On quantization of interacting velocity gauge fields

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Abstract

The many velocity gauge fields interaction Lagrangian density is extended to a continuous spectrum Lagrangian then quantized and normalized. The Euler-Lagrange equation is transformed to the Yang-Mills field, curvature and integral operators. The Yang-Mills field operator quantization gives the equivalent Dirac equation, the integral operator quantization gives perturbation series similar to the path integral formulation of Quantum mechanics and the curvature operator gives the Einstein-Hilbert Lagrangian. In all quantization cases, the field form operators have mass eigenvalue. Finally, the normalization of the wave function gives the Lagrangian of massive particle Lagrangian with many different representations.

I. Introduction

The many velocity gauge fields interaction Lagrangian density introduced in article [1] is extended to a continuous spectrum Lagrangian then quantized and normalized. First, the source terms in the Lagrangian density of N interacting fields are included in the connection and summations are converted to integral over a small value $d\epsilon$. Second, the solution of the Euler-Lagrange equation is converted to the Yang-Mills field, curvature and integral operators and quantized with mass eigenvalue. The Yang-Mills field operator quantization gives the Dirac equation, the quantization of the integral operator gives the perturbation series similar to the path integral formulation of Quantum mechanics and the curvature operator gives the Einstein-Hilbert Lagrangian. Finally, the normalization of the wave function gives the massive particle Lagrangian with many different representations.

II. Lagrangian

From the Lagrangian density in article [1], including the source term in the connection and summing over a small value $d\epsilon$, the Lagrangian is evaluated to

$$L = \int d\epsilon \left[\alpha^{\mu}\left[\partial_{\mu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\phi\right]^{T}\left[\alpha^{\nu}\left[\partial_{\nu} + \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi\right] \tag{1}$$

Solving the Euler-Lagrange equation for the Lagrangian in equation (1), yields

$$\alpha^{\mu T}\left[\partial_{\mu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\alpha^{\nu}\left[\partial_{\nu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi = \frac{1}{i\hbar}\int d\epsilon\left[\alpha^{\mu T}\phi_{\mu}\alpha^{\nu}\left[\partial_{\nu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi\right] \tag{2}$$

III. Yang-Mills equations form

Let us define the field F and substitute in equation (2) as

$$\alpha^{\nu}\left[\partial_{\nu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi=F \tag{3}$$

$$\alpha^{\mu T}\left[\partial_{\mu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]F=\alpha^{\mu T}\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}F \tag{4}$$

Equation (3) and (4) have the Yang-Mills equations form with source term, but they are different as they are operator identities to be quantized in the next sections.

IV. Dirac equation form

Let us quantize the differential operator in equation (4) by introducing the mass eigenvalue as

$$\alpha^{\mu T}\left[\partial_{\mu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]F=-\frac{m^2c^2}{\hbar^2}\phi \tag{5}$$

Defining the wave function as

$$\psi=\begin{bmatrix} i\hbar F \\ mc\phi \end{bmatrix} \tag{6}$$

Using equation (6), equations (3) and (4) can be written as

$$i\hbar\Gamma^{\mu}\left[\partial_{\mu}-\frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\psi=mc\psi \tag{7}$$

Equations (7) is the equivalent to the Dirac equation with interaction term and different gamma matrices introduced in article [1].

V. Integral equation form

Similarly to equation (5), an integral equation with mass eigenvalue part of the equation (4) is given by

$$\alpha^{\mu T}\frac{1}{i\hbar}\int d\epsilon\phi_{\mu}F=-\frac{m^2c^2}{\hbar^2}\phi \tag{8}$$

From article [2], equations (8) can be written as

$$\frac{i\hbar}{mc}\int e^{-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}}F=\phi \tag{9}$$

Expending equation (9) in power series, yields

$$\frac{i\hbar}{mc}\int\sum_{n=0}^{\infty}\frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n}F=\phi \tag{10}$$

Substituting equation (10) in the wave function in equation (6), gives

$$\psi\ = i\hbar\left[\frac{F}{\int\sum_{n=0}^{\infty}\frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n}F}\right] \tag{11}$$

