An approach for analysing time dilation in the TSR

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Abstract. We investigate time dilation under the conditions of the theory of special relativity (TSR). The arguments apply a direct comparison of clock readings at identical positions. As observations of time depends on (the location of) the clocks used for time registrations we investigate various observational principles. Three principles are in focus: Reference frame applying single clock (SC); reference frame applying multiple clocks (MC); and the completely symmetric situation. We also study variants of the Lorentz transformation. Finally, we apply the approach to present a thorough discussion of the travelling twin example.

Key words: Time dilation, Lorentz transformation; observational principle; positional time; simultanity by symmetry; travelling twin.

1 Introduction and basic assumptions

The present work explores the concept of time dilation within the theory of special relativity (TSR). Chapter 2 presents an abridged and modified version of material previously posted on ViXra, see [1], [2]. We pinpoint the importance of the observational principle, that is, the specification of which clocks to apply for the required time comparisons, and present a unified framework for the various observational principles.

Further, a modified version of the Lorentz transformation is given in Chapter 3. Overall, the approach provides a tool for investigating time dilation within the framework of the TSR. In Chapter 4 we apply the results to give a lengthy discussion on the 'travelling twin' example; claiming a concise conclusion regarding the twin's ages.

1.1 Background and problem formulation

First of all I find the literature somewhat ambiguous regarding the very interpretation of *time dilation*. For instance, how should we interpret the common statement: 'Moving clock goes slower'? Many authors apply the expression 'as seen' by the observer on the other reference system, perhaps indicating that it is an apparent effect, not a physical reality, but without elaborating on the interpretation of 'as seen'. Others stress that 'everything goes slower' on the 'moving system', not only the clocks; truly stating that the time dilation represents a physical reality also under the conditions of TSR (*i.e.* no gravitation *etc*.). On the other side Giulini [3] in Section 3.3 of his book states: 'Moving clocks slow down' is 'potentially misleading and should not be taken too literally'. However, the expression 'not be taken too literally' is not very precise. So in what sense – and under which precise conditions – is time dilation to be considered a true physical phenomenon?

We stress that rather than specifying *one* single time dilation formula – which is typically based on a somewhat arbitrary definition of simultaneity – we will in the present work look at the total picture of *all* expressions for time dilation.

Definition of *simultaneity* becomes crucial when reference frames are moving relative to each other, and the convention seems to define simultaneity across reference frames by use of light rays. However, in the present paper we restrict to consider simultaneity of events, which occur at the same location *and* time. So each reference frame has a set of calibrated clocks, located at virtually any position, and in principle we can at any position compare the clocks of the two reference frames.

The question of *symmetry* is also essential. The TSR essentially describes a symmetric situation for the two systems/observers moving relative to each other, but this does not always seem to be fully utilized. And some references are not found that explicit, describing situations apparently involving some asymmetry. The present paper intends to fully account for the symmetry, and also introduces the concept of 'simultaneity by symmetry'.

The considerations of the present work are mainly mathematical, and we essentially discusses rather well known results. However, I believe that the presentation deviates from the main narratives on the topic. In particular, rather than focusing on a specific time dilation formula – which is often based on a somewhat arbitrary definition of simultaneity – we will in the present work look at the total picture of *all* expressions for time dilation.

1.2 Basic assumptions and some notation

The basis for the discussions is the standard theoretical experiment, two co-ordinate systems (reference frames), K and K_v moving relative to each other at speed, v. We investigate the relation between space and time parameters, (x, t) on system K and the corresponding parameters (x_v, t_v) on the system K_v. Thus, we restrict to consider just one space co-ordinate, (x-axis) and will base the discussions on the following specifications:

- There is a complete *symmetry* between the two co-ordinate systems, K and K_v; the systems being identical in all respects.
- On both reference frames there is an arbitrary number of identical, synchronized clocks, located at any positions where it is required to measure time.
- At time $t = t_v = 0$, clocks at the location $x = 0$ on K and location $x_v = 0$ on K_v are synchronized. This represents the defining starting point, from which all events are measured: the 'point of initiation'.
- When we consider two different reference system, simultaneity of events will mean that they occur at the same time *and* at the same location So if clocks are on different reference frames, we only compare them at an instant when they are at the same location. Note that we will relax on this condition in Chapter 3 by introducing the concept ' simultaneity by symmetry'.
- We will choose the *perspective* of one of the systems, (here usually K), and refer to this as the *primary* system. The time on this 'primary' system is at any position, x given as a constant, $t(x) \equiv t$, independent of x, (all clocks being synchronized). In contrast, at a certain time, t on the primary system, the observed time, t_v on the other ('secondary') system(s), (here K_v), will depend on the location where the time reading is carried out. When there are several reference frames, we are free to choose any one as the primary.
- We use a notation where *SC* refers to a reference frame utilizing a 'single clock' (or 'same clock'), for the time comparisons with other reference frames, and *MC* refers to a reference frame utilizing 'multiple' (several) clocks for time comparisons.

2 The Lorentz transformation and special cases

We here present the Lorentz transformation, and further investigate a variant of this.

2.1 The standard formulation

The Lorentz transformation represents the fundament for our discussion of time dilation. We now first introduce a change of the standard notation. Rather than t' and x' we will write t_v and x_v. Then the Lorentz transformation takes the form

$$t_v = \frac{t - \frac{v}{c^2}x}{\sqrt{1-(\frac{v}{c})^2}} \tag{1}$$

$$x_v = \frac{x - vt}{\sqrt{1-(\frac{v}{c})^2}} \tag{2}$$

Thus the position, x_v corresponds to (has the same location as) x when the clocks at this positions show time t and t_v, respectively. The formulas include the length contraction along the x-axis (Lorentz factor):

$$k_x = \sqrt{1-(\frac{v}{c})^2} \tag{3}$$

So this transformation relates simultaneous time readings, t and t_v performed at identical locations x on K and x_v on K_v.

