TODIM Method for Group Decision Making under Bipolar Neutrosophic Set Environment

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ABSTRACT

Classical TODIM (an acronym in Portuguese for Interactive Multi criteria Decision Making) method works on crisp numbers to solve multi-attribute group decision making problems. In this paper, we define TODIM method in bipolar neutrosophic set environment to handle multi-attribute group decision making problems, which means we combine the TODIM with bipolar neutrosophic number to deal with multi-attribute group decision making problems. We have proposed a new method for solving multi-attribute group decision making problems. Finally, we solve multi-attribute group decision making problem using our newly proposed TODIM method to show the applicability and effectiveness of the proposed method.

Keywords: Bipolar neutrosophic sets, TODIM method, Multi attribute group decision making.

1. Introduction

There exist many decision making methods (Triantaphyllou, 2000; Hwang & Yoon, 1981; Shanian & Savadogo, 2009; Chan & Tong, 2007; Rao & Davim, 2008; Gomes & Lima, 1992; Zadeh, 1995) in the literature to deal with multi attribute group decision making (MAGDM) problems which are frequently meet in many fields such as politics, economy, military, etc. In classical methods for MAGDM attribute values are assumed as crisp numbers. In realistic decision making problem uncertainty involves due to the complexity of the problem. So crisp numbers are not sufficient to characterize attribute values. To handle this type of difficulties, Zadeh (1965) introduced the concept of fuzzy set by defining membership function. Atanassov (1986) incorporated non-membership function as independent component and defined intuitionistic fuzzy set to deal with uncertainty. Intuitionistic fuzzy set has been rapidly applied to many MADM fields (Gumus et al., 2016; Mondal & Pramanik, 2014c; Mondal & Pramanik, 2015a; Dey et al., 2015; Pramanik & Mukhopadhyaya, 2011; Xu, 2007; Xu &Yager, 2008; Atanassov et al., 2005; Wei, 2010).

Smarandache (1998) introduced the notion of neutrosophic set by incorporating indeterminacy as independent component to intuitionistic fuzzy set. For dealing with the imperfection knowledge received from real world decision making problems, Wang et al. (2010) defined single valued neutrosophic set (SVNS), which is an instance of neutrosophic set.

Neutrosophic sets and SVNSs are essential topics for research in different route of research such as conflict resolution (Pramanik & Roy, 2014), clustering analysis (Ye, 2014a, 2014b), decision making (Biswas et al., 2014a, 2014b, 2015a, 2015b, 2016a, 2016b, 2017; Deli & Subas, 2016; Ji, Wang et al., 2016; Kharal, 2014; Pramanik, Biswas et al., 2017; Pramanik, Banerjee et el., 2016; Pramanik, Dalapati et al., 2016; Ye, 2013a, 2013b, 2014c, 2014d, 2015a, 2015b, 2016), educational problem (Mondal & Pramanik 2014b, 2015b), medical diagnosis (Ye, 2015c), optimization (Pramanik, 2016a, 2016b; Roy & Das, 2015), social problem (Mondal & Pramanik, 2014a; Pramanik & Chakrabarti, 2013), and so on.

Deli et al. (2015) defined bipolar neutrosophic sets and applied it to MCDM problems. Pramanik and Mondal (2016) defined bipolar rough neutrosophic set. Dey et al. (2016) defined TOPSIS for solving MADM problems under bipolar neutrosophic set environment.

Firstly, Gomes and Lima (1992) introduced TODIM method on the basis of the prospect theory (Kahneman &Tversky, 1979) in crisp environment.

Krohling & De Souza (2012) developed a generalized version of TODIM called fuzzy TODIM to deal with fuzzy information. Researchers presented fuzzy TODIM methods in varied fuzzy MADM or MAGDM problems (Liu & Teng, 2014; Tosun & Akyu, 2015; Gomes et al., 2013). Fan et al. (2013) extended TODIM method to deal with the hybrid MADM problems where attribute values are crisp numbers, interval numbers and fuzzy information.

Krohling et al. (2013) studied intuitionistic fuzzy TODIM for MCDM problems. Lourenzutti & Krohling (2013) proposed intuitionistic fuzzy random TODIM method which deals intuitionistic fuzzy information and an underlying random vector that affects the performance of the alternatives. Krohling, R. A., & Pacheco proposed interval-valued intuitionistic fuzzy TODIM to tackle MCDM problems involving uncertainty characterized by interval-valued intuitionistic fuzzy numbers.

