The wave function of the electron

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Introduction

The basic equation, which implies the hypothesis of de Broglie wave properties of matter, was the formula, recorded by him on the basis of the hypothesis of Planck quantum of the least energy and Einstein's equation relating mass and energy. From this it is followed that every portion of the energy having mass corresponds to a batch process, the frequency of which is equal to $\frac{m_0 c^2}{\hbar}$.

Applying the hypothesis of de Broglie to elementary particles, in particular, to an electron, will mean that the electron has an internal wave process. But at that time this wave process was not detected and de Broglie waves only become associated with mechanical motion of the particles.

Modern physics interprets de Broglie waves as waves of probability, not having material embodiment.

Recently, there were experimental evidence to support the initial hypothesis of the existence of de Broglie matter waves [3], [4].

These experimental results have stimulated the emergence of numerous attempts at a theoretical explanation [7], [8].

In this paper, by solving the equations of the relativistic M2 [2] it will show that elementary particles, such as electrons, can contain the wave process with very specific properties. The wave model of a stationary electron is represented in the form of a spherical wave process.

Unsteady equation M2

We will transform the stationary relativistic equation M2 (1.1) in the work [2] in the form of non-stationary.

$$\Delta\Psi - \frac{1}{\hbar^2}\left[\frac{m^4 c^6}{(E - U(\vec{r}))^2} - m^2 c^2\right]\Psi = 0 \qquad (1.1)$$

To do this, remove from the equation the potential energy $U(\vec{r})$ and substitute value of the square energy , on time-dependent wave function $E^2 = -\frac{\hbar^2}{\Psi}\frac{\partial^2\Psi}{\partial t^2}$

The result will be: $$\frac{m^4c^6}{\frac{\hbar^2}{\Psi}\frac{\partial^2\Psi}{\partial t^2}} + \frac{\hbar^2}{\Psi}\Delta\Psi + m^2c^2 = 0 \qquad (1.2)$$

Now we need to find the dispersion relation for the resulting equation (1.2).

To do this, we substitute the spherical wave $\Psi = \frac{1}{r}e^{-i(\omega t - kr)}$ (1.3) into equation.

We define the second derivative with respect to time: $$\frac{\partial^2\Psi}{\partial t^2} = -\frac{\omega^2 e^{-i(\omega t - kr)}}{r} \qquad (1.4)$$

The Laplace operator in spherical coordinates is as follows:

$$\Delta_{r,\theta,\varphi} = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{\Delta_{\theta,\varphi}}{r^2} \qquad (1.5)$$

Define the result of the Laplace operator on a spherical wave: $\Delta\Psi = -\frac{k^2 e^{-i(\omega t - kr)}}{r}$ (1.6)

Substitute spherical wave (1.3) and the values obtained (1.4) and (1.6) to the original time-dependent equation (1.2).

As a result, we obtain the dispersion relation in the form: $-\frac{m^4c^6}{\hbar^2\omega^2} - \hbar^2k^2 + m^2c^2 = 0$ (1.7)

Wave without dispersion and without spreading of the wave packet

To obtain the dispersion relation, let us define the phase and group velocity of wave propagation.

