) : v \in V_2, (w_1,w_2) \in E_1\} \cup \{((w_1,v_1),(w_2,v_2)) : (w_1,w_2) \in E_1, v_1 \neq v_2\}$ defined as

(i) $T_{F(e_1,e_2)}(w,v) = T_{F_1(e_1)}(w) \wedge T_{F_2(e_2)}(v)$,
$I_{F(e_1,e_2)}(w,v) = I_{F_1(e_1)}(w) \wedge I_{F_2(e_2)}(v)$,
$F_{F(e_1,e_2)}(w,v) = F_{F_1(e_1)}(w) \vee F_{F_2(e_2)}(v)\ \forall\ (w,v) \in V, (e_1,e_2) \in N_1 \times N_2$,

(ii) $T_{K(e_1,e_2)}((w,v_1),(w,v_2)) = T_{F_1(e_1)}(w) \wedge T_{K_2(e_2)}(v_1,v_2)$,
$I_{K(e_1,e_2)}((w,v_1),(w,v_2)) = I_{F_1(e_1)}(w) \wedge I_{K_2(e_2)}(v_1,v_2)$,
$F_{K(e_1,e_2)}((w,v_1),(w,v_2)) = F_{F_1(e_1)}(w) \vee F_{K_2(e_2)}(v_1,v_2)\ \forall\ w \in V_1, (v_1,v_2) \in E_2$,

(iii) $T_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = T_{F_2(e_2)}(v) \wedge T_{K_1(e_1)}(w_1,w_2)$,
$I_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = I_{F_2(e_2)}(v) \wedge I_{K_1(e_1)}(w_1,w_2)$,
$F_{K(e_1,e_2)}\big((w_1,v),(w_2,v)\big) = F_{F_2(e_2)}(v) \vee F_{K_1(e_1)}(w_1,w_2)\ \forall\ v \in V_2, (w_1,w_2) \in E_1$,

(iv) $T_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = T_{F_1(e_1)}(w_1,w_2) \wedge T_{F_2(e_2)}(v_1) \wedge T_{F_2(e_2)}(v_2)$,
$I_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = I_{F_1(e_1)}(w_1,w_2) \wedge I_{F_2(e_2)}(v_1) \wedge I_{F_2(e_2)}(v_2)$,
$F_{K(e_1,e_2)}\big((w_1,v_1),(w_2,v_2)\big) = F_{F_1(e_1)}(w_1,w_2) \vee F_{F_2(e_2)}(v_1) \vee F_{F_2(e_2)}(v_2)\ \forall\ (w_1,w_2) \in E_1$, where $v_1 \neq v_2, v_1, v_2 \in V_2$.

$H(e_1, e_2) = H_1(e_1)[H_2(e_2)]$ for all $(e_1, e_2) \in N_1 \times N_2$ are INGs.

Proposition 3.1. The Cartesian product, cross product, lexicographic product, strong product and composition of two INSGs is an INSG.

Definition 3.8. Let $G_1 = (F_1, K_1, N_1)$ and $G_2 = (F_2, K_2, N_2)$ be two INSGs. The *intersection* of G_1 and G_2 is an INSG denoted by $G = G_1 \cap G_2 = (F, K, N_1 \cup N_2)$, where $(F, N_1 \cup N_2)$ is an INSS over $V = V_1 \cap V_2$, $(K, N_1 \cup N_2)$ is an INSS over $E = E_1 \cap E_2$, the truth-membership, indeterminacy-membership, and falsity-membership functions of G for all $w, v \in V$ defined by,

(i)
$$T_{F(e)}(v) = \begin{cases} T_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ T_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ T_{F_1(e)}(v) \wedge T_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$I_{F(e)}(v) = \begin{cases} I_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ I_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ I_{F_1(e)}(v) \wedge I_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$F_{F(e)}(v) = \begin{cases} F_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ F_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ F_{F_1(e)}(v) \vee F_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

(ii)
$$T_{K(e)}(wv) = \begin{cases} T_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ T_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ T_{K_1(e)}(wv) \wedge T_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$I_{K(e)}(wv) = \begin{cases} I_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ I_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ I_{K_1(e)}(wv) \wedge I_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$F_{K(e)}(wv) = \begin{cases} F_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ F_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ F_{K_1(e)}(wv) \vee F_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

Definition 3.9. Let $G_1 = (F_1, K_1, N_1)$ and $G_2 = (F_2, K_2, N_2)$ be two INSGs. The *union* of G_1 and G_2 may or may not be INSG denoted by $G = G_1 \cup G_2 = (F, K, N_1 \cup N_2)$, where $(F, N_1 \cup N_2)$ is an INSS over $V = V_1 \cup V_2$, $(K, N_1 \cup N_2)$ is an INSS over $E = E_1 \cup E_2$, the truth-membership, indeterminacy-membership, and falsity-membership functions of G for all $w, v \in V$ defined by,

(i)
$$T_{F(e)}(v) = \begin{cases} T_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ T_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ T_{F_1(e)}(v) \vee T_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$I_{F(e)}(v) = \begin{cases} I_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ I_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ I_{F_1(e)}(v) \wedge I_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

$$F_{F(e)}(v) = \begin{cases} F_{F_1(e)}(v) & \text{if } e \in N_1 - N_2; \\ F_{F_2(e)}(v) & \text{if } e \in N_2 - N_1; \\ F_{F_1(e)}(v) \wedge F_{F_2(e)}(v), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

(ii) $$T_{K(e)}(wv) = \begin{cases} T_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ T_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ T_{K_1(e)}(wv) \vee T_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$
$$I_{K(e)}(wv) = \begin{cases} I_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ I_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ I_{K_1(e)}(wv) \wedge I_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$
$$F_{K(e)}(wv) = \begin{cases} F_{K_1(e)}(wv) & \text{if } e \in N_1 - N_2; \\ F_{K_2(e)}(wv) & \text{if } e \in N_2 - N_1; \\ F_{K_1(e)}(wv) \wedge F_{K_2(e)}(wv), & \text{if } e \in N_1 \cap N_2. \end{cases}$$

Remark. Let G_1 and G_2 be two INSG over G^* then $G_1 \cup G_2$ may or may not be INSG.

Definition 3.10. Let G_1 and G_2 be two INSGs. The *join* of G_1 and $\mathbb{G}_2$ may or may not be intuitionistic neutrosophic soft graph denoted by $G_1 + G_2 = (F_1 + F_2, K_1 + K_2, N_1 \cup N_2)$, where $(F_1 + F_2, N_1 \cup N_2)$ is an intuitionistic neutrosophic soft set over $V_1 \cup V_2$, $(K_1 + K_2, N_1 \cup N_2)$ is an INSS over $E_1 \cup E_2 \cup \acute{E}$ defined by

(i) $(F_1 + F_2, N_1 \cup N_2) = (F_1, N_1) \cup (F_2, N_2)$,

(ii) $(K_1 + K_2, N_1 \cup N_2) = (K_1, N_1) \cup (K_2, N_2)$ if $wv \in E_1 \cup E_2$, where $e \in N_1 \cap N_2, wv \in \acute{E}$, and $\acute{E}$ is the set of all edges joining the vertices of V_1 and V_2, the truth-membership, indeterminacy-membership, and falsity-membership functions are defined by

$$\begin{aligned} T_{K_1+K_2(e)}(wv) &= \min\{T_{F_1(e)}(w), T_{F_2(e)}(v)\}, \\ I_{K_1+K_2(e)}(wv) &= \min\{I_{F_1(e)}(w), I_{F_2(e)}(v)\}, \\ F_{K_1+K_2(e)}(wv) &= \max\{F_{F_1(e)}(w), F_{F_2(e)}(v)\} \ \forall wv \in \acute{E}. \end{aligned}$$

Proposition 3.2. If G_1 and G_2 are two INSGs then their join $G_1 + G_2$ may or may not be intuitionistic neutrosophic soft graph.