Wang (2015) extended TODIM method for MCDM in multi-valued neutrosophic set environment. Ji, Zhang et al. (2016) define projection based TODIM method under multi-valued neutrosophic environment and applied it to personal selection. Zhang et al. (2016) proposed TODIM method for group decision making in neutrosophic environment using neutrosophic numbers (Smarandache, 1998) in the form a + bI, where 'a' denotes real part and 'bI' denotes indeterminate part. Bipolar neutrosophic numbers are more suitable to deal with the uncertain information and the TODIM is a good decision making method based on prospect theory, so our objective is to propose an extended TODIM method to deal with multi-criteria group decision making problems in which the evaluation information is expressed by bipolar neutrosophic numbers.

Literature review suggests that TODIM method in bipolar neutrosophic set is yet to appear. To fill the gap, we develop a novel TODIM method for MAGDM in bipolar neutrosophic environment. A numerical example of MAGDM problem in bipolar neutrosophic set environment is solved to show the effectiveness of the proposed method.

Rest of the paper is presented as follows: Section 2 recalls some basic definitions of neutrosophic sets, single valued neutrosophic sets, bipolar neutrosophic set. Section 3 develops a novel MAGDM method based on TODIM method in bipolar neutrosophic set environment. Section 4 solves an illustrative example of MAGDM based on proposed TODIM method in bipolar neutrosophic environment. Finally, section 5 presents concluding remarks and future scope of research.

2. Preliminaries

In this section we recall some basic definitions related to neutrosophic sets, bipolar neutrosophic sets and TODIM method.

Definition 2.1: Neutrosophic set (Smarandache, 1998)

Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets A in U is characterized by a truth-membership function $\mu_A(u)$, an indeterminacy-membership function $\upsilon_A(u)$ and a falsity-membership function $\delta_A(u)$,
where, $\mu_A(u)$, $\upsilon_A(u)$, $\delta_A(u): U \to]^-0, 1^+[$.
Neutrosophic set A can be written as:
A = {< u, ($\mu_A(u)$, $\upsilon_A(u)$, $\delta_A(u)$) >: u ∈U, $\mu_A(u)$, $\upsilon_A(u)$, $\delta_A(u) \in]^-0, 1^+[$ }. There is no restriction on the sum of $\mu_A(u)$, $\upsilon_A(u)$, $\delta_A(u)$ so $^-0 \le \mu_A(u) + \upsilon_A(u) + \delta_A(u) \le 3^+$.

Definition 2.2: Single valued neutrosophic set (Wang et al., 2010)

Let U be a space of points (objects) with a generic element in U denoted by u. A single valued neutrosophic set H in U is characterized by a truth-membership function $\mu_H(u)$, an indeterminacy-membership function $\upsilon_H(u)$ and a falsity-membership function $\delta_H(u)$, where, $\mu_H(u)$, $\upsilon_H(u)$, $\delta_H(u): U \to [0, 1]$. A single valued neutrosophic set H can be expressed by H = {<u, ($\mu_H(u)$, $\upsilon_H(u)$, $\delta_H(u)$)>, u ∈ U}. Therefore for each u ∈ U, $\mu_H(u)$, $\upsilon_H(u)$, $\delta_H(u) \in [0, 1]$ the sum of three functions lies between 0 and 1, i.e. $0 \le \mu_H(u) + \upsilon_H(u) + \delta_H(u) \le 3$.

Definition 2.3: Bipolar neutrosophic set (Deli et al., 2015)
Let U be a space of points (objects) with a generic element in U denoted by u. A bipolar neutrosophic set B in U is defined as an object of the form
$B = \{<u, \mu^+(u), \upsilon^+(u), \delta^+(u), \mu^-(u), \upsilon^-(u), \delta^-(u)>: u \in U\}$, where, $\mu^+(u), \upsilon^+(u), \delta^+(u): U \to [0,1]$ and $\mu^-(u), \upsilon^-(u), \delta^-(u): U \to [-1, 0]$. We denote $B = \{<u, \mu^+(u), \upsilon^+(u), \delta^+(u), \mu^-(u), \upsilon^-(u), \delta^-(u)>: u \in U\}$ simply b = $<\mu^+, \upsilon^+, \delta^+, \mu^-, \upsilon^-, \delta^->$ as a bipolar neutrosophic number (BNN).

Definition 2.4: Containment of two bipolar neutrosophic sets (Deli et al., 2015)
Let $B_1 = \{<u, \mu_1^+(u), \upsilon_1^+(u), \delta_1^+(u), \mu_1^-(u), \upsilon_1^-(u), \delta_1^-(u)>: u \in U\}$ and
$B_2 = \{<u, \mu_2^+(u), \upsilon_2^+(u), \delta_2^+(u), \mu_2^-(u), \upsilon_2^-(u), \delta_2^-(u)>: u \in U\}$ be any two bipolar neutrosophic sets in U.

