Gravitoelectromagnetism. I. Gravitational Fraday's Law and Gravitational Ampère's Law

Zbigniew Osiak

E-mail: zbigniew.osiak@gmail.com

http://orcid.org/0000-0002-5007-306X

http://vixra.org/author/zbigniew_osiak

Abstract

From the modified Maxwell-Hertz equations in three-dimensional form, it appears that gravitational waves have an effect on electromagnetic phenomena. Therefore, a very simple method of detection of gravitational fields with a time-varying value of the determinant of the metric tensor is possible.

Keywords: Maxwell-Hertz equations, determinant of the metric tensor, gravitational waves.

01. Introduction

To take a look at what physical contents contain generally covariant Maxwell-Hertz equations, we will write them in a modified three-dimensional form. We will discuss the interaction of the non-stationary gravitational field with a stationary magnetic field and a stationary electric field.

02. Maxwell-Hertz equations

Equations describing the electromagnetic field were formulated by Maxwell in 1865 [1]. The modern form of Maxwell equations was given by Hertz in 1890 [2].

$$\mathrm{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \mathrm{div}\mathbf{B} = 0$$

$$\mathrm{rot}\mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \qquad \mathrm{div}\mathbf{D} = \rho$$

E – vector of electric field intensity
D – vector of electric induction
B – vector of magnetic induction
H – vector of magnetic field intensity
j – vector of current density
ρ – electric charge density

03. Maxwell-Hertz equations in special relativity

Maxwell-Hertz equations were written in 1908 by Minkowski in a four-dimensional tensor form within special relativity [3].

$$\left.\begin{array}{l}\mathrm{rot}\mathbf{E}=-\dfrac{\partial\mathbf{B}}{\partial t}\\ \mathrm{div}\mathbf{B}=0\end{array}\right\}\to\sum_{\nu=1}^{4}\frac{\partial E^{\mu\nu}}{\partial x^{\nu}}=0\,,\qquad \left.\begin{array}{l}\mathrm{rot}\mathbf{H}=\mathbf{j}+\dfrac{\partial\mathbf{D}}{\partial t}\\ \mathrm{div}\mathbf{D}=\rho\end{array}\right\}\to\sum_{\nu=1}^{4}\frac{\partial H^{\mu\nu}}{\partial x^{\nu}}=J^{\mu}\,,\qquad(\mu=1,2,3,4)$$

$$E^{\mu\nu}=\begin{bmatrix}0 & iE_z & -iE_y & -cB_x\\ -iE_z & 0 & iE_x & -cB_y\\ iE_y & -iE_x & 0 & -cB_z\\ cB_x & cB_y & cB_z & 0\end{bmatrix}\qquad H^{\mu\nu}=\begin{bmatrix}0 & H_z & -H_y & -icD_x\\ -H_z & 0 & H_x & -icD_y\\ H_y & -H_x & 0 & -icD_z\\ icD_x & icD_y & icD_z & 0\end{bmatrix}$$

J^{μ} – component of the four-vector current density

04. Maxwell-Hertz equations in general relativity

Maxwell-Hertz equations in the generally covariant form were presented independently by Kottler in 1912 [4], Einstein in 1913 [5], 1914 [6], and 1916 [7], Cartan in 1923-1924 [8, 9], and van Dantzig in 1934 [10].

$$\boxed{\frac{\partial\left(\sqrt{g}E^{\mu\nu}\right)}{\partial x^{\nu}}=0}\qquad\boxed{\frac{\partial\left(\sqrt{g}H^{\mu\nu}\right)}{\partial x^{\nu}}=\sqrt{g}J^{\mu}}$$

$$\begin{aligned}g=\;&g_{11}\left(g_{22}g_{33}g_{44}+g_{23}g_{34}g_{42}+g_{24}g_{32}g_{43}-g_{42}g_{33}g_{24}-g_{43}g_{34}g_{22}-g_{44}g_{32}g_{23}\right)+\\ &+g_{12}\left(g_{41}g_{33}g_{24}+g_{43}g_{34}g_{21}+g_{44}g_{31}g_{23}-g_{21}g_{33}g_{44}-g_{23}g_{34}g_{41}-g_{24}g_{31}g_{43}\right)+\\ &+g_{13}\left(g_{21}g_{32}g_{44}+g_{22}g_{34}g_{41}+g_{24}g_{31}g_{42}-g_{41}g_{32}g_{24}-g_{42}g_{34}g_{21}-g_{44}g_{31}g_{22}\right)+\\ &+g_{14}\left(g_{41}g_{32}g_{23}+g_{42}g_{33}g_{21}+g_{43}g_{31}g_{22}-g_{21}g_{32}g_{43}-g_{22}g_{33}g_{41}-g_{23}g_{31}g_{42}\right)\end{aligned}$$

g – determinant of the metric tensor of space-time

05. Determinant of the Schwarzschild metric tensor

The determinant of the Schwarzschild space-time metric tensor equals one [11]. The stationary gravitational field of the Earth can be described in the first approximation with Schwarzschild metric and therefore we do not observe the noticeable influence of our field on electromagnetic phenomena.

