# Test of General Relativity Theory by Investigating the Conservation of Energy in a Relativistic Free Fall in the Uniform Gravitational Field

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**Abstract:** This paper investigates the General Relativity Theory (GRT) by studying the relativistic free fall of a small test body in a uniform gravitational field. The paper compares the predictions of energy loss, perhaps by radiation, in a free fall obtained from the GRT and from the Metric Theory of Gravity (MTG). It is found that the gravitational mass dependence on velocity in GRT is not correct, because it predicts a negative loss of energy while the MTG predicts correctly a positive loss.

**Introduction:** The theories describing the free fall are well understood in both; the GRT and the MTG. In the GRT the inertial mass and the gravitational mass are assumed identical with identical dependencies on velocity. In the MTG, on the other hand, the gravitational mass depends on velocity differently than the inertial mass [1, 2]. It is thus simple for both theories to derive equations describing the free fall velocity and from that the energy loss of a small test body that falls in a uniform gravitational field.

**Theories:** In the GRT the relation between the velocity *v* and time is somewhat more complicated but can be easily derived as follows:

$$\frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right] = g\frac{m_0}{\sqrt{1-v^2/c^2}} \tag{1}$$

where $m_0$ is the rest mass and *c* the speed of light in a vacuum. The left hand side of Eq.1 is the relativistic formula for the inertial force and the right hand side is the formula for the gravitational force that includes the gravitational mass dependence on velocity. The formula in Eq.1 can be rearranged and simplified resulting in the following relation for the small body acceleration:

$$\frac{dv}{dt} = g(1-v^2/c^2) \tag{2}$$

The energy loss will be calculated by comparing the potential energy that is obtained by lifting the small test body very slowly in the uniform field by a distance *z* to the energy of the falling body. The test body potential energy is simply expressed as follows:

$$E = m_0 g \cdot z \tag{3}$$

This relation will be kept as reference energy even if the test body may move fast. For the actual falling body energy the incremental energy gain by a fall can be expressed in terms of the velocity *v* and the gravitational force *F* as follows:

$$dE = F \cdot v \cdot \frac{dv}{\left(\dfrac{dv}{dt}\right)} = \frac{g \cdot m_0}{\sqrt{1-\dfrac{v^2}{c^2}}} \frac{v \cdot dv}{g\left(1-\dfrac{v^2}{c^2}\right)} \tag{4}$$

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After rearrangements and integration the expression for the energy as a function of velocity becomes:

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \qquad (5)$$

This result is expected and it is a nice confirmation of methodology used in Eq.1.

In the next step we will evaluate the energy difference $\Delta E$ given by Eq.3 and Eq.5. However, for the convenience of calculations it will be useful to first evaluate the time derivative of this difference.

$$\frac{d\Delta E}{dt} = m_0 g \cdot v - \frac{m_0 v}{\left(1 - v^2/c^2\right)^{3/2}} \frac{dv}{dt} \qquad (6)$$

In this formula the variable $v$ in the first term was substituted for the time derivative of $z$. By substituting for the acceleration from Eq.2 the result becomes:

$$\frac{d\Delta E}{dt} = m_0 g \cdot v \left( 1 - \frac{1}{\sqrt{1 - v^2/c^2}} \right) \cong -\frac{1}{2} m_0 g \cdot \frac{v^3}{c^2} \qquad (7)$$

By integrating this result in the velocity domain, similarly as it was shown in Eq.4, we obtain the expression for the total energy difference loss during the fall:

$$\Delta E \cong -\frac{m_0 c^2}{8} \left( \frac{v}{c} \right)^4 \qquad (8)$$

This is a very strange result. It seems that the falling body is gaining some additional energy on top of the energy that is predicted from the free fall by Eq.5. This is not reasonable and it is pointing to a problem that exists in the GRT for a long time. The gravitational mass cannot depend on velocity the same way as the inertial mass. This problem will become clear from the result presented next.

The similar expression introduced in Eq.1 is used, but with the gravitational mass depending on velocity as follows [1, 2]:

$$m_g = m_0 \sqrt{1 - v^2/c^2} \qquad (9)$$

This leads to the following formula:

$$\frac{d}{dt} \left[ \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \right] = g \cdot m_0 \sqrt{1 - \frac{v^2}{c^2}} \qquad (10)$$

After rearrangements the formula is simplified with the result as follows:

$$\frac{dv}{dt} = g(1 - v^2/c^2)^2 \qquad (11)$$

Following the same procedure as above for the GRT case the differential of energy will be:

$$dE = g \cdot m_0 \sqrt{1 - \frac{v^2}{c^2}} \; \frac{v \cdot dv}{g\left(1 - \frac{v^2}{c^2}\right)^2} \tag{12}$$

This becomes, after integration, identical to formula in Eq.5. Both theories, the GRT and MTG, thus give the same expression for the energy, which is expected and confirms once more that the calculating procedure is correct. For the energy loss the same procedure is also followed with the result:

$$\frac{d\Delta E}{dt} = m_0 g \cdot v - \frac{m_0 v}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \frac{dv}{dt} = m_0 g \cdot v \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right) \cong \frac{1}{2} m_0 g \frac{v^3}{c^2} \tag{13}$$