2.2 An alternative formulation

Taking the perspective of K, we may at any time t choose an 'observational position' equal to $x = wt$, (for an arbitrarily chosen w). By inserting $x = wt$ in (1) we directly get that time on K_v at this position equals:

$$t_v(w) = \frac{1-\frac{vw}{c^2}}{\sqrt{1-(\frac{v}{c})^2}}t \tag{4}$$

Thus to pinpoint the dependence on w we here -and when appropriate- write $t_v(w)$ rather than t_v. The new time dilation formula (4) will – for a given time, t, on the primary system, K - give the time, $t_v(w)$ on the secondary system, K_v, as a linear, decreasing function of w; *cf.* Fig. 1 at the end of the paper; (and a similar figure in [1] and [2]).

Now we, similarly, define w_v so that $x_v = w_v \cdot t_v = w_v \cdot t_v(w)$. By inserting both $x = wt$ and $x_v = w_v \cdot t_v$, in (2), we will after some manipulations obtain

$$w_v = \frac{x_v}{t_v(w)} = \frac{w-v}{1-\frac{w}{c}\cdot\frac{v}{c}} \tag{5}$$

So equations (4), (5) represent the alternative version of Lorentz transformation, here being expressed by parameters (t, w) rather than (t, x). Next, we introduce

$$\gamma_v(w) = (1-\frac{vw}{c^2})/\sqrt{1-(\frac{v}{c})^2} \tag{6}$$

as the 'generalized time dilation factor', valid for any location, (any $w=x/t$), *i.e.* any observational principle. That is, we can write (4) as

$$t_v(w) = \gamma_v(w)\, t.$$

Note that we do not need to think of w as a velocity; rather as a way to specify a certain position $x=wt$, representing the location of the clocks being applied at time t. However, we will later see that it can also be fruitful to interpret w as the velocity of a third 'observational reference frame'.

2.3 Special cases

Focusing on time, (4) there are various interesting special cases. First, if the specific clock at $x_v = 0$ on K_v is compared with various clocks on K, these clocks must have position $x = vt$, and thus we choose $w=v$ and get the relation

$$t_v(v) = t\sqrt{1-(v/c)^2} \tag{7}$$

However, when a specific clock at position $x = 0$, on K is used for comparisons with various clocks on K_v, we must choose $w=0$ and get

$$t_v(0) = t\,/\sqrt{1-(v/c)^2} \tag{8}$$

as the relation between t and t_v, (also giving $x_v = -vt\,/\sqrt{1-(v/c)^2} = -vt_v$). Two other special cases are obtained by choosing $w = c$ and $w = -c$. First

$$t_v(c) = \frac{1-v/c}{\sqrt{1-(\frac{v}{c})^2}}t = \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}t \tag{9}$$

For observing this we apply two clocks on both system: one at $x = 0$ and one at $x = ct$ on K; and similarly, one at $x_v = 0$ and one at $x_v = ct_v$ on K_v.

So eq. (9) is valid when the light ray is emitted in the positive direction ($x > 0$); *i.e.* c having the same direction as the velocity v, as seen from K. In the negative direction, (choosing $x = - ct$) we similarly get another well-known result:

$$t_v(-c) = \frac{1+v/c}{\sqrt{1-(\frac{v}{c})^2}}t \; = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}t \qquad (10)$$

The principles (9), (10) seems essentially to be applied for two ways light flashes ('round trips'), *e.g.* see [1]. We return to a fifth special case in Section 2.5.

2.4 "The moving clock". SC vs. MC

The two special cases, (7) and (8) require special attention.

In (7), the clock at $x_v \equiv 0$ on K_v passes a position, $x = vt$ (on K) at time t. We now let SC indicate a reference frame utilizing a single (same) clock for time comparisons, and by writing $t_v(v) = t_v^{SC}$ *eq.* (7) becomes

$$t_v^{SC} = t\sqrt{1-(\frac{v}{c})^2}$$

In (8) we follow a clock at $x \equiv 0$ on K, and at this position we make comparisons with various clocks on K_v as they pass along. Now we let MC indicate a reference frame utilizing multiple clocks, and so by writing $t_v(0) = t_v^{MC}$ we get:

$$t_v^{MC} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}t$$

Now when K_v is SC then K becomes MC and *vie versa.* Thus the two symmetric results, (7), (8) could be presented in a compact form as

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2} \qquad (11)$$

So in this notation t^{MC} is time measured on a MC reference frame K^{MC}, and we let x^{MC} be the position of measurements on this system. Thus, there are two clocks on K^{MC}, located at $x^{MC} = 0$ and $x^{MC} = v\, t^{MC}$, respectively. The SC reference frame, K^{SC} has time, t^{SC}, and we utilizes just one clock on its system, located at $x^{SC} = 0$.

In (11) we could add a subscript, v at either of the two time parameters, (as done above) to indicate secondary system. Actually, we might say that the two special cases (7), (8) in a way represent the same observational approach. The only difference between these cases is the choice of which of the systems is SC and which is MC, and so they are more effectively expressed by *eq.* (11).

Some comments are relevant here. First we stress that observers on both reference frames agree on this result (11). Thus, I find it rather misleading here to apply the phrase 'as seen' regarding the clock reading on 'the other' system; which is a formulation used by some authors. The time readings are objective, and all observers (observational equipment) on the location in question will 'see' the same thing. The main point is rather that observers at *different reference frames* will not agree regarding simultaneity of events.