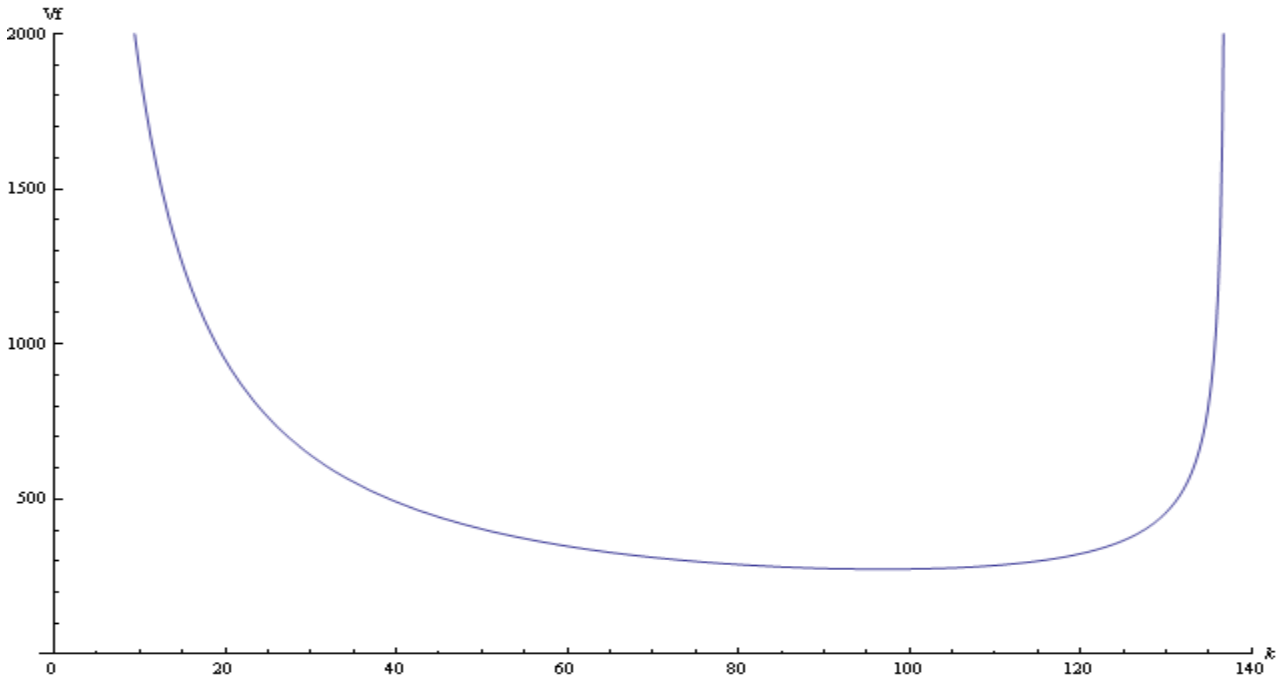
As we know the phase velocity is determined by the angular frequency and wave number of the formula: $V_f = \frac{\omega}{k}$ (2.1)

From equation (1.7) let us define the circular frequency ω. $\omega = \sqrt{\frac{m^4c^6}{\hbar^2\left(c^2m^2 - \hbar^2k^2\right)}}$ (2.2)

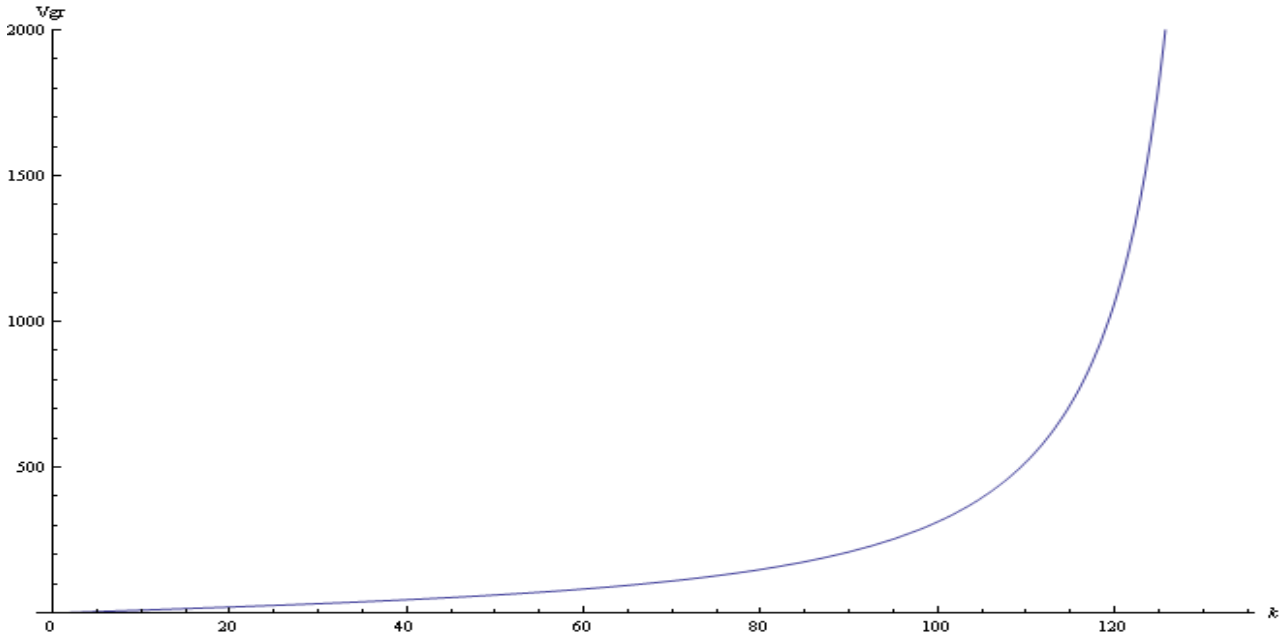
Substituting the obtained value (2.2) in formula (2.1) $V_f = \sqrt{\frac{m^4 c^6}{\hbar^2\left(k^2 c^2 m^2 - \hbar^2 k^4\right)}}$ (2.3)