$$\begin{aligned}ds^2=&\left\{\frac{x^2}{r^2}\left[\left(1-\frac{r_S}{r}\right)^{-1}-1\right]+1\right\}dx^2+\left\{\frac{y^2}{r^2}\left[\left(1-\frac{r_S}{r}\right)^{-1}-1\right]+1\right\}dy^2+\\ &+\left\{\frac{z^2}{r^2}\left[\left(1-\frac{r_S}{r}\right)^{-1}-1\right]+1\right\}dz^2-\left(1-\frac{r_S}{r}\right)c^2dt^2+\\ &+\frac{2}{r^2}\left[\left(1-\frac{r_S}{r}\right)^{-1}-1\right]\left(xy\,dxdy+xz\,dxdz+yz\,dydz\right),\qquad r_S=\frac{2GM}{c^2},\end{aligned}$$

$g\;=\;1.$

06. Modified Maxwell-Hertz equations

$$\left.\begin{matrix}\mathrm{rot}\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \mathrm{div}\mathbf{B} = 0\end{matrix}\right\} \to \sum_{\nu=1}^{4} \frac{\partial E^{\mu\nu}}{\partial x^{\nu}} = 0 \to \sum_{\nu=1}^{4} \frac{\partial \sqrt{g} E^{\mu\nu}}{\partial x^{\nu}} = 0 \to \left\{\begin{matrix}\mathrm{rot}\sqrt{g}\mathbf{E} = -\dfrac{\partial \sqrt{g}\mathbf{B}}{\partial t} \\ \mathrm{div}\sqrt{g}\mathbf{B} = 0\end{matrix}\right.$$

$$\left.\begin{matrix}\mathrm{rot}\mathbf{H} = \mathbf{j} + \dfrac{\partial \mathbf{D}}{\partial t} \\ \mathrm{div}\mathbf{D} = \rho\end{matrix}\right\} \to \sum_{\nu=1}^{4} \frac{\partial H^{\mu\nu}}{\partial x^{\nu}} = J^{\mu} \to \sum_{\nu=1}^{4} \frac{\partial \sqrt{g} H^{\mu\nu}}{\partial x^{\nu}} = \sqrt{g} J^{\mu} \to \left\{\begin{matrix}\mathrm{rot}\sqrt{g}\mathbf{H} = \sqrt{g}\mathbf{j} + \dfrac{\partial \sqrt{g}\mathbf{D}}{\partial t} \\ \mathrm{div}\sqrt{g}\mathbf{D} = \sqrt{g}\rho\end{matrix}\right.$$

$(\mu = 1,2,3,4)$

$$E^{\mu\nu} = \begin{bmatrix} 0 & iE_z & -iE_y & -cB_x \\ -iE_z & 0 & iE_x & -cB_y \\ iE_y & -iE_x & 0 & -cB_z \\ cB_x & cB_y & cB_z & 0 \end{bmatrix} \qquad H^{\mu\nu} = \begin{bmatrix} 0 & H_z & -H_y & -icD_x \\ -H_z & 0 & H_x & -icD_y \\ H_y & -H_x & 0 & -icD_z \\ icD_x & icD_y & icD_z & 0 \end{bmatrix}$$

Below we will further analyze the modified homogeneous Maxwell-Hertz equations.

$$\boxed{\begin{matrix}\mathrm{rot}\left(\sqrt{g}\mathbf{E}\right) = -\dfrac{\partial}{\partial t}\left(\sqrt{g}\mathbf{B}\right) \\ \mathrm{div}\left(\sqrt{g}\mathbf{B}\right) = 0\end{matrix}}$$

$$\mathrm{rot}(\varphi\mathbf{a}) = \varphi\,\mathrm{rot}\,\mathbf{a} + (\mathrm{grad}\,\varphi)\times\mathbf{a}$$
$$\mathrm{div}(\varphi\mathbf{a}) = \varphi\,\mathrm{div}\,\mathbf{a} + \mathbf{a}\cdot\mathrm{grad}\,\varphi$$

$$\boxed{\begin{matrix}\mathrm{rot}\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} - \dfrac{\mathbf{B}}{2g}\dfrac{\partial g}{\partial t} - \dfrac{1}{2g}(\mathrm{grad}\,g)\times\mathbf{E} \\ \mathrm{div}\mathbf{B} = -\dfrac{1}{2g}\mathbf{B}\cdot(\mathrm{grad}\,g)\end{matrix}}$$