VI. Curvature Form

Evaluating equation (2) with

$$\alpha^{\mu^T}\alpha^{\nu}+\alpha^{\nu^T}\alpha^{\mu}=2\eta^{\mu\nu} \tag{12}$$

$$\Delta=\eta^{\mu\nu}\left[\partial_\mu\partial_\nu-2\left[\frac{1}{i\hbar}\int d\epsilon\phi_\mu\right]\partial_\nu\right] \tag{13}$$

$$R=\alpha^{\mu^T}\alpha^{\nu}\left[\partial_\mu\left[\frac{1}{i\hbar}\int d\epsilon\phi_\nu\right]-\left[\frac{1}{i\hbar}\int d\epsilon\phi_\mu\right]\left[\frac{1}{i\hbar}\int d\epsilon\phi_\nu\right]\right] \tag{14}$$

Using equations (3), (12), (13) and (14), equation (2) can be evaluated to

$$[\Delta-R]\phi=\alpha^{\mu^T}\frac{1}{i\hbar}\int d\epsilon\phi_\mu F \tag{15}$$

Quantizing the Laplace-Beltrami and Curvature operator as in equations (5) and (8) gives

$$[\Delta-R]\phi=-\frac{m^2c^2}{\hbar^2}\phi \tag{16}$$

VII. Normalization

Since $\psi^{\dagger}\psi$ is the probability density function, it is normalized as

$$\int dx^4\psi^{\dagger}\psi=1 \tag{17}$$

The variation of equation (17) yields

$$\delta\int dx^4\psi^{\dagger}\psi=0 \tag{18}$$

Using equation (3) and (6) in differential form, equation (18) is solved by the Euler-Lagrange equation with the Lagrangian density

$$\mathcal{L}=\frac{m^2c^2}{\hbar^2}\phi^{\dagger}\phi+\left[\alpha^{\mu}\left[\partial_\mu-\frac{1}{i\hbar}\int d\epsilon\phi_\mu\right]\phi\right]^{\dagger}\left[\alpha^{\nu}\left[\partial_\nu-\frac{1}{i\hbar}\int d\epsilon\phi_\nu\right]\phi\right] \tag{19}$$

Using equation (11) in integral form, equation (18) is solved by the Euler-Lagrange equation with the Lagrangian density

$$\mathcal{L} = \left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n} F\right]^{\dagger}\left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n} F\right] + F^{\dagger}F \tag{20}$$

Combining differential and integral forms representation using equation (3) and (10) gives the Lagrangian density

$$\mathcal{L} = \left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n} F\right]^{\dagger}\left[\int \sum_{n=0}^{\infty} \frac{1}{n!}\left[-\frac{d\epsilon}{mc}\boldsymbol{\alpha}\cdot\boldsymbol{\phi}\right]^{n} F\right] + \left[\alpha^{\mu}\left[\partial_{\mu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\phi\right]^{\dagger}\left[\alpha^{\nu}\left[\partial_{\nu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi\right] \tag{21}$$

Using equation (3) and (10) without operators, equation (18) is solved by the Euler-Lagrange equation with the Lagrangian density

$$\mathcal{L} = \frac{m^2c^2}{\hbar^2}\phi^{\dagger}\phi + F^{\dagger}F \tag{22}$$

Using equation (16), (19) and (22) in curvature form, equation (18) is solved by the Euler-Lagrange equation with the Lagrangian density

$$\mathcal{L} = \phi^{\dagger}R\phi - \phi^{\dagger}\Delta\phi + \left[\alpha^{\mu}\left[\partial_{\mu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\mu}\right]\phi\right]^{\dagger}\left[\alpha^{\nu}\left[\partial_{\nu} - \frac{1}{i\hbar}\int d\epsilon\phi_{\nu}\right]\phi\right] \tag{23}$$

$$\mathcal{L} = \phi^{\dagger}R\phi - \phi^{\dagger}\Delta\phi + F^{\dagger}F \tag{24}$$

Equations (19) to (24) are different representations of a charged massive particle Lagrangian density.

VIII. Conclusion

In summary, the velocity gauge fields were quantized with Yang-Mills field, curvature and integral operators and normalized. The Yang-Mills field operator quantization gave the equivalent Dirac equation, the integral operator gave perturbation series similar to the path integral formulation of Quantum mechanics and the curvature operator gave the Einstein-Hilbert Lagrangian. During quantization, it is shown that Yang-Mills field, curvature and integral operators have mass eigenvalue. Finally, the normalization for the wave function gave the Lagrangian of massive particle with many different representations.

IX. References

[1] Rukundo, JPR, 2017. On velocity gauge field approach of interactions. Quaternion Physics, [Online]. 1, 5. Available at: https://quaternionphysics.files.wordpress.com/2017/11/on-velocity-gauge-field-approach-of-interactions7.pdf [Accessed 26 November 2017].

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