The Pauli Objection Addressed in a Logical Way

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Abstract

One of the greatest unsolved problems in quantum mechanics is related to time operators. Since the Pauli objection was first raised in 1933, time has only been considered a parameter in quantum mechanics and not as an operator. The Pauli objection basically asserts that a time operator must be Hermitian and self-adjoint, something the Pauli objection points out is actually not possible. Some theorists have gone so far as to claim that time between events does not exist in the quantum world. Others have explored various ideas to establish an acceptable type of time operator, such as a dynamic time operator, or an external clock that stands just outside the framework of the Pauli objection. However, none of these methods seem to be completely sound. We think that a better approach is to develop a deeper understanding of how elementary particles can be seen, themselves, as ticking clocks, and to examine more broadly how they relate to time.

Key words: Time operator, Pauli objection, momentum time uncertainty.

1 A New Consistent Time Operator

Time operators have not been commonly used in quantum mechanics. The main resistance against a time operator can be traced back to Wolfgang Pauli's strong objection [1] regarding the existence of a self-adjoint time operator. Pauli's objections have encountered several counterexamples, criticisms, and discussions; see, for example, [2–16]. Some have taken the Pauli objection to the extreme, and argued that time between two events is meaningless in quantum mechanics, [17], "I prove that quantum theory rules out the possibility of any quantity that one might call 'the time interval between two events.'". Others have creatively tried to come up with acceptable time operators by introducing dynamic time operators or clocks that are outside the quantum system and therefore may be able to bypass the Pauli objection. Here we will suggest a logicial, new time operator. Modern physics, despite enormous progress in understanding time (in particular through the work of Larmor [18] and Einstein's special relativity theory), does not have an in-depth understanding of what time is or is not at the deepest quantum level.

Haug has suggested a model where mass is closely related to the tick of time; see [19, 20]. (This is my first working paper on the subject and provides useful background information).

In this paper, we will here suggest a new way to look at particles that is related to Schrödinger's [21] hypothesis in 1930 of a ("trembling motion" in German) in the electron. Schrödinger indicated that the electron was in a sort of trembling motion $\frac{2mc^2}{\hbar} \approx 1.55269 \times 10^{21}$ per second. We will suggest that the electron is in a Planck mass state $\frac{c}{\bar{\lambda}_e} \approx 7.76344 \times 10^{20}$ per second (exactly half of that of Schrödinger's "Zitterbewegung" frequency). However, each Planck mass state only lasts for one Planck second and we therefore get the normal electron mass from

$$\frac{c}{\bar{\lambda}_e} m_p \frac{l_p}{c} = \frac{\hbar}{\bar{\lambda}_e} \frac{1}{c} \approx 9.10938 \times 10^{-31} \text{ kg} \tag{1}$$

We can also look at the same idea from a slightly different angle. It is well-known that the mass of any elementary particle can be expressed as

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{2}$$

This can be rewritten as

$$
\begin{aligned}
m &= \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \\
m &= \frac{\frac{\hbar}{c^2}}{\frac{\bar{\lambda}}{c^2}}\frac{1}{c} \\
m &= \frac{\hbar}{c^2}\frac{1}{\frac{\bar{\lambda}}{c}}
\end{aligned} \tag{3}
$$

The part $\frac{\bar{\lambda}}{c}$ we can call the reduced Compton time t, and we have

$$
m = \frac{\hbar}{c^2}\frac{1}{t} \tag{4}
$$

Be also aware that $\frac{\hbar}{c^2}$ indeed is identical to the Planck mass times one Planck second. Further the plane wave function of the Klein–Gordon equation can be written as

$$
\begin{aligned}
\Psi &= e^{\frac{i}{\hbar}(px-Et)} \\
\Psi &= e^{\frac{i}{\hbar}\left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}x-\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}t-mc^2t\right)} \\
\Psi &= e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}x-\frac{\frac{\hbar}{c^2}\frac{1}{t}c^2}{\sqrt{1-\frac{v^2}{c^2}}}t-\frac{\hbar}{c^2}\frac{1}{t}c^2t\right)} \\
\Psi &= e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}x-\frac{\hbar}{\sqrt{1-\frac{v^2}{c^2}}}-\hbar\right)}
\end{aligned} \tag{5}
$$

Now, taking the partial derivative with respect to the plane wave function with respect to time we get

$$
\begin{aligned}
\frac{\partial\Psi}{\partial t} &= -\frac{ix}{\hbar t}\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}\Psi \\
\frac{\partial\Psi}{\partial t} &= -\frac{ix}{\hbar t}p\Psi
\end{aligned} \tag{6}
$$

since $t = \frac{\bar{\lambda}}{c}$ and if we assume $x = \bar{\lambda}$ we get

$$
i\frac{\hbar}{c}\frac{\partial\Psi}{\partial t} = p\Psi \tag{7}
$$

This means that the time momentum operator is

$$
\hat{p} = i\frac{\hbar}{c}\frac{\partial\Psi}{\partial t} \tag{8}
$$

and the time operator we suggest is simply $\hat{t} = t$. These two operators are time operators: the momentum time operator and the time operator. Based on its construction, this time operator we are quite sure must be Hermitian and self-adjoint. In other words, the Pauli objection likely does not hold in this instance. What quantum mechanics seems to have been missing is that elementary particles are functions of time; they are quantum clocks that tick in every reduced Compton time period. Each tick is the Planck mass that last for one Planck second $m_pt_p = \frac{\hbar}{c^2}$.

Next we will check to see whether the momentum operator and time operator commute or not

$$
\begin{aligned}
[\hat{p},\hat{t}]\Psi &= [\hat{p}\hat{t}-\hat{t}\hat{p}]\Psi \\
&= \left(i\frac{\hbar}{c}\frac{\partial}{\partial t}\right)(t)\Psi-(t)\left(i\frac{\hbar}{c}\frac{\partial}{\partial t}\right)\Psi \\
&= i\frac{\hbar}{c}\left(\Psi+t\frac{\partial\Psi}{\partial(t)}\right)-i\hbar t\frac{\partial\Psi}{\partial(t)} \\
&= i\frac{\hbar}{c}\left(\Psi+t\frac{\partial\Psi}{\partial(t)}-\frac{\partial\Psi}{\partial(t)}\right) \\
&= i\frac{\hbar}{c}\Psi
\end{aligned} \tag{9}
$$

As we can see they do not commute. Further, we get the following uncertainty relation

$$