Then $B_1 \subseteq B_2$ iff $\mu_1^+(u) \le \mu_2^+(u)$, $\upsilon_1^+(u) \ge \upsilon_2^+(u)$, $\delta_1^+(u) \ge \delta_2^+(u)$ and $\mu_1^-(u) \ge \mu_2^-(u)$, $\upsilon_1^-(u) \le \upsilon_2^-(u)$, $\delta_1^-(u) \le \delta_2^-(u)$ for all $u \in U$.

Definition 2.5: Equality of two bipolar neutrosophic sets (Deli et al., 2015)

Let $B_1 = \{<u, \mu_1^+(u), \upsilon_1^+(u), \delta_1^+(u), \mu_1^-(u), \upsilon_1^-(u), \delta_1^-(u)>: u \in U\}$ and $B_2 = \{<u, \mu_2^+(u), \upsilon_2^+(u), \delta_2^+(u), \mu_2^-(u), \upsilon_2^-(u), \delta_2^-(u)>: u \in U\}$ be any two bipolar neutrosophic sets in U. Then, $B_1 = B_2$ iff $\mu_1^+(u) = \mu_2^+(u)$, $\upsilon_1^+(u) = \upsilon_2^+(u)$, $\delta_1^+(u) = \delta_2^+(u)$ and $\mu_1^-(u) = \mu_2^-(u)$, $\upsilon_1^-(u) = \upsilon_2^-(u)$, $\delta_1^-(u) = \delta_2^-(u)$ for all $u \in U$.

Definition 2.6: Union of two bipolar neutrosophic sets (Deli et al., 2015)

Let $B_1 = \{<u, \mu_1^+(u), \upsilon_1^+(u), \delta_1^+(u), \mu_1^-(u), \upsilon_1^-(u), \delta_1^-(u)>: u \in U\}$ and

$B_2 = \{<u, \mu_2^+(u), \upsilon_2^+(u), \delta_2^+(u), \mu_2^-(u), \upsilon_2^-(u), \delta_2^-(u)>: u \in U\}$ be any two bipolar neutrosophic sets in U.

Then, their union is defined as

$$B_3(u) = B_1(u) \cup B_2(u) = \{<u, \max(\mu_1^+(u), \mu_2^+(u)), \max(\upsilon_1^+(u), \upsilon_2^+(u)), \min(\delta_1^+(u), \delta_2^+(u)),$$
$$\min(\mu_1^-(u), \mu_2^-(u)), \min(\upsilon_1^-(u), \upsilon_2^-(u)), \max(\delta_1^-(u), \delta_2^-(u))>: u \in U\}, \text{for all } u \in U.$$

Definition 2.7: Intersection of two bipolar neutrosophic sets (Deli et al., 2015)

Let $B_1 = \{<u, \mu_1^+(u), \upsilon_1^+(u), \delta_1^+(u), \mu_1^-(u), \upsilon_1^-(u), \delta_1^-(u)>: u \in U\}$ and

$B_2 = \{<u, \mu_2^+(u), \upsilon_2^+(u), \delta_2^+(u), \mu_2^-(u), \upsilon_2^-(u), \delta_2^-(u)>: u \in U\}$ be any two bipolar neutrosophic sets in U.

Then, their intersection is defined as

$$B_4(u) = B_1(u) \cap B_2(u) = \{<u, \min(\mu_1^+(u), \mu_2^+(u)), \min(\upsilon_1^+(u), \upsilon_2^+(u)), \max(\delta_1^+(u), \delta_2^+(u)),$$
$$\max(\mu_1^-(u), \mu_2^-(u)), \max(\upsilon_1^-(u), \upsilon_2^-(u)), \min(\delta_1^-(u), \delta_2^-(u))>: u \in U\} \text{for all } u \in U.$$

Definition 2.8: Compliment of a bipolar neutrosophic set (Deli et al., 2015)

Let $B_1 = \{<u, \mu_1^+(u), \upsilon_1^+(u), \delta_1^+(u), \mu_1^-(u), \upsilon_1^-(u), \delta_1^-(u)>: u \in U\}$ be a bipolar neutrosophic set in U. Then the compliment of B_1 is denoted by B_1^c and is defined by

$$B_1^c = \{<u, 1-\mu_1^+(u), 1-\upsilon_1^+(u), 1-\delta_1^+(u), \{-1\}-\mu_1^-(u), \{-1\}-\upsilon_1^-(u), \{-1\}-\delta_1^-(u)>: u \in U\} \text{ for all } u \in U.$$

