Classify Positive Integers to Prove Collatz Conjecture by Mathematical Induction

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Abstract

Positive integers which are able to be operated to 1 by the leftwards operational rule and generating positive integers which start with 1 to operate by the rightwards operational rule are one-to-one correspondence and the same. So, we refer to the bunch of integers' chains to apply the mathematical induction, next classify positive integers to get comparable results via operations, such that finally summarize out a proof at substep according to beforehand prepared two theorems as judgmental criteria.

AMS subject classification: 11P81, 11A25 and 11Y55

Keywords: mathematical induction; two-way operational rules; classify positive integers; the bunch of integers' chains; operational routes

1. Introduction: The Collatz conjecture is also variously well-known 3n+1 conjecture, the Ulam conjecture, Kakutani's problem, the Thwaites conjecture, Hasse's algorithm, and the Syracuse problem etc.

Yet it is still both unproved and un-negated a conjecture ever since named after Lothar Collats in 1937.

2. Basic Concepts and Criteria

The Collatz conjecture states that take any positive integer n, if n is an

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even number, then divide n by 2; if n is an odd number, then multiply n by 3 and add 1. Repeat the above process indefinitely, then no matter which positive integer you start with, it will eventually reach a result of 1.

We regard above-mentioned operational stipulations of the conjecture as the leftwards operational rule. Also, regard the operational rule versus the leftwards operational rule as the rightwards operational rule.

The rightwards operational rule stipulates that for any positive integer n, uniformly multiply n by 2. In addition, when n is an even number, if divide the difference of n minus 1 by 3 to get an odd number, then, must operate this step, and the operational route via here go on.

First let us make a statement that thereinafter emerging integers, odd numbers, even numbers and expressions thereof are all positive.

Start with any integer to operate successive emerging integers by either operational rule, afterwards, we regard consecutive integers plus synclastic arrowheads among them as an operational route.

If an integer (or an integer's expression) P_{ie} exists at an operational route, then we may term the operational route "an operational route of P_{ie}" or "the operational route P_{ie}". Two operational routes of P_{ie} branch from an integer (or an integer's expression) after pass the operation of P_{ie}.

Start with 1 to operate successive emerging integers by the rightwards operational rule, whereafter, it will form a bunch of operational routes. We term such a bunch of operational routes "a bunch of integers' chains".

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That is not difficult to understand that the whole bunch of integers' chains must consist of infinite many operational routes.

See also an initial part of the whole bunch of integers' chains as follows.

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                        …           …      452→…
                       168↑        680↑    226↑→75→…
                      84↑        340↑→113↑     227→…
                    42↑         170↑          682↑→…
      1            21↑         85↑          341↑
1→2→4↑→8→16↓→32→64↑→128→256↑→512→1024↑→2048→…
              5→10↓→20→40↓→80→160↓→320→…
                  3↓       13↓      53→106↓→212→…
                   6↓       26↓         35→70↓→140→…
                    12↓       52↓→104→…     23→46↓→15→…
                     24↓       17↓               92→…
                      …         34↓→11→22↓→7→14→28↓→56…
                                 68↓      44→88↓→29…9→18→…
                                  136↓→…      29↓
                                     45→…      58↓→19→…
                                                  …    …
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First Illustration

Since each of integers at a bunch of integers' chains comes from an adjacent integer before itself, also every two integers that root in an even number are an odd number and an even number, thus each of integers except for 1 at the whole bunch of integers' chains is unique.

Since two kinds of operations reverse for each other, so integers at the bunch of integers' chains and integers which are able to be operated to 1 by the leftwards operational rule are one-to-one correspondence and the same.

Such being the case, we shall refer to the bunch of integers' chains to apply the mathematical induction to prove the conjecture.

Before make the proof, we are necessary to prepare two theorems concerned, so as to reach certain conclusions by them.

Theorem 1* If an integer or an integer's expression suits the conjecture,

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and that it exists at operational route P_{ie}, then P_{ie} suits the conjecture.

For examples: (1) If $27+2^3\eta$ suits the conjecture, and $P_{ie}=31+3^2\eta$, where $\eta\geq0$, then from $27+2^3\eta\rightarrow82+3\times2^3\eta\rightarrow41+3\times2^2\eta\rightarrow124+3^2\times2^2\eta\rightarrow62+3^2\times2\eta\rightarrow31+3^2\eta>27+2^3\eta$, conclude that $31+3^2\eta$ suits the conjecture. (2) If $4+3\mu$ suits the conjecture, and $P_{ie}=5+2^2\mu$ where $\mu\geq0$, then from $5+2^2\mu\rightarrow16+3\times2^2\mu\rightarrow8+3\times2\mu\rightarrow4+3\mu<5+2^2\mu$, conclude that $5+2^2\mu$ suits the conjecture.