Secondly, we have the formulation 'moving clock goes slower'. It is true that an observer on a reference frame (K^{SC}), observing a *specific clock* passing by, will see this clock going slower when it is compared to his own clocks. So in a certain sense this confirms the standard phrase 'moving clock goes slower'. However, we could equally well take the perspective of the single clock, considering this to be at rest, implying that the clocks on K^{MC} are moving. The point is definitely not that clock(s) on K^{SC} are moving and clocks on K^{MC} are not. Rather, we could look at the symmetry of the situation: We are starting out with two clocks at origin, $x^{SC} = x^{MC} = 0$, moving relative to each other. And the decision to either compare the clock at $x^{SC} = 0$ *or* the clock at $x^{MC} = 0$ with clocks on the other system, (a decision that can be interchanged at random!), will decide which of the two clocks comes out as the fast one!

So, first of all, none of the clocks are more moving than the other. Further, it is the observational principle that decides which of the two clocks initially at the origin, which we see to move slower. Therefore I find the talk about the 'moving clock' rather misleading.

This choice on which reference frame shall apply just a single clock is obviously crucial, and it introduces an asymmetry between the two reference frames.

2.5 The symmetric case

Now returning to our version of the Lorentz transformation, (4), (5), we may ask which value of w (and thus w_v) would results in $t_v(w)= t$. we easily derive that this equality is obtained by choosing

$$w = w' = \frac{c^2}{v}\left(1 - \sqrt{1 - \left(\frac{v}{c}\right)^2}\right) = \frac{v}{1+\sqrt{1-(v/c)^2}} \tag{12}$$

By this choice of w we further get $w_v = -w'$. This means that if we consistently consider the positions where simultaneously $x = w't$ and $x_v = -w't_v = - w't$, then no time dilation will be observed at these positions. In other words

$$t_v(w') \equiv t \tag{13}$$

Since here $x_v = -x$, we consider this to be the midpoint between the origins of the two reference frames; in total providing a nice symmetry. Observe that when we choose this observational principle, then absolutely everything is symmetric, and it should be no surprise that this gives $t_v = t$.

Note that we could give w' a nice interpretation. Now assume that there is also a third reference frame. This moves relative to the reference frame K with velocity, v_1, and relative to K_v with velocity, v_2. Then, according to standard results of TSR (*cf.* [1]) – expressed by v_1 and v_2 - the relative velocity between K and K_v is given by the formula

$$\frac{v_2 - v_1}{1 - \frac{v_1}{c} \cdot \frac{v_2}{c}}$$

If we here insert $v_1 = - w'$ and $v_2 = w'$, where w' is given by (12), then we obtain the (desired) result v. So if both K and K_v move relative to this third reference frame at speed, w' (in opposite direction), then the velocity between K and K_v equals v. So there is a fixed point on this new reference frame where we make the time registrations of K and K_v. Not surprisingly, the symmetric case corresponds to K and K_v having same speed w' relative to this, and the two w' then 'add up' to the speed v between K and K_v.

We mention that the third reference frame we introduce here is a SC system; (there is a single observational point applied on this). For our purpose we actually do not need a clock at this position; we just observe the clock readings on K and K_v. However, both K and K_v become MC systems; so a series of clocks are required for these clock comparisons. We return to this in Chapter 3.

In summary, we consider the observational principles of Sections 2.4 and 2.5 - specified by *eq.* (11) and *eqs.* (12)-(13), respectively - as the main observational principles, and further investigate these in Chapter 3.

3 Introducing an auxiliary reference frame

In this section we elaborate further on the special cases represented by (7), (8), (11); but for completeness first present a short comment on length contraction which is so closely related to the time dilation.

3.1 Length contraction

The interpretation of x and x_v in (1) and (2) is rather straightforward. The position x_v on K_v corresponds exactly to the position x on K at an instant where the clock located at x_v shows time t_v and the clock at x shows time t.

However, x and x_v could also have a slightly different interpretation. Consider again a 'SC system', K_v moving at relative speed along a system, K. Now let a distance, x be marked out on K in the same direction as this movement. As known, the time measured on K_v for its single clock to pass this distance will imply that – as measured from K_v – the length of the distance x equals

$$x_v^{SC} = x\sqrt{1-(\frac{v}{c})^2} \tag{14}$$

So now x_v^{SC} equals the length of x on K 'as seen from' K_v. (In order to utilize the clock reading to observe x_v^{SC} one first have to establish the relative speed, v, between the reference frames.) So this length contraction (14) corresponds exactly to the time dilation observed for a single clock moving relative to a fixed distance on the other reference frame. Thus, anyone on K_v observes the distance travelled to be shorter, and so the required time to travel this distance will be observed to be shorter (both on K and K_v). Therefore, the length contraction and time dilation are indeed two aspects of the same phenomenon.

3.2 The auxiliary reference frame having a fixed point of observation ('SC system')

We go back to relations (7), (8), and the combined result, (11). This treats the case where we follow one clock on a SC system, consistently comparing it with the adjacent clock on the other system (MC), which thus, applies several clocks. One way to write this result is:

$$t_v^{MC}\sqrt{1-(v/c)^2} = t^{SC}$$

(notation here indicating that the SC reference frame is seen as the 'primary system'). Now consider a slightly different situation. If we have two systems, K_1 and K_2 moving at relative speeds, v_1 and v_2 with respect to a new auxiliary reference frame denoted K^{SC}, then in total we have

$$t_{v_i}^{MC}\sqrt{1-(v_i/c)^2} = t^{SC},\ i = 1, 2 \tag{15}$$

So as the notation indicates, here the auxiliary system, K^{SC} is SC and K_1 and K_2 are MC, and we specify a single point on the auxiliary reference frame, where we carry out all clock readings/comparisons on K_1 and K_2. We can of course eliminate t^{SC} (*i.e.* time on the auxiliary system) from these two relations in (15), and obtain

$$t_{v_2}^{MC} = \frac{\sqrt{1-(v_1/c)^2}}{\sqrt{1-(v_2/c)^2}}t_{v_1}^{MC} \tag{16}$$