Assumption
$\mathrm{grad}\,g = 0$

$$\boxed{\begin{matrix}\mathrm{rot}\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} - \dfrac{\mathbf{B}}{2g}\dfrac{\partial g}{\partial t} \\ \mathrm{div}\mathbf{B} = 0\end{matrix}}$$

$$\iint_S \mathrm{rot}\mathbf{A}\cdot d\mathbf{S} = \oint_l \mathbf{A}\cdot d\mathbf{l}, \quad \iiint_V \mathrm{div}\mathbf{A}\, dV = \oiint_S \mathbf{A}\cdot d\mathbf{S}, \quad \iint_S \frac{\partial \mathbf{A}}{\partial t}\cdot d\mathbf{S} = \frac{\partial}{\partial t}\iint_S \mathbf{A}\cdot d\mathbf{S}$$

$$\oint_l \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S} - \iint_S \left(\frac{\mathbf{B}}{2g}\frac{\partial g}{\partial t}\right)\cdot d\mathbf{S} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S} - \frac{1}{2g}\left(\iint_S \mathbf{B}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

$$\oiint_S \mathbf{B}\cdot d\mathbf{S} = 0$$

$$\mathrm{SEM} \overset{\mathrm{df}}{=} \oint_l \mathbf{E}\cdot d\mathbf{l}$$

$$\mathrm{SEM} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S} - \frac{1}{2g}\left(\iint_S \mathbf{B}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

Below we will further analyze the modified inhomogeneous Maxwell-Hertz equations.

$$\mathrm{rot}\left(\sqrt{g}\,\mathbf{H}\right) = \sqrt{g}\,\mathbf{j} + \frac{\partial}{\partial t}\left(\sqrt{g}\,\mathbf{D}\right)$$

$$\mathrm{div}\left(\sqrt{g}\,\mathbf{D}\right) = \sqrt{g}\,\rho$$

$$\mathrm{rot}(\varphi\mathbf{a}) = \varphi\,\mathrm{rot}\,\mathbf{a} + (\mathrm{grad}\,\varphi)\times\mathbf{a}$$

$$\mathrm{div}(\varphi\mathbf{a}) = \varphi\,\mathrm{div}\,\mathbf{a} + \mathbf{a}\cdot\mathrm{grad}\,\varphi$$

$$\mathrm{rot}\mathbf{H} = \mathbf{j} + \frac{\partial\mathbf{D}}{\partial t} + \frac{\mathbf{D}}{2g}\frac{\partial g}{\partial t} - \frac{1}{2g}(\mathrm{grad}\,g)\times\mathbf{H}$$

$$\mathrm{div}\mathbf{D} = \rho - \frac{1}{2g}\mathbf{D}\cdot\mathrm{grad}\,g$$

Assumption

$\mathrm{grad}\,g = 0$

$$\mathrm{rot}\mathbf{H} = \mathbf{j} + \frac{\partial\mathbf{D}}{\partial t} + \frac{\mathbf{D}}{2g}\frac{\partial g}{\partial t}$$

$$\mathrm{div}\mathbf{D} = \rho$$

$$\iint_S \mathrm{rot}\mathbf{A}\cdot d\mathbf{S} = \oint_l \mathbf{A}\cdot d\mathbf{l},\quad \iiint_V \mathrm{div}\mathbf{A}\,dV = \oiint_S \mathbf{A}\cdot d\mathbf{S},\quad \iint_S \frac{\partial\mathbf{A}}{\partial t}\cdot d\mathbf{S} = \frac{\partial}{\partial t}\iint_S \mathbf{A}\cdot d\mathbf{S}$$

$$\oint_l \mathbf{H}\cdot d\mathbf{l} = \iint_S \mathbf{j}\cdot d\mathbf{S} + \frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S} + \iint_S \left(\frac{\mathbf{D}}{2g}\frac{\partial g}{\partial t}\right)\cdot d\mathbf{S} = \iint_S \mathbf{j}\cdot d\mathbf{S} + \frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S} + \frac{1}{2g}\left(\iint_S \mathbf{D}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

$$\oiint_S \mathbf{D}\cdot d\mathbf{S} = \iiint_V \rho\,dV$$

07. Gravitational Faraday's law

Gravitational Faraday's law states that as a result of the interaction of the non-stationary gravitational field with a stationary magnetic field, an electric field is created.