Proof* Suppose C_{ie} suit the conjecture. At an identical operational route by the leftwards operational rule, if C_{ie} appears before P_{ie}, then the operations of C_{ie} via P_{ie} reached 1 already, naturally P_{ie} was operated into 1. If C_{ie} appears behind P_{ie}, then the operations of P_{ie} pass C_{ie}, afterwards continue along operational route of C_{ie} to reach 1.

In addition, at an identical operational route by the rightwards operational rule, C_{ie} and P_{ie} root in 1, of course, can operate either of them to reach 1 by the leftward operational rule reversely.

Theorem 2* If an integer or an integer's expression suits the conjecture, and that it exists only at operational route Q_{ie}, also operational route P_{ie} and the operational route Q_{ie} intersect, then P_{ie} suits the conjecture, where $P_{ie}\neq Q_{ie}$. Such as $71+3^3\times2^5\varphi$ suits the conjecture, and $P_{ie}=95+3^2\times2^7\varphi$, where $\varphi\geq0$, so from $95+3^2\times2^7\varphi\rightarrow286+3^3\times2^7\varphi\rightarrow143+3^3\times2^6\varphi\rightarrow430+3^4\times2^6\varphi\rightarrow215+3^4\times2^5\varphi\rightarrow646+3^5\times2^5\varphi\rightarrow323+3^5\times2^4\varphi\rightarrow970+3^6\times2^4\varphi\rightarrow485+3^6\times2^3\varphi\rightarrow1456+3^7\times2^3\varphi\rightarrow728+3^7\times2^2\varphi\rightarrow364+3^7\times2\varphi\rightarrow182+3^7\varphi\rightarrow\ldots$

$\uparrow121+3^6\times2\varphi\leftarrow242+3^6\times2^2\varphi\leftarrow484+3^6\times2^3\varphi\leftarrow$

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$161+3^5\times2^3\varphi\leftarrow 322+3^5\times2^4\varphi\leftarrow 107+3^4\times2^4\varphi\leftarrow 214+3^4\times2^5\varphi\leftarrow 71+3^3\times2^5\varphi <$ $95+3^2\times2^7\varphi$, we conclude that $95+3^2\times2^7\varphi$ suits the conjecture.

Proof* Let D_{ie} suit the conjecture, and two operational routes intersect at A_{ie}, then D_{ie} and A_{ie} exist at operational route Q_{ie}, so A_{ie} suits the conjecture according to the theorem 1. Like the reason, P_{ie} and A_{ie} exist at operational route P_{ie}, of course, P_{ie} suits the conjecture.

Actually all integers or integer's expressions at successively intersecting operational routes suit the conjecture, so long as one therein is suitable.

3. A Classified Proof by Mathematical Induction

We set to prove the conjecture by the mathematical induction by now, next classify integers to get comparable results via operations first.

1. All integers at the initial part of the whole bunch of integers' chains in the preceding chapter suit the conjecture, and that we are not difficult to find that there are consecutive positive integers$\leq$24 therein.

2. After further operate integers at the initial part of the whole bunch of integers' chains by the rightwards operational rule, suppose that there are consecutive integers$\leq$ n within all integers got, where n$\geq$ 24.

3. After continue to operate above appeared integers by the rightwards operational rule, prove that there are consecutive integers$\leq$ 2n within all integers got, i.e. prove that integers from n+1 to 2n suit the conjecture too.

First, let us divide limits of consecutive positive integers at the number axis into segments according to 2^Xn as greatest integer per segment, where X≥0 and n≥24, so as to accord with the mathematical induction.

A simple segmenting illustration is as follows.

0-1——**2**^{0}n———2^{1}n——————**2**^{2}n————————**2**^{3}n———→

Second Illustration

Proof * Since there are consecutive positive integers≤ n with n≥24 at an initial part of the whole bunch of integers' chains according to the supposition of the mathematical induction, thereby multiply each and every positive integer≤ n by 2 by the rightwards operational rule, then we get all even numbers between 2^0n and 2^1n+1 at the bunch of integers' chains, irrespective of repeated even numbers≤ n.

So all even numbers between 2^0n and 2^1n+1 exist at the bunch of integers' chains, and each of them suits the conjecture according to the theorem 1.

For odd numbers between 2^0n and 2^1n+1, we orderly classify them, and that for each of classifications, find out a smaller integer's expression< itself via operations, up to confirm that they are proved in the end.

In any case, first let us divide all odd numbers between 2^0n and 2^1n+1 into two genera, i.e. 5+4k and 7+4k, where k ≥5.