In summary, v_1 and v_2 are the velocities of the two MC reference frames K_1 and K_2 relative to a common system, K^{SC}, and (16) now gives the relation between the times of these two reference frame measured at a fixed observational point on this common auxiliary system, K^{SC}. Here, the special case, $v_1 = 0$, reduces to the standard situations, (7). (When $v_1 = 0$ the observational point on K^{SC} is at rest with respect to K_1, and thus K_1 reduces to a SC system in this case.) Further, the special case $v_2 = 0$ reduces to the other standard situation, (8).

Of course the two times, $t_{v_i}^{MC}$ of (16) are identical when $v_1 = v_2$. Further, also by inserting $v_2 = -v_1$ we obviously get the same time reading, *i.e.*

$$t_{v_1}^{MC} = t_{-v_1}^{MC} \tag{17}$$

In particular, we can also choose $v_1 = -w'$ in *eq.* (17), and then we get the velocity, v, between K_1 and K_2. So by this choice of v_1 the symmetric case, (*cf.* (13)), comes out as a special case of (17):

$$t_{-w'}^{MC} = t_{w'}^{MC} \tag{18}$$

Further, *eq.* (15) gives the relation to the clock reading of the auxiliary reference frame:

$$t_{w\prime}^{MC} = t^{SC}/\sqrt{1-(w'/c)^2} \tag{19}$$

Fig. 2a illustrates the clock readings of the three reference frames. This demonstrates that the clocks on K_1 and K_2, which show the same time, are those clocks, which at that time are located at the midpoint in between the origins of K_1 and K_2.

Fig. 3 is a generalization of Fig. 1, presenting the time readings as a function of position w on the SC primary system (K). We indicates the three clock readings of (18), (19) with a circle marked 'a'.

We conclude this section by pointing out that *eq* (16) includes all the three major special cases of time dilation, referred in Chapter 2. In spite of this, (16) is not quite as general as the Lorentz transformation, (4). But using (16) in combination with the sum of velocity formula of TSR, *i.e.* $(u+v)/(1+uv/c^2)$, we can actually derive (4). *Eq.* (16) has one advantage, however. It gives the relation between the time measurements of the two reference frames at a specified position in space, but these observations are completely independent of the position, x (or equivalently of $w = x/t$).

3.3 The auxiliary reference frame being a MC system

In Section 3.2 we applied a fixed position on an auxiliary reference frame K^{SC} to observe time on the two reference frames K_1 and K_2, and thus these two reference frames both had to be MC. Of course we could also do it the other way. The two reference frames K_1 and K_2 could both be SC, and the 'primary' (auxiliary) system would thus be MC. Now this means, that one is able to 'follow' single clocks on K_1 and K_2, and at any time on K^{MC} read these clocks on K_1 and K_2 (wherever they are located). In analogy with (15) and (16) we now get

$$t_{v_i}^{SC}/\sqrt{1-(v_i/c)^2} = t^{MC}, \; i = 1, 2 \tag{20}$$

$$t_{v_2}^{SC} = \frac{\sqrt{1-(v_2/c)^2}}{\sqrt{1-(v_1/c)^2}} t_{v_1}^{SC} \tag{21}$$

Again the observational principles (7) and (8) come out as special cases. Also a variant of the symmetric case appears by choosing $v_2 = -v_1$, and again we can choose $v_1 = -w'$ to achieve the relative velocity, v, between K_1 and K_2. Thus, the analogy to (18, (19) equals

$$t_{-w\prime}^{SC} = t_{w\prime}^{SC} = t^{MC}\sqrt{1-(w'/c)^2} \tag{22}$$

So here we specify one position on K_1 and one on K_2 (*i.e.* the origins of these systems), and all clock comparisons with the auxiliary system are carried out at these two locations, see Fig. 2.b, and circles marked 'b' in Fig. 3.

Thus, the auxiliary reference frame is now a MC system with time, t^{MC}, and the result, (22) opens for a definition of 'simultaneity' at different – but symmetric – locations. Here $t_{-w\prime}^{SC}$ and $t_{w\prime}^{SC}$ are the times at the origin of K_1 and K_2, and as these origins have moved apart (after time 0), we will definitely *not* consider these times to be identical. If the clock at the origin of K_1 is compared with the adjacent one on K_2, the reading would be according to *eq.* (4). And of course exactly the same holds when we compare the clock at the origin of K_2 with the corresponding clock at K_1.

However, *eq.* (22) tells that when we compare both the clocks at the origin with the adjacent clock on the auxiliary system, which at the time is at the same location, then the time readings are identical, (and further, the clock reading at the time on the auxiliary system is also given in (22)).

This suggests that we in a certain sense could consider any time ($t_{-w\prime}^{SC}$) at the origin of K_1 to be simultaneous with the same time ($t_{w\prime}^{SC}$) observed at the origin of K_2. (Possibly, we may also extend this argument to other locations on K_1 and K_2?)

Due to symmetry reasons, the result is not surprising. In the next chapter we apply it in the argument regarding the 'travelling twin'.

4 The travelling twin

We now utilize the above approach for analysing time dilation to treat the so-called travelling twin example. As stated for instance in [4] the travelling twin paradox shall illustrate that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together.