Definition 2.9: Score function of a BNN (Deli et al., 2015)

The score function of a bipolar neutrosophic number b = $<\mu^+, \upsilon^+, \delta^+, \mu^-, \upsilon^-, \delta^->$ is denoted by $Sc(b)$ and is defined by

$$Sc(b) = \frac{(\mu^+ + 1 - \upsilon^+ + 1 - \delta^+ + 1 + \mu^- - \upsilon^- - \delta^-)}{6}. \qquad (1).$$

Definition 2.10: Accuracy function of a BNN (Deli et al., 2015)

The accuracy function of a bipolar neutrosophic number b = $<\mu^+, \upsilon^+, \delta^+, \mu^-, \upsilon^-, \delta^->$ is denoted by $Ac(b)$ and is defined by

$$Ac(b) = \mu^+ - \delta^+ + \mu^- - \delta^-. \qquad (2).$$

Definition 2.11: Certainty function of a BNN (Deli et al., 2015)
The certainty function of a bipolar neutrosophic number b = $<\mu^{+}, \upsilon^{+}, \delta^{+}, \mu^{-}, \upsilon^{-}, \delta^{-}>$ is denoted by C(b) and is defined by C(b) = $\mu^{+} - \delta^{-}$ (3).

Definition 2.12: Comparison procedure of two BNNs (Deli et al., 2015)

Let $b_1 = <\mu_1^{+}, \upsilon_1^{+}, \delta_1^{+}, \mu_1^{-}, \upsilon_1^{-}, \delta_1^{-}>$ and $b_2 = <\mu_2^{+}, \upsilon_2^{+}, \delta_2^{+}, \mu_2^{-}, \upsilon_2^{-}, \delta_2^{-}>$ be any two bipolar neutrosophic numbers in U. The comparison procedure is stated as follows:

1. If $Sc(b_1) > Sc(b_2)$, then b_1 is greater than b_2, denoted by $b_1 > b_2$.
2. If $Sc(b_1) = Sc(b_2)$ and $Ac(b_1) > Ac(b_2)$, then b_1 is greater than b_2, denoted by $b_1 > b_2$.
3. If $Sc(b_1) = Sc(b_2)$, $Ac(b_1) = Ac(b_2)$ and $C(b_1) > C(b_2)$, then b_1 is greater than b_2, denoted by $b_1 > b_2$.
4. If Sc(b) = Sc(b), Ac (b) = Ac (b) and $C(b_1) = C(b_2)$, then b_1 is equal to b_2, denoted by $b_1 = b_2$.

Definition 2.13: Distance measure between two BNNs

Let $b_1 = <\mu_1^{+}, \upsilon_1^{+}, \delta_1^{+}, \mu_1^{-}, \upsilon_1^{-}, \delta_1^{-}>$ and $b_2 = <\mu_2^{+}, \upsilon_2^{+}, \delta_2^{+}, \mu_2^{-}, \upsilon_2^{-}, \delta_2^{-}>$ be any two bipolar neutrosophic numbers in U. Hamming distance measure between b_1 and b_2 is denoted by $d_H(b_1, b_2)$ and defined as:

$$d_H(b_1, b_2) = \frac{1}{6}[|\mu_1^{+} - \mu_2^{+}| + |\upsilon_1^{+} - \upsilon_2^{+}| + |\delta_1^{+} - \delta_2^{+}| + |\mu_1^{-} - \mu_2^{-}| + |\upsilon_1^{-} - \upsilon_2^{-}| + |\delta_1^{-} - \delta_2^{-}|\,] \qquad (4)$$

Definition 2.14: Procedure of normalization

Assume that b_{ij} be a BNN to assess i-th alternative with regarding to j-th criterion. A criterion may be benefit type or cost type. To normalize the BNN b_{ij}, we use the following formula.

$$b_{ij}^{*} = <\{1\} - \mu_{ij}^{+}, \{1\} - \upsilon_{ij}^{+}, \{1\} - \delta_{ij}^{+}, \{-1\} - \mu_{ij}^{-}, \{-1\} - \upsilon_{ij}^{-}, \{1\} - \delta_{ij}^{-}> \qquad (5)$$