$$\mathrm{rot}\mathbf{E} = -\cancel{\frac{\partial \mathbf{B}}{\partial t}} - \frac{\mathbf{B}}{2g}\frac{\partial g}{\partial t}$$

Differential form

$$\oint_l \mathbf{E}\cdot d\mathbf{l} = -\cancel{\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S}} - \frac{1}{2g}\left(\iint_S \mathbf{B}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

Integral form

08. Gravitational Ampère's law

Gravitational Ampère's law states that as a result of the interaction of the non-stationary gravitational field with a stationary electric field, a magnetic field is created.

$$\mathrm{rot}\mathbf{H} = \cancel{\mathbf{j}} + \cancel{\frac{\partial \mathbf{D}}{\partial t}} + \frac{\mathbf{D}}{2g}\frac{\partial g}{\partial t}$$

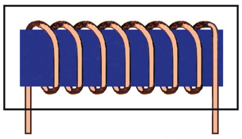
Differential form

$$\oint_l \mathbf{H}\cdot d\mathbf{l} = \cancel{\iint_S \mathbf{j}\cdot d\mathbf{S}} + \cancel{\frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S}} + \frac{1}{2g}\left(\iint_S \mathbf{D}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

Integral form

09. Gravito-magnetic interaction

The gravito-magnetic interaction is a phenomenon in which the gravitational wave, passing through a circuit located in a constant magnetic field (immobile relative to the magnetic induction vector of this field), induces an electromotive force in it.



$$\mathrm{SEM} = -\cancel{\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S}} - \frac{1}{2g}\left(\iint_S \mathbf{B}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

$$\iint_S \mathbf{B}\cdot d\mathbf{S} = BnS$$

$$\mathrm{SEM} = -\cancel{\frac{\partial}{\partial t}\iint_S \mathbf{B}\cdot d\mathbf{S}} - \frac{BnS}{2g}\frac{\partial g}{\partial t}$$

B – value of the magnetic induction vector of a permanent magnet
n – number of coil scrolls
S – surface of one scroll
g – determinant of the metric tensor of space-time

10. Gravito-magnetic method of gravitational waves detection

The proposed gravito-magnetic method of gravitational waves detection, using the phenomenon of gravito-magnetic interaction, consists in placing a coil with a permanent magnet core in a Faraday cage.

11. Example of the "heart" of the gravito-magnetic detector

The photograph shows a coil wound between magnets (photo by Jarosław Konieczny).

Construction of the detector's "heart":
– 2 neodymium magnets, cylindrical with a diameter of 33 mm and a height of 30 mm
– seal separating the magnets having a diameter of about 8 ÷ 10 mm and a thickness of about 2 mm
– coil with about 210 scrolls wound between magnets with wire (in enamel) with a diameter of 0.2 mm

12. Gravito-electric interaction

The gravito-electric interaction is a phenomenon in which a gravitational wave, passing through a stationary homogeneous electric field, for example existing between the plates of a charged flat capacitor, induces a magnetic field in it.

$$\oint_l \mathbf{H}\cdot d\mathbf{l} = \cancel{\iint_S \mathbf{j}\cdot d\mathbf{S}} + \cancel{\frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S}} + \frac{1}{2g}\left(\iint_S \mathbf{D}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

$$\iint_S \mathbf{D}\cdot d\mathbf{S} = DS$$

$$\oint_l \mathbf{H}\cdot d\mathbf{l} = \cancel{\iint_S \mathbf{j}\cdot d\mathbf{S}} + \cancel{\frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S}} + \frac{DS}{2g}\frac{\partial g}{\partial t}$$

H – vector of magnetic field strength
j – vector of current density
D – vector of electric induction
D – value of the electric induction vector
E – value of the electric field intensity vector
S – plate surface of the flat capacitor
g – determinant of the metric tensor of space-time

13. Gravito-electric method of gravitational wave detection

$$\oint_l \mathbf{H}\cdot d\mathbf{l} = \cancel{\iint_S \mathbf{j}}\cdot d\mathbf{S} + \cancel{\frac{\partial}{\partial t}\iint_S} \mathbf{D}\cdot d\mathbf{S} + \frac{DS}{2g}\frac{\partial g}{\partial t}$$

The proposed gravito-electric method of gravitational wave detection, using the phenomenon of gravito-electric interaction, consists in the fact that a charged flat capacitor with a dielectric between its plates should be placed in a Faraday cage. The gravitational wave, passing through the capacitor, will create a magnetic field in it.