Definition 3.11. The *complement* of an INSG $G = (F, K, N)$ denoted by $G^c = (F^c, K^c, N^c)$ is defined as follows:

(i) $N^c = N$,

(ii) $F^c(e) = F(e)$,

(iii) $T_{K^c(e)}(w, v) = T_{F(e)}(w) \wedge T_{F(e)}(v) - T_{K(e)}(w, v)$,

(iv) $I_{K^c(e)}(w, v) = I_{F(e)}(w) \wedge I_{F(e)}(v) - I_{K(e)}(w, v)$, and

(v) $F_{K^c(e)}(w, v) = F_{F(e)}(w) \vee F_{F(e)}(v) - F_{K(e)}(w, v)$, for all $w, v \in V, e \in N$.

Example 3.2. Let $G^* = (V, E)$ be a crisp graph with $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_4, v_1v_3, v_2v_3, v_3v_4\}$. Let $N = \{e_1, e_2\}$ be a set of parameters and let (F, N) be an INSS over V with intuitionistic neutrosophic approximation function $F : N \to \mathcal{N}(V)$ defined by
$F(e_1) = \{(v_1, 0.4, 0.6, 0.1), (v_2, 0.5, 0.4, 0.7), (v_3, 0.5, 0.3, 0.4), (v_4, 0.5, 0.6, 0.2)\}$,
$F(e_2) = \{(v_1, 0.4, 0.2, 0.2), (v_2, 0.5, 0.3, 0.4), (v_3, 0.6, 0.3, 0.5), (v_4, 0.5, 0.4, 0.2)\}$.
Let (K, N) be an INSS over E with intuitionistic neutrosophic approximation function $K : N \to \mathcal{N}(E)$ defined by
$K(e_1) = \{(v_1v_2, 0.3, 0.3, 0.5), (v_1v_4, 0.2, 0.5, 0.2), (v_1v_3, 0.4, 0.3, 0.4), (v_2v_3, 0.5, 0.3, 0.5)\}$,
$K(e_2) = \{(v_1v_3, 0.3, 0.2, 0.5), (v_1v_4, 0.4, 0.1, 0.1), (v_3v_4, 0.5, 0.3, 0.4), (v_3v_2, (0.5, 0.3,$

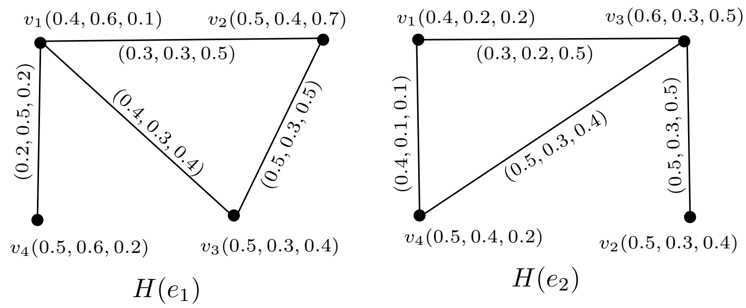

FIGURE 3.2. INSG $G = \{H(e_1), H(e_2)\}$.

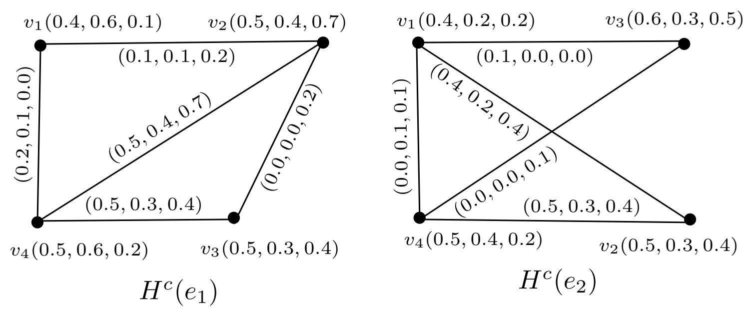

FIGURE 3.3. Complement of INSG $G^c = \{H^c(e_1), H^c(e_2)\}$

0.5)}.
Clearly, $G = \{H(e_1) = (F(e_1), K(e_1)), H(e_2) = (F(e_2), K(e_2))\}$ is intuitionistic neutrosophic soft graph, $H(e_1)$ and $H(e_2)$ are intuitionistic neutrosophic graphs corresponding to the parameters e_1 and e_2, respectively as shown in Figure 3.2. Now, the complement of INSG $G = \{H(e_1), H(e_2)\}$ is the complement of INGs $H(e_1)$ and $H(e_2)$ which are shown in Figure 3.3.

Definition 3.12. An INSG G is a *complete INSG* if $H(e)$ is a complete ING for all $e \in N$, i.e.,

$$T_{K(e)}(wv) = \min(T_{F(e)}(w), T_{F(e)}(v)),$$
$$I_{K(e)}(wv) = \min(I_{F(e)}(w), I_{F(e)}(v)),$$
$$F_{K(e)}(wv) = \max(F_{F(e)}(w), F_{F(e)}(v))$$

$\forall\ w, v \in V, e \in N$.

Definition 3.13. An INSG G is a *strong INSG* if $H(e)$ is a strong ING for all $e \in N$.

Example 3.3. Consider the simple graph $G^* = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{v_1v_2, v_2v_5, v_3v_5, v_1v_3, v_1v_4, v_3v_6, v_5v_6\}$. Let $N = \{e_1, e_2\}$. Let (F, N) be an INSS over V with its approximation function $F : N \to \mathcal{N}(V)$ defined by
$F(e_1) = \{(v_1, 0.4, 0.5, 0.7), (v_2, 0.6, 0.5, 0.5), (v_3, 0.6, 0.3, 0.5), (v_4, 0.7, 0.5, 0.4), (v_5, 0.7, 0.4, 0.5), (v_6, 0.3, 0.5, 0.7)\}$,
$F(e_2) = \{(v_1, 0.6, 0.4, 0.3), (v_2, 0.5, 0.3, 0.8), (v_3, 0.5, 0.6, 0.3), (v_4, 0.8, 0.5, 0.4), (v_5, 0.6,$

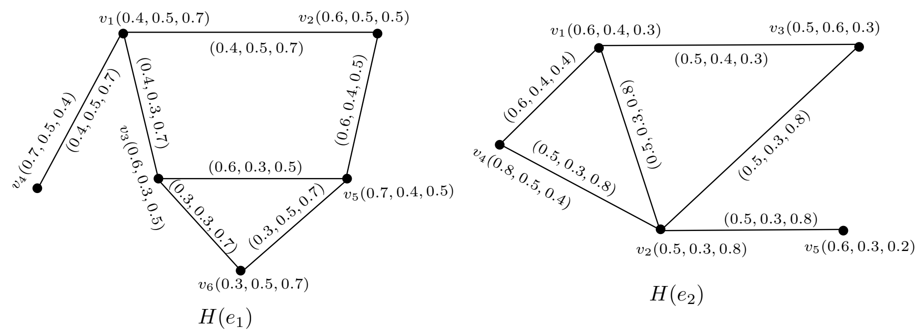

FIGURE 3.4. Strong INSG $G = \{H(e_1), H(e_2)\}$.