For 5+4k, from 5+4k→16+12k→8+6k→4+3k, it shows that 5+4k →…→smaller 4+3k.

For 7+4k, divide it into 3 sorts: 15+12c, 19+12c and 23+12c, where c≥1.

For 23+12c, from 15+8c→46+24c→23+12c, it shows that smaller 15+8c→…→23+12c.

For 15+12c and 19+12c, we need to operate them right along.

Firstly, operate 15+12c by the leftwards operational rule as follows.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)   e=2f: 142+486f→71+243f ♥
♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)
c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)
d=2e:160+486e♦   e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)   …
♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ …
f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→…
g=2h: 322+4374h→… …

g=2h: 86+243h (**5**)
♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→…
f=2g: 304+1458g→152+729g ↓→…
…
…
♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ …   …
e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ …
f=2g: 20+243g↓→g=2h: 10+243h (**6**)   …
g=2h+1:880+1458h→440+729h↑→ …

Annotation:
(1) Each of letters c, d, e, f, g, h …etc at listed above operational routes expresses each of natural numbers plus 0, similarly hereinafter.
(2) Also, there are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦.
(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.

We are necessary to define a terminology before analyzing operational results of 15+12c/19+1c.

Namely, if an operational result is smaller than a kind of 15+12c/19+12c, and that the operational result first appears at an operational route of 15+12c/19+12c by the leftwards operational rule, then we term the operational result “№1 satisfactory operational result”.

Then, each kind of 15+12c/19+12c derives from a №1 satisfactory operational result, and that the former is greater than the latter always, also both of them coexist at an operational route of 15+12c/19+12c.

Thereupon, we conclude six kinds of 15+12c derived monogamously from six №1 satisfactory operational results at the above bunch of operational routes of 15+12c, ut infra.

From c=2d+1 and d=2e+1, we get c=2d+1=2(2e+1)+1=4e+3, and $15+12c=15+12(4e+3)=51+48e=51+3\times2^4e\to154+3^2\times2^4e\to77+3^2\times2^3e\to232+3^3\times2^3e\to116+3^3\times2^2e\to58+3^3\times2e\to29+27e$ where mark (1), so it shows that 51+48e→…→smaller 29+27e.

From c=2d+1, d=2e and e=2f+1, we get c=2d+1=4e+1=4(2f+1)+1=8f+5, and $15+12c=15+12(8f+5)=75+96f=75+3\times2^5f\to226+3^2\times2^5f\to113+3^2\times2^4f\to340+3^3\times2^4f\to170+3^3\times2^3f\to85+3^3\times2^2f\to256+3^4\times2^2f\to128+3^4\times2^1f\to64+81f$ where mark (2) , so it shows that 75+96f→…→smaller 64+81f.

From c=2d, d=2e+1 and e=2f+1, we get c=2d= 4e+2= 4(2f+1)+2=8f+6, and $15+12c=15+12(8f+6)=87+96f=87+3\times2^5f\to262+3^2\times2^5f\to131+3^2\times2^4f\to394+3^3\times2^4f\to197+3^3\times2^3f\to592+3^4\times2^3f\to296+3^4\times2^2f\to148+3^4\times2^1f\to74+81f$ where mark (3) , so it shows that 87+96f→…→smaller 74+81f.

From c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1, we get c=2d+1=4e+1=8f+1=8(2g+1)+1=16g+9=16(2h+1)+9=32h+25, and $15+12c=15+12(32h+25)=315+384h=315+3\times2^7h\to946+3^2\times2^7h\to473+3^2\times2^6h\to1420+3^3\times2^6h\to710+3^3\times2^5h\to355+3^3\times2^4h\to1066+3^4\times2^4h\to533+3^4\times2^3h\to1600+3^5\times2^3h\to800+3^5\times2^2h\to400+3^5\times2^1h\to200+243h$ where mark (4), so it shows that 315+384h→…→smaller 200+243h.

From c=2d, d=2e+1, e=2f, f=2g+1 and g=2h, we get c=2d=2(2e+1)=4e+2=8f+2=8(2g+1)+2=16g+10=32h+10,and15+12c=15+12(32h+10)=135+384h

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$=135+3\times2^7h\rightarrow406+3^2\times2^7h\rightarrow203+3^2\times2^6h\rightarrow610+3^3\times2^6h\rightarrow305+3^3\times2^5h\rightarrow916$ $+3^4\times2^5h\rightarrow458+3^4\times2^4h\rightarrow229+3^4\times2^3h\rightarrow688+3^5\times2^3h\rightarrow344+3^5\times2^2h\rightarrow86+243h$ where mark (5), so it shows that 135+384h→…→smaller 86+243h.