Reference [4] also gives the following numerical example, (Chapter 10): “If one twin goes to a star 3 light years away in a super rocket that travels at 3/5 the speed of light, the journeys out and back each takes 5 years in the frame of the earth. But since the slowing-down factor is $\sqrt{1-(3/5)^2} = 4/5$ the twin on the rocket will age only 4 years on the outward journey, and another 4 years on the return journey. When she gets back home, she will be 2 years younger than her stay-at-home sister, who has aged the full 10 years.” So the claim is that the referred difference in ageing occurs during the periods of the journey with a constant speed; *i.e.* under the conditions of the TSR, (ignoring the acceleration/deceleration periods). After all the whole argument relies on the Lorentz transformation! Thus, our discussion will fully restrict to the periods of constant velocity. Now throughout this chapter

t = time on clock of earthbound twin

t_v = time on clock of travelling twin

The distance between earth and the ‘star’ equals x = 3 light years, and the rocket has speed, $v = (3/5)c$, giving $\sqrt{1-(v/c)^2} = 4/5$. It follows that in the reference frame of the earth/star, the rocket reaches the star at time, $t = x/v$ = 5 years. And the clock on the rocket is then located on $x_v = 0$, corresponding to $x = vt$, and thus the Lorentz transformation gives that at the arrival at the star this clock reads $t_v = t \cdot \sqrt{1-(v/c)^2} = 4$ years; so obviously, $t_v/t = 0.8$; (and the argument is also valid both ways of the travel).

This is a rather convincing argument. It does follow from the Lorentz transformation that the returning clock shows 8 years when he/she returns. However, recalling the discussion of Chapters 2-3 the case is perhaps not that straightforward, and since we have made no assumption of asymmetry regarding the periods of constant velocity, we seem to have a true paradox.

Thus, we will not question the clock of the travelling twin, but take a new look at the clock of the earthbound twin, trying to look at the total situation. First, we observe - following the notation of Chapter 3 - that the above presentation describes the travelling twin as a ‘SC system’, and so the earthbound twin is located on a ‘MC system’. So, actiually we could just look at *eq.* (11) to obtain the above result. (And as observed in Section 3.1 this is related to the length contraction: Seen from the perspective of the travelling twin, the distance between earth and the star does not equal x = 3 light years but just $x_v = 3 \cdot 0.8 = 2.4$ light years; fully ‘explaining’ the reduction in travelling time.)

Now the question is: Could we not similarly describe the situation as the travelling twin being located on a MC system, and the earthbound twin on a SC system (which would then be the clock located on the earth). If we insist on the symmetry of the situation, the answer must be yes. Thus, we simply assume that there is also a reference frame of the travelling twin with the required number of clocks. Say, he is equipped with rockets at appropriate and fixed distances from his own rocket, all moving with constant speed in the same direction as himself, and all equipped with a synchronized clock showing the same time, t_v. Whether this is practically feasible is not relevant here. We are referring to the model of the TSR, and point out what this theory tells about clock readings, *if* we provide such an arrangement.

By making this assumption, we could consider the earthbound twin as travelling back and forth along the reference frame of the travelling twin. This will now give that the one way ‘travelling time’ of the earthbound twin equals 4 years; while the time required for the travelling twin is 5 years. Due to this symmetry of results, we find it required to give a further discussion. There is both a lengthy and a short argument on this paradox. We first take the lengthy.

We follow up on the assumption that the travelling twin is located on a MC system, and the earthbound twin is a SC system; (*i.e.* just applying his clock at the earth for time comparisons with the travelling clocks.). Now *eq.* (11) gives the result, $t_v / t = 1/0.8 = 1.25$ whatever instant we consider after departure. Now consider various moments at which we observe the clocks positioned at (and passing by) the earth:

1. ('Perspective of travelling twin.') When the travelling twin arrives to the star, the clock on his rocket shows 4 years. So all clocks on the reference frame of the travelling twin show time, $t_v = 4$ years. This is also the case for the clock which as this moment is passing the earth, *i.e.* at $x = 0$. Thus, the clock on the earth at this instant shows time $t = t_v \cdot 0.8 = 3.2$ years.
2. ('Perspective of the earth/star system.') At the instant when the twin arrives at the star, the time of the earthbound system equals $t = 5$ years. (Earthbound twin could verify this by installing a clock at the 'star'.) When he performs a clock comparison at the earth ($x = 0$) at this moment, it gives that $t_v = t \cdot 1.25 = 6.25$ years for the clock which pass the earth at this moment.
3. ('Symmetric perspective.') The above two cases demonstrate that the two twins completely disagree about which event at the earth is simultaneous with the travelling twin's arrival at the star. Now consider the moment when the clock on the earth shows $t = 4$ years and the passing 'travelling clock' shows $t_v = t \cdot 1.25 = 5$ years. This instant obviously occurs in between the previous two moments, and also represents a moment being 'symmetric' to the event of the twin's arrival at the star (regarding clock readings!). And more important: It is the instant when the earthbound twin have carried out a 'travel' equivalent to the distance of the travelling twin!

In summary, when we now let the earthbound twin represent the 'SC system', and thus, carry out the clock comparison at the earth, we always get $t_v/t = 1.25$; *i.e.* it is the clock on the earth that 'goes slower'. We summarize the findings in Table 1. These three 'perspectives' in some way all 'correspond to' the arrival of the travelling twin at the star, and thus, demonstrate the problem we have to define the 'simultaneous' event on the earth.

Table 1. Various clock readings (light years) at/on the earth, potentially 'corresponding to' the arrival of the travelling twin at the star.