$0.3, 0.2)\}$.
Let (K, N) be an INSS over E with its approximation function $K : N \to \mathcal{N}(E)$ defined by
$K(e_1) = \{(v_1v_2, 0.4, 0.5, 0.7), (v_1v_3, 0.4, 0.3, 0.7), (v_1v_4, 0.4, 0.5, 0.7), (v_2v_5, 0.6, 0.4, 0.5), (v_3v_5, 0.6, 0.3, 0.5), (v_3v_6, 0.3, 0.3, 0.7), (v_5v_6, 0.3, 0.5, 0.7)\}$,
$K(e_2) = \{(v_1v_3, 0.5, 0.4, 0.3), (v_1v_4, 0.6, 0.4, 0.4), (v_1v_2, 0.5, 0.3, 0.8), (v_2v_3, 0.5, 0.3, 0.8), (v_2v_4, 0.5, 0.3, 0.8), (v_2v_5, 0.5, 0.3, 0.8)\}$.
$H(e_1) = (F(e_1), K(e_1))$, and $H(e_2) = (F(e_2), K(e_2))$ are strong INGs corresponding to the parameters e_1, and e_2, respectively as shown in Figure 3.4. Hence $G = \{H(e_1), H(e_2)\}$ is a strong INSG of G^*.

Proposition 3.3. If G_1 and G_2 are strong INSGs, then $G_1 \times G_2$, and $G_1[G_2]$ are strong INSGs.

Remark. The union of two strong INSGs is not necessarily strong INSG.

Example 3.4. Let $N_1 = \{e_1\}$ and $N_2 = \{e_1, e_2\}$ be the parameter sets. Let G_1 and G_2 be the two strong INSGs defined as follows:
$G_1 = \{H_1(e_1), H_1(e_2)\} = \{(\{(w_1, 0.5, 0.6, 0.4), (w_2, 0.7, 0.4, 0.5), (w_3, 0.5, 0.8, 0.4)\}, \{(w_1w_2, 0.5, 0.4, 0.5), (w_2w_3, 0.5, 0.4, 0.5)\}), (\{(w_1, 0.4, 0.6, 0.5), (w_3, 0.5, 0.7, 0.4)\}, \{(w_1w_3, 0.4, 0.6, 0.5)\})\}$,
$G_2 = \{H_2(e_1)\} = \{(w_1, 0.4, 0.9, 0.3), (w_2, 0.5, 0.6, 0.4), (w_1w_2, 0.4, 0.6, 0.4)\}$.
The union of G_1 and G_2 is $G = G_1 \cup G_2 = (H, N_1 \cup N_2)$, where $N_1 \cup N_2 = \{e_1, e_2\}$, $H(e_1) = H_1(e_1) \cup H_2(e_1)$ and $H(e_2) = H_1(e_2)$ are as shown in Figure. 3.5. Clearly, $G = \{H(e_1), H(e_2)\}$ is not a strong INSG as shown in Figure. 3.6.

Proposition 3.4. If $G_1 \times G_2$ is strong INSG, then at least G_1 or G_2 must be strong INSG.

Proposition 3.5. If $G_1[G_2]$ is strong INSG, then at least G_1 or G_2 must be strong INSG.

Definition 3.14. The complement of a strong INSG $G = (F, K, N)$ is an INSG $G^c = (F^c, K^c, N^c)$ defined by

(i) $N^c = N$,
(ii) $F^c(e)(w) = F(e)(w)$ for all $e \in N$ and $w \in V$,

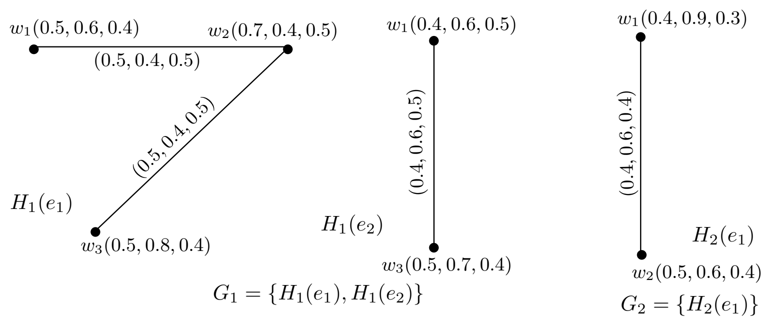

Figure 3.5. Strong INSGs G_1 and G_2.

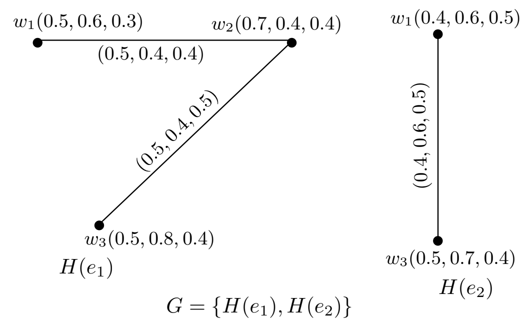

Figure 3.6. Union of two strong intuitionistic neutrosophic soft graphs.

(iii) $T_{K^c(e)}(w,v) = \begin{cases} 0 & \text{if } T_{K(e)}(w,v) > 0, \\ \min\{T_{F(e)}(w), T_{F(e)}(v)\}, & \text{if } T_{K(e)}(w,v) = 0, \end{cases}$

$$I_{K^c(e)}(w,v) = \begin{cases} 0 & \text{if } I_{K(e)}(w,v) > 0, \\ \min\{I_{F(e)}(w), I_{F(e)}(v)\}, & \text{if } I_{K(e)}(w,v) = 0, \end{cases}$$

$$F_{K^c(e)}(w,v) = \begin{cases} 0 & \text{if } F_{K(e)}(w,v) > 0, \\ \max\{F_{F(e)}(w), F_{F(e)}(v)\}, & \text{if } F_{K(e)}(w,v) = 0, \end{cases}$$

Proposition 3.6. If G is a strong INSG over G^*, then G^c is also a strong intuitionistic neutrosophic soft graph.

Theorem 3.1. If G and G^c are strong INSGs of G^*. Then $G \cup G^c$ is a complete intuitionistic neutrosophic soft graph.

4. Isomorphism of intuitionistic neutrosophic soft graphs

Definition 4.1. Let $G_1 = (F_1, K_1, N)$ and $G_2 = (F_2, K_2, N)$ be two INSGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. A *homomorphism* $f_N : G_1 \to G_2$ is a mapping $f_N : V_1 \to V_2$ which satisfies the following conditions:

(i) $T_{F_1(e)}(v_1) \leq T_{F_2(e)}(f_e(v_1))$, $I_{F_1(e)}(v_1) \leq I_{F_2(e)}(f_e(v_1))$, $F_{F_1(e)}(v_1) \geq F_{F_2(e)}(f_e(v_1))$,
(ii) $T_{K_1(e)}(v_1v_2) \leq T_{K_2(e)}(f_e(v_1)f_e(v_2))$, $I_{K_1(e)}(v_1v_2) \leq I_{K_2(e)}(f_e(v_1)f_e(v_2))$, $F_{K_1(e)}(v_1v_2) \geq F_{K_2(e)}(f_e(v_1)f_e(v_2))$, for all $e \in N, v_1 \in V_1, v_1v_2 \in E_1$.

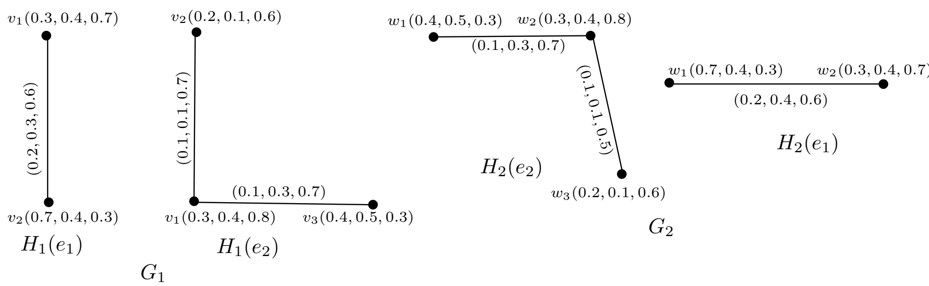

FIGURE 4.1. $G_1 = \{H_1(e_1), H_1(e_2)\}$, and $G_2 = \{H_2(e_1), H_2(e_2)\}$.