From c=2d, d=2e, e=2f, f=2g and g=2h, we get c=2d =32h, and 15+12c= $15+12(32h)=15+3\times2^7h\rightarrow46+3^2\times2^7h\rightarrow23+3^2\times2^6h\rightarrow70+3^3\times2^6h\rightarrow35+3^3\times2^5h$ $\rightarrow106+3^4\times2^5h\rightarrow53+3^4\times2^4h\rightarrow160+3^5\times2^4h\rightarrow80+3^5\times2^3h\rightarrow40+3^5\times2^2h\rightarrow10+$ 243h where mark (6), so it shows that 15+384h→…→smaller 10+243h.

Secondly, operate 19+12c by the leftwards operational rule as follows.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**) e=2f:37+81f (**β**)
♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥
c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠
d=2e+1:94+81e↓→e=2f:47+81f (γ)
e=2f+1:175+162f ♦

g=2h: 119+243h (**δ**) **…**
f=2g+1:238+243g↑→g=2h+1:1504+1458h→752+729h↑→**…**
♥466+486f→233+243f↑→f=2g:700+1458g→350+729g↓→g=2h+1:3238+4374h↓
g=2h: 175+729h↓→**…** **…**
…

g=2h+1:172+243h (**ε**)
f=2g: 101+243g↑→g=2h: 304+1458h→**…**
e=2f+1:202+243f↑→f=2g+1:1336+1458g→**…**
♠322+486e→161+243e↑→e=2f:484+1458f→**…**

♦175+162f→263+243f↓→f=2g:263+486g→**…**
f=2g+1: 253+243g↓→g=2h+1: 248+243h (ζ)
g=2h: 253+486h→**…**

Like that, we conclude six kinds of 19+12c derived monogamously from six №1 satisfactory operational results at the above bunch of operational routes of 19+12c, ut infra.

From c=2d and d=2e, we get c=2d=4e, and 19+12c=19+12(4e)=19+48e=

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$19+3\times2^4e\to58+3^2\times2^4e\to29+3^2\times2^3e\to88+3^3\times2^3e\to44+3^3\times2^2e\to22+3^3\times2e$ $\to11+27e$ where mark (α), so it shows that $19+48e\to\ldots\to$smaller $11+27e$.

From $c=2d$, $d=2e+1$ and $e=2f$, we get $c=2d = 2(2e+1) = 4e+2 = 8f+2$, and $19+12c= 19+12(8f+2)= 43+96f= 43+3\times2^5f\to130+3^2\times2^5f\to65+3^2\times2^4f\to 196+3^3\times2^4f\to98+3^3\times2^3f\to49+3^3\times2^2f\to148+3^4\times2^2f\to74+3^4\times2^1f\to37+81f$ where mark (β), so it shows that $43+96f\to\ldots\to$smaller $37+81f$.

From $c=2d+1$, $d=2e+1$ and $e=2f$, we get $c=2d+1=4e+3=8f+3$, and $19+12c =19+12(8f+3)=55+96f=55+3\times2^5f\to166+3^2\times2^5f\to83+3^2\times2^4f\to250+3^3\times2^4f \to125+3^3\times2^3f\to 376+3^4\times2^3f\to 188+3^4\times2^2f\to 94+3^4\times2^1f\to47+81f$ where mark (γ), so it shows that $55+96f\to\ldots\to$smaller $47+81f$.

From $c=2d$, $d=2e+1$, $e=2f+1$, $f=2g+1$ and $g=2h$, we get $c=2d=2(2e+1)= 4e+2=4(2f+1)+2=8f+6=8(2g+1)+6=16g+14=32h+14$, and $19+12c= 19+12(32h+14)=187+384h=187+3\times2^7h\to562+3^2\times2^7h\to281+3^2\times2^6h\to 844+3^3\times2^6h\to422+3^3\times2^5h\to211+3^3\times2^4h\to634+3^4\times2^4h\to317+3^4\times2^3h\to 952+3^5\times2^3h \to476+3^5\times2^2h\to238+3^5\times2^1h\to119+243h$ where mark (δ), so it shows that $187+384h\to\ldots\to$ smaller $119+243h$.

From $c=2d+1$, $d=2e$, $e=2f+1$, $f=2g$ and $g=2h+1$, we get $c=2d+1=4e+1= 4(2f+1)+1=8f+5=16g+5=16(2h+1)+5=32h+21$, and $19+12c=19+12(32h+21) =271+384h=271+3\times2^7h\to814+3^2\times2^7h\to407+3^2\times2^6h\to1222+3^3\times2^6h\to 611+3^3\times2^5h\to1834+3^4\times2^5h\to917+3^4\times2^4h\to2752+3^5\times2^4h\to1376+3^5\times2^3h \to688+3^5\times2^2h\to344+3^5\times2^1h\to172+243h$ where mark (ε), so it shows that $271+384h\to\ldots\to$smaller $172+243h$.

