Michelson and Morley experiment contains a hidden error.

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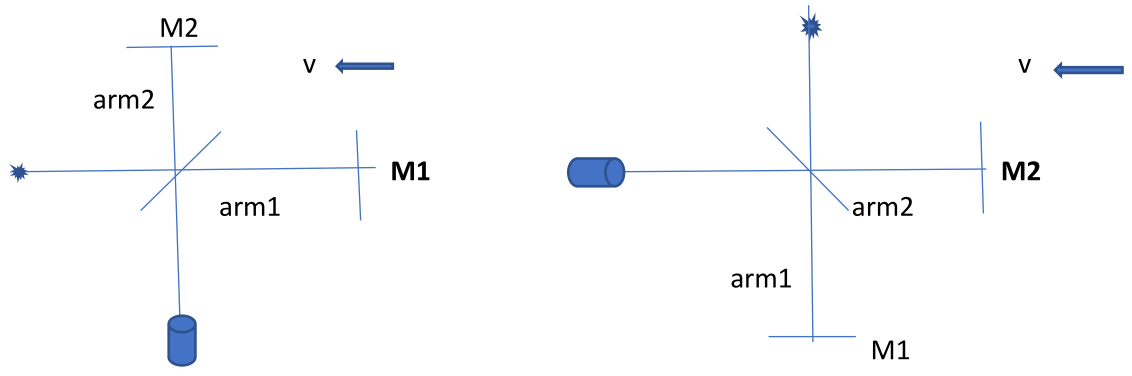
Abstract
In this paper, I endeavour to show why Michelson and Morley experiment did not give the expected result and to show that their disappointment was due to a hidden error in their assumptions.

Introduction

Physicists before the advent of the theory of relativity, believed that space contained a substance called the ether that was the medium for the propagation of the electromagnetic waves. In 1887, Michelson and Morley tried to detect it and measure the velocity the earth through it by means of the Michelson interferometer. They performed the experiment many times but failed to get the expected result. Since then many other physicists tried the experiment but they also failed. This experiment became known as the "famous failed experiment." It was then concluded that ether did not exist. It became the groundwork for some aspects of modern physics including the Einstein's theory of relativity. The explanation of the experiment is found in all university physics books. The reader may refer to them or to the original paper of Michelson and Morley experiment [1] for more details. Here, only a brief explanation is given.

Michelson-Morley experiment

A beam of light from a source is split by a half-silvered mirror and sent through two perpendicular arms of the Michelson interferometer which is fixed on earth. The earth moves through the ether with a velocity v relative to the ether and therefore experiences an ether wind. One beam is sent in the direction of the motion of the earth and the other perpendicular to it. These beams are then reflected back by mirrors fixed at the end of each arm and after being reflected and transmitted by the half-silvered mirror, enter a region behind the half-silvered mirror where they interfere and produce an interference pattern. The interferometer is then rotated through 90^0. The direction of v is unchanged, but the two paths in the interferometer are interchanged. This (according to Michelson) will introduce a path difference in the opposite sense to that obtained before. A fringe shift therefore is expected to take place but in fact no shift was observed.



Original Orientation

Rotated Orientation

Looking at the experiment from the point of view of the wave theory of light, we can see that the two beams of light have the same situations in both the original and rotated positions of the apparatus. In both situations one arm is parallel to the direction of the earth's motion and the other is perpendicular to it. The rays from the source are split and sent in the two directions. The two rays reach the field of vision with a time difference. The position of the light source differs in the two orientations. In the original orientation, it is in the direction of the earth's motion and in the rotated orientation it is perpendicular to it. But this is of no consequence, as the beam from the source in each orientation, is split and sent in the two directions.

Let the length of each arm be l. In the original orientation, the time for beam 1 to travel from the half-silvered mirror to M1 and back (transit time in horizontal direction) is given by

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2l}{c}\left(\frac{1}{1-v^2/c^2}\right) = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)$$

Where c is the speed of light in the ether and v is the velocity of the earth relative to the ether. The time t_2 for beam 2 to travel from the half-silvered mirror to M2 and back (transit time in perpendicular direction) is given by

$$2\sqrt{\left[t_2^2 + \left(\frac{vt_2}{2}\right)^2\right]} = ct_2$$

therefore

$$t_2 = \frac{2l}{c}\frac{1}{\sqrt{1-v^2/c^2}} = \frac{2l}{c}\ (1+\frac{v^2}{2c^2})$$