3. TODIM METHOD FOR SOLVING MAGDM PROBLEM UNDER BIPOLAR NEUTROSOPHIC ENVIRONMENT

In this section, we propose a MAGDM method under bipolar neutrosophic environment. Assume that $P = \{p_1, p_2, p_3, ..., p_r\}$ be a set of r alternatives and $C = \{c_1, c_2, c_3, ..., c_s\}$ be a set of s criteria. Assume that $W = \{w_1, w_2, w_3, ..., w_s\}$ be the weight vector of the criteria, where $w_k > 0$ and $\sum_{k=1}^{s} w_k = 1$. Let $D = \{D_1, D_2, D_3, ..., D_t\}$ be the set of t decision makers and $\lambda = \{\lambda_1, \lambda_2, \lambda_3, ..., \lambda_t\}$ be the set of weight vector of decision makers, where $\lambda_1 > 0$ and $\sum_{L=1}^{t} \lambda_L = 1$.

In the following sub section, we describe the TODIM based MAGDM method under bipolar neutrosophic set environment. The proposed method is described using the following steps:

Step1- Construction of the decision matrix

Assume that $M^L = (b_{ij}^L)_{r\times s}$ (L = 1, 2, 3, …, t) be the L-th decision matrix, where information about the alternative p_i provided by the decision maker D_L with respect to attribute c_j (j = 1, 2, 3, …, s). The L-th decision matrix denoted by M^L (see Equation 6) is constructed as follows:

$$M^L = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_s \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ . \\ p_r \end{array} & \begin{pmatrix} b_{11}^L & b_{12}^L & \cdots & b_{1s}^L \\ b_{21}^L & b_{22}^L & & b_{2s}^L \\ . & \cdots & & . \\ b_{r1}^L & b_{r2}^L & \cdots & b_{rs}^L \end{pmatrix} \end{array} \quad (6)$$

where L = 1, 2, 3,…, t; i = 1, 2, 3,…, r; j = 1, 2, 3,…, s.

Step 2-Normalization of the decision matrix

In decision making situation cost criteria and benefit criteria play an important role to choose the best alternative. Cost criteria and benefit criteria exist together, so the decision matrix needs to be normalized. We use Equation 5 to normalize the cost criteria. Benefit criteria need not be normalized. Using Equation 5 the normalize decision matrix (see Equation 6) is represented below (see Equation 7).

$$M^L = \begin{array}{cc} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_s \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ . \\ p_r \end{array} & \begin{pmatrix} \tilde{b}_{11}^L & \tilde{b}_{12}^L & \cdots & \tilde{b}_{1s}^L \\ \tilde{b}_{21}^L & \tilde{b}_{22}^L & & \tilde{b}_{2s}^L \\ . & \cdots & & . \\ \tilde{b}_{r1}^L & \tilde{b}_{r2}^L & \cdots & \tilde{b}_{rs}^L \end{pmatrix} \end{array} \quad (7)$$

Here L = 1, 2, 3,…, t; i = 1, 2, 3,…, r; j = 1, 2, 3,…, s.

Step 3- Determination of the relative weight of each criterion

We find relative weight of each criterion with respect to criterion with maximum weight. Relative weight is presented as:

$$W_{RC_j} = \frac{w_{C_j}}{w_m}, \text{ where } w_m = \max\{w_1, w_2, w_3, ..., w_s\}. \quad (8)$$

Step 4- Calculation of score values

If the criteria are benefit criteria, then score values of Equation 6 are calculated by Equation 1, otherwise score values of Equation 7 are calculated by Equation 1.

Step 5- Calculation of accuracy values

If the criteria are benefit type, then accuracy values of Equation 6 are calculated by Equation 2, otherwise score values of Equation 7 are calculated by Equation 2.

Step 6- Construction of the dominance matrix remove

We construct the dominance matrix of each alternative p_i with respect to the criterion C_j of the L-th decision maker D_L (see Equation 9).

(For cost criteria)

$$\left.\begin{aligned} \alpha_c^L(p_i,p_j) &= \sqrt{\frac{W_{RC}}{\sum_{C=1}^{s} W_{RC}} d_H(\tilde{b}_{ic}^L, \tilde{b}_{jc}^L)}, \text{if } \tilde{b}_{ic}^L > \tilde{b}_{jc}^L \\ &= 0 \qquad\qquad\qquad\qquad , \text{if } \tilde{b}_{ic}^L = \tilde{b}_{jc}^L \\ &= -\frac{1}{\xi}\sqrt{\frac{W_{RC}}{\sum_{C=1}^{s} W_{RC}} d_H(\tilde{b}_{ic}^L, \tilde{b}_{jc}^L)}, \text{if } \tilde{b}_{ic}^L < \tilde{b}_{jc}^L \end{aligned}\right\} \tag{9}$$