	'Perspective'		
Time reading	1. Travelling twin	2. Earthbound twin	3. Symmetric
Travelling twin system (MC): t_v	4	6.25	5
Earth/star system (SC): t	3.2	5	4

So how should we conclude regarding the time (clock readings) at the turning of the rocket? When now the information of Table 1 is available, let us assume that the earthbound twin is in charge. He could control his twin's travel by sending a light signal to the star, which on arrival initiates the return of his travelling twin. How should he do this to be sure the signal arrives at the right moment? One possibility that he might consider is to send a signal that arrives at the star when his earthbound clock shows 5 years (*i.e.* perspective 2 in Table 1). The problem is that at this moment the clock on the travelling twin system passing the earth shows 6.25 years. Thus one would suspect the travelling twin when he returns have aged 12.5 years (and not 8). The reason being that if he turns when his twin's signal reaches the rocket, he may have travelled a longer distance than the intended 3 light years. A similar objection applies to using perspective 1.

Actually, if the earthbound twin should be in charge, I guess the following strategy should be the most ingenious. Knowing about the length contraction, he will know that the travelling twin will observe a travelling distance to the star that equals just 2.4 light years. So the earthbound twin will adopt strategy

3: he sends a signal ordering to turn, such that the travelling twin will receive this signal when the clock on his own earthbound system shows 4 years, (*cf.* 'strategy 3' of Table 1).

Following this strategy, we conclude that at the local time when each of the twins now consider to be the turning of the rocket, the twins will agree on the following facts: Their own clock shows 4 years, and the adjacent clock on the other system shows 5 years. So by the direct measurements, they observe that the other twin at this moment apparently has aged more than himself by a factor 1.25. This gives a completely symmetric and consistent answer to the paradox.

Following this argument, both clocks show 4 years at the point of return. The same argument applies for the return travel, and we should conclude that by the reunion *both* clocks show 8 years.

Another way to put. We can choose between three options.

1. Either the travelling twin being on a SC system giving travelling times 8 years for him and 10 for the earthbound, *eq*. (11), or
2. The earthbound twin is located on a SC system, giving 8 years for him and 10 for the travelling *eq* (11);

both these options being consistent and in full agreement with the Lorentz transformation, or we could choose

3. The symmetric solution, *eq* (22), treating both systems as SC, giving a total duration of 8 years for both twins.

Here we are close to the short argument for our solution to the paradox. According to the symmetry of the situation, we would adopt the concept 'simultaneity by symmetry' (*cf.* Section 3.3 and Figs. 2-3). According to this, the event (t_v = 4 years, t =5 years) on the star is 'simultaneous' with the event (t_v = 5 years, t = 4 years) on the earth, and considering *both* systems as single clock, the conclusion is that both twins have aged 8 years by the return of the travelling twin.

So what is then wrong with the standard argument. It is obviously about simultaneity. The standard narrative seems implicitly to be based on the assumption that the arrival of the twin at the star is 'simultaneous' with the earthbound twin having aged 5 years. I disagree with this claim. The Lorentz transformation tells that the clock of the earthbound system located at the 'star' shows 5 years by the traveling twins arrival. But that does not imply that we can say that the earthbound twin has aged 5 years 'at the same time'. In my understanding simultaneity in TSR does not work that way.

Further, by focusing on the full symmetry of the situation, it should actually be rather meaningless to claim that one ages faster than the other does. And if we ignore the effects of the acceleration/deceleration periods, it is hard to see any asymmetry here that could justify a claim of a true difference in ageing.

Following up on the 'simultaneity by symmetry' we mention that we in this numerical example get w' $= c/3$, giving $\sqrt{1-(w'/c)^2} = \frac{2}{3}\sqrt{2} \approx 0.94$. So $4/\sqrt{1-(w'/c)^2} \approx 4.24$ years is the time of the auxiliary reference frame at the travelling twins arrival at the star; *cf. eq.* (22).

Actually, the travelling twin example suggests that the concept of 'simultaneity by symmetry' could be fruitful in some applications, and therefore such simultaneity considerations might supplement the pure comparison of clock readings.

5 Conclusions

We have used the Lorentz transformation to discuss a number of results on time dilation between two reference frames moving relative to each other at constant speed v. Basic features of the approach are:

- There is a complete *symmetry* between the two reference frames, and we synchronize all clocks on the same reference frame.

- In the outset we do not utilize any definition of *simultaneity* across systems. The basic approach restricts to explore direct comparisons of clocks being at the same location at the same time.
- We do not use the expression '*as seen'* (from the other reference frame). Observers (observational equipment) on both reference frames agree on the time readings; as they are carried out 'on location'.
- We specify that one of the reference frames is the 'primary', meaning that time equals t all over this reference frame, independent of position.
- We always specify the applied *observational principle*, which means specifying the location of the clocks that are used for time comparisons between the reference frames. Thus, an essential aspect of the approach is to specify how the observed time, t_v, on the 'other' ('secondary') system, K_v depends on the position, x on the primary reference frame, K.
- We do not describe time dilation by the expression 'moving clock goes slower'. It seems irrelevant which of the two reference frames we consider to be moving, as it is rather the observational principle that matters.
- We rather stress the distinct difference between 'single clock' (SC) systems – where one and the same clock is used for time comparisons, and 'multiple clock' (MC) systems – where several clocks along the x- axis are applied.

Thus, it is an important fact that at a given time, t on K, the time, t_v observed on K_v will depend on the position, x on K. The standard result $t_v = t\sqrt{1-(v/c)^2}$ comes out as a rather special case, where one of the reference frames applies just a single clock for the time comparisons.

An interesting observation is that if we choose the midpoint between $x = 0$ and $x_v = 0$ as the location for time comparisons, then we will observe $t_v = t$. Therefore, this choice represents an observational principle being symmetric with respect to the two reference frames. So when we observe $t \neq t_v$, in an otherwise symmetric situation, we claim that this is caused by the asymmetry of the chosen observational principle.