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From c=2d+1, d=2e+1, e=2f+1, f=2g+1 and g=2h+1, we get c=2d+1= 2(2e+1)+1=4e+3=4(2f+1)+3=8f+7=8(2g+1)+7=16(2h+1)+15=32h+31 and 19+12c=19+12(32h+31)=391+384h=391+3×2^7h→1174+3^2×2^7h→587+3^2×2^6h→1762+3^3×2^6h→881+3^3×2^5h→2644+3^4×2^5h→1322+3^4×2^4h→661+3^4×2^3h→1984+3^5×2^3h→992+3^5×2^2h→496+3^5×2^1h→248+243h where mark (ζ), so it shows that 391+384h→…→ smaller 248+243h.

Refer to №1 satisfactory operational results at above two bunches of operational routes of 15+12c and 19+12c and farther verify each kind of 15+12c and 19+12c derived from a №1 satisfactory operational result, we summarize out two points of objective reality as follows.

Firstly, each kind of 15+12c/19+12c derived from a №1 satisfactory operational result and the №1 satisfactory operational result coexist at an operational route of 15+12c /19+12c, and that both share a variable.

Secondly, the greatest exponent of factor 2 of coefficient of variable within each kind of 15+12c/19+12c is exactly the number of times that divided by 2 in the operational course from each kind of 15+12c/19+12c to №1 satisfactory operational result which derives the kind.

Thereinafter, we shall explain the actual state of operational routes, №1 satisfactory operational results and kinds of odd numbers in relation to 15+12c/19+12c and the mutual relation amongst them. Furthermore, emphatically expound that why №1 satisfactory operational results which directly monogamously derive all kinds of 15+12c/19+12c between 2^0n and 2^1n+1 always first appear at operational routes of 15+12c/19+12c?

First, let χ represent intensively variables d, e, f, g, h … etc. within integer's expressions at operational routes of 15+12c/19+12c by the leftwards operational rule. But, χ represents not c.

Then, the odevity of part integer's expressions that contain variable χ at operational routes of 15+12c/19+12c is still indeterminate.

That is to say, for every such integer's expression, both consider it as an odd number to operate, and consider it as an even number to operate. Thus, let us label such integer's expressions "odd-even expressions".

For any odd-even expression at operational routes of 15+12c/19+12c, two kinds of operations synchronize at itself.

After regard an odd-even expression as an odd number to operate, we get a greater operational result>itself. Yet, after regard it as an even number to operate, we get a smaller operational result< itself.

Begin with any odd-even expression to operate by the leftwards operational rule continuously, every such operational route via consecutive greater operational results will be getting longer and longer up to elongate infinitely, and that orderly- emerging odd-even expressions therein will be getting greater and greater up to infinities.

On the other, for a smaller operational result in synchronism with a greater operational result, if it can be divided by 2^μ to get an even smaller integer's expression where $\mu \geq 2$, then, when the even smaller integer's expression is first smaller than a kind of 15+12c/19+12c, the even smaller integer's expression is exactly the №1 satisfactory operational result. Accordingly operations at the operational route may stop at here.

When the even smaller integer's expression is greater than any kind of 15+12c /19+12c still, or the smaller operational result itself is an odd expression, this needs us to continue to operate it.

In other words, at the bunch of operational routes of 15+12c/ 19+12c, on the one hand, odd-even expressions are getting both greater and greater, and more and more, along the continuation of operations, up to infinities and infinitely many.

Accordingly there are infinitely many operational routes of 15+12c /19+12c.

On the other hand, there endlessly stop operations of branches therein, since №1 satisfactory operational results always appear ceaselessly at branches therein.

So there are infinitely many stopped operational routes of 15+12c/19+12c, including infinitely many №1 satisfactory operational results at them.

By this token, 15+12c/19+12c must be divided into infinitely many kinds, just enable that infinitely many №1 satisfactory operational results correspond with infinitely many kinds of 15+12c/19+12c monogamously.

As expected, the variable c of 15+12c /19+12c is able to be endowed with infinitely many natural numbers plus 0, one by one, thus there are

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infinitely many kinds of 15+12c /19+12c authentically.

Nevertheless, what we need is merely to prove all kinds of 15+12c/ 19+12c between 2^0n and 2^1n+1 according to the requirement of third step of the mathematical induction, yet it is not to require all of them.

If let 15+12c=2n+1, figure out c= (n-7)/6; if let 19+12c=2n+1, figure out c=(n-9)/6, so it follows that the number of kinds of 15+12c between 2^0n and 2^1n+1 is smaller than (n-7)/6, and the number of kinds of 19+12c between 2^0n and 2^1n+1 is smaller than (n-9)/6.