(For benefit criteria)

$$\left.\begin{aligned} \alpha_c^L(p_i,p_j) &= \sqrt{\frac{W_{RC}}{\sum_{C=1}^{s} W_{RC}} d_H(b_{ic}^L, b_{jc}^L)}, \text{if } b_{ic}^L > b_{jc}^L \\ &= 0 \qquad\qquad\qquad\qquad , b_{ic}^L = b_{jc}^L \\ &= -\frac{1}{\xi}\sqrt{\frac{W_{RC}}{\sum_{C=1}^{s} W_{RC}} d_H(b_{ic}^L, b_{jc}^L)}, b_{ic}^L < b_{jc}^L \end{aligned}\right\} \tag{9a}$$

Here, 'ξ' denotes decay factor of loss and $\xi > 0$.

Step 7- Construction of the individual final dominance matrix

Using the Equation 10, individual final dominance matrix is constructed as follows:

$$\eta_L = \sum_{c=1}^{s} \alpha_c^L(p_i, p_j) \tag{10}$$

Step 8- Aggregation of all dominance matrix

Using the Equation 11, the aggregated dominance matrix is obtained as:

$$\eta(p_i, p_j) = \sum_{L=1}^{t} \lambda_L \eta_L(p_i, p_j) \tag{11}$$

Step 9- Calculation of the global values

Using Equation 12, the global value p_i is obtained as:

$$\beta_i = \frac{\sum_{j=1}^{s} \eta(p_i, p_j) - \min_{1 \le i \le r} \left(\sum_{j=1}^{s} \eta(p_i, p_j) \right)}{\max_{1 \le i \le r} \left(\sum_{j=1}^{s} \eta(p_i, p_j) \right) - \min_{1 \le i \le r} \left(\sum_{j=1}^{s} \eta(p_i, p_j) \right)} \tag{12}$$

Step 10- Ranking of the alternatives

Ranking of the alternatives is done based on descending order of global values. The highest global value β_i reflects the best alternative p_i.

4. ILLUSTRATIVE EXAMPLE

To demonstrate the applicability and effectiveness of the proposed method, we solve a MAGDM problem adapted from (Ye, 2014d, Zhang et al., 2016). We assume that an investment company wants to invest a sum of money in the best option. The investment company forms a decision making board involving of three members (D_1, D_2, D_3) who evaluate the four alternatives to invest money. The alternatives are:

1. Car company (p_1),
2. Food company (p_2),
3. Company (p_3), and
4. Arms company (p_4).

Decision makers take decision to evaluate alternatives based on the criteria namely, risk factor (c_1), growth factor (c_2), environment impact (c_3). We consider three criteria as benefit type based on Zhang et al. (2016). Assume that the weight vector of attributes is $W = (.37, .33, .3)^T$ and weight vector of decision makers is $\lambda = (.38, .32, .3)^T$. Now, we apply the proposed MAGDM method to solve the problem using the following steps.

Step1- Construction of the decision matrix

We construct the decision matrix based on information provided by the decision makers in terms of BNN with respect to the criteria as follows:

Decision matrix for D_1

$$M^1 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} c_1 & C_2 & C_3 \\ (.5, .6, .7, -.3, -.6, -.3) & (.8, .5, .6, -.4, -.6, -.3) & (.9, .4, .6, -.1, -.6, -.5) \\ (.6, .2, .2, -.4, -.5, -.3) & (.6, .3, .7, -.4, -.3, -.5) & (.7, .5, .3, -.4, -.3, -.3) \\ (.8, .3, .5, -.6, -.4, -.5) & (.5, .2, .4, -.1, -.5, -.3) & (.4, .2, .8, -.5, -.3, -.2) \\ (.7, .5, .3, -.6, -.3, -.3) & (.8, .7, .2, -.8, -.6, -.1) & (.6, .3, .4, -.3, -.4, -.7) \end{pmatrix}$$

Decision matrix for D_2

$$M^2 = \begin{array}{cc} & \begin{array}{ccc} c_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} & \begin{pmatrix} (.6,.3,.4,-.5,-.3,-.7) & (.5,.3,.4,-.3,-.3,-.4) & (.1,.5,.7,-.5,-.2,-.6) \\ (.7,.4,.5,-.3,-.2,-.1) & (.8,.4,.5,-.7,-.3,-.2) & (.6,.2,.7,-.5,-.2,-.9) \\ (.8,.3,.2,-.5,-.2,-.6) & (.3,.2,.1,-.6,-.3,-.4) & (.7,.5,.4,-.4,-.3,-.2) \\ (.3,.5,.2,-.5,-.5,-.2) & (.5,.6,.4,-.3,-.6,-.7) & (.4,.3,.8,-.5,-.6,-.5) \end{pmatrix} \end{array}$$