This is a similar result as in Eq.7 except that the energy loss differential is now positive as it should be. This confirms the correctness of the gravitational mass dependence on velocity and therefore disproves the validity of GRT. The energy loss is likely due to the gravitational radiation, because the falling body is accelerated. By integrating the result from Eq.13 the energy loss is equal to:

$$\Delta E \cong \frac{m_0 c^2}{8}\left(\frac{v}{c}\right)^4 \tag{14}$$

The result in Eg.14 is derived again in Appendix with more calculation details:

For a better understanding of the magnitude of radiated energy the graph of the energy loss as a function of the fall time for a mass of 100kg is shown in FIG.1.

**The relativistic energy conservation test for a Free Falling body in uniform g field**

$g = 9.80665 \, \dfrac{m}{s^2}$    $c = 2.997925 \times 10^8 \, \dfrac{m}{s}$    $t := 0, 0.1 .. 30$    $vnt(t) := g \cdot t$    $mo := 100.0 \cdot kg$

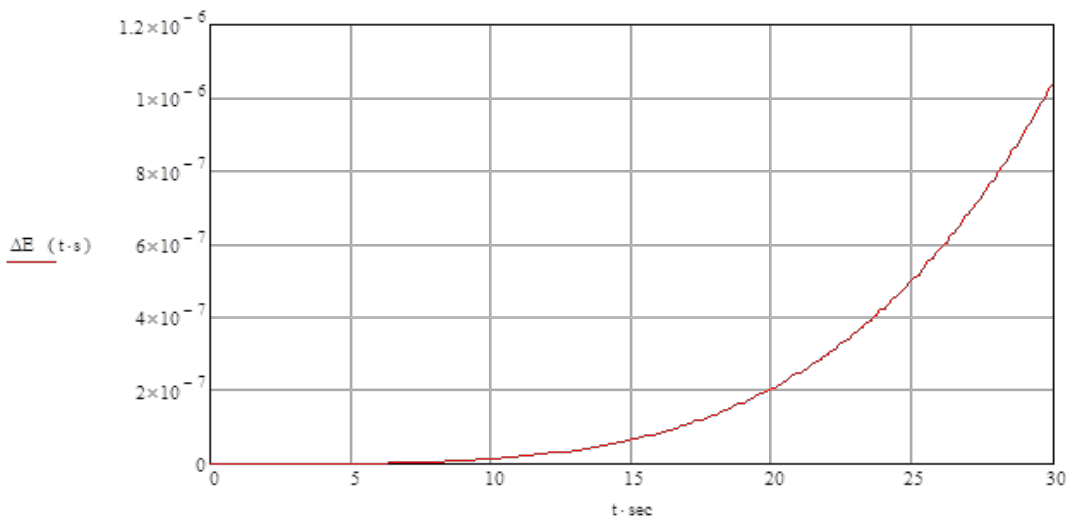$\Delta E\,(t) := \dfrac{mo \cdot c^2}{8} \cdot \left(\dfrac{g \cdot t}{c}\right)^4$



FIG.1 the dependency of energy loss due to radiation for a 100kg test body free falling in a uniform gravitational field equal to Earth's gravity. The loss for a 30 sec fall is about 1.0 micro Joules.

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It may sometimes be useful to express the energy loss in terms of the fallen distance instead of the velocity. This can be derived starting from Eq.11 as follows:

$$\frac{dv}{\left(1-v^2/c^2\right)^2} = g\frac{dt}{dz}dz \tag{15}$$

After integration and rearrangements the relation between the fallen distance and the velocity becomes equal to:

$$1+\frac{2g\cdot z}{c^2} = \frac{1}{\left(1-v^2/c^2\right)} \tag{16}$$

By substituting this result into the formula for the energy difference and considering the mass dependence on velocity as given in Eq.9 the result becomes as follows:

$$\Delta E = m_0 g\cdot z + m_0 c^2 - m_0 c^2 \sqrt{1+\frac{2g\cdot z}{c^2}} \tag{17}$$

In this equation the rest mass equivalent energy was added to the reference energy to be consistent with the rest energy of the falling body. It is necessary that for the zero distance the energy difference and the velocity of falling body are both zero. By expanding the result from Eq.17 into a power series and neglecting higher order terms the energy loss will be as follows:

$$\Delta E \cong \frac{m_0 c^2}{8}\left(\frac{2g\cdot z}{c^2}\right)^2 \cong \frac{m_0 c^2}{8}\left(\frac{v^2}{c^2}\right)^2 \tag{18}$$

The graph of this dependency is shown for convenience in FIG.2.