From known n≥24, to my way of thinking, if regard n as an infinity, then there are infinitely many positive integers including all odd numbers of 15+12c and 19+12c to suit the conjecture. Although the infinitely more are unequal to the all, but if enter into an infinite field, then positive integers inside the infinite field have not the distinction of large and small, or many and little. Thus we have no occasion to do the proof in all senses.

If regard n as a finite-large positive integer, then 2n+1 is a finite-large odd number, thereupon each of odd numbers of 15+12c and 19+12c between 2^0n and 2^1n+1 is a finite-large odd number. Of course, their number of kinds is a finite number too, i.e. a positive integer which is smaller than (n-7)/6 or (n-9)/6 is a finite number.

At the bunch of operational routes of 15+12c/19+12c by the leftwards operational rule, for each operational route therein, either it is operated to get a №1 satisfactory operational result, or it is operated up to an infinity on and on, and that in the latter case, integer’s expressions are getting greater and greater along the continuation of operations.

Undoubtedly, odd numbers of 15+12c/19+12c between 2^0n and 2^1n+1 are smallest or smaller as compared with kindred odd numbers. Or say, the coefficient of χ and the constant term of each kind of 15+12c/19+12c between 2^0n and 2^1n+1 are smallest or smaller as compared with other kinds of 15+12c/19+12c.

As thus, we can determine all kinds of 15+12c/19+12c between 2^0n and 2^1n+1 so as operate for finite times by the leftwards operational rule to first get №1 satisfactory operational results concerned, then derive them from these №1 satisfactory operational results concerned monogamously.

Apply the leftwards operational rule to operate, for №1 satisfactory operational results which derive all kinds of 15+12c /19+12c between 2^0n and 2^1n+1 monogamously, since they have smaller constant terms and smaller coefficients of χ as compared with others, so they always first appear at operational routes of 15+12c /19+12c.

When the number of smaller №1 satisfactory operational results reaches the number of kinds of 15+12c/19+12c between 2^0n and 2^1n+1 just, we can deduce exactly all kinds of 15+12c/19+12c between 2^0n and 2^1n+1 from these №1 satisfactory operational results monogamously.

Such being the case, how know that a smaller №1 satisfactory operational result is inside suitable limits? So we act in accordance with such a norm to judge right or wrong, i.e. compare the sum of coefficient of χ plus constant term of a kind of 15+12c/19+12c derived from the smaller №1 satisfactory operational result whether is not greater than 2n.

If the sum is not greater than 2n, then the smaller №1 satisfactory operational result is desirable.

If the sum is greater than 2n, then the smaller №1 satisfactory operational result can not be counted in the number of kinds of 15+12c/19+12c between 2^0n and 2^1n+1.

Therefore, all kinds of 15+12c/19+12c between 2^0n and 2^1n+1 derived monogamously from smaller №1 satisfactory operational results are always to get first them as compared with others.

So much for, the explanation that first get all kinds of 15+12c/19+12c between 2^0n and 2^1n+1 in the course of operations should be said clearly.

Yet, in order to impress upon everyone the aforesaid conclusion and dispel minority doubts, we might as well again give a concrete example in which case suppose $2^Xn+1=193$ where X=3 and n=24, to practically explain that odd numbers of 15+12c and 19+12c between 24 and $2^3\times24+1$ always are first got within kindred odd numbers in the course of operations by the leftwards operational rule.

First, on the basis of preceding proven 8 kinds of 15+12c and 19+12c concerned, let variable χ be endowed with suitable values, such that each such odd number exists between 24 and $2^3\times24+1$, as listed below.

χ,	51+48e,	75+96f,	87+96f,	135+384h,	19+48e,	43+96f,	55+96f,	187+384h
0:	51,	75,	87,	135,		43,	55,	187
1:	99,	171,	183,		67,	139,	151	
2:	147,				115,			
3:					163,			

Listed above odd numbers of 15+12c and 19+12c between 24 and $2^3\times24+1$ are 43, 51, 55, 67, 75, 87, 99, 115, 135, 139, 147, 151, 163, 171, 183 and 187.

Yet, odd numbers of 15+12c and 19+12c between 24 and $2^3\times24+1$ have altogether 27, 31, 39, 43, 51, 55, 63, 67, 75, 79, 87, 91, 99, 103, 111, 115, 123, 127, 135, 139, 147, 151, 159, 163, 171, 175, 183 and 187, therein 12 underlined odd numbers are absentees in the above list.

The absent reason is due to unable to show overlong operational routes at listed above two bunches of operational routes of 15+12c and 19+12c.