Decision matrix for D_3

$$M^3 = \begin{array}{cc} & \begin{array}{ccc} c_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} & \begin{pmatrix} (.9,.6,.4,-.7,-.3,-.2) & (.7,.5,.3,-.6,-.2,-.5) & (.4,.2,.3,-.2,-.5,-.7) \\ (.5,.3,.2,-.6,-.4,-.1) & (.5,.2,.7,-.3,-.2,-.5) & (.6,.3,.2,-.7,-.6,-.3) \\ (.2,.5,.6,-.4,-.5,-.7) & (.3,.2,.7,-.2,-.3,-.5) & (.8,.2,.4,-.2,-.3,-.6) \\ (.8,.5,.5,-.4,-.6,-.3) & (.9,.3,.4,-.5,-.6,-.7) & (.7,.4,.3,-.2,-.5,-.7) \end{pmatrix} \end{array}$$

Step 2-Normalization of the decision matrix

Since all the criteria are considered as benefit type, we do not need to normalize the decision matrix (M^1, M^2, M^3).

Step 3- Determination of the relative weight of each criterion

Using Equation 8, the relative weights of the criteria are obtained as:

$$W_{RC_1} = 1,\ W_{RC_2} = .89,\ W_{RC_3} = .81.$$

Step 4- Calculation of score values

Using Equation 1, we calculate the score values of each alternative with respect to each criterion (see Table 1, 2, and 3).

Table 1 Score value for M^1

	C_1	C_2	C_3
p_1	.47	.53	.70
p_2	.60	.50	.52
p_3	.55	.60	.40
p_4	.48	.50	.58

Table 2 Score value for M^2

	C_1	C_2	C_3
p_1	.60	.53	.37
p_2	.47	.45	.55
p_3	.60	.52	.48
p_4	.46	.58	.48

Table 3 Score value for M^3

	C_1	C_2	C_3
p_1	.45	.50	.55
p_2	.48	.50	.55
p_3	.48	.50	.65
p_4	.55	.67	.67

Step 5-Calculate accuracy values

Using Equation 2, we calculate the accuracy values of each alternative with respect to each criterion (see Table 4, 5, and 6.)

Table 4 Accuracy value for M^1

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline p_1 & -.2 & .1 & .7 \\ p_2 & .3 & 0 & .3 \\ p_3 & .2 & .3 & -.7 \\ p_4 & .1 & -.1 & .6 \end{array}$$

Table 5 Accuracy value for M^2

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline p_1 & .4 & .2 & -.5 \\ p_2 & 0 & -.2 & .3 \\ p_3 & .7 & 0 & .1 \\ p_4 & -.2 & .5 & -.4 \end{array}$$

Table 6 Accuracy value for M^3

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline p_1 & 0 & .3 & .3 \\ p_2 & -.2 & 0 & 0 \\ p_3 & -.1 & -.1 & .8 \\ p_4 & .2 & .7 & .9 \end{array}$$

Step 6- Construction of the dominance matrix

Here, using Equation 9, we construct dominance matrix (Taking $\xi = 1$). The dominance matrices are represented in Table 7, 8, 9, 10, 11, 12, 13, 14, and 15.

Table 7 Dominance matrix α_1^1

$$\alpha_1^1 = \begin{array}{c|cccc} & p_1 & p_2 & p_3 & p_4 \\ \hline p_1 & 0 & .27 & -.82 & -.77 \\ p_2 & -.73 & 0 & -.70 & -.64 \\ p_3 & .30 & .26 & 0 & .22 \\ p_4 & .28 & .24 & -.59 & 0 \end{array}$$

Table 8 Dominance matrix α_2^1

$$\alpha_2^1 = \begin{array}{c|cccc} & p_1 & p_2 & p_3 & p_4 \\ \hline p_1 & 0 & .24 & .27 & .26 \\ p_2 & -.72 & 0 & -.78 & 0 \\ p_3 & -.82 & .26 & 0 & .33 \\ p_4 & -.78 & 0 & -1 & 0 \end{array}$$