**MTG energy conservation test in a Free Fall in terms of the fallen distance z**

$$g = 9.80665\frac{m}{s^2} \qquad z := 0\cdot m, 0.2\cdot m.. 100\cdot m \qquad mo := 100.0\cdot kg \qquad \Delta E(z) := \frac{mo\cdot c^2}{2}\cdot\left(\frac{g\cdot z}{c^2}\right)^2$$
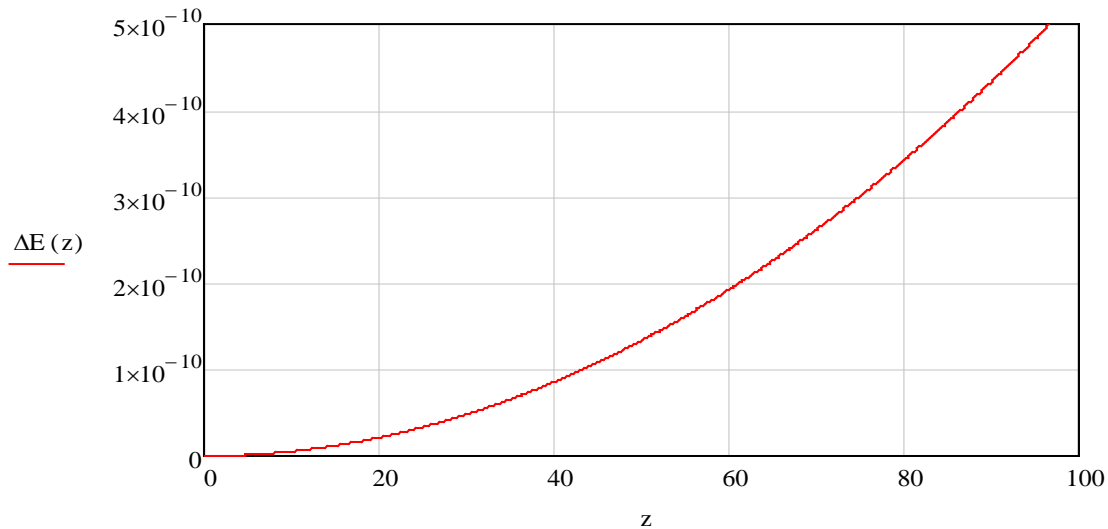


Fig.2 the dependency of energy loss for a 100kg test body free falling in a uniform gravitational field.

**Conclusions:** The paper derived simple expressions for the energy loss during the small body free fall in a uniform gravitational field. It was shown that the loss derived according to the GRT is negative. This is unacceptable and this fact thus disproves the validity of GRT. This problem has its root cause in the identical dependency of inertial mass and gravitational mass on velocity. When the correct dependency of gravitational mass on velocity, as derived in the MTG, is used the correct positive energy loss is calculated.

This result has fatal consequences for the GRT, because unquestionably proves its incorrectness. This finding thus has a significant impact on all the theories based on the GRT such as the Big Bang and similar ridiculous models of the Universe.

The author hopes that the main stream relativists finally recognize this problem and abandon the GRT with all its ridiculous claims of existence of Black Holes, Event Horizons, and the Big Bang Universe with its accelerating expansion to infinity from nothing.

**References:**

[1] http://gsjournal.net/Science-Journals/Research%20Papers/View/7070

[2] http://physicsessays.org/browse-journal-2/product/904-8-jaroslav-hynecek-remarks-on-the-equivalence-of-inertial-and-gravitational-masses-and-on-the-accuracy-of-einstein-s-theory-of-gravity.html

**Appendix:** From Eq.3 we have for the time derivative of the reference energy the following:

$$\frac{dE}{dt} = m_0 g \frac{dz}{dt} = m_0 g \cdot v \tag{A1}$$

The reference energy time derivative should remain unchanged for small velocities as well as for large velocities. However, for the energy derivative of the falling body we must include the mass dependence on velocity from Eq.9. This results in the following expression for the derivative of $\Delta E$.

$$\frac{d\Delta E}{dv}\frac{dv}{dt} = m_0 g \cdot v \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right) \tag{A2}$$

Using the formula from Eq.11 and substituting it into Eq.A2, the result becomes as follows:

$$\frac{d\Delta E}{dv} = \frac{m_0 v}{\left(1 - v^2/c^2\right)^2} - \frac{m_0 v}{\left(1 - v^2/c^2\right)^{3/2}} \tag{A3}$$

After integration the energy difference is found to be:

$$\Delta E = \int_0^v \frac{m_0 v\, dv}{\left(1 - v^2/c^2\right)^2} - \int_0^v \frac{m_0 v\, dv}{\left(1 - v^2/c^2\right)^{3/2}} = \frac{m_0 c^2}{2\left(1 - v^2/c^2\right)} - \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} + \frac{m_0 c^2}{2} \tag{A4}$$

This result can be expanded into a power series leading to the following final expression:

$$\Delta E = \frac{m_0 c^2}{2}\left[1 + \left(1 + v^2/c^2 + v^4/c^4 + ...\right) - \left(2 + v^2/c^2 + \frac{3}{4} v^4/c^4 + ...\right)\right] \cong \frac{m_0 c^2}{8}\left(\frac{v}{c}\right)^4 \tag{A5}$$