Investigations of time dilation should take this overall view, clearly accounting for the effect of the observational principle. The approach proceeds to introduce an auxiliary reference frame that serves as a primary system. That is, we relate all time observations to this auxiliary system; thus providing a link between the original two reference frames moving at relative speed, v. This provides a needed support in lack of a complete definition of simultaneity across systems.

The distinction/specification of which system(s) are SC and which are MC becomes a crucial element of the approach. Another element is the concept 'simultaneity by symmetry'. This concept applies for events that actually are not simultaneous in any of the two original reference frames. Thus, the clock readings of synchronized clocks will differ. However, by symmetry considerations and by the contribution of the auxiliary system, it nevertheless seems appropriate to refer to a kind of simultaneity.

We have further applied the suggested approach to discuss the so-called travelling twin case. As the standard example goes, the travelling twin will – at a speed of $0.6 \cdot c$ – age only 8 years during his trip, as opposed to the 10 years passed on the earth. Our claim is that the observational principle – in combination with symmetry considerations - is essential to explain the phenomenon. It seems no doubt that the trip actually takes just 8 years, (*cf.* the length contraction). But by taking an overall view of the situation (including 'simultaneity by symmetry' our claim is that the earthbound twin has aged the same number of years (*i.e.* 8).

It is further my opinion that under the conditions of having strict symmetry in all respects it would be rather meaningless to claim a 'true' time dilation, causing different ageing on the two systems. So it should be interesting to identify the conditions – in particular departures from symmetry - that would cause time dilation to represent such a physical reality. (In the travelling twin case I can not see that such conditions are identified.)

A further comment is that an observer moving relative to a reference frame where the actual event takes place might be a rather 'unreliable' observer regarding time. The various observational principles will provide him with different results; so one should be careful to let such an 'outside' observer define the phenomenon, (without properly considering his position).

The main results given here are a rather direct consequence of the Lorentz transformation, and many of the results are of course well known. However, I believe that the suggested approach for investigating the phenomenon of time dilation has some distinct differences, compared to current narratives on the topic.

References

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Appendix Some trigonometric relations

As suggested in some versions of [2], we may introduce φ by the identity

$$\sin\varphi = v/\mathrm{c}$$

Thus, the Lorentz factor $\sqrt{1-(v/c)^2}$ becomes

$$\cos\varphi = \sqrt{1-(v/c)^2}$$

This might be of some interest as the speed v obviously enters the time dilation formulas only through the angle φ. In particular, a fundamental formula like (11) becomes

$$t^{SC} = t^{MC}\cos\varphi$$

More generally, if we also let θ be defined by

$$\sin\theta = w/\mathrm{c}$$

then our version of the Lorentz transformation, (4), (5) takes the form

$$t_v = \frac{1-\sin\theta\,\sin\varphi}{\cos\varphi}t$$

$$\frac{w_v}{c} = \frac{\sin\theta-\sin\varphi}{1-\sin\theta\,\sin\varphi}$$

Further, by a standard trigonometric formula we have that the w' of (12) equals

$$w' = tg\left(\frac{\varphi}{2}\right)$$

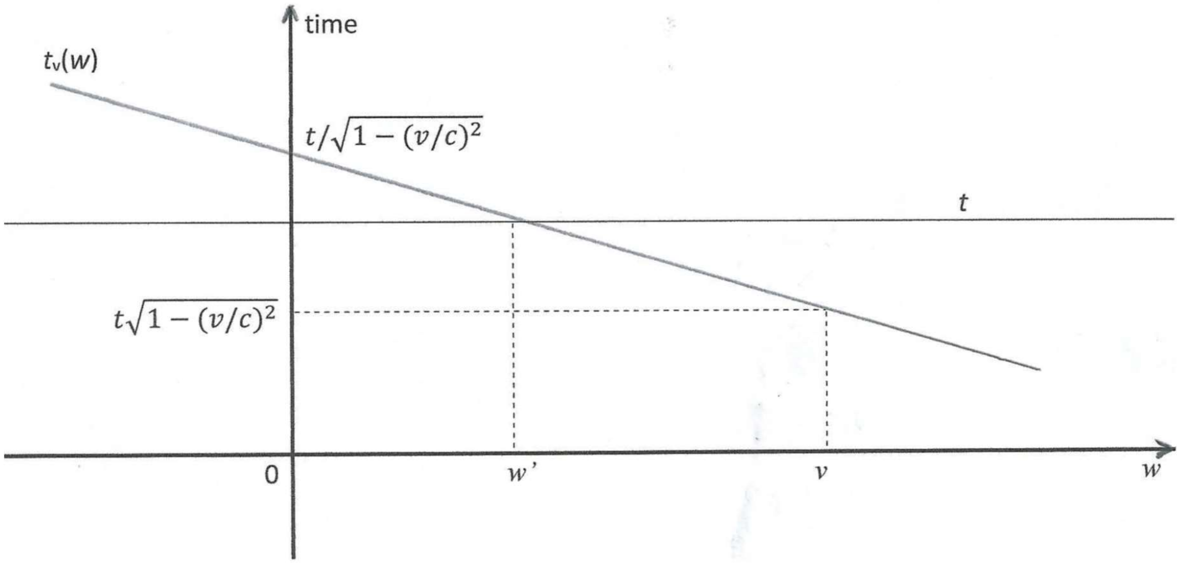
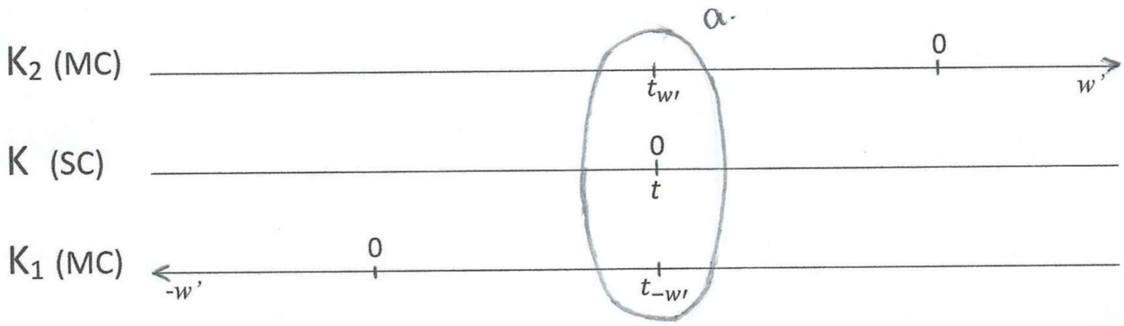
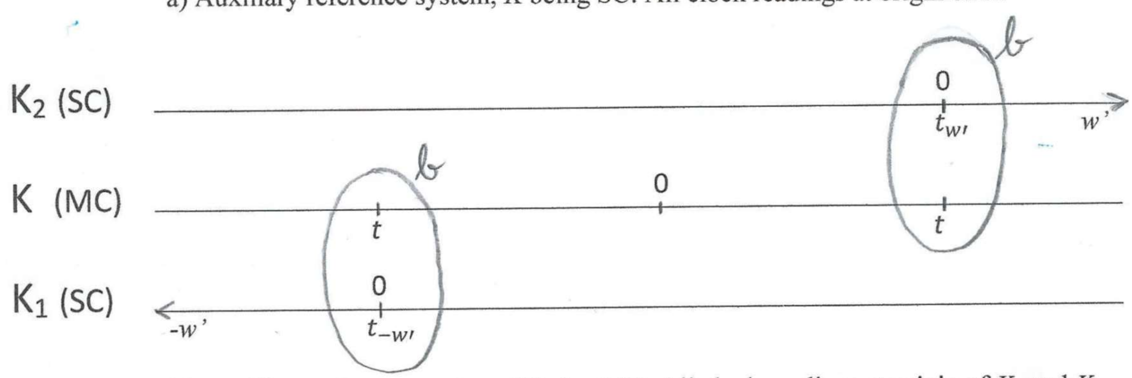


