Quantum mechanics expressed in terms of the approach "Emission & Regeneration" UFT.

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Abstract

Quantum mechanics differential equations are based on the de Broglie postulate. This paper presents the repercussions on quantum mechanics differential equations when the de Broglie wavelength is replaced by a relation between the radius and the energy of a particle. This relation results from the theoretical work [11] about the interaction of charged particles, where the particles are modelled as focal points of rays of fundamental particles with longitudinal and transversal angular momentum. Interaction of subatomic particles is described as the interaction of the angular momenta of their fundamental particles. The relationship between the solution of the differential equation for a radial Coulomb field and the Correspondence Principle is presented. The potential well of an atomic nucleus is shown with the regions that are responsible for the four type of interactions defined in quantum mechanics.

Contents

1 Introduction.

Quantum mechanics differential equations are based on the de Broglie postulate. In the theoretical work [3] about the interaction of charged particles, where particles are represented by a non local model emitting and absorbing continuously fundamental particles, a relation between the radius r_o and the energy of a particle is derived.

$$r_o = \frac{\hbar\, c}{E} \qquad with \qquad E = \sqrt{E_o^2 + E_p^2} \quad the\ relativistic\ energy. \tag{1}$$

This relation is used instead of the de Broglie wavelength, to build wave packages with a Gauss distribution, and to derive the corresponding probability differential equations of quantum mechanics.

The effects on the uncertainty relations and the most important quantum mechanics operators are presented.

1.1 General considerations.

To make use of the of Fourier-Transformation, the movement of a particle is first described as a sequence of particles represented by a sinus wave, having a wavelength λ equal to $2\pi r_o$. Then the Fourier-Transformation of a wave package of sinus waves with a Gauss shaped amplitude is build.

We have that

$$\lambda = 2\pi r_o = 2\pi \frac{\hbar\, c}{E_{rel}} \qquad with \qquad E_{rel} = \sqrt{E_o^2 + E_p^2} \tag{2}$$

We define

$$E_{rel} = E_o + E_{kin} \tag{3}$$

which can be written as

$$E_{rel} = E_o + E_{kin} = E_o + E_o \left[\frac{1}{2}\, x + \frac{1 \cdot 3}{2 \cdot 4}\, x^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\, x^3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\, x^4 + \cdots \right] \tag{4}$$

with

$$E_o = m_o\, c^2 \qquad E_p = p\, c \qquad p = \frac{m_o\, c}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x = \frac{v^2}{c^2} \tag{5}$$

The sinus wave on the x-axis is

$$\xi_x = A\ e^{i(k_x\ x - \omega_x\ t)} \qquad with \qquad k_x = \frac{2\pi}{\lambda_x} \qquad and \qquad \omega_x = 2\pi\ \frac{v_x}{\lambda_x} \tag{6}$$

If we now introduce in the expression that $\lambda_x = 2\pi r_{o_x} = 2\pi\hbar c/E_{rel_x}$ we get

$$\xi_x = A\ exp\left[i\frac{c}{\hbar}\left(\frac{E_{rel_x}}{c^2}\ x - \ \frac{v_x}{c^2}\ E_{rel_x}\ t\right)\right] \tag{7}$$

or

$$\xi_x = A\ exp\left[i\frac{c}{\hbar}\left(\frac{E_{rel_x}}{c^2}\ x - \ p_x\ t\right)\right] \tag{8}$$

with

$$E_{rel_x} = E_o + E_{kin_x} = m_o\ c^2\left(1 - \frac{v_x^2}{c^2}\right)^{-1/2} \qquad and \qquad p_x = \frac{v_x}{c^2}\ E_{rel_x} \tag{9}$$

with E_{kin_x} the relativistic kinetic energy of the particle on the x-axis.

Note: The wave-length used by Schroedinger is based exclusively on the kinetic energy E_{kin_x} for the non-relativistic case as follows.

$$\lambda = 2\pi r_o = 2\pi\frac{\hbar\ c}{E_{rel}} \quad with \quad E_o = 0 \quad and \quad E_p = p\ c \quad where \quad p = m\ v \tag{10}$$

The proposed approach includes for the calculation of the wave-length the total energy with the rest energy of a particle. For the relativistic cases we get

$$\lambda = 2\pi r_o = 2\pi\frac{\hbar\ c}{E_{rel}} = 2\pi\frac{\hbar}{m\ v\ \gamma} \quad with \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{11}$$

For $v \to c$ we get that $r_o \to 0$.

1.2 The wave package.

We define the Fourier-Transformation of a wave package [1,2]; on the x-axis as

$$\phi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \kappa_x(p_x)\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}(p_x)\ x - \ p_x\ t\right]\right\} dp_x \tag{12}$$

with a Gauss distribution $\kappa_x(p_x)$ on the p_x-axis

$$\kappa_x(p_x) = B\ exp\ \left\{-\frac{(p_x - p_{x_o})^2}{4(\Delta p_x)^2}\right\} \tag{13}$$

and the dispersion $m_{rel_x} = m_{rel_x}(p_x)$ with

$$m_{rel_x} = \frac{E_{rel_x}}{c^2} \qquad m_{rel_x} = m_{rel_x}(p_x) = \frac{1}{c^2}\sqrt{E_o^2 + p_x^2 c^2} \qquad and \qquad E_o = m_o c^2 \tag{14}$$

Because of symmetry reasons we also have a wave package

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}\ x - \ p_x(m_{rel_x})\ t\right]\right\} dm_{rel_x} \tag{15}$$

with the Gauss distribution on the m_{rel_x}-axis

$$\chi_x(m_{rel_x}) = A\ exp\ \left\{-\frac{(m_{rel_x} - m_{rel_{x\,o}})^2}{4(\Delta m_{rel_x})^2}\right\} \tag{16}$$

and the dispersion

$$p_x(m_{rel_x}) = c\ \sqrt{m_{rel_x}^2\ -\ m_o^2} \qquad and \qquad m_o = \frac{E_o}{c^2} \tag{17}$$

1.3 Unrestricted differential equations.

In this and the following section the probability differential equations are derived. The differential equations are classified into unrestricted and non-relativistic. Then they are subclassified in groups of general, time or space independent.

The unrestricted differential equations are valid for the whole range of speed $0 \leq v \leq c$.

We start with the wave package

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}\ x - \ p_x(m_{rel_x})\ t\right]\right\} dm_{rel_x} \tag{18}$$

with

$$m_{rel_x} = \frac{E_{rel_x}}{c^2} \qquad and \qquad p_x(m_{rel_x}) = c\ \sqrt{m_{rel_x}^2 - m_o^2} \tag{19}$$

with

$$E_{rel_x} = E_o + E_{kin_x} = \sqrt{E_o^2 + E_{p_x}^2} \qquad E_o = m_o c^2 \qquad E_{p_x} = p_x\ c \tag{20}$$

For the unrestricted range of velocities $0 \leq v \leq c$ we have that

$$p_x = \frac{v_x}{c^2} E_{rel_x} \tag{21}$$

and E_{kin_x} represents the kinetic energy for the whole range of speed.

1.3.1 The wave equation.

The wave differential equation we obtain by derivation of ψ_x two times versus t and two times versus x. The results are then connected through

$$p_x = \frac{v_x}{c^2} E_{rel_x} \tag{22}$$

We get

$$\frac{\partial^2}{\partial x^2}\psi_x \;=\; \frac{1}{v_x^2}\,\frac{\partial^2}{\partial t^2}\psi_x \tag{23}$$

For $v_x \to c$ we have

$$\frac{\partial^2}{\partial x^2}\psi_x(x,t) \;=\; \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2}\psi_x(x,t) \tag{24}$$

the well known wave equation

1.3.2 The time independent differential equation.

Time independent differential equations are deduced derivating one time and two times the wave function ψ_x.

a) We derivate the wave function ψ_x one time versus x and get the following time independent differential equation on the x coordinate

$$\frac{\partial}{\partial x}\psi_x = \frac{i}{\hbar\, c}\; E_{rel_x}\; \psi_x = \frac{i}{\hbar\, c}\; (E_o + E_{kin_x})\; \psi_x \tag{25}$$

E_{kin_x} represents the kinetic energy for the whole range of speed, relativistic and non-relativistic.

b) We derivate the wave function ψ_x two times versus x and get the following time independent differential equation on the x coordinate

$$\frac{\partial^2}{\partial x^2}\psi_x = -\;\frac{c^2}{\hbar^2}\; m_{rel_x}^2\; \psi_x \tag{26}$$

With

$$m_{rel_x} = \frac{1}{c^2}\sqrt{E_o^2 + E_{p_x}^2} \qquad\qquad E_o = m_o c^2 \qquad and \qquad E_{p_x} = p_x\, c \tag{27}$$

we get

$$\frac{\partial^2}{\partial x^2}\psi_x = -\;\frac{1}{\hbar^2\; c^2}\; (E_o^2 + E_{p_x}^2)\; \psi_x \tag{28}$$

1.3.3 The space independent differential equation.

We derivate the wave function ψ_x two times versus t

$$\frac{\partial^2}{\partial t^2}\psi_x = -\frac{c^2}{\hbar^2}\, p_x^2\, \psi_x \tag{29}$$

and with

$$E_{p_x} = p_x\, c \qquad and \qquad E_p^2 = E_{p_x}^2 + E_{p_y}^2 + E_{p_z}^2 \tag{30}$$

we get

$$-\ \hbar^2\, \frac{\partial^2}{\partial t^2}\psi_x = E_{p_x}^2 \psi_x \tag{31}$$

and for the space

$$-\ \hbar^2\, \Delta_{\mathbf{t}}\psi = E_p^2 \psi \tag{32}$$

with the operator $\Delta_{\mathbf{t}}$ defined in sec. 1.6.