A bijective homomorphism is called a *weak isomorphism* if
$T_{F_1(e)}(v_1) = T_{F_2(e)}(f_e(v_1)), I_{F_1(e)}(v_1) = I_{F_2(e)}(f_e(v_1)), F_{F_1(e)}(v_1) = F_{F_2(e)}(f_e(v_1))$, $\forall e \in N, v_1 \in V_1$.
A bijective homomorphism $f_N : G_1 \to G_2$ such that
$T_{K_1(e)}(v_1v_2) = T_{K_2(e)}(f_e(v_1)f_e(v_2)), I_{K_1(e)}(v_1v_2) = I_{K_2(e)}(f_e(v_1)f_e(v_2)), F_{K_1(e)}(v_1v_2) = F_{K_2(e)}(f_e(v_1)f_e(v_2))$, for all $e \in N, v_1v_2 \in E_1$ is called a *co-weak isomorphism.*
An *endomorphism* of INSG G with V as the underlying set is a homomorphism of G into itself.

Definition 4.2. Let $G_1 = (F_1, K_1, N)$ and $G_2 = (F_2, K_2, N)$ be two INSGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. An *isomorphism* $f_N : G_1 \to G_2$ is a mapping $f_N : V_1 \to V_2$ which satisfies the following conditions:

(i) $T_{F_1(e)}(v_1) = T_{F_2(e)}(f_e(v_1)), I_{F_1(e)}(v_1) = I_{F_2(e)}(f_e(v_1)), F_{F_1(e)}(v_1) = F_{F_2(e)}(f_e(v_1))$,
(ii) $T_{K_1(e)}(v_1v_2) = T_{K_2(e)}(f_e(v_1)f_e(v_2)),\ I_{K_1(e)}(v_1v_2) = I_{K_2(e)}(f_e(v_1)f_e(v_2))$, $F_{K_1(e)}(v_1v_2) = F_{K_2(e)}(f_e(v_1)f_e(v_2))$, for all $e \in N, v_1 \in V_1, v_1v_2 \in E_1$.

Example 4.1. Let $N = \{e_1, e_2\}$ be a parameter set. $G_1 = (F_1, K_1, N)$ and $G_2 = (F_1, K_2, N)$ be two INSGs defined as follows:
$G_1 = \{H_1(e_1), H_1(e_2)\} = \{(\{(v_1, 0.3, 0.4, 0.7), (v_2, 0.7, 0.4, 0.3)\}, \{(v_1v_2, 0.2, 0.3, 0.6)\}), (\{(v_1, 0.3, 0.4, 0.8), (v_2, 0.2, 0.1, 0.6), (v_3, 0.4, 0.5, 0.3)\}, \{(v_1v_2, 0.1, 0.1, 0.7), (v_1v_3, 0.1, 0.3, 0.7)\})\}$,
$G_2 = \{H_2(e_1), H_2(e_2)\} = \{(\{(w_1, 0.7, 0.4, 0.3), (w_2, 0.3, 0.4, 0.7)\}, \{(w_1w_2, 0.2, 0.4, 0.6)\}), (\{(w_1, 0.4, 0.5, 0.3), (w_2, 0.3, 0.4, 0.8), (w_3, 0.2, 0.1, 0.6)\}, \{(w_1w_2, 0.1, 0.3, 0.7), (w_2w_3, 0.1, 0.1, 0.5)\})\}$.
A mapping $f_N : V_1 \to V_2$ defined by $f_{e_1}(v_1) = w_2$, $f_{e_1}(v_2) = w_1$ and $f_{e_2}(v_1) = w_2$, $f_{e_2}(v_2) = w_3$, and $f_{e_2}(v_3) = w_1$, then $T_{F_1(e_1)}(v_1) = T_{F_2(e_1)}(w_2)$, $I_{F_1(e_1)}(v_1) = I_{F_2(e_1)}(w_2)$, $F_{F_1(e_1)}(v_1) = F_{F_2(e_1)}(w_2)$, and $T_{F_1(e_1)}(v_2) = T_{F_2(e_1)}(w_1)$, $I_{F_1(e_1)}(v_2) = I_{F_2(e_1)}(w_1)$, $F_{F_1(e_1)}(v_2) = F_{F_2(e_1)}(w_1)$, but $T_{K_1(e_1)}(v_1v_2) = T_{K_2(e_1)}(w_2w_1)$, $I_{K_1(e_1)}(v_1v_2) \neq I_{K_2(e_1)}(w_2w_1)$, $F_{K_1(e_1)}(v_1v_2) = F_{K_2(e_1)}(w_2w_1)$. Clearly, $H_1(e_1)$ is weak isomorphic to $H_2(e_1)$. By routine computation, we can see that $H_1(e_2)$ is weak isomorphic to $H_2(e_2)$.
Hence G_1 is weak isomorphic to G_2 but not isomorphic as shown in Figure 4.1.

Example 4.2. Let $N = \{e_1, e_2\}$ be a parameter set. $G_1 = (F_1, K_1, N)$ and $G_2 = (F_1, K_2, N)$ be two INSGs as shown in Figure 4.2. A mapping $f_N : V_1 \to V_2$ defined by $f_{e_1}(w_1) = v_2$, $f_{e_1}(w_2) = v_1$, $f_{e_1}(w_3) = v_4$, $f_{e_1}(w_4) = v_3$ and $f_{e_2}(w_1) = v_1$, $f_{e_2}(w_2) = v_2$, and $f_{e_2}(w_3) = v_3$. By routine computations, we can see that G_1 is co-weak isomorphic to G_2 but not isomorphic as $T_{F_1(e_1)}(w_2) = T_{F_2(e_1)}(v_1)$,

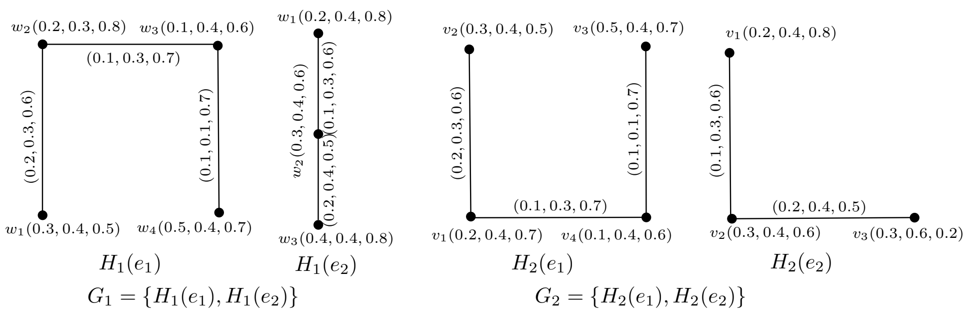

FIGURE 4.2. $G_1 = \{H_1(e_1), H_1(e_2)\}$, and $G_2 = \{H_2(e_1), H_2(e_2)$.

$I_{F_1(e_1)}(w_2) \neq I_{F_2(e_1)}(v_1)$, $F_{F_1(e_1)}(w_2) \neq F_{F_2(e_1)}(v_1)$ and $T_{F_1(e_2)}(w_3) \neq T_{F_2(e_2)}(v_3)$, $I_{F_1(e_2)}(w_3) \neq I_{F_2(e_2)}(v_3)$, $F_{F_1(e_2)}(w_3) \neq F_{F_2(e_2)}(v_3)$.

Theorem 4.1. For any two isomorphic INSGs their orders and sizes are same.