So, we operate each of 12 such odd numbers all alone to an integer<itself, and that point out which kind of belongingness of each of them, ut infra.

(1) From 27→82→41→124→62→$31^{\&}$→94→47→142→71→214→107→322→161→484→242→121→364*→182→$91^{\#}$→274→137→412→206→$103^{\#\#}$→ 310→ 155→466→233→700→350→$175^{\$\$}$→526→263→790→ 395→ 1186→ 593→ 1780→890→445→1336→668→334**→167→502→ 251→ 754→ 377→ 1132→566→283→850→425→1276→638→319→958→479→1438→719→2158→1079→3238→1619→4858→2429→

7288→3644→1822***→911→2734→1367→4102→2051→6154→3077→ 9232→ 4616→ 2308→1154→577→1732→866→433→ 1300→ 650→ 325→ 976→ 488→ 244→122→ 61→184→92→46→23, it shows that 27→…→ smaller 23, and 27 belongs within $27+2^{59}\times3y$.

In addition, several signs except for arrowheads at above operational route of 27 will be cited by latter certain operational routes respectively.

(2) From 31&— connect to the operational route of 27 →… →23, it shows that 31→…→smaller 23, and 31 belongs within $31+2^{56}\times3w$.

(3) From 39→118→59→178→89→268→134→67→202→101→304→ 152→76→38, it shows that 39→…→smaller 38, and 39 belongs within $39+2^{8}\times3k$.

(4) From 63→190→95→286→143→430→215→646→323→970→485 →1456→728→364*—connect to the operational route of 27→…→ 61, it shows that 63→…→smaller 61, and 63 belongs within $63+2^{54}\times3w$.

(5) From 79→238→119→358→179→538→269→808→404→202←67, it shows that 79→202←smaller 67, and 79 belongs within $79+2^{5}\times3j$.

If odd number 67 suits the conjecture, then the theorem 2 is cited here.

(6) From 91#—connect to the operational route of 27→…→ 61, it shows that 91→…→smaller 61, and 91 belongs within $91+2^{45}\times3v$.

(7) From 103##—connect to the operational route of 27→…→ 61, it shows that 103→…→smaller 61, and 103 belongs within $103+2^{42}\times3u$.

(8) From 111→334**—connect to the operational route of 27→…→61, it

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shows that 111→…→smaller 61, and 111 belongs within $111+2^{31}\times3q$.

(9) From 123→370→185→556→278→139→418→209→628→314→157→472→236→118, it shows that 123→…→smaller 118, and 123 belongs within $123+2^{8}\times3m$.

(10) From 127→382→191→574→287→862→431→1294→647→1942→971 →2914→1457→4372→2186→1093→3280→1640→820→410→205→616→308→154→77, it shows that 127→…→smaller 77, and 127 belongs within $127+2^{15}\times3q$.

(11) From 159→478→239→718→359→1078→539→1618→809→2428→1214→607→1822***—connect to the operational route of 27→…→ 122, it shows that 159→…→smaller 122, and 159 belongs within $159+2^{21}\times3s$.

(12) From 175$^{\$\$}$—connect to the operational route of 27→…→167, it shows that 175→…→smaller 167, and 175 belongs within $175+2^{8}\times3m$.

To sum up, there are altogether 28 odd numbers of 20 kinds of 15+12c and 19+12c between 24 and $2^{3}\times24+1$, and that they are first found out within kindred odd numbers. They are: 19+48e (contains 67, 115 and 163), $27+2^{59}\times3y$(contains 27), $31+2^{56}\times3w$(contains 31), $39+2^{8}\times3k$(contains 39), 43+96f (contains 43 and 139), 51+48e (contains 51, 99 and 147), 55+96f (contains 55 and 151), $63+2^{54}\times3w$(contains 63), 75+96f (contains 75 and 171), $79+2^{5}\times3j$(contains 79), 87+96f (contains 87 and 183), $91+2^{45}\times3v$(contains 91), $103+2^{42}\times3u$(contains 103), $111+2^{31}+3q$(contains 111), $123+2^{8}\times3m$(contains 123), $127+2^{15}\times3q$(contains 127), 135+384h(contains 135), $159+2^{21}\times3s$(contains

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159), $175+2^8\times3m$(contains 175) and $187+384n$(contains 187).

Of course, χ represents variables within odd numbers' belongingness too.

If continue to operate foregoing two bunches of operational routes of 15+12c and 19+12c, then you will get inevitably all №1 satisfactory operational results which derive listed above absent kinds of 15+12c and 19+12c monogamously, so long as the display surface is enough big.

Evidently, a constant term of each such №1 satisfactory operational result is exactly first emerging smallest integer at above each operational route, excepting 79→ …→202←67.