Figure 1. Time, $t_v(w)$,on K_v as a function of w. Have the perspective of K: The time all over K equals, t, and, w gives the position on K.



a) Auxiliary reference system, K being SC: All clock readings at origin of K.



b) Auxiliary reference system, K being MC: All clock readings at origin of K_1 and K_2.

Figure 2. Time observations on reference frames K_1 and K_2 at relative speed $\pm\, w'$ relative to auxiliary system, K, which serves as 'primary system'. So time equals t all over K.

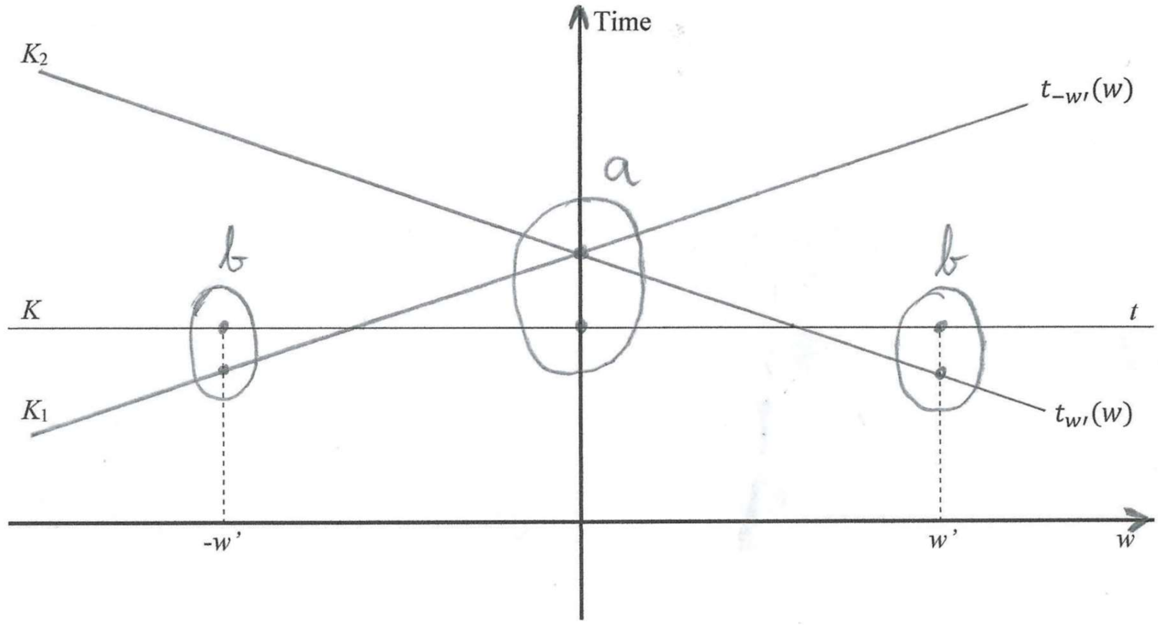


Figure 3. Times $t_{-w'}(w)$ on K_1 and $t_{w'}(w)$ on K_2 as a function of *w*. Here K_1 and K_2 have speed ±*w'* relative to auxiliary system *K*. Time equals *t* all over *K*. Simultaneous time readings 'a' and 'b' marked with circles as in Fig. 2.