Table 9 Dominance matrix α_3^1

$$\alpha_3^1 = \begin{array}{c|cccc} & p_1 & p_2 & p_3 & p_4 \\ \hline p_1 & 0 & -.88 & .31 & -.82 \\ p_2 & .26 & 0 & .26 & -.75 \\ p_3 & -1 & -.86 & 0 & -.91 \\ p_4 & .25 & .23 & .27 & 0 \end{array}$$

Table 10 Dominance matrix α_1^2

$$\alpha_1^2 = \begin{array}{c|cccc} & p_1 & p_2 & p_3 & p_4 \\ \hline p_1 & 0 & .27 & -.52 & .29 \\ p_2 & -.73 & 0 & -.73 & .29 \\ p_3 & .19 & .27 & 0 & .29 \\ p_4 & -.79 & -.79 & -.79 & 0 \end{array}$$

Table 11 Dominance matrix α_2^2

$$\alpha_2^2 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & .24 & .22 & -.67 \\ -.74 & 0 & -.84 & -.95 \\ -.67 & .28 & 0 & -.95 \\ .22 & .31 & .31 & 0 \end{pmatrix}$$

Table 12 Dominance matrix α_3^2

$$\alpha_3^2 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & -.77 & -.91 & -.77 \\ .23 & 0 & .28 & .24 \\ .27 & -.95 & 0 & .28 \\ .23 & -.82 & -.95 & 0 \end{pmatrix}$$

Table 13 Dominance matrix α_1^3

$$\alpha_1^3 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & -.73 & -.94 & -.68 \\ .27 & 0 & -.90 & -.79 \\ .35 & .33 & 0 & -.73 \\ .25 & .29 & .27 & 0 \end{pmatrix}$$

Table 14 Dominance matrix α_2^3

$$\alpha_2^3 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & .26 & .30 & -.45 \\ -.78 & 0 & .14 & -.91 \\ -.91 & -.43 & 0 & -.95 \\ .26 & .29 & .31 & 0 \end{pmatrix}$$

Table 15 Dominance matrix α_3^3

$$\alpha_3^3 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & .26 & -.91 & -.86 \\ -.88 & 0 & -.95 & -.86 \\ .27 & .28 & 0 & -.63 \\ .26 & .26 & .91 & 0 \end{pmatrix}$$

Step 7- Construction of the individual final dominance matrix

Using Equation 10, the individual final dominance matrices are constructed (see Table 16, 17, and 18).

Table 16 Final dominance matrix η_1

$$\eta_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & -.37 & -.24 & -1.33 \\ -1.19 & 0 & -1.22 & -1.39 \\ -1.52 & -.34 & 0 & -.36 \\ -.25 & .47 & -1.32 & 0 \end{pmatrix}$$

Table 17 Final dominance matrix η_2

$$\eta_2 = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ 0 & -.26 & -1.21 & -1.15 \\ -1.24 & 0 & -1.29 & -.42 \\ -.21 & -.40 & 0 & -.38 \\ -.34 & -1.3 & -1.43 & 0 \end{pmatrix}$$

Table 18 Final dominance matrix η_3

$$\eta_3 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -.21 & -1.5 & -1.9 \\ p_2 & -1.39 & 0 & -1.7 & -2.6 \\ p_3 & -.29 & .18 & 0 & -2.3 \\ p_4 & .77 & .84 & .77 & 0 \end{pmatrix}$$

Step 8- Aggregation of all dominance matrix

Using Equation 11, the aggregated dominance matrix is represented in Table 19.

Table 19 Aggregated dominance matrix η

$$\eta = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -.29 & -.94 & -1.47 \\ p_2 & -1.26 & 0 & -1.07 & -1.43 \\ p_3 & -.73 & -.20 & 0 & -.95 \\ p_4 & .03 & .01 & -.73 & 0 \end{pmatrix}$$

Step 9- Calculation of the global values

Using Equation 12, the global values β_i are calculated as:

$\beta_1 = .34, \beta_2 = 0, \beta_3 = .61, \beta_4 = 1.$

Step 10- Ranking of the alternatives

Here $\beta_4 > \beta_3 > \beta_1 > \beta_2$.

Thus the Arm company (p_4) is the best option to invest money.

Section 5. CONCLUSION

In real decision making, the evaluation information of alternatives provided by the decision maker is often incomplete, indeterminate and inconsistent. Bipolar neutrosophic set can describe this kind of information. In this paper, we have developed a new group decision making method based on TODIM under bipolar neutrosophic set environment. Finally, a numerical example is shown to demonstrate its practicality and effectiveness. We hope that the proposed method can be extended for solving multi criteria group decision making in other neutrosophic hybrid environment.

ACKNOWLEDGMENT

We thank both editors for their useful suggestions.

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