We can see that

$$t_1 > t_2$$

That is, it takes longer for the beam to travel from the half-silvered mirror to M1 and back (through the arm parallel to the motion of the earth) than it takes for the beam to travel from the half-silvered mirror to M2 and back (travelling through the perpendicular arm). Therefore, the time difference Δt is

$$\Delta t = \ t_1 - t_2 = \frac{2}{c}\left[\frac{l}{1-v^2/c^2} - \frac{l}{\sqrt{1-v^2/c^2}}\right] = \frac{l}{c}\frac{v^2}{c^2}$$

This corresponds to a path difference of

$$\Delta l = l\frac{v^2}{c^2}$$

Michelson simply and without any justification stated that "if now the whole apparatus be turned through 90° the difference will be in the opposite direction. Hence the displacement of the interference fringes should be $2l\frac{v^2}{c^2}$". This assumption is incorrect.
Michelson must have assumed that when the apparatus is rotated through 90° thereby the arms switching their positions from horizontal to vertical and vice versa, the corresponding transit times also switch with them. This is incorrect, but let us continue and see how Michelson would have arrived at the above shift.

If we now assume (according to Michelson) that the transit time corresponding to each arm remain the same as apparatus is rotated, we will have

$$t_2'' = \frac{2l}{c}\frac{1}{\sqrt{1-v^2/c^2}} = \frac{2l}{c}\left(1+\frac{v^2}{2c^2}\right)$$

Where t_2'' is the transit time in the horizontal arm in the rotated orientation and

$$t_1'' = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2l}{c}\left(\frac{1}{1-v^2/c^2}\right) = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)$$

Where t_1'' is the transit time in the vertical arm. Hence

$$\Delta t'' = t_2'' - t_1'' = -\frac{l}{c}\frac{v^2}{c^2}$$

And therefore

$$\Delta t - \Delta t'' = \frac{l}{c}\frac{v^2}{c^2} - \left(-\frac{l}{c}\frac{v^2}{c^2}\right) = \frac{2l}{c}\frac{v^2}{c^2}$$

The above demonstration shows that the shift predicted by Michelson can only be achieved if we assume that when the arms switch positions from vertical to horizontal and vice versa the corresponding times of transit of the beams in them switch as well, which obviously is not the case. **The time a beam of light takes to travel a given distance in the direction of the earth's motion or perpendicular to it, through an arm, has nothing to do with what the position of the arm has been in the previous orientation. This is the fact that was overlooked by Michelson.**

Now let us see why the experiment gives the null result. When the apparatus is turned through 90^0, arm2 will be in the direction of the motion of the earth and arm1 perpendicular to it. Let the transit time for the beam to travel from the half-silvered mirror to M2 and back in arm2 which is now parallel to the motion of the earth in the rotated position be t_2'

$$t_2' = \frac{l}{c-v} = \frac{l}{c+v} = \frac{2l}{c}\left(\frac{1}{1-v^2/c^2}\right) = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)$$

And let the transit time for the beam travelling in arm1, which is now perpendicular to the direction of the motion of the earth, be t_1'

$$t_1' = \frac{2l}{c}\frac{1}{\sqrt{1-v^2/c^2}} = \frac{2l}{c}\left(1+\frac{v^2}{2c^2}\right)$$

It can be seen that t_2' is greater than $t_{1.}'$ Again it takes longer for the beam to travel along arm2, which is now parallel to the direction of the earth's motion, than it takes the beam travelling in arm1. Therefore, the time difference between the arrival of the two beams in the region of interference in the rotated orientation is

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left[\frac{l}{1-v^2/c^2} - \frac{l}{\sqrt{1-v^2/c^2}}\right] = \frac{l}{c}\frac{v^2}{c^2}$$

Which is the same as Δt.

Hence

$$\Delta t' - \Delta t = 0$$

So, the rotation should not cause a shift in the fringe pattern since it does not change the phase relationship between the two beams. Since $\Delta t'$ - Δt = 0, it makes no difference how long the arms of the interferometer are.

Conclusion

Michelson's calculation of the shift is ***equivalent to assuming that as the arms switch positions, the travel time of the beams in them also switch with them*****. In other words, it takes the same time for the light to travel in the direction parallel to the motion of the earth in the rotated position, as it takes to travel in the perpendicular direction when the apparatus is in the original position. This obviously is not the case.**

Reference

1. A. A. Michelson and E. Morley, *Am. J. Sci.,* 34, 333 (1887)