The group velocity is defined as: $V_{gr} = \frac{d\omega}{dk}$ (2.4). Let's define the derivative using the formula (2.2). $V_{gr} = \frac{\hbar^2 k \sqrt{\frac{m^4 c^6}{\hbar^2\left(c^2 m^2 - \hbar^2 k^2\right)}}}{c^2 m^2 - \hbar^2 k^2}$ (2.5).

Plot the phase and group velocity of the wave number. Next, we will use the Hartree atomic units. $a_0 = 1, m = 1, e = 1, \hbar = 1, c = 137.03599971, 4\pi\varepsilon_0 = 1$



Picture 1. A plot of the phase velocity V_f from the wave number k.



Picture 2. A plot of the phase velocity V_{gr} from the wave number k.

Now let us combine both graphs in one figure

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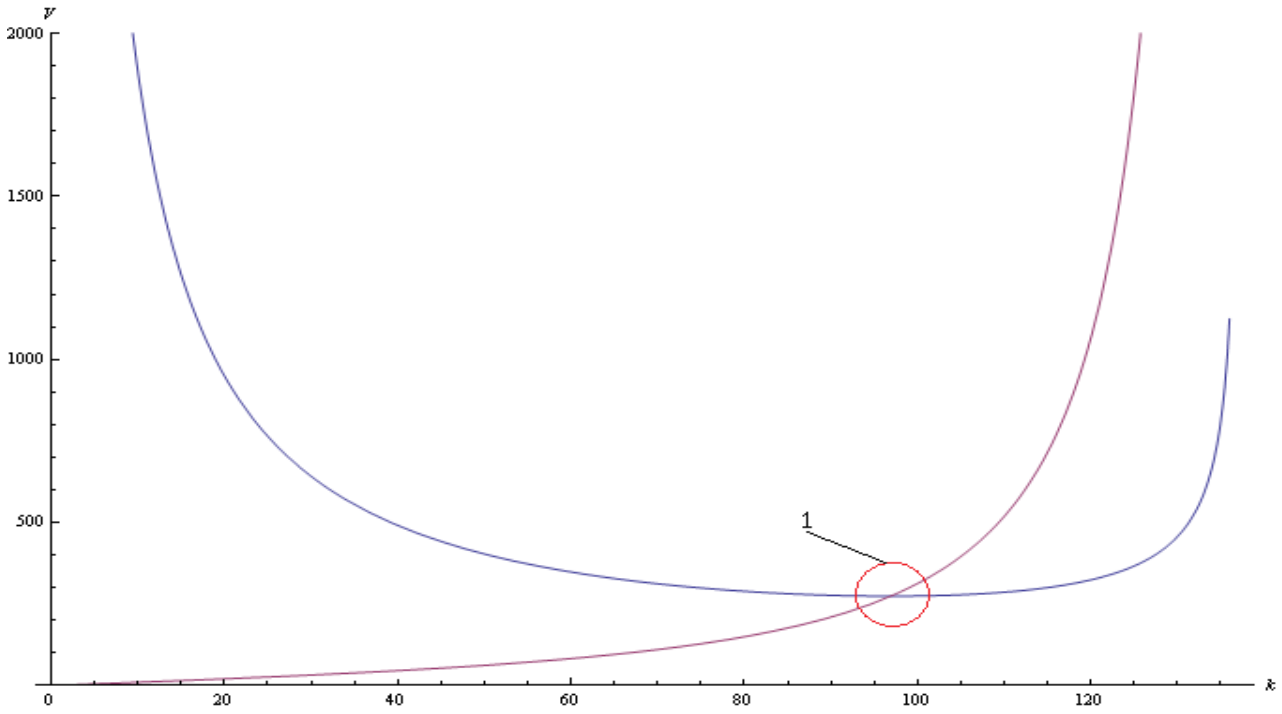


Picture 3. Joint chart phase and group velocity.