Definition 4.3. Let G be an INSG with V as the underlying set. A one-to-one, onto map $f_N : V \to V$ is an *automorphism* of G if

(i) $T_{F_1(e)}(v_1) = T_{F_1(e)}(f_e(v_1))$, $I_{F_1(e)}(v_1) = I_{F_1(e)}(f_e(v_1))$, $F_{F_1(e)}(v_1) = F_{F_1(e)}(f_e(v_1))$,
(ii) $T_{K_1(e)}(v_1v_2) = T_{K_1(e)}(f_e(v_1)f_e(v_2))$, $I_{K_1(e)}(v_1v_2) = I_{K_1(e)}(f_e(v_1)f_e(v_2))$, $F_{K_1(e)}(v_1v_2) = F_{K_1(e)}(f_e(v_1)f_e(v_2))$, for all $e \in N, v_1, v_2 \in V$.

Definition 4.4. An INSG $G = (F, K, N)$ of $G^* = (V, E)$ is an *ordered intuitionistic neutrosophic soft graph* if it satisfies the following condition:

$$T_{F(e)}(v_1) \leq T_{F(e)}(v_2), I_{F(e)}(v_1) \leq I_{F(e)}(v_2), F_{F(e)}(v_1) \geq F_{F(e)}(v_2),$$

$$T_{F(e)}(w_1) \leq T_{F(e)}(w_2), I_{F(e)}(w_1) \leq I_{F(e)}(w_2), F_{F(e)}(w_1) \geq F_{F(e)}(w_2),$$

for $v_1, v_2, w_1, w_2 \in V, v_1 \neq w_1$, $v_2 \neq w_2$, for all $e \in N$, imply

$$T_{K(e)}(v_1w_1) \leq T_{K(e)}(v_2w_2), I_{K(e)}(v_1w_1) \leq I_{K(e)}(v_2w_2), F_{K(e)}(v_1w_1) \geq F_{K(e)}(v_2w_2)$$
.

Proposition 4.1. Let G_1, G_2 and G_3 are INSGs. Then the isomorphism between these intuitionistic neutrosophic soft graphs is an equivalence relation.

Proof. Let $G_1 = (F_1, K_1, N)$, $G_2 = (F_2, K_2, N)$, and $G_3 = (F_3, K_3, N)$ are three INSGs with the underlying sets V_1, V_2 and V_3, respectively.

(1) Reflexive: Consider identity mapping $f_N : V_1 \to V_1$, $f_e(v) = v$ for all $v \in V_1$, satisfying
$T_{F_1(e)}(v) = T_{F_1(e)}(f_e(v))$, $I_{F_1(e)}(v) = I_{F_1(e)}(f_e(v))$, $F_{F_1(e)}(v) = F_{F_1(e)}(f_e(v))$,
$T_{K_1(e)}(uv) = T_{K_1(e)}(f_e(u)f_e(v))$, $I_{K_1(e)}(uv) = I_{K_1(e)}(f_e(u)f_e(v))$, $F_{K_1(e)}(uv) = F_{K_1(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$. Hence f_N is an isomorphism of intuitionistic neutrosophic soft graph to itself.
(2) Symmetric: Let $f_N : V_1 \to V_2$ be an isomorphism of G_1 onto G_2, $f_e(v) = v'$ for all $v \in V_1$, such that
$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v))$, $I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v))$, $F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$,

$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v)), I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v)), F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$.
As f_N is a bijective mapping, $f^{-1}(v') = v$ for all $v' \in V_2$, then
$T_{F_2(e)}(v') = T_{F_1(e)}(f^{-1}(v')), I_{F_2(e)}(v') = I_{F_1(e)}(f^{-1}(v')), F_{F_2(e)}(v') = F_{F_1(e)}(f^{-1}(v'))$,
$T_{K_2(e)}(u'v') = T_{K_1(e)}(f^{-1}(u')f^{-1}(v')), I_{K_2(e)}(u'v') = I_{K_1(e)}(f^{-1}(u')f^{-1}(v'))$,
$F_{K_2(e)}(u'v') = F_{K_1(e)}(f^{-1}(u')f^{-1}(v'))$ for all $u', v' \in V_2, e \in N$.
Hence $f^{-1} : V_2 \to V_1$ is an isomorphism from G_2 to G_1, that is $G_1 \cong G_2$ implies $G_2 \cong G_1$.

(3) Transitive: Let $f_N : V_1 \to V_2$ and $g_N : V_2 \to V_3$ are isomorphisms of the intuitionistic neutrosophic soft graphs G_1 onto G_2 and G_2 onto G_3, respectively. For transitive relation we consider a bijective mapping $g_N \circ f_N : V_1 \to V_3$ such that $(g_N \circ f_N)(u) = g_e(f_e(u))$ for all $u \in V_1$.
As $f_N : V_1 \to V_2$ is an isomorphism from G_1 onto G_2, such that $f_e(v) = v'$ for all $v \in V_1$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v'), I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v')$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v')$, and
$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v'), I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v')$,
$F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v')$, for all $u, v \in V_1, e \in N$.

As $g_N : V_2 \to V_3$ is an isomorphism from G_2 onto G_3 such that $g_e(v') = v''$ for all $v' \in V_2$, then

$T_{F_2(e)}(v') = T_{F_3(e)}(g_e(v')) = T_{F_2(e)}(v''), I_{F_2(e)}(v') = I_{F_3(e)}(g_e(v')) = I_{F_3(e)}(v'')$,
$F_{F_2(e)}(v') = F_{F_3(e)}(g_e(v')) = F_{F_3(e)}(v'')$, and
$T_{K_2(e)}(u'v') = T_{K_3(e)}(g_e(u')g_e(v')) = T_{K_3(e)}(u''v''), I_{K_2(e)}(u'v') = I_{K_3(e)}(g_e(u')g_e(v')) = I_{K_2(e)}(u''v'')$,
$F_{K_2(e)}(u'v') = F_{K_3(e)}(g_e(u')g_e(v')) = F_{K_3(e)}(u''v'')$, for all $u', v' \in V_2, e \in N$.

For transitive relation we consider a bijective mapping $g_N \circ f_N : V_1 \to V_3$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v') = T_{F_3(e)}(g_e(f_e(v)))$,
$I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v') = I_{F_3(e)}(g_e(f_e(v)))$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v') = F_{F_3(e)}(g_e(f_e(v)))$, and
$T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v') = T_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v') = I_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v') = F_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$
for all $u, v \in V_1, e \in N$.
Therefore $g_N \circ f_N$ is an isomorphism between G_1 and G_3.

Hence isomorphism between INSGs by (1), (2) and (3) is an equivalence relation. □

Proposition 4.2. Let G_1, G_2 and G_3 are INSGs. Then the weak isomorphism between these INSGs is a partial order relation

Proof. Let $G_1 = (F_1, K_1, N)$, $G_2 = (F_2, K_2, N)$, and $G_3 = (F_3, K_3, N)$ be three INSGs with the underlying sets V_1, V_2 and V_3, respectively.

(1) Reflexive: Consider identity mapping $f_N : V_1 \to V_1$, $f_e(v) = v$ for all $v \in V_1$, satisfying
$T_{F_1(e)}(v) = T_{F_1(e)}(f_e(v)), I_{F_1(e)}(v) = I_{F_1(e)}(f_e(v)), F_{F_1(e)}(v) = F_{F_1(e)}(f_e(v))$,
$T_{K_1(e)}(uv) = T_{K_1(e)}(f_e(u)f_e(v)), I_{K_1(e)}(uv) = I_{K_1(e)}(f_e(u)f_e(v)), F_{K_1(e)}(uv) = F_{K_1(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$. Hence f_N is a weak isomorphism of intuitionistic neutrosophic soft graph to itself. Thus G_1 is a weak isomorphic to itself.