1.4 Non relativistic differential equations

For non relativistic speeds we have that $v \ll c$ and that $E_{kin_x} \approx p^2/(2m_o)$.

1.4.1 General non relativistic differential equation.

The general non relativistic differential equation we obtain by deriving ψ_x two times versus t and one time versus x. The results are then connected through $E_{kin_x} \approx p^2/(2m_o)$. We get

$$-\ i\,\hbar\, c\frac{\partial}{\partial x}\psi_x(x,t)\ -\ E_o\,\psi_x(x,t) \approx\ -\frac{\hbar^2}{2\, m_o\, c^2}\,\frac{\partial^2}{\partial t^2}\psi_x(x,t) \quad with \quad E_o = m_o\, c^2 \tag{33}$$

The differential equation with the constant energy E_o describes the movement of a non-accelerated particle in a cero potential energy field.

We define

$$E_{rel} = E_o + E_{kin} \approx E_o + \frac{1}{2m_o}p^2 = E_{tot}^{'} \qquad for \qquad v << c \tag{34}$$

With $E_{tot}^{'}$ the total non-relativistc energy for a non-accelerated particel and E_{kin} the corresponding kinetic energy. Equation (33) is equivalent to $E_{tot}^{'} - E_o = E_{kin}$.

For an accelerated particle we must add an accelerating potential $E_{pot} = U_x(x,t)$ to the kinetic energy E_{kin} to get the total energy. The result is

$$-\ i\,\hbar\,c\frac{\partial}{\partial x}\psi_x(x,t)\ -\ E_o\,\psi_x(x,t)\ =\ E_{kin}\psi_x(x,t)\ +\ U_x(x,t)\psi_x(x,t) \tag{35}$$

$$-\ \frac{\hbar^2}{2\,m_o\,c^2}\,\frac{\partial^2}{\partial t^2}\psi_x(x,t) = E_{kin}\psi_x(x,t) \tag{36}$$

Comparing equation (33) with the **General Schroedinger** differential equation, the main difference is that equation (33) derives one time versus space and two times versus time, in other words, time and space are interchanged.

1.4.2 The time independent non relativistic differential equation.

Differential equations are deduced in derivating one time or two times the wave function ψ_x.

a) We derivate the wave function ψ_x one time versus x

$$\frac{\partial}{\partial x}\psi_x = \frac{i}{\hbar\,c}\,E_{rel_x}\,\psi_x = \frac{i}{\hbar\,c}\,(E_o + E_{kin_x})\,\psi_x \tag{37}$$

For a conservative field $U_x = q_e\,V_x$ with a total energy E_{tot_x} we have

$$E_{tot_x} = E_{kin_x} + U_x \qquad and\ \ with \qquad E_{kin_x} \approx \frac{1}{2\,m_o}\,p_x^2 \tag{38}$$

we get

$$\left\{-\ i\,\hbar\,c\,\frac{\partial}{\partial x} + U(x)\ \right\}\psi(x) \approx E_x\,\psi(x) \tag{39}$$

with

$$E_x = E_{tot_x} + E_o \tag{40}$$

the Eigenvalue.

b) For the time independent differential equation deduced derivating the wave function ψ_x two times versus x see sec. 1.7.

1.4.3 Space independent non relativistic differential equation.

We take two times the derivate of the wave function ψ_x versus t

$$\frac{\partial^2}{\partial t^2}\psi_x = -\ \frac{c^2}{\hbar^2}\,p_x^2\,\psi_x \tag{41}$$

and with eq. (32)

$$- \hbar^2 \Delta_{\mathbf{t}} \psi = E_p^2 \psi \tag{42}$$

and $v \ll c$ and a conservative potential U

$$E_{kin} \approx \frac{1}{2\, m_o}\, p^2 = \frac{E_p^2}{2\, E_o} \qquad and \qquad E_{tot} = E_{kin} + U \tag{43}$$

we obtain the space independent non relativistic differential equation

$$\left\{ - \frac{\hbar^2}{2\, E_o}\, \Delta_{\mathbf{t}} + U \right\} \psi \approx E_{tot}\, \psi \tag{44}$$

which is equivalent to the time inependent equation from Schroedinger.

1.5 Uncertainty principle.

In the proposed model the pairs of canonical conjugated variables lead to the following uncertainty relations

$$(\Delta E) \cdot (\Delta x) \geq \frac{1}{2}\, \hbar\, c \tag{45}$$

and

$$(\Delta p) \cdot (\Delta t) \geq \frac{1}{2}\, \frac{\hbar}{c} \tag{46}$$

Noticeable at this point is the relation

$$E\, r_o = h\, c \tag{47}$$

for a particle, that connects the radius r_o and the relativistic energy E through $h\, c$.

1.6 Operators.

1.6.1 Relativistic operator for the linear momentum.

The relativistic operator for the linear momentum of a particle is

$$\hat{p} = \; i\, \frac{\hbar}{c}\, \frac{\partial}{\partial t} \tag{48}$$

The linear momentum we get with

$$\bar{p}\, \chi = \; i\, \frac{\hbar}{c}\, \nabla_t\, \chi \tag{49}$$

where χ is the total mass-probability function

$$\chi = \psi_x \, \psi_y \, \psi_z \tag{50}$$

and $\nabla_{\mathbf{t}}$

$$\nabla_{\mathbf{t}} = \frac{\partial}{\partial t}|_x \, \mathbf{e}_x \; + \; \frac{\partial}{\partial t}|_y \, \mathbf{e}_y \; + \; \frac{\partial}{\partial t}|_z \, \mathbf{e}_z \tag{51}$$

1.6.2 Relativistic operators for the energy.

For the relativistic energy of a non-accelerated particle we obtain the operator

$$\hat{E}_{rel_x} = - \, i \, \hbar \, c \, \frac{\partial}{\partial x} \tag{52}$$

Application example.
If we apply the relativistic operators to the relativistic energy of a particle

$$E_x^2 = m_o^2 \, c^4 \; + \; p_x^2 \, c^2 \tag{53}$$

we get

$$- \; \hbar^2 \, c^2 \, \frac{\partial^2}{\partial x^2}\psi_x = m_o^2 \, c^4 \, \psi_x \; - \; \hbar^2 \, \frac{\partial^2}{\partial t^2}\psi_x \tag{54}$$

the **Klein-Gordon** equation.
With $m_o = 0$ we have

$$\frac{\partial^2}{\partial x^2}\psi_x \; = \; \frac{1}{c^2} \, \frac{\partial^2}{\partial t^2}\psi_x \tag{55}$$

1.6.3 Non-relativistic operator for the kinetic energy.

The non-relativistic operator for the kinetic energy on the x coordinate is

$$\hat{E}_{kin_x} = - \, \frac{\hbar^2}{2 \, m_o \, c^2} \, \frac{\partial^2}{\partial t^2}|_x \tag{56}$$

and the total kinetic energy E_{kin} in the three dimensional space

$$E_{kin} = E_{kin_x} + E_{kin_y} + E_{kin_z} = - \, \frac{\hbar^2}{2 \, m_o \, c^2} \, \Delta_{\mathbf{t}}\chi \tag{57}$$

with

$$\Delta_{\mathbf{t}} = \frac{\partial^2}{\partial t^2}|_x \; + \; \frac{\partial^2}{\partial t^2}|_y \; + \; \frac{\partial^2}{\partial t^2}|_z \tag{58}$$

1.6.4 Non-relativistic Hamilton operator.

The operator for the non-relativistic total energy on the x coordinate has the form

$$\hat{E}_x = \frac{1}{2\,m_o}\left(\, i\,\frac{\hbar}{c}\,\frac{\partial}{\partial t}|_x\right)^2 \;+\; \hat{U}_x \tag{59}$$

or

$$\hat{E}_x = \frac{\hat{p}_x^{\,2}}{2\,m_o} \;+\; \hat{U}_x \tag{60}$$

which is equal to the Hamilton operator $\hat{H}_x$.
The general non-relativistic differential equation thus takes the form

$$-\,i\,\hbar c\,\frac{\partial}{\partial x}\psi_x(x,t) - E_o\,\psi_x(x,t) = \hat{H}_x\,\psi_x(x,t) \tag{61}$$

with

$$\hat{H}_x = \frac{\hat{p}_x^{\,2}}{2\,m_o} \;+\; \hat{U}_x \tag{62}$$

the non-relativistic Hamilton operator.