As you can see, at a certain value of the wave number, the phase velocity is equal to the group velocity.

Equating the corresponding formulas $V_f = V_{gr}$ and solving the resulting equation, we determine the value of the wave of steady state: $k_0 = \frac{cm}{\sqrt{2}\hbar}$ (2.6). This value will speed $V_f = V_{gr} = 2c$. Corresponding circular frequency value we get: $\omega_0 = \frac{2mc^2}{\sqrt{2}\hbar}$ (2.7). Thus, we have obtained a spherical wave without dispersion. Since, under the condition of equality of the phase and group velocity dispersion disappears.

The resulting speed value is twice the speed of light. But it is not the speed of motion of the electron. Since we consider an electron at rest. This is the speed of propagation in the space of a spherical wave of de Broglie (for the original version of the hypothesis), that is, matter waves. And because the resulting wave of the matter is that there is an unknown nature, that is not going to restrict its characteristics and in particular the speed of propagation.

Before continuing, let us note the following. We define the electron rest energy in accordance with the formula $E = \hbar\omega_0$. Then we get $E = \sqrt{2}mc^2$. As you can see, the resulting value of the energy in $\sqrt{2}$ times more than the expected value, the electron rest energy $E = mc^2$.

To eliminate the resulting discrepancies, we introduce the notion of bare electron mass which entering in the internal wave processes, forms the rest energy $E = mc^2$. From these considerations,

we obtain the value of the seed weight $m_0 = \frac{m}{\sqrt{2}}$. So in further calculations instead of the electron mass we will apply a lot of clipping.

Then we finally get to a steady state value of the wave number $k_0 = \frac{cm}{2\hbar}$ (2.8) and the value of the angular frequency $\omega_0 = \frac{mc^2}{\hbar}$ (2.9).

Integrating spherical waves in a small neighborhood of the steady state (region 1 Picture 3).

We can get a spherical wave packet, without the spreading of (2.10).

$$\Psi = \int_{k_0-\alpha}^{k_0+\alpha} A(k)\frac{e^{-i(\omega_0 t - kr)}}{r}dk \qquad (2.10)$$

Now, after receiving the parameters of sustainable state of the electron, it makes sense to go to a stationary equation and finally get the wave function of an electron of a naked motionless.

To do this, we substitute the resulting value of the seed weight $m_0 = \frac{m}{\sqrt{2}}$ and the value of the rest energy $E = mc^2$ in the original stationary equation (1.1) with no potential energy. Then we get :

$$\Delta\Psi + \frac{1}{\hbar^2}\frac{m^2c^2}{4}\Psi = 0 \qquad (2.11)$$

The resulting equation in the theory of differential equations, known as the Helmholtz equation.

The solution of the Helmholtz equation

To solve the equation (2.11) using the standard technique of separation of variables in spherical coordinates.

Let us represent the wave function as a product of the radial and angular parts $\Psi(r,\theta,\varphi) = R(r)Y(\theta,\varphi)$ (3.1)

The Laplace operator in spherical coordinates looks like this:

$\Delta_{r,\theta,\varphi} = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{\Delta_{\theta,\varphi}}{r^2}$ (3.2) Substitute product (3.1) in the original equation (2.11).

$Y\frac{\partial^2 R}{\partial r^2} + Y\frac{2}{r}\frac{\partial R}{\partial r} + R\frac{\Delta_{\theta,\varphi}Y}{r^2} + \frac{1}{\hbar^2}\frac{m^2c^2}{4}RY = 0$ Multiplying the resulting equation at the root $\frac{r^2}{RY}$.