(2) Anti symmetric: Let $f_N : V_1 \to V_2$ be an isomorphism of G_1 onto G_2, $f_e(v) = v'$ for all $v \in V_1$, such that
$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)), I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)), F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$,
$T_{K_1(e)}(uv) \leq T_{K_2(e)}(f_e(u)f_e(v)), I_{K_1(e)}(uv) \leq I_{K_2(e)}(f_e(u)f_e(v)), F_{K_1(e)}(uv) \geq F_{K_2(e)}(f_e(u)f_e(v))$,
for all $u, v \in V_1, e \in N$.
Let $g_N : V_2 \to V_1$ be an isomorphism of G_2 onto G_1, $g_e(v') = v$ for all $v' \in V_2$, such that
$T_{F_2(e)}(v') = T_{F_1(e)}(g_e(v')), I_{F_2(e)}(v') = I_{F_1(e)}(g_e(v')), F_{F_2(e)}(v') = F_{F_2(e)}(g_e(v'))$,
$T_{K_2(e)}(u'v') \leq T_{K_1(e)}(g_e(u')g_e(v')), I_{K_2(e)}(u'v') \leq I_{K_1(e)}(g_e(u')g_e(v')), F_{K_2(e)}(u'v') \geq F_{K_1(e)}(g_e(u')g_e(v'))$,
for all $u', v' \in V_2, e \in N$.
Both weak isomorphisms f_N from G_1 onto G_2 and g_N from G_2 onto G_3, are holds when G_1 and G_2 have same number of edges and the corresponding edges have same truth-membership degree, indeterminacy-membership degree and falsity-membership degree corresponding to the parameter to the set of parameters. Hence G_1 and G_2 are identical.

(3) Transitive: Let $f_N : V_1 \to V_2$ and $g_N : V_2 \to V_3$ are weak isomorphisms of the intuitionistic neutrosophic soft graphs G_1 onto G_2 and G_2 onto G_3, respectively. For transitive relation we consider a bijective mapping $g_N \circ f_N : V_1 \to V_3$ such that $(g_N \circ f_N)(u) = g_e(f_e(u))$ for all $u \in V_1$.
As $f_N : V_1 \to V_2$ is a weak isomorphism from G_1 onto G_2, such that $f_e(v) = v'$ for all $v \in V_1$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v'), I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v')$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v')$, and
$T_{K_1(e)}(uv) \leq T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v'), I_{K_1(e)}(uv) \leq I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v')$,
$F_{K_1(e)}(uv) \geq F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v')$, for all $u, v \in V_1, e \in N$.

As $g_N : V_2 \to V_3$ is an isomorphism from G_2 onto G_3 such that $g_e(v') = v''$ for all $v' \in V_2$, then

$T_{F_2(e)}(v') = T_{F_3(e)}(g_e(v')) = T_{F_3(e)}(v''), I_{F_2(e)}(v') = I_{F_3(e)}(g_e(v')) = I_{F_3(e)}(v'')$,
$F_{F_2(e)}(v') = F_{F_3(e)}(g_e(v')) = F_{F_3(e)}(v'')$, and
$T_{K_2(e)}(u'v') \leq T_{K_3(e)}(g_e(u')g_e(v')) = T_{K_3(e)}(u''v''), I_{K_2(e)}(u'v') \leq I_{K_3(e)}(g_e(u')g_e(v')) = I_{K_3(e)}(u''v'')$,
$F_{K_2(e)}(u'v') \geq F_{K_3(e)}(g_e(u')g_e(v')) = F_{K_3(e)}(u''v'')$, for all $u', v' \in V_2, e \in N$.

For transitive relation we consider a bijective mapping $g_N \circ f_N : V_1 \to V_3$, then

$T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)) = T_{F_2(e)}(v') = T_{F_3(e)}(g_e(f_e(v)))$,
$I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)) = I_{F_2(e)}(v') = I_{F_3(e)}(g_e(f_e(v)))$,
$F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v)) = F_{F_2(e)}(v') = F_{F_3(e)}(g_e(f_e(v)))$, and
$T_{K_1(e)}(uv) \leq T_{K_2(e)}(f_e(u)f_e(v)) = T_{K_2(e)}(u'v') \leq T_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$I_{K_1(e)}(uv) \leq I_{K_2(e)}(f_e(u)f_e(v)) = I_{K_2(e)}(u'v') \leq I_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$,
$F_{K_1(e)}(uv) \geq F_{K_2(e)}(f_e(u)f_e(v)) = F_{K_2(e)}(u'v') \geq F_{K_3(e)}(g_e(f_e(u))g_e(f_e(v)))$
for all $u, v \in V_1, e \in N$.
Therefore $g_N \circ f_N$ is a weak isomorphism between G_1 and G_3, i.e., weak isomorphism satisfying transitivity.

Hence isomorphism between INSGs by (1), (2) and (3) is a partial order relation. □

Definition 4.5. An INSG G is *self complementary* if $G \approx G^c$.

Proposition 4.3. Let G_1 and G_2 be INSGs. Then $G_1 \cong G_2$ if and only if $G_1^c \cong G_2^c$.

Proof. Let G_1, G_2 be the two INSGs. Suppose that $G_1 \cong G_2$, then there exist a bijective mapping $f_N : V_1 \to V_2$ such that $f_e(v) = v'$ for all $v \in V_1$, $T_{F_1(e)}(v) = T_{F_2(e)}(f_e(v)), I_{F_1(e)}(v) = I_{F_2(e)}(f_e(v)), F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$, and $T_{K_1(e)}(uv) = T_{K_2(e)}(f_e(u)f_e(v)), I_{K_1(e)}(uv) = I_{K_2(e)}(f_e(u)f_e(v))$,
$F_{K_1(e)}(uv) = F_{K_2(e)}(f_e(u)f_e(v))$, for all $u, v \in V_1, e \in N$. By the definition of complement of INSGs

$$\begin{aligned}
T^c_{K_1(e)}(uv) &= T_{F_1(e)}(u) \wedge T_{F_1(e)}(v) - T_{K_1(e)}(uv),\\
&= T_{F_2(e)}(f_e(u)) \wedge T_{F_2(e)}(f_e(v)) - T_{K_2(e)}(f_e(u)f_e(v))\\
&= T^c_{K_2(e)}(f_e(u)f_e(v)),\\
I^c_{K_1(e)}(uv) &= I_{F_1(e)}(u) \wedge I_{F_1(e)}(v) - I_{K_1(e)}(uv),\\
&= I_{F_2(e)}(f_e(u)) \wedge T_{F_2(e)}(f_e(v)) - I_{K_2(e)}(f_e(u)f_e(v))\\
&= I^c_{K_2(e)}(f_e(u)f_e(v)),\\
F^c_{K_1(e)}(uv) &= F_{F_1(e)}(u) \vee F_{F_1(e)}(v) - F_{K_1(e)}(uv),\\
&= F_{F_2(e)}(f_e(u)) \wedge F_{F_2(e)}(f_e(v)) - F_{K_2(e)}(f_e(u)f_e(v))\\
&= F^c_{K_2(e)}(f_e(u)f_e(v))
\end{aligned}$$

Hence $G_1^c \cong G_2^c$.
Conversely, assume that $G_1^c \cong G_2^c$, then there exist an isomorphism $g_N : V_1 \to V_2$ such that $g_e(v) = v'$,
$T_{F_1(e)}(v) = T_{F_2(e)}(g_e(v)), I_{F_1(e)}(v) = I_{F_2(e)}(g_e(v)), F_{F_1(e)}(v) = F_{F_2(e)}(f_e(v))$, for all $v \in V_1, e \in N$,$T^c_{K_1(e)}(uv) = T^c_{K_2(e)}(g_e(u)g_e(v)), I^c_{K_1(e)}(uv) = I^c_{K_2(e)}(g_e(u)g_e(v)), F^c_{K_1(e)}(uv) = F^c_{K_2(e)}(g_e(u)g_e(v))$, for all $u, v \in V_1, e \in N$.