14. LC circuit in non-stationary gravitational field

$$\oint_l \mathbf{E}\cdot d\mathbf{l} = -\cancel{\frac{\partial}{\partial t}\iint_S \mathbf{B}}\cdot d\mathbf{S} - \frac{1}{2g}\left(\iint_S \mathbf{B}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

$$\mathrm{rot}\mathbf{E} = -\cancel{\frac{\partial \mathbf{B}}{\partial t}} - \frac{\mathbf{B}}{2g}\frac{\partial g}{\partial t}$$

$$\oint_l \mathbf{H}\cdot d\mathbf{l} = \cancel{\iint_S \mathbf{j}\cdot d\mathbf{S}} + \cancel{\frac{\partial}{\partial t}\iint_S \mathbf{D}\cdot d\mathbf{S}} + \frac{1}{2g}\left(\iint_S \mathbf{D}\cdot d\mathbf{S}\right)\frac{\partial g}{\partial t}$$

$$\mathrm{rot}\mathbf{H} = \cancel{\mathbf{j}} + \cancel{\frac{\partial \mathbf{D}}{\partial t}} + \frac{\mathbf{D}}{2g}\frac{\partial g}{\partial t}$$

The above equations can be used to describe the influence of the gravitational wave on phenomena in the LC circuit placed in the Faraday cage, formed from the coil with the core in the form of a permanent magnet and a capacitor with a dielectric between its plates.

15. Examples of magnetic induction values

The induction of Earth's magnetic field has a value of several tens of microtesla. In the International Laboratory of Strong Magnetic Fields and Low Temperatures in Wroclaw pulsed fields with a magnetic induction of 50 T are generated.

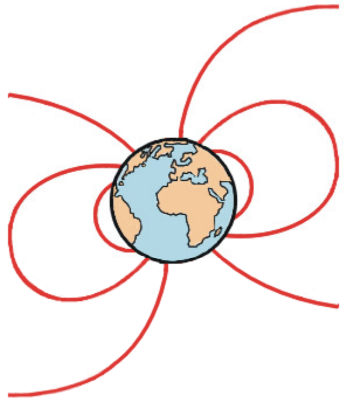
16. Noises in electronic circuits

Gravitational waves have an effect on electromagnetic phenomena, among others they can cause some noises in electronic circuits. It may turn out that we have been receiving signals from space for a long time, but we did not know about it.

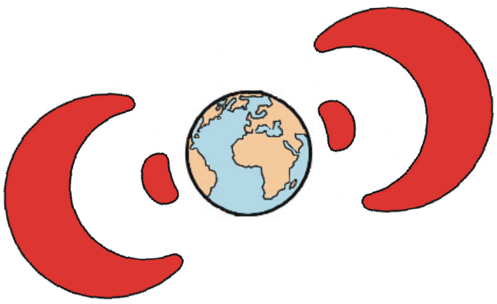
17. Gravitational waves and the problem of "white bears"

The problem of "white bears" is, how to hide from the public the true purpose of scientific research data. Information is available from everywhere that scientists are sent to the poles to study the fauna and flora of those areas and the thickness of the ice cover. Meanwhile, the main purpose of their work is to study the properties of cosmic radiation that reaches there without any obstacles.

Earth's magnetic field



Van Allen radiation belts



Why do some countries devote enormous financial resources to building gravity wave detectors? The answer to this question is simple, gravitational waves are the only sensible carrier of information possibly transmitted by civilizations more developed than our. These Earthlings, who will be the first to have access to knowledge and new technologies, will gain an advantage over the others. The principle of operation of existing gravitational wave detectors is of little use for this purpose, but it is so simple that it is understandable for greedy sponsors. Science should be free of ideology and politics – otherwise the work of scientists can do more harm than good.

18. Final remarks

Electromagnetism "enters" into the Einstein's field equations of gravity through the tensor of energy-momentum of the electromagnetic field [11]. Gravity "enters" into the Maxwell-Hertz equations through the determinant of the metric space-time tensor.

For low frequencies of gravitational waves and small size detectors, the assumption about the zeroing of the gradient of determinant of the metric tensor of space-time is not a significant limitation for the proposed methods of gravitational waves detection.

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