$\frac{r^2}{R}\frac{\partial^2 R}{\partial r^2} + \frac{r^2}{R}\frac{2}{r}\frac{\partial R}{\partial r} + \frac{1}{\hbar^2}\frac{m^2c^2}{4}r^2 = -\frac{\Delta_{\theta,\varphi}Y}{Y}$ (3.3) As you can see the left-hand side of equation (3.3) depends only on the variable r , and the right of the variables θ and φ. Therefore, both sides are

equal to some constant number λ. What allows to select a real part of the equation from the corner portion. $\frac{\partial^2 R}{\partial r^2}+\frac{2}{r}\frac{\partial R}{\partial r}+\frac{1}{\hbar^2}\frac{m^2c^2}{4}R-\frac{\lambda}{r^2}R=0$ (3.4) $\Delta_{\theta,\varphi}Y=-\lambda Y$ (3.5)

Next, we represent the function $Y(\theta,\varphi)$ as a product $Y(\theta,\varphi)=\Theta(\theta)\Phi(\varphi)$ (3.6).

The angular part of the Laplace operator has the following form: $\Delta_{\theta,\varphi}=\frac{\partial^2}{\partial\theta^2}+ctg\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$ (3.7). By substituting product (3.6) in the angular equation (3.5) we get: $\Phi\frac{\partial^2\Theta}{\partial\theta^2}+\Phi ctg\theta\frac{\partial\Theta}{\partial\theta}+\Theta\frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2}=-\lambda\Theta\Phi$ (3.8).

Multiplying equation (3.8) on the root $\frac{\sin^2\theta}{\Theta\Phi}$ we get: $\frac{\sin^2\theta}{\Theta}\frac{\partial^2\Theta}{\partial\theta^2}+\frac{\sin^2\theta}{\Theta}ctg\theta\frac{\partial\Theta}{\partial\theta}+\lambda\sin^2\theta=-\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2}$ (3.9) The left side of equation (3.9) depends only on the variable θ while the right side only on the variable φ. Therefore, both sides are equal to some constant number, which we denote m^2 (not to be confused with the mass of an electron). As a result, we obtain the two equations: $\frac{\partial^2\Theta}{\partial\theta^2}+ctg\theta\frac{\partial\Theta}{\partial\theta}+\lambda\Theta-\frac{m^2}{\sin^2\theta}\Theta=0$ (3.10) and $\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2}+m^2=0$ (3.11)

The solution of equation (3.11) is well known $\Phi(\varphi)=A\exp(\pm im\varphi)$ (3.12). Since identical values of the angle φ (0 and 2π) the function must have the same value, which is $A\exp(\pm im0)=A\exp(\pm im2\pi)=A$ and $\exp(\pm im2\pi)=1$. Using Euler's formula for complex numbers: $\cos(2\pi m)\pm i\sin(2\pi m)=1$, we get $m=0,\pm1,\pm2,\pm3,.....$ In this way, m can only take integer values. The constant is determined from the normalization condition for the function Φ. However, since the standard normalization and probabilistic interpretation of the wave function is now no longer valid, this issue will not be discussed.

To solve the Θ equation (3.10), we use the mathematical online resource **WolframAlpha** http://www.wolframalpha.com/

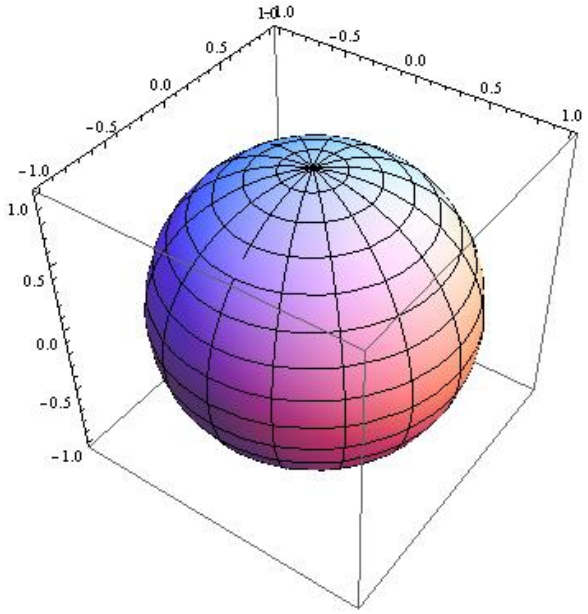
The solution gives: $\Theta(\theta)=c_1P^m_{\frac{1}{2}\left(\sqrt{4\lambda+1}-1\right)}\cos(\theta)+c_2Q^m_{\frac{1}{2}\left(\sqrt{4\lambda+1}-1\right)}\cos(\theta)$ (3.13)