By using the definition of complement of intuitionistic neutrosophic soft graph

$$
\begin{aligned}
T^c_{K_1(e)}(uv) &= T^c_{F_1(e)}(u) \wedge T^c_{F_1(e)}(v) - T_{K_1(e)}(uv),\\
T^c_{K_2(e)}(g_e(u)g_e(v)) &= T^c_{F_2(e)}(g_e(u)) \wedge T^c_{F_2(e)}(g_e(v)) - T_{K_2(e)}(g_e(u)g_e(v)),\\
I^c_{K_1(e)}(uv) &= I^c_{F_1(e)}(u) \wedge I^c_{F_1(e)}(v) - I_{K_1(e)}(uv),\\
I^c_{K_2(e)}(g_e(u)g_e(v)) &= I^c_{F_2(e)}(g_e(u)) \wedge I^c_{F_2(e)}(g_e(v)) - I_{K_2(e)}(g_e(u)g_e(v)),\\
F^c_{K_1(e)}(uv) &= F^c_{F_1(e)}(u) \vee F^c_{F_1(e)}(v) - F_{K_1(e)}(uv),\\
F^c_{K_2(e)}(g_e(u)g_e(v)) &= F^c_{F_2(e)}(g_e(u)) \vee F^c_{F_2(e)}(g_e(v)) - F_{K_2(e)}(g_e(u)g_e(v)).
\end{aligned}
$$

As $T^c_{K_1(e)}(uv) = T^c_{K_2(e)}(g_e(u)g_e(v)), I^c_{K_1(e)}(uv) = I^c_{K_2(e)}(g_e(u)g_e(v)), F^c_{K_1(e)}(uv) = F^c_{K_2(e)}(g_e(u)g_e(v))$, for all $u, v \in V_1, e \in N$, $g_N : V_1 \to V_2$ is an isomorphism between G_1 and G_2, that is $G_1 \cong G_2$. □

Proposition 4.4. If G_1 is co-weak isomorphic to G_2, then there can be a homomorphism between G_1^c and G_2^c.

Proposition 4.5. If G_1 is weak isomorphic to G_2, then G_1^c and G_2^c are weak isomorphic intuitionistic neutrosophic soft graphs.

5. Applications

Intuitionistic neutrosophic soft graph has several applications in decision making problems and used to deal with uncertainties from our different daily life problems. In this section we apply the concept of INSSs in a decision making problems. Many practical problems can be represented by graphs. We present an application of INSG to a multiple criteria decision-making problem. We present an algorithm for most appropriate selection of an object in a multiple criteria decision-making problem.

Algorithm 5.1.

(1) Input the set of parameters $e_1, e_2, \dots, e_k$.
(2) Input the INSSs (F, N) and (K, N).
(3) Input the INGs $H(e_1), H(e_2), \dots, H(e_k)$.
(4) Calculate the score values of INGs $H(e_1), H(e_2), \dots, H(e_k)$ using formula

$$S_{ij} := \sqrt{(T_j)^2 + (I_j)^2 + (1 - F_j)^2} \tag{5.1}$$

Tabular representation of score values of INGs $H(e_k)$, $\forall\ k$.

(5) Compute the choice values of $C_p = \sum_j S_{ij}$ for all $i = 1, 2, \dots, n$ and $p = 1, 2, \dots, k$.
(6) The decision is S_i if $S_i = \max_{i=1}^{n}\{\min_{p=1}^{k} C_p\}$.
(7) If i has more than one value then any one of S_i may be chosen.

An algorithm for the selection of optimal object based upon given set of information.

(1) An appropriate selection of a machine for a specific task is an important decision-making problem for a machine manufacturing corporation. The performance of a manufacturing corporation is badly affected by the wrong selection. The main purpose in machine selection is that machine will achieve the require tasks within possible short time and minimum cost.

The main purpose is to select the machine that will complete the required task within the time available for the lowest possible cost. Rate of productivity, automatic system and price are important aspects considered in selection of a machine. The rate of productivity, value of product and charge of manufacturing depends upon the performance of machine. Mr. X should be an expert or at least familiar with the machine properties, to select a best machine among the parameters (alternatives), i.e., "price", "rate of productivity" and "automatic system". Let $V = \{m_1, m_2, m_3, m_4, m_5, m_6\}$, set of six machines to be consider as the universal set and $N = \{e_1, e_2, e_3\}$ be the set of parameters that characterize the machine, the parameters e_1, e_2 and e_3 stands for "price", "rate of productivity" and "automatic system", respectively. Consider the INSS (F, N) over V which define the "efficiency of machines" corresponding to the given parameters that Mr. X want to select. (K, N) is an INSS over $E = \{m_1m_2, m_2m_3, m_6m_1, m_1m_3, m_1m_4, m_1m_5, m_2m_4, m_2m_5, m_2m_6, m_3m_4, m_3m_5, m_3m_6, m_4m_5, m_4m_6, m_5m_6\}$ define degree of truth membership, degree of indeterminacy, and degree of falsity membership of the connection between two machines corresponding to the selected attributes e_1, e_2 and e_3. The INGs $H(e_1)$, $H(e_2)$ and $H(e_3)$ of INSG $G = \{H(e_1), H(e_2), H(e_3)\}$ corresponding to the parameters "price", "rate of productivity" and "automatic system", respectively are shown in Figure 5.1.

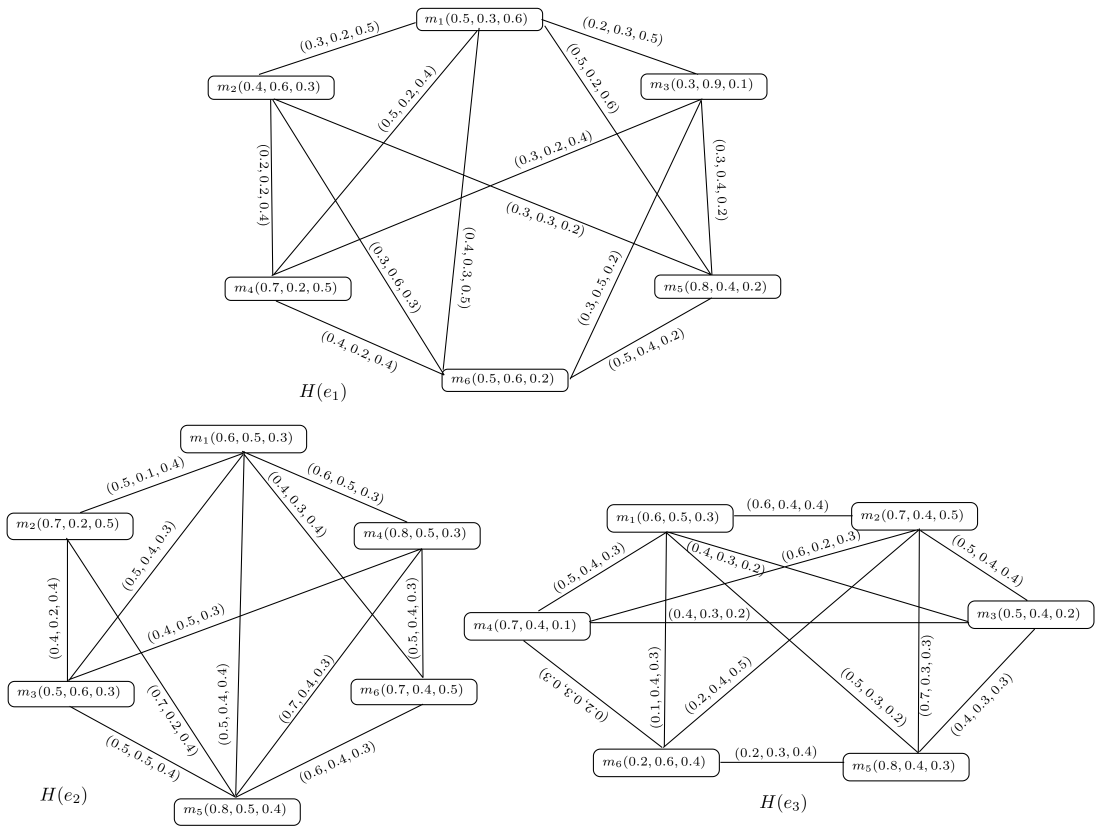

FIGURE 5.1. Intuitionistic neutrosophic soft graph $G = \{H(e_1), H(e_2), H(e_3)\}$