After variables of all kinds of 15+12c and 19+12c between 24 and $2^3\times24+1$ are endowed with 0, 1, 2 and 3 for more or less, we get all odd numbers of 15+12c and 19+12c between 24 and $2^3\times24+1$.

By this token, each such odd number of 15+12c and 19+12c between 2^0n and 2^1n+1 is smallest or smaller one within kindred odd numbers.

Besides, we are not difficult to discover that many odd numbers coexist at operational routes of 15+12c and 19+12c, such as certain odd numbers at operational route of 27.

If X>3 and/or n>24, likewise, we can prove that odd numbers of 15+12c/19+12c between 2^0n and 2^Xn+1 are got first in the same way.

Overall, №1 satisfactory operational results relating to kinds of 15+12c/ 19+12c between 2^0n and 2^1n+1 in operations appear first at the bunch of operational routes of 15+12c/19+12c by the leftwards operational rule, then first derive kinds of 15+12c/19+12c between 2^0n and 2^1n+1 from №1 satisfactory operational results got, next first get odd numbers of 15+12c /19+12c between 2^0n and 2^1n+1within kindred odd numbers.

Erenow, we have proven that every genus, every sort and every kind of odd numbers between 2^0n and 2^1n+1 are operated to get a smaller integer's expression respectively, or begin with a smaller integer's expression via operations to get a classification of odd numbers, and that each and every classification of odd numbers and a homologous smaller integer's expression share a variable.

As thus, after variables within these genera, sorts, kinds of odd numbers and homologous smaller integer's expressions are endowed with 0 and from small to large part natural numbers respectively, if therein at least one value of homologous smaller integer's expression exists between 0 and 2^0n+1, then at the least one concerned genus, sort, or kind of odd numbers between 2^0n and 2^1n+1 suits the conjecture according to the theorem 1 or the theorem 2. Here, why refer to the theorem 2? As there may are such cases like 79→ …→202←smaller 67, similarly hereinafter.

Now that each and every odd number between 2^0n and 2^1n+1 belongs within a classification, thus, anyhow, at least the smallest odd number n+1 or n+2 between 2^0n and 2^1n+1 like its belongingness has been operated to get an even smaller integer, so n+1 or n+2 suits the conjecture for the classification of belongingness of n+1 or n+2 suits the conjecture.

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Thereinbefore, we have found out all kinds of odd numbers between 2^0n and 2^1n+1 according as the sum of coefficient of χ plus constant term is from small to large, and that their variables have been endowed with 0 and from small to large part natural numbers respectively.

Thus, on the basis of which at least n+1 or n+2 suits the conjecture, we can orderly extract odd numbers between 2^0n and 2^1n+1, one by one, according to the order from small to large until 2n-1.

Even though smallest value of some smaller integer's expression exists between 2^0n and 2^1n+1, likewise, we can extract it according to the order from small to large. Because the smallest value belongs within a kind of odd numbers between 2^0n and 2^1n+1 too, then the kind of odd numbers has been operated to an even smaller integer's expression according to preceding way of doing. If the even smaller integer's expression exists still between 2^0n and 2^1n+1, then, as like reason, the rest may be deduced by analogy, until an even smaller integer's expression exists under 2^0n+1.

By this token, all odd numbers between 2^0n and 2^1n+1 exist at the bunch of integers' chains, for they were proven to suit the conjecture.

In addition, all even numbers between 2^0n and 2^1n+1 exist at the bunch of integers' chains originally.

Therefore, all integers between 2^0n and 2^1n+1 exist at the bunch of integers' chains. Consequently, all integers between 2^0n and 2^1n+1 suit the conjecture according to the inference that generating positive integers by two-way operational rules are one-to-one correspondence and the same.

Hereto, we have proven that positive integers$\leq 2^1$n suit the conjecture by consecutive positive integers$\leq 2^0$n. Like that, we can too prove that positive integers$\leq 2^2$n suit the conjecture by consecutive positive integers$\leq 2^1$n, and so on and so forth, up to prove positive integers$\leq 2^X$n suit the conjecture, in the light of the same way, where X≥3, and n≥24.

For greatest positive integer 2^Xn per segment, X begins with 0, next it is endowed with 1, 2, 3 etc natural numbers in proper order, then consecutive positive integers$\leq 2^X$n are getting both more and more, and greater and greater at longer and longer segments. After X is equal to each of natural numbers plus 0, all positive integers are proven to suit the conjecture.

Namely every positive integer is proven to suit the conjecture, so that the Collatz conjecture is proven as the true fully.

The proof was thus brought to a close. As a consequence, the Collatz conjecture holds water.

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