That is, the solution is the amount of associated Legendre functions of the first and second kind. Let us make designation $s=\frac{1}{2}\left(\sqrt{4\lambda+1}-1\right)$ (3.14). Variable s we will be associated with the spin quantum number. We define the range of possible values of the quantum number s. From formula (3.14) it follows that $4\lambda+1\geq0$ so $s\geq-\frac{1}{2}$ (3.15). Rewriting relationship (3.14) relatively s we get $\lambda=s(s+1)$ (3.16) as expected. Thus the angular part of the wave function looks like:

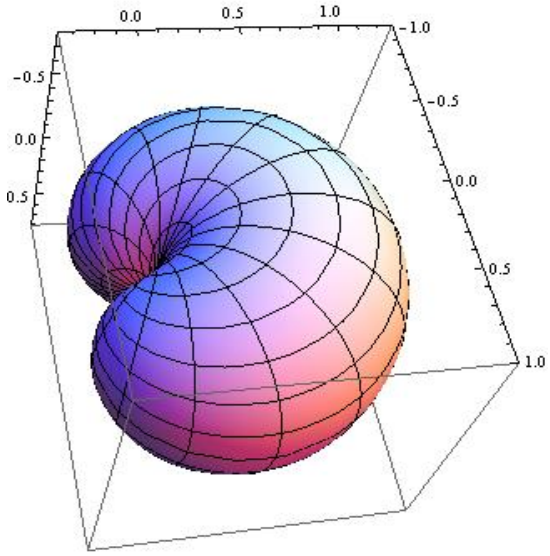
$$Y(\theta,\varphi)=\left[c_1P^m_s\cos(\theta)+c_2Q^m_s\cos(\theta)\right]\exp(\pm im\varphi) \quad (3.17)$$

Since we have the electron is stationary, then the usual conditions of quantization of orbital angular momentum does not hold. So while we can not say that the quantum number s can take only integer or half-integer values. The area of possible values s will be limited only by the ratio (3.15) and the condition of continuity of the angular part of the wave function (3.17). Specific values of the quantum numbers m and s and integration constants c_1 and c_2 will depend on external factors. That is, the boundary conditions of a specific task. In the works of the author [5] is an approach that the quantum numbers m and s can take as whole and half-integer values. We believe this approach is reasonable.

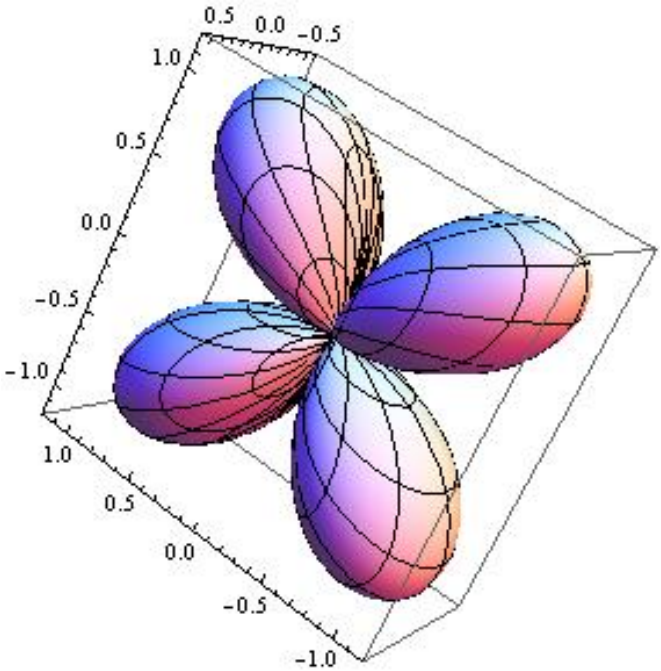
Here, for clarity, some graphic examples of the angular part of the wave function at different values of the quantum numbers m and s.