Tabular representation of score values of INGs $H(e_1)$, $H(e_2)$, and $H(e_3)$ with normalized score function $S_{ij} = \sqrt{(T_j)^2 + (I_j)^2 + (1 - F_j)^2}$ and choice value for each machine m_i for $i = 1, 2, 3, 4, 5, 6$.

TABLE 2. Tabular representation of score values and choice values of $H(e_1)$.

	m_1	m_2	m_3	m_4	m_5	m_6	$\acute{m}_k$
m_1	0	0.62	0.62	0.80	0.67	0.71	3.42
m_2	0.62	0	0	0.66	0.91	0.97	3.16
m_3	0.62	0	0	0.70	0.94	0.99	3.25
m_4	0.80	0.66	0.70	0	0	0.75	2.91
m_5	0.67	0.91	0.94	0	0	1.0	3.52
m_6	0.71	0.97	0.94	0.75	1.0	0	4.37

TABLE 3. Tabular representation of score values and choice values of $H(e_2)$.

	m_1	m_2	m_3	m_4	m_5	m_6	$\acute{m}_k$
m_1	0	0.79	0.94	1.0	0.88	0.78	4.39
m_2	0.79	0	0.75	0	0.94	0	2.48
m_3	0.94	0.75	0	0.95	0.93	0	3.57
m_4	1.0	0	0.95	0	1.0	0.95	3.9
m_5	0.88	0.94	0.93	1.0	0	1.0	4.75
m_6	0.78	0	0	0.95	1.0	0	2.73

TABLE 4. Tabular representation of score values and choice values of $H(e_3)$.

	m_1	m_2	m_3	m_4	m_5	m_6	$\acute{m}_k$
m_1	0	0.94	0.94	0.95	0.99	0.81	4.63
m_2	0.94	0	0.94	0.94	1.0	0.67	4.49
m_3	0.94	0.94	0	0.94	0.86	0	3.68
m_4	0.95	0.94	0.94	0	0	0.79	3.62
m_5	0.99	1.0	0.86	0	0	0.70	3.55
m_6	0.81	0.67	0	0.79	0.70	0	2.97

The decision is S_i if $S_i = \max\limits_{i=1}^{6}\{\min\limits_{p=1}^{3} m_p\} = \max\limits_{i=1}^{6}\{3.42, 2.48, 3.25, 2.91, 3.52, 2.73\} = 3.52$. Clearly, the maximum score value is 3.52, scored by the m_5. Mr. X will buy the machine m_5.

(2) We present a multi-criteria decision making problem for product marketing if there are multiple brands of a product, product marketing has intuitionistic neutrosophic behaviour. Consider Mr. X who is a retail owner wants to maximize his profit by selling some electronic items which meets all the requirements set by a retail outlet owner. Let $V = \{S_1, S_2, S_3, S_4, S_5\}$ be a set of five brands of an item to be sold in an international market, and let $N = \{e_1 =$"price", $e_2 =$"quality"$\}$ be a set of parametric factors in product marketing. Let (F, N) be the INSS over V, which describes the effectiveness of the brands, $T_{F(e_k)}(S_i)$, $T_{F(e_k)}(S_i)$, and $T_{F(e_k)}(S_i)$, for $i = 1, 2, \ldots, 5, k = 1, 2$ represent the degree of membership (goodness), degree of indeterminacy and degree of non-membership (poorness) of the brands corresponding to the parameters $e_1 =$"price" and $e_2 =$"quality", respectively and (K, N) be the INSS on $E = \{S_1S_2, S_1S_4, S_1S_3, S_2S_3, S_3S_4, S_2S_5, S_3S_5, S_1S_5, S_4S_5\}$ describes the relationship between brands corresponding to the parameters $e_1 =$"price" and $e_2 =$"quality". The INSG is shown in Figure 5.2. The method for selection of brand in product marketing is presented in Algorithm 5.2.

Algorithm 5.2.

(a) Input the set of parameters $e_1, e_2, \ldots, e_k$.
(b) Input the INSSs (F, N) and (K, N).

(c) Construct ING $H(e_1) \cap H(e_2) \cap \ldots \cap H(e_k)$.

(d) Calculate the average score values of INGs $H(e)$ using formula

$$\zeta_{ij} := \frac{T_{j_{F(e)}} + I_{j_{F(e)}} + 1 - F_{j_{F(e)}}}{3}, \tag{5.2}$$

Tabular representation of score values of INGs $H(e)$.

(e) Compute the choice values of $C_i = \sum_j \zeta_{ij}$ for all $i = 1, 2, \ldots, n$.

(f) The decision is S_i if $S_i = \max_{i=1}^{n} C_i$.

(g) If i has more than one value then any one of S_i may be chosen.

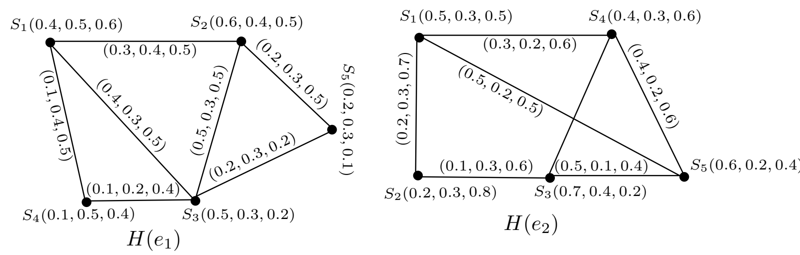

FIGURE 5.2. Intuitionistic neutrosophic soft graph.

The ING $H(e_1) \cap H(e_2)$ is shown in Figure 5.3. and tabular representation of average score values of ING is shown in Table 5.

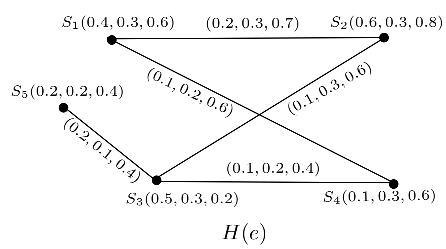

FIGURE 5.3. $H(e_1) \cap H(e_2)$

TABLE 5. Tabular representation of score values with choice values.

	S_1	S_2	S_3	S_4	S_5	$\acute{C}_i$
S_1	0	0.27	0	0.23	0	0.5
S_2	0.27	0	0.27	04	0	0.54
S_3	0	0.27	0	0.30	0.30	0.87
S_4	0.23	0	0.30	0	0	0.53
S_5	0	0	0.30	0	0	0.30

Clearly, the maximum score value is 0.87, scored by the S_3. Mr. X will choose the brand S_3.

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