1.6.5 Non-relativistic operator for the orbital-angular-momentum.

The wave function for the three dimentional space is

$$\psi_x(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\chi(m_{rel})\;exp\left\{i\frac{c}{\hbar}\left[m_{rel}\;\mathbf{r} - \;\mathbf{p}(m_{rel})\;t\right]\right\}dm_{rel} \tag{63}$$

with

$$\mathbf{r} = x\;\mathbf{e}_x + y\;\mathbf{e}_y + z\;\mathbf{e}_z \qquad and \qquad \mathbf{p} = p_x\;\mathbf{e}_x + p_y\;\mathbf{e}_y + p\;\mathbf{e}_z \tag{64}$$

We define the linear momentum operator for the different coordinates as:

$$\hat{p}_k = \;i\,\frac{\hbar}{c}\,\frac{\partial}{\partial t}|_k \tag{65}$$

The orbital-angular-momentum-operator can be expressed as

$$\mathbf{M}\left(\mathbf{r},\; i\,\frac{\hbar}{c}\,\nabla_{\mathbf{t}}\right) = \left(\mathbf{r}\;\times\;i\,\frac{\hbar}{c}\,\nabla_{\mathbf{t}}\right) \tag{66}$$

with

$$\nabla_{\mathbf{t}} = \;\frac{\partial}{\partial t}|_x\,\mathbf{e}_x \;+\; \frac{\partial}{\partial t}|_y\,\mathbf{e}_y \;+\; \frac{\partial}{\partial t}|_z\,\mathbf{e}_z \tag{67}$$

The operators for the vectorcomponents are:

$$\hat{M}_x = \hat{y}\ \hat{p}_z - \hat{z}\ \hat{p}_y \qquad \hat{M}_y = \hat{z}\ \hat{p}_x - \hat{x}\ \hat{p}_z \qquad \hat{M}_z = \hat{x}\ \hat{p}_y - \hat{y}\ \hat{p}_z \tag{68}$$

The conmutations are as known

$$[\hat{M}_k, \hat{M}_{k+1}] \neq 0 \qquad [\hat{M}_k, \hat{Q}] = 0 \qquad with \qquad \hat{Q} = \hat{M}_x^2 + \hat{M}_y^2 + \hat{M}_z^2 \tag{69}$$

1.7 The proposed theory and the Correspondence Principle.

The present theory is based on the radius-energy relation that substitutes the de Broglie wavelength.

The accordance of the proposed theory with the correspondence principle of quantum mechanics is ensured, in that the time independent differential equation from Schroedinger, deduced from the wave package constructed with the de Broglie wavelength, can be derived from the wave package constructed with the radius-energy relation presented in this work.

We start derivating the wave function ψ_x two times versus space, to get the time independent differential equation

$$\frac{\partial^2}{\partial x^2}\psi_x = -\ \frac{c^2}{\hbar^2}\ m_{rel_x}^2\ \psi_x \tag{70}$$

With

$$m_{rel_x} = \frac{1}{c^2}\sqrt{E_o^2 + E_{p_x}^2} \qquad E_o = m_o c^2 \qquad and \qquad E_{p_x} = p_x\ c \tag{71}$$

we get

$$\frac{\partial^2}{\partial x^2}\psi_x = -\ \frac{1}{\hbar^2\ c^2}\ (E_o^2 + E_{p_x}^2)\ \psi_x \tag{72}$$

For non-relativistic velocities $v \ll c$ we have that

$$E_{kin_x} = \frac{p_x^2}{2\ m_o} \qquad and \qquad E_{p_x}^2 = p_x^2\ c^2 = 2\ m_o\ c^2\ E_{kin_x} \tag{73}$$

and we get

$$\frac{\partial^2}{\partial x^2}\psi_x = -\ \frac{2\ m_o}{\hbar^2}\ \left[\frac{1}{2}E_o + E_{kin_x}\right]\ \psi_x \tag{74}$$

With a conservative potential $E_{tot_x} = U_x + E_{kin_x}$ we get finally

$$\left[-\ \frac{\hbar^2}{2\ m_o}\frac{\partial^2}{\partial x^2}\ +\ U_x\right]\psi_x\ =\ E_x\ \psi_x \qquad with \qquad E_x = \frac{1}{2}\left[E_o + 2\ E_{tot_x}\right] \tag{75}$$

For the three dimensional space we have

$$\left[-\frac{\hbar^2}{2\,m_o}\,\Delta_{\mathbf{r}} \;+\; U \right]\,\chi = E\,\chi \tag{76}$$

with $\Delta_{\mathbf{r}}$ the Laplace operator and

$$E = \frac{1}{2}\,[E_o + 2\;E_{tot}] \tag{77}$$

If we make $E_o = 0$ we get

$$\left[-\frac{\hbar^2}{2\,m_o}\,\Delta_{\mathbf{r}} \;+\; U \right]\,\chi = E_{tot}\,\chi \tag{78}$$

Eq. (78) is exactly the time independent differential equation constructed by **Schroedinger** with E_{tot} the Eigenvalue.

1.8 The mass conservation equation.

The mass conservation differential equation we obtain by derivating ψ_x one time versus t and one time versus x. The results are then connected through

$$p_x = \frac{v_x}{c^2} E_{rel_x} \tag{79}$$

We get

$$\frac{\partial}{\partial t}\psi_x(x,t) \;=\; -\,v_x\,\frac{\partial}{\partial x}\psi_x(x,t) \tag{80}$$

We define the mass probability density as

$$\rho_x(x,t) = \psi_x^*(x,t)\;\psi_x(x,t) \qquad or \qquad \rho(\mathbf{r},t) = \psi^*(\mathbf{r},t)\;\psi(\mathbf{r},t) \tag{81}$$

We derive the mass probability density versus time

$$\frac{\partial}{\partial t}\rho_x(x,t) = \frac{\partial}{\partial t}\,[\psi_x^*(x,t)\;\psi_x(x,t)] = \frac{\partial}{\partial t}\psi_x^*(x,t)\;\psi_x(x,t) \;+\; \psi_x^*(x,t)\;\frac{\partial}{\partial t}\psi_x(x,t) \tag{82}$$

With eq. (80) we get

$$\frac{\partial}{\partial t}\rho_x(x,t) = -v_x\left[\frac{\partial}{\partial x}\psi_x^*(x,t)\;\psi_x(x,t) \;+\; \psi_x^*(x,t)\;\frac{\partial}{\partial x}\psi_x(x,t)\right] \tag{83}$$

or

$$\frac{\partial}{\partial t}\rho_x(x,t) = -v_x \frac{\partial}{\partial x}\left[\psi_x^*(x,t)\ \psi_x(x,t)\right] = -\frac{\partial}{\partial x}\left[v_x\ \rho_x(x,t)\right] = -\frac{\partial}{\partial x} j(x,t) \tag{84}$$

or

$$\frac{\partial}{\partial t}\rho(\mathbf{r},t) \ = \ -\nabla_{\mathbf{r}}\,\mathbf{j}(\mathbf{r},t) \qquad with \qquad \mathbf{j}(\mathbf{r},t) = \mathbf{v}\ \psi^*(\mathbf{r},t)\ \psi(\mathbf{r},t) \tag{85}$$

where $\mathbf{j}(\mathbf{r},t)$ is the mass-current probability density.

1.9 The wave equation for relativistic speeds.

We start with the wave eq. (15) from sec. 1.2

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left[i\frac{c}{\hbar}\left(m_{rel_x}\ x - \ p_x(m_{rel_x})\ t\right)\right] dm_{rel_x} \tag{86}$$

and analyze the equation for relativistic speeds where $\Delta v = c - v \ll c$. We get

$$E_{rel} = E_p = p\ c = \frac{m\ v}{\beta}\ c \qquad \beta = \sqrt{1 - \frac{v^2}{c^2}} \qquad \lambda = \frac{h}{p} \tag{87}$$

The resulting wave equation is

$$\psi_x(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \chi_x(m_{rel_x})\ exp\left[\frac{i}{\hbar}\left(p\ x - \ E_{pv}\ t\right)\right] dm_{rel_x} \tag{88}$$

where

$$E_{pv} = p\ v = \frac{m\ v}{\beta}\ v \tag{89}$$

With $E_{rel} = pc^2/v$ and $E_o^2 \ll E_p^2$ we get

$$E_{pv} = p\ v = \frac{p^2\ c^2}{E_{rel}} = \frac{p^2\ c^2}{\sqrt{E_o^2 + E_p^2}} \approx pc = E_p \tag{90}$$

We now derive the wave equation one time versus space and one time versus time and connect the results with $E_{pv} = pc$. We get

$$\frac{\partial}{\partial t}\psi_x = -\ c\ \frac{\partial}{\partial x}\psi_x \tag{91}$$

1.10 Applications of the non-relativistic differential equation

The solutions of the time independent non-relativistic differential equation (35) for a potential pot, the harmonic oscillator and the hydrogen atom are derived.

1.10.1 Potential pot

The non-relativistic time independent differential equation is

$$-\ i\,\hbar\,c\,\frac{\partial}{\partial x}\psi_x(x)\ +\ U_x(x)\,\psi_x(x) = [E_{tot}\ +\ E_o]\,\psi_x(x) = E\,\psi_x(x) \tag{92}$$

With $y = \psi_x(x)$ we can write

$$-\ i\,\hbar\,c\,\frac{dy}{y} = [E - U]\,dx \tag{93}$$

After integration we get

$$-\ i\,\hbar\,c\ [\ln|y| + \ln C_y] = \int [E - U]\,dx \tag{94}$$

resulting

$$|y| = \frac{1}{C_y}\,exp\left\{\ \frac{i}{\hbar\,c}\ \int [E - U]\,dx\right\} \tag{95}$$

Equation (95) is valid for all potential energies U and gives real values for y if

$$\left\{\ \frac{i}{\hbar\,c}\ \int [E - U]\,dx\right\} = k\,\pi \qquad and \qquad k = 0,\ \pm 1,\ \pm 2,\ \pm 3, \cdots \tag{96}$$

defining the quantization condition, which together with the normalization condition allows the calculation of the eigenfunctions.

The potential pot is defined as

$$U = \begin{cases} \infty & \text{for } x \le 0 \\ 0 & \text{for } 0 < x < a \\ \infty & \text{for } x \ge a \end{cases}$$

and we have for $U = 0$

$$\frac{1}{\hbar\,c}\,E\,x = k\,\pi \qquad resulting\ with\ x = a \qquad E_k = \pi\,\frac{\hbar\,c}{a}\,k \tag{97}$$

with $k = 0,\ \pm 1,\ \pm 2,\ \pm 3, \cdots$.

The total energy is with $E_k = E_{tot} + E_o$

$$E_{tot} = E_k - E_o = \pi\,\frac{\hbar\,c}{a}\,k\ -\ E_o \tag{98}$$

and for $E_{tot} = 0$ we get

$$a_o = k\,\frac{\pi\hbar\,c}{E_o} = k\,\pi\,r_o \qquad with \qquad \frac{\hbar\,c}{E_o} = r_o \tag{99}$$

the radius of of a rest electron or positron.

The eigenfunction is

$$y_k = \frac{1}{C_y}\ exp\left\{\frac{i}{\hbar\ c}\ E_k\ x\right\} \tag{100}$$

The integration constant C_y we get with the normalization condition

$$\int_{-\infty}^{\infty} y_{k'}^{*}\ y_k\ dx = \delta_{(k',k)} \tag{101}$$

For $k^{'} = k$ we get

$$\frac{1}{C_y^2}\ \int_0^a exp\left\{\frac{i}{\hbar\ c}\ [E_{k'} - E_k]\ x\right\}\ dx = 1 \tag{102}$$

resulting

$$\frac{1}{C_y^2}\ = a \qquad or \qquad C_y = \sqrt{a} \tag{103}$$

The normalized eigenfunction is

$$y_k = \frac{1}{\sqrt{a}}\ exp\left\{\frac{i}{\hbar\ c}\ E_k\ x\right\} \tag{104}$$

The main differences compared with the solution obtained with the Schroedinger equation is that the quantization of the energy is proportional to k instead of k^2 and for defined values of a the total energy E_{tot} becomes zero.

1.10.2 Harmonic oscillator

The potential energy for the harmonic oscillator is

$$U(x) = \frac{K}{2}\ x^2 = \frac{m\ \omega^2}{2}\ x^2 \qquad with \qquad \omega^2 = K/m \tag{105}$$

With eq. (95) we get

$$|y| = \frac{1}{C_y}\ exp\left\{\ \frac{i}{\hbar\ c}\ \int\left[E - \frac{K}{2}\ x^2\right]\ dx\right\} \tag{106}$$

With the quantization condition we get

$$\frac{1}{\hbar\ c}\ \int_0^a \left[E - \frac{K}{2}\ x^2\right]\ dx = \frac{1}{\hbar\ c}\ \left[E\ a - \frac{K}{6}\ a^3\right] = k\ \pi \tag{107}$$

resulting for the quantized energy with $E_{tot} = E_k - E_o$

$$E_{tot} = \pi\ \frac{\hbar\ c}{a}\ \left[\ k\ +\ \frac{1}{6}\ \frac{m\ \omega^2}{\pi\ \hbar\ c}\ a^3\right] - E_o \tag{108}$$

The minimum quantum change between two adjacent energy levels is

$$\Delta E_{tot} = \pi \ \frac{\hbar \ c}{a} \tag{109}$$

For $E_{tot} = 0$ we get

$$a \ \left[E_o - \frac{1}{6} \ m \ \omega^2 \ a^2 \right] = k \ \pi \ \hbar \ c \tag{110}$$

which for $k = 0$ gives

$$a_1 = 0 \qquad or \qquad a_{2,3} = \pm \sqrt{\frac{6 \ E_o}{m \ \omega^2}} \qquad for \qquad k = 0 \tag{111}$$

With the normalization condition given by equation (101) we have that

$$\int_{-\infty}^{\infty} y_{k'}^* \ y_k \ dx = \frac{1}{C_y^2} \ \int_{-\infty}^{\infty} exp \left\{ \frac{i}{\hbar \ c} \ [E_{k'} - E_k] \ x \right\} \ dx \tag{112}$$

or

$$\frac{\hbar \ c}{C_y^2} \ \int_{-\infty}^{\infty} exp \{ \ i \ [E_{k'} - E_k] \ \eta \} \ d\eta = \frac{\hbar \ c}{C_y^2} \ \delta_{(k',k)} \quad with \quad \eta = \frac{x}{\hbar \ c} \tag{113}$$

With $k^{'} = k$ we get the integration constant $C_y = \sqrt{\hbar \ c}$ resulting the normalized eigenfunctions

$$y_k = \frac{1}{\sqrt{\hbar \ c}} \ exp \left\{ \ \frac{i}{\hbar \ c} \ \left[E_k \ x - \frac{K}{6} \ x^3 \right] \right\} \tag{114}$$

The main differences compared with the solution obtained with the Schroedinger equation is, that the minimum quantum energy change between two adjacent energy levels is constant and independent of the oscillation frequency ω and, that for defined values of a the total energy E_{tot} becomes zero.

1.10.3 Hydrogen atom

We start with the deduction of the quantization conditions with eq. (35)

$$- \ i \ \hbar \ c \ \frac{\partial}{\partial x} \psi_x(x) \ + \ U_x(x) \ \psi_x(x) = [E_{tot} \ + \ E_o] \ \psi_x(x) = E \ \psi_x(x) \tag{115}$$

We define the operator

$$\vec{\nabla} \ \cdot \ \vec{E} = \nabla E = \frac{\partial}{\partial x} \ + \ \frac{\partial}{\partial y} \ + \ \frac{\partial}{\partial z} \quad with \quad \vec{E} = \vec{e}_x \ + \ \vec{e}_y \ + \ \vec{e}_z \tag{116}$$

$$\nabla E\ \psi(x,y,z) = \frac{\partial}{\partial x}\psi(x,y,z)\ +\ \frac{\partial}{\partial y}\psi(x,y,z)\ +\ \frac{\partial}{\partial z}\psi(x,y,z) \tag{117}$$

For polar coordinates we write

$$-\ i\,\hbar\,c\,\nabla\,\chi(r,\theta,\varphi)\ +\ U\,\chi(r,\theta,\varphi) = E\,\chi(r,\theta,\varphi) \tag{118}$$

with the ∇ operator expressed in polar coordinates

$$\nabla = \frac{\partial}{\partial r}\ +\ \frac{2}{r}\ +\ \frac{1}{r\ \sin\theta}\,\frac{\partial}{\partial\varphi}\ +\ \frac{1}{r}\,\frac{\partial}{\partial\theta}\ +\ \frac{1}{r}\ \cot\theta \tag{119}$$

The differential equation has now the form

$$\left[\ \nabla\ +\ \frac{i}{\hbar\,c}\,U\ \right]\ \chi = \frac{i}{\hbar\,c}\,E\,\chi \tag{120}$$

We now assume that the wave function χ can be expressed as a product of a function exclusively of the distance r and a function of the angular variables θ and φ.

$$\chi(r,\theta,\varphi) = R(r)\ Y(\theta,\varphi) \tag{121}$$

We get

$$\left[\frac{d}{r}+\frac{4}{r}\right] R\cdot Y + \frac{1}{r}\ \Lambda Y\cdot R + \frac{i}{\hbar\,c}U\cdot R\cdot Y = \frac{i}{\hbar\,c}E\cdot R\cdot Y \tag{122}$$

with the operator Λ

$$\Lambda = \frac{1}{\sin\theta}\,\frac{\partial}{\partial\varphi}\ +\ \frac{\partial}{\partial\theta}\ +\ 2\ \cot\theta \tag{123}$$

We now assume that

$$\Lambda Y = -\lambda Y \tag{124}$$

and get two separate differential equations for $R(r)$ and $Y(\theta,\varphi)$.

$$\frac{d}{r}R - \frac{i}{\hbar\,c}\,[E-U]\,R + \frac{1}{r}\,[4-\lambda]\,R = 0 \tag{125}$$

and

$$\left[\frac{1}{\sin\theta}\,\frac{\partial}{\partial\varphi}\ +\ \frac{\partial}{\partial\theta}\ +\ 2\ \cot\theta\right] Y = -\lambda Y \tag{126}$$

Eq. (125) gives

$$\ln R = \frac{i}{\hbar\,c}\int_{r_o}^{r}[E-U]\,dr - [4-\lambda]\ln\frac{r}{r_o}\ +\ C_R \tag{127}$$

with $C_R = C_r + i\ C_i$ a complex integration constant.

From the solution of eq. (126) results that $\lambda = i\ l$ with $l = \ 0,\ \pm 1,\ \pm 2; \cdots$ as will be shown later on. We get

$$R = \ exp\left\{-4\ \ln\frac{r}{r_o} + C_r\right\}\ exp\left\{\frac{i}{\hbar\ c}\left[\int_{r_o}^{r}(E-U)dr + l\ \hbar\ c\ \ln\frac{r}{r_o} + C_i\ \hbar\ c\right]\right\} \quad (128)$$

The quantization condition requires that

$$\frac{1}{\hbar\ c}\left[\int_{r_o}^{r}(E-U)dr + l\ \hbar\ c\ \ln\frac{r}{r_o} + C_i\ \hbar\ c\right] = k\ \pi \quad with \quad k = \ 0,\ \pm 1,\ \pm 2; \cdots \quad (129)$$

Equation (129) is valid for all point symmetrical potentials U. We now introduce the potential of an atomic nucleus

$$U = \ -\frac{Z\ e^2}{4\pi\ \epsilon_o\ r} = \ -\frac{K_u}{r} \qquad with \qquad K_u = \frac{Z\ e^2}{4\pi\ \epsilon_o} \quad (130)$$

where Z is the atomic number, and get for the quantization condition with $E = E_k$

$$E_k = \left[\ k\ \pi\ \hbar\ c - (K_u + l\ \hbar\ c)\ \ln\frac{r}{r_o} - C_i\ \hbar\ c\right]\ \frac{1}{r} \quad (131)$$

or

$$\ln\frac{r}{r_o} = \left(k\ \pi - \frac{E_k\ r}{\hbar\ c} - C_i\right) / \left(\frac{K_u}{\hbar\ c} + l\right) \quad (132)$$

The energy E_k of eq. (131) must be equal to one term of the series of the hydrogen spectrum empirically introduced by Ballmer, namely

$$E_n = h\ c\ R_H\ \frac{1}{n^2} \qquad and \qquad \Delta E_n = h\ c\ R_H\ \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \quad (133)$$

with R_H the Rydberg constant and $n = 1,\ 2, ...$

This is only possible if the product $E_k\ r$ in eq. (132) is quantized. We define that $E_k\ r = A_q(k,l)$ and get

$$E_k = \frac{A_q}{r_o}\ exp\left\{\ -\ \left(k\ \pi - \frac{A_q}{\hbar\ c} - C_i\right) / \left(\frac{K_u}{\hbar\ c} + l\right)\right\} \quad (134)$$

The problem reduces now to find the function $A_q(k,l)$ that makes $E_k = E_{tot} + E_0$ for the hydrogen spectrum, with $E_{tot} = E_n$.

Before we continue we will deduce the condition $\lambda = i\ l$ introduced previously. We assume that

$$Y(\theta,\phi) = \Theta(\theta)\ \Phi(\varphi) \qquad and \qquad \frac{d}{d\varphi}\Phi = m\ \Phi \tag{135}$$

and with $\Phi(\varphi) = \Phi(\varphi + 2\pi)$ we get

$$\Phi = exp\{m\ \varphi\} \qquad with \qquad m = i\ m_l \qquad and \qquad m_l = \pm 0,\ \pm 1,\ \pm 2; \cdots \tag{136}$$

With eq. (135) we have that eq. (126) transforms to

$$\frac{m}{\sin\theta}\ \Theta + \frac{d}{d\theta}\Theta\ +\ 2\ \cot\theta\ \Theta = -\ \lambda\ \Theta \tag{137}$$

and

$$\frac{d\Theta}{\Theta} = -\ \left[\ \frac{m}{\sin\theta}\ +\ 2\ \cot\theta\ +\ \lambda\ \right]\ d\theta \tag{138}$$

which gives the solution

$$\Theta = \frac{1}{C_\Theta}\ exp\left\{-\int\left[\ \frac{i\ m_l}{\sin\theta}\ +\ 2\ \cot\theta\ +\ \lambda\ \right]d\theta\ \right\} \tag{139}$$

With $\Theta(\theta) = \Theta(\theta + 2\pi)$ we conclude that

$$\Theta = \frac{1}{C_\Theta}\ exp\left\{-2\ln\sin\theta\right\}\ exp\left\{-i\ \left[\ m_l \ln(\csc\theta - \cot\theta)\ +\ l\ \theta\ \right]\right\} \tag{140}$$

with $\lambda = i\ l$ and $l = \pm 0,\ \pm 1,\ \pm 2; \cdots$ what we have anticipated for eq. (128).
Eq.(137) we can now write as

$$\frac{d}{d\theta}\Theta\ +\ i\ \frac{m_l}{\sin\theta}\ \Theta\ =\ -\ 2\ \cot\theta\ \Theta\ -\ i\ l\ \Theta \tag{141}$$

In this equation the real and the imaginary terms must be equal, and we get from the imaginary terms that

$$\frac{m_l}{l} = -\sin\theta \qquad with \qquad m_l = \pm 0,\ \pm 1,\ \pm 2; \cdots \quad and \quad l = \pm 0,\ \pm 1,\ \pm 2; \cdots \tag{142}$$

We conclude, that the relation between the orbital quantum number l and the magnetic quantum number m_l is

$$\left|\frac{m_l}{l}\right| \leq 1 \qquad or \qquad |m_l|\ \leq\ |l| \tag{143}$$

Now we have to find $A_q(k,l)$ that makes $E_k = E_n + E_o$.

1.10.4 Calculations made for the Hydrogen atom

A numerical calculation is required to obtain $E_k\ r = A_q(k,l)$ from eq. (134)

$$E_k = \frac{A_q}{r_o}\ exp\left\{ -\ \left(k\ \pi - \frac{A_q}{\hbar\ c} - C_i \right) / \left(\frac{K_u}{\hbar\ c} + l \right) \right\} \quad with \quad E_k = E_{tot} + E_o \tag{144}$$

and the help of

$$E_{tot} = E_n \qquad and \qquad E_k = -(h\ c\ R_H \frac{1}{k^2} + E_o) \tag{145}$$

where the sign for E_k was changed because eq. (144) gives only acceptable results with $r > 0$ if $E_k\ r = A_q(k,l) < 0$. That is because the external potential U was defined negative with $U \to 0$ for $r \to \infty$. After obtaining A_q we can calculate the radius $r = A_q/E_k$. The calculations were made with $l = 1$ and with $C_i = 500$.

If $E_k\ r = A_q(k,l)$ could be express analytically it would be possible to calculate the differences $\Delta E_n = \Delta E_k$ directly with

$$E_k(r,k,l,Ci) = \left[k\ \pi\ \hbar\ c - (K_u + l\ \hbar\ c)\ \ln\frac{r}{r_o} - C_i\ \hbar\ c \right]\ \frac{1}{r} \tag{146}$$

end the help of

$$r(k,l,Ci) = r_o\ exp\ \left\{ \left(k\ \pi - \frac{A_q}{\hbar\ c} - C_i \right) / \left(\frac{K_u}{\hbar\ c} + l \right)\ \right\} \tag{147}$$

which was derived from eq. (132).

Results of calculations:

- The energy E_k tends to the negative Energy of a rest electron instead to zero for $k \to \infty$ as it is for the Rydberg term (see Fig. 1). The energy-radius product A_q is shown in Fig. 2.
- The radius of the Hydrogen atom decreases with increasing energy E_k for constant orbital number l (see Fig.3). This is because for a constant orbital number l the energy can only increase by contracting the orbital radius r, increasing the tangential speed of the electron. The orbital radius r is of the order of $10^{-10}m$ with $l = 1$, which is also the order of magnitude of the inter-atomic distances and the Bohr radius $a_o \approx 0.5\ \overset{o}{A}$
- The radius increases with constant energy E_k or constant n and increasing orbital quantum number l (see Fig.4). The tangential speed of the electron decreases with the increasing radius r.

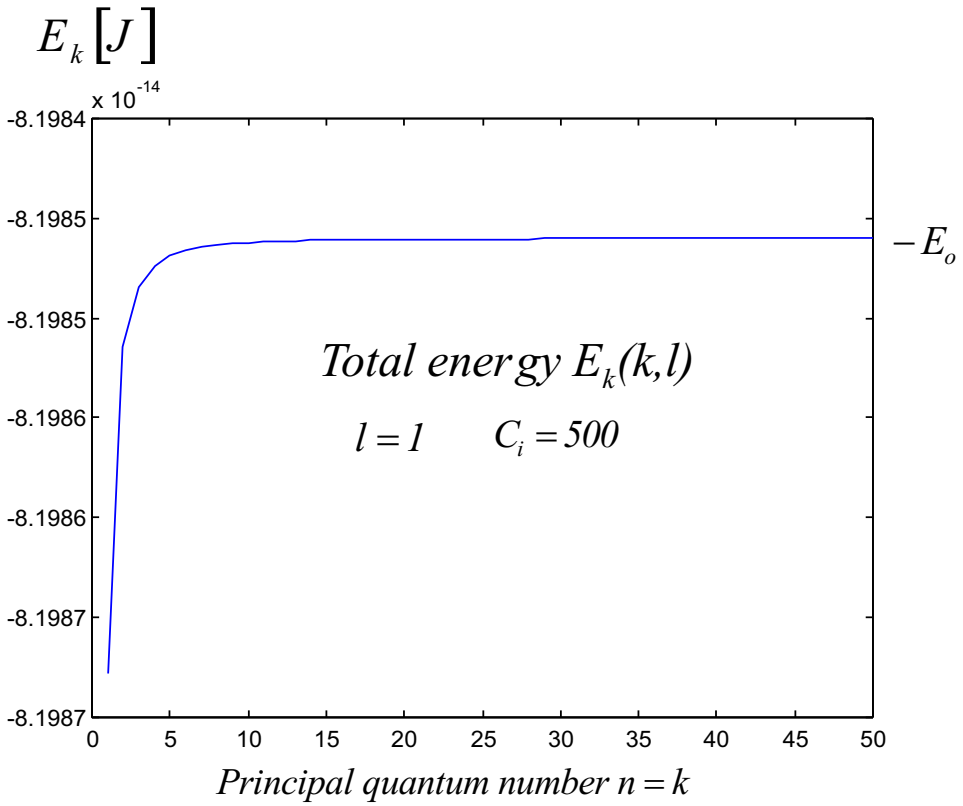

Figure 1: Energy E_k of Hydrogen atom

- The energy eigenvalues don't depend exclusively from the principal quantum number $n = k$. There is no degeneracy of the principal quantum state n, because the eigenvalues depend also from the orbital quantum number l. It is important to note, that the principal quantum state n in the Hydrogen solution of the Schroedinger equation is also a function of the orbital quantum number l because it was defined as $n = n_r + l + 1$.

- The proposed theoretical approach explains the two spin states with two types of electrons and positrons, namely, the accelerating and decelerating electrons and positrons. The fine-structure splitting of the Hydrogen spectrum is thus produced by the orbital quantum state l, the finite mass of the nucleus and probably by relativistic effects, and not by the supposed magnetic spin moment attributed to the electron by standard theory.

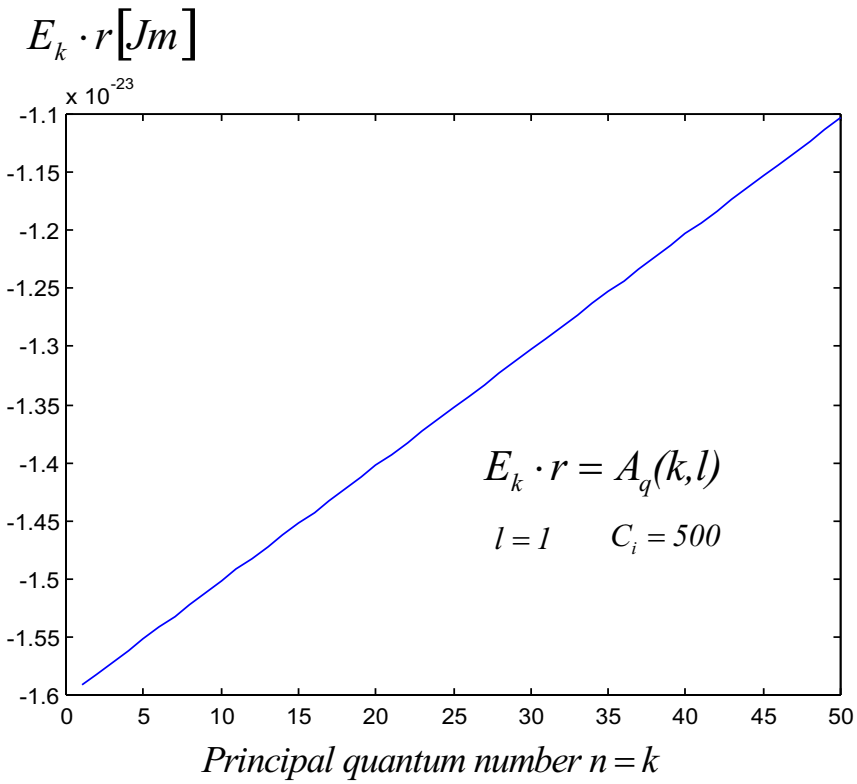

Figure 2: Energy-radius product Aq of Hydrogen atom

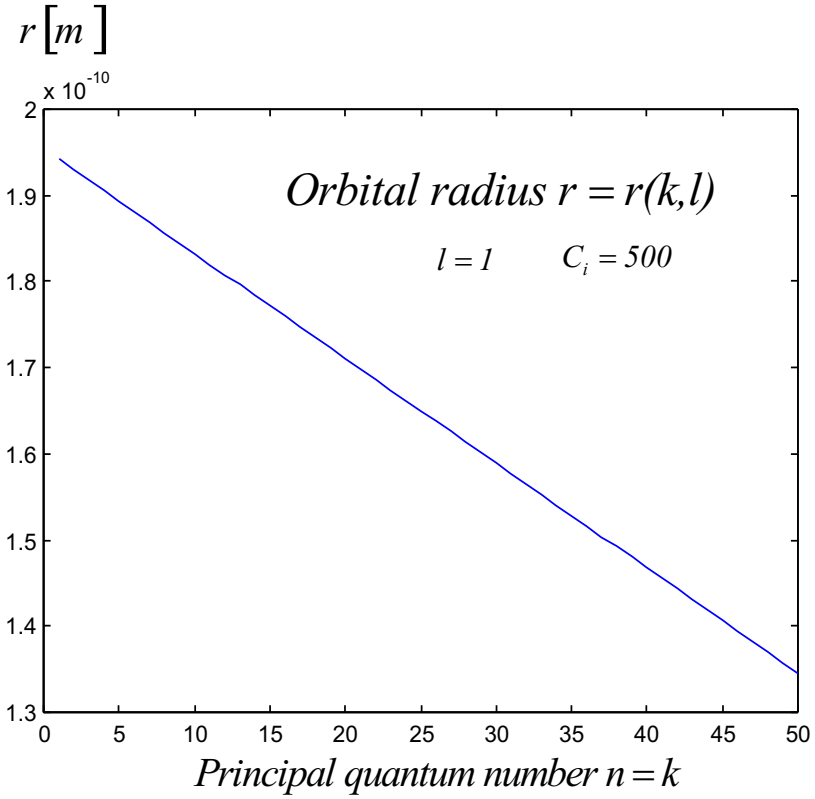

Figure 3: Orbital radius r of Hydrogen atom for constant $l = 1$.

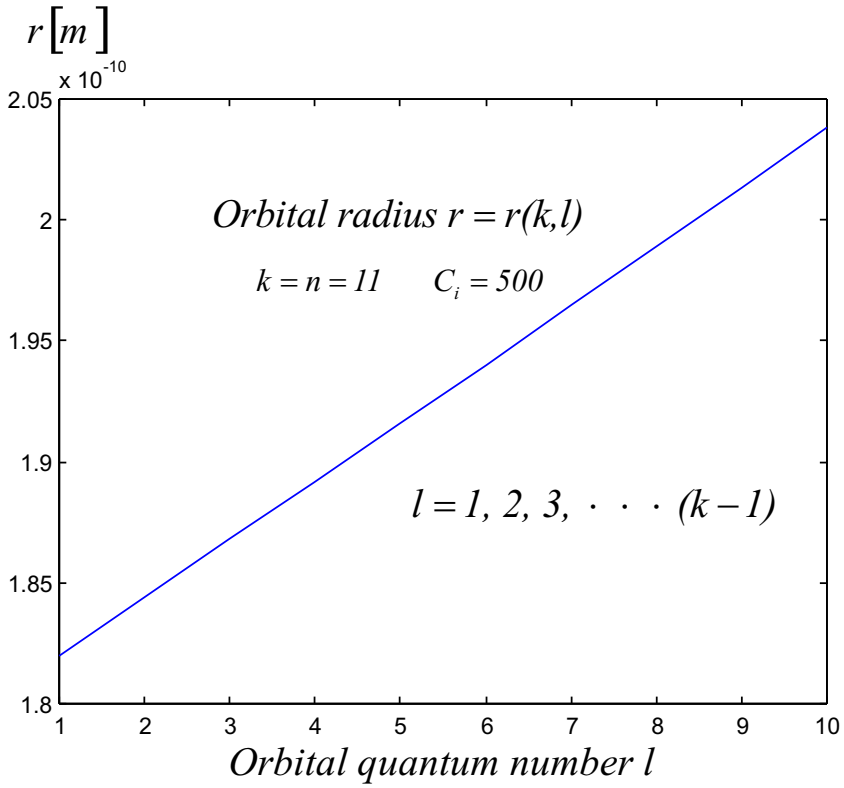

Figure 4: Orbital radius r of Hydrogen atom for constant $k = 11$.

2 Wave equations for free moving particles.

2.1 The relativistic wave equation for the free moving particle.

We start with the dispersion equations for the relativistic mass m_{rel_x} of sec. 1.2. In what follows we omit the sub-index x and writhe m_{rel} instead of m_{rel_x}.

$$m_{rel} = \frac{E_{rel}}{c^2} \qquad m_{rel} = m_{rel}(p) = \frac{1}{c^2}\sqrt{E_o^2 + p^2c^2} \qquad and \qquad E_o = m_o c^2 \tag{148}$$

which can be transformed to

$$m_{rel} = \frac{1}{c}\left[p^2 + \frac{E_o^2}{c^2}\right]^{1/2} = \frac{1}{c}\ \left[p + p^{'}\right] \tag{149}$$

with

$$p^{'}_{1,2} = -p \pm \sqrt{p^2 + \frac{E_o^2}{c^2}} \tag{150}$$

We also transform

$$p(m_{rel}) = c\ \sqrt{m_{rel}^2\ -\ m_o^2} \qquad and \qquad m_o = \frac{E_o}{c^2} \tag{151}$$

to

$$p = \frac{1}{c}\left[E_{rel}^2 - m_o^2\ c^4\right]^{1/2} \qquad with \qquad E_{rel} = E_o + E_{kin} \tag{152}$$

and

$$p = \frac{1}{c}\left[E_{kin}^2 + 2\ E_o\ E_{kin}\right]^{1/2} = \frac{1}{c}\left[E_{kin} + E^{'}\right] \tag{153}$$

with

$$E^{'}_{1,2} = -E_{kin} \pm \sqrt{E_{kin}^2 + 2\ E_o\ E_{kin}} \tag{154}$$

Note: In what follows we changed the symbol for the wave function from ϕ to Ψ to follow the convention.

If we now introduce (149) and (153) in eq. (12)

$$\Psi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \kappa_x(p_x)\ exp\left\{i\frac{c}{\hbar}\left[m_{rel_x}(p_x)\ x -\ p_x\ t\right]\right\} dp_x \tag{155}$$

we get

$$\Psi(x,t) \propto \ exp\left\{\frac{i}{\hbar}\left[\ [p+p^{'}]x - [E_{kin}+E^{'}]t\ \right]\right\} \tag{156}$$

what we can write in the form

$$\Psi(x,t) \propto \ exp\left\{\frac{i}{\hbar}\left[p^{'}x - E^{'}t\ \right]\right\} \cdot exp\left\{\frac{i}{\hbar}\left[p\ x - E_{kin}\ t\ \right]\right\} \tag{157}$$

2.2 The non-relativistic wave equation for the free moving particle

If we make $E_o = 0$ because we want an equation that describes only the kinetic energy we get $p^{'} = 0$ and $E^{'} = 0$, and if we reduce our observation to non-relativistic speeds with $v << c$ we have from eq. (157)

$$\Psi(x,t) \propto exp\left\{\frac{i}{\hbar}\left[p\ x - E_{kin}\ t\ \right]\right\} \qquad with \qquad E_{kin} = \frac{1}{2}\ \frac{p^2}{m} = E_{kin}(p) \tag{158}$$

$$\Psi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \kappa_x(p_x)\ exp\left\{\frac{i}{\hbar}\left[p\ x - E_{kin}(p)\ t\right]\right\} dp_x \tag{159}$$

The wave function derived two times versus x and one time versus t gives the differential equation of the free moving particle of mass m. If we introduce an external potencial U we have the Schrödinger equation for an accelerated particle.

$$i\ \hbar\ \frac{\partial}{\partial t}\Psi(x,t)\ \approx \left[\ -\ \frac{\hbar^2}{2\ m_o}\ \frac{\partial^2}{\partial x^2} + U\right]\Psi(x,t) \tag{160}$$

3 Stable and unstable particles.

Particles in the SM are classified as Gauge Bosons, Leptons, Quarks, Baryons and Mesons. The classification makes no difference between stable and unstable particles. Unstable particles with energies much grater than the energies of the stable electron ($0.511\ MeV/c^2$), positron or neutrino are defined as Basic Subatomic Particles (BSPs), violating the concept of basic particles which must be the constituents of all not basic particles. The result is the search for basic particles like the unstable Quarks with energies above $0.35\ GeV/c^2$.

The approach "Emission and Regeneration" UFT [11]

1. defines as BSPs the electron, positron and the neutrino which are stable particles, and defines all particles with higher energies, stable or unstable, as Composed Subatomic Particles (CSPs) which are integrated by BSPs.

2. defines electrons and positrons as focal points of rays of Fundamental Particles (FPs) which go from infinite to infinite and have longitudinal and transversal angular momenta. Interactions between electrons and positrons are the result of the interactions of the angular momenta of their FPs. No carrier bosons are required to describe interactions between subatomic particles.

3. defines neutrinos as pairs of FPs with opposed angular momenta which generate linear momenta, and photons as a sequence of pairs of FPs with opposed angular momenta that generate a sequence of opposed linear momenta.

4. shows that no strong forces are required to hold electrons and positrons together, which are the constituents of protons and neutrons. The forces between the constituents electrons and positrons tend to zero for the distance between them tending to zero.

5. shows that weak forces which are responsible for the decay of atomic nuclei are electromagnetic forces.

6. shows that gravitation forces are also electromagnetic forces.

The conclusion is, that all interactions between subatomic particles are electromagnetic interactions and described by QED. Interactions as described by QCD and Gauge/Gravity Duality are simply the product of the deficiencies of the SM and not required.

4 The potentials of the four interactions.

Our SM differentiates between the following potentials to explain interactions between particles.

- Strong
- Weak
- Gravitation
- Electromagnetic

In [11] the momentum curve between two static charged BSAs (electron/positron) was derived resulting Fig. 5 and the following regions were defined:

1. From $0 \ll \gamma \ll 0.1$ where $p_{stat} = 0$

2. From $0.1 \ll \gamma \ll 1.8$ where $p_{stat} \propto d^{\,2}$

3. From $1.8 \ll \gamma \ll 2.1$ where $p_{stat} \approx constant$

4. From $2.1 \ll \gamma \ll 518$ where $p_{stat} \propto \frac{1}{d}$

5. From $518 \ll \gamma \ll \infty$ where $p_{stat} \propto \frac{1}{d^{\,2}}$ (Coulomb)

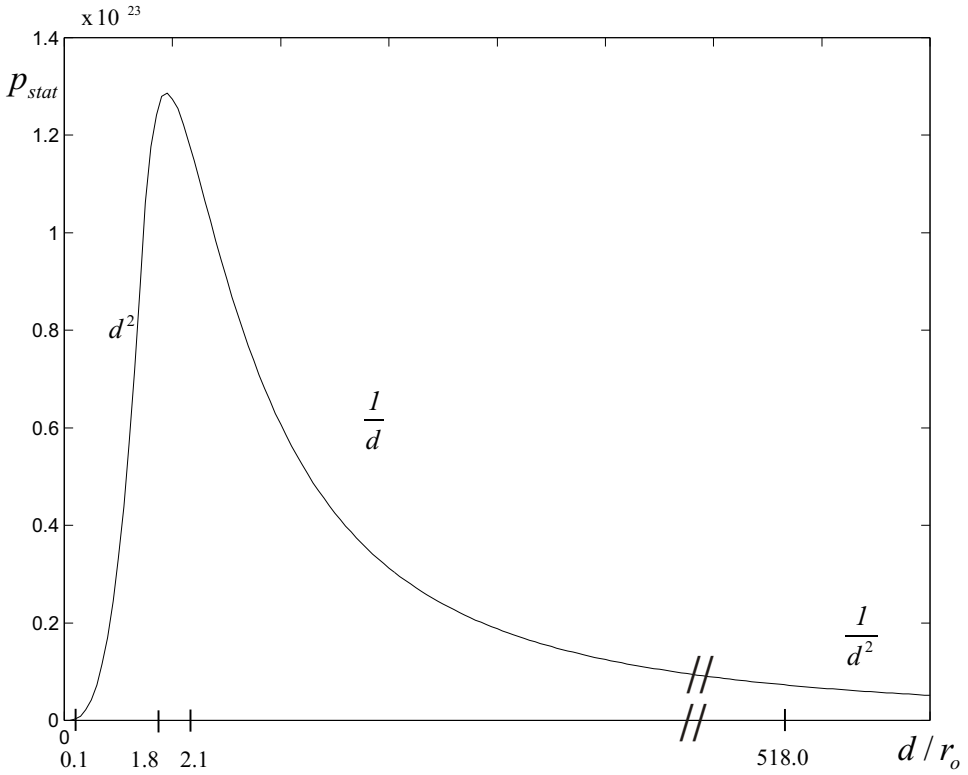

Figure 5: Linear momentum p_{stat} as function of $\gamma = d/r_o$ between two static BSPs with equal radii $r_{o_1} = r_{o_2}$

The static momentum curve of Fig. 5 is part of the potential well of an atomic nucleus as shown in Fig. 6, which can be approximated by a piecewise constant potential for the analytical analysis in quantum mechanics.

The force on electrons or positrons that move in the defined regions of the potential well is given by the following equations:

$$d\bar{F}_{i_n} = \frac{1}{8\,\pi}\,\sqrt{m_p}\,r_{o_p}\,rot\,\frac{d}{dt}\int_{r_r}^{\infty} d\bar{H}_n \qquad with \tag{161}$$

$$\frac{d}{dt}\int_{r_r}^{\infty} d\bar{H}_n = \frac{1}{2}\,\frac{d}{dt}[H_n]\,\frac{r_o}{r_r}\,\sin\varphi\,d\varphi\,\bar{s}_\gamma \;-\; H_n\,v\,\frac{r_o}{r_r^2}\,\sin\varphi\,\cos\varphi\,d\varphi\,\bar{s}_\gamma \tag{162}$$

$$+\,\frac{1}{2}\,H_n\,\frac{1}{r_r}\,\sin\varphi\,d\varphi\,\frac{dr_o}{dt}\,\bar{s}_\gamma$$

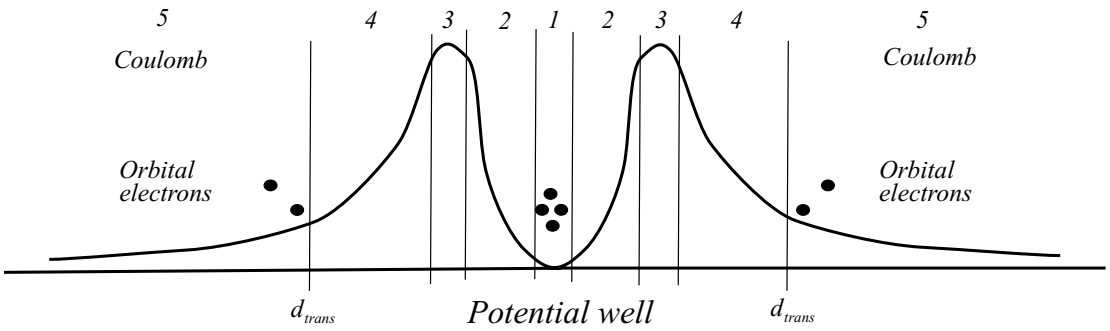

Figure 6: Potential well of an atom.

For the regions we have that:

- BSPs that are in region **1** don't attract nor repel each other. The static force is zero and no binding Gluons nor **strong forces** are needed to hold them together.
- BSPs that have migrated slowly from region **1** to region **2** where the potential groves approximately with d^2, are accelerated to or away from the potential wall by the static force according the charge of the particle and the charge of the remaining particles in region **1**. The force on these moving particles is given by eq. (161). We can differentiate between:
  - BSPs that are accelerated away from the potential wall (region **3**) induce on BSPs of other atoms the **gravitation force**. The accelerated BSPs transmit their acquired momentum to BSPs of other atoms (induction) and stop their movement immediately according the conservation law of momentum. The force on accelerated BSPs is given by the first term of the right side of eq. (162) with $\frac{d}{dt}[H_n] = \sqrt{m}\frac{dv}{dt}$.
  - BSPs that are accelerated to the potential wall may tunnel the wall what results in the decay of the atom with the corresponding radiations. No special **weak force** is required.
- BSPs in the region **5** where the Coulomb force exists, orbit around the atomic nucleus. This force is called in the SM the **electromagnetic force**.

The "Emission & Regeneration" UFT approach shows that all forces are derived from one Field, the dH field. It also shows that all interactions are of electromagnetic type and described by QEDs (Quantum Electrodynamics) and that no other type of interactions are required. It shows that all particles are composed of electrons, positrons and neutrinos and that particles of very short lifetime are composed particles.

5 Summery of main characteristics of the proposed model.

The main characteristics of the proposed model described in [11] are:

- The approach is Lorentz invariant, quantification and probability are inherent to it.
- The interacting particles for all types of interactions (electromagnetic, strong, weak, gravitation) are the FPs with their longitudinal and transversal angular momenta.
- The energy of a BSP is stored as rotation in the emitted and regenerating FPs. The rotation sense of the longitudinal angular momenta of the emitted FPs defines the charge of the BSP.
- All known forces are derived as rotors from one vector field generated by the longitudinal and transversal angular momenta of fundamental particles.
- All forces are expressed as quantized elementary linear momenta acting in a quantized time.
- All the basic laws of physics (Coulomb, Ampere, Lorentz, Maxwell, Gravitation, bending of particles and interference of photons, Bragg, Schroedinger) are mathematically derived from the proposed model, making sure that the approach is in accordance with experimental data.
- The coexistence of BSPs of equal charge in the atomic nucleus does not require the definition of a special strong force nor additional mediating particles (gluons).
- The emission of particles from a heavy atomic nucleus does not require the definition of a special weak force nor additional mediating particles.
- Gravitation has its origin in the linear momenta induced by the reintegration of migrated BSPs to their nuclei. No special mediating particles are required (gravitons).
- The gravitation force is composed of an induced component and a component due to parallel currents of reintegrating BSPs. For galactic distances the induced component can be neglected, what explains the flattening of galaxies' rotation curve (No dark matter is required).
- The inertia of particles is explained with the time delay between the emission and the regeneration of FPs. No special mediating particles are required.

- The wave character of the photon is defined as a sequence of BSPs with potentially opposed transversal linear momenta, which are generated by transversal angular momenta of FPs that comply with specific symmetry conditions.
- Light that moves trough a gravitation field can only lose energy, what explains the red shift of light from far galaxies (no expansion of the universe is required).
- Diffraction of particles such as the Bragg diffraction of electrons is now the result of the quantized interaction of parallel currents.
- Permanent magnets are explained with the synchronization along a closed path of the reintegration of BSPs to their nuclei.
- As the model relies on BSPs permitting the transmission of linear momenta at infinite speed via FPs, it is possible to explain that entangled photons show no time delay when they change their state.
- Relativity based on speed variables instead of space-time variables doesn't present time dilatation and length contraction.
- The addition of a wave to a particle (de Broglie) is effectively replaced by a relation between the particles radius and its energy.
- The Schroedinger equation is replaced by an equation where the wave function is derived one time versus space and two times versus time in analogy to Newton's second law.
- The uncertainty relation of quantum mechanics derived with the new wave function form pairs of canonical conjugated variables between "energy and space" and "momentum and time".
- The time independent Schroedinger equation results deriving the new wave function two times versus space, the same as for the established wave function.
- The new quantum mechanics theory, based on wave functions derived from the radius-energy relation, is in accordance with the quantum mechanics based on the correspondence principle.
- All interactions are of electromagnetic type and described by QEDs (Quantum Electrodynamics) and that no other type of interactions are required.
- Finally the hypothesis is made that the apparent CMB radiation is a radiation induced by gravitation explaining the isotropy of the radiation.

Bibliography

Note: The present approach is based on the concept that fundamental particles are constantly emitted by electrons and positrons and constantly regenerate them. As the concept is not found in mainstream theory, no existing paper can be used as reference.

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