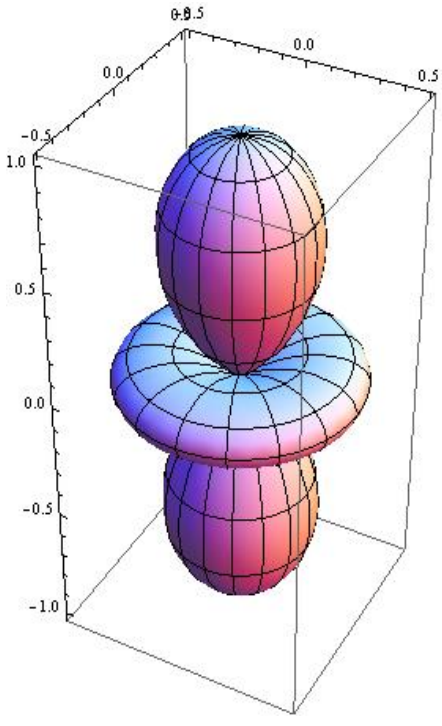
$m = 0 \qquad s = 0$



$m = \frac{1}{2} \qquad s = \frac{1}{2}$



$m = 1 \qquad s = 2$



$m = 0 \qquad s = 2$

We now turn to the solution of the radial equation (3.4). For this we use a mathematical online resource **WolframAlpha** http://www.wolframalpha.com/

We rewrite the equation (3.4) with the notation (3.16).

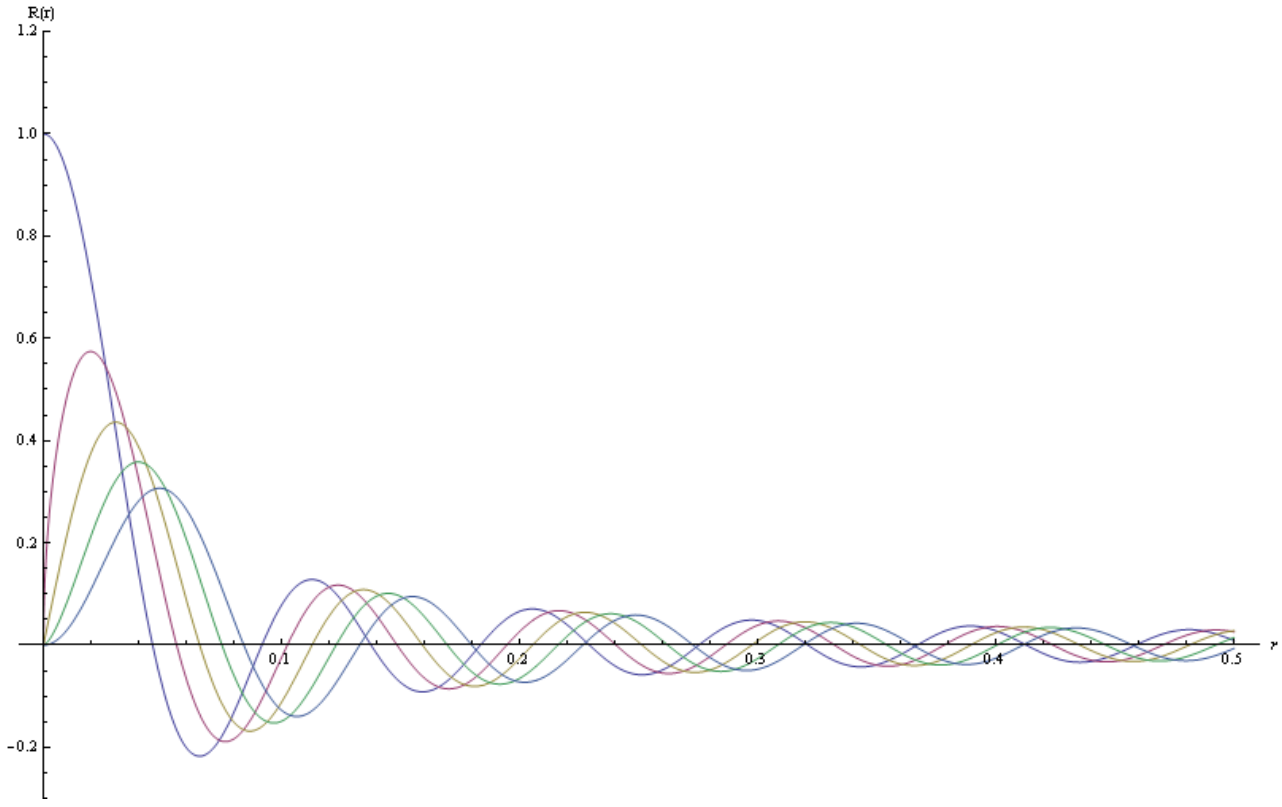
$$\frac{\partial^2 R}{\partial r^2}+\frac{2}{r}\frac{\partial R}{\partial r}+\frac{1}{\hbar^2}\frac{m^2c^2}{4}R-\frac{s(s+1)}{r^2}R=0 \qquad (3.18)$$

The solution has the form: $R(r)=k_1 SphericalBesselJ[s,\frac{cmr}{2\hbar}]+k_2 SphericalBesselY[s,\frac{cmr}{2\hbar}]$ (3.19)

That is, a combination of the spherical Bessel functions of the first and second kind. As it is known, a second-order spherical Bessel function is unbounded at $r\to 0$. Therefore we accept $k_2=0$. And finally the radial part of the wave function is of the form:

$$R(r)=k_1 SphericalBesselJ[s,\frac{cmr}{2\hbar}] \qquad (3.20)$$

Here is a graph of the radial wave function in the Hartree atomic units for the first few values of the quantum numbers $s=0,\frac{1}{2},1,\frac{3}{2},2$. Since the question of normalization has not been discussed, for the reasons given, it will take $k_1=1$. In terms of mass of the electron Hartree $m=1$, Planck's constant $\hbar=1$ and the speed of light $c=137.03599971$.



Picture 4. The graph of the radial part of the wave function for values $s=0,\frac{1}{2},1,\frac{3}{2},2$

Thus, the total electron wave function will be, a work of radial, angular and time-dependent parts.

$$\Psi(r,\theta,\varphi,t) = R(r)\Theta(\theta)\Phi(\varphi)\Psi(t)$$

$$\Psi(r,\theta,\varphi,t) = SphericalBesselJ[s,\frac{cmr}{2\hbar}]*\left[c_1 P_s^m \cos(\theta) + c_2 Q_s^m \cos(\theta)\right]\exp(\pm im\varphi)\exp(-i\frac{mc^2}{\hbar}t) \quad (3.21)$$

Results and discussion

As shown, an internal electron wave process with frequency $\omega_0 = \frac{mc^2}{\hbar}$. The wave process goes to the outside and forms of matter waves. This number is equal to the wave $k_0 = \frac{cm}{2\hbar}$, hence the wavelength is $\lambda = \frac{2\pi}{k_0} = \frac{4\pi\hbar}{cm}$, and the velocity of propagation in the space of two-fold greater than the speed of light. Consequently, the matter waves are not electromagnetic in nature.

As it is known, electron characterized by experimentally observed value: Compton wavelength $\lambda_{Comp} = \frac{2\pi\hbar}{cm}$. Consequently, the resulting wavelength is double-size Compton wavelength $\lambda = 2\lambda_{Comp}$. It can be assumed that the Compton wavelength is related to the wave properties of electrons.

At his time, Schrodinger did not agree with a purely corpuscular explanation of the Compton effect. And published his description of the wave [9].

"Schrodinger argued that X-rays can be diffracted by a standing" charge density wave "created by the incident and reflected electron, as well as the light is diffracted by a standing wave ultrasound (Born and Wolf 1959)."

Further it was found that the oscillatory process involved is not the whole mass of the electron. As a result, the notion of seed weight is entered $m_0 = \frac{m}{\sqrt{2}}$. That is, the electron mass is formed, as a result of the entry of the seed weight of the inner wave process. By analogy, we can assume that the same seed mass is formed by the process of the second stage of the wave. And therefore, the matter is a multi-stage, nested wave structure as matrioshka. It is found that the spin of the electron does not have to make a difference $\frac{1}{2}$. The region of possible values of the quantum numbers of the electron remains to be studied.

These results open up new horizons for theoretical and experimental research. Let a fresh look at many of the experimental results, which appear in the wave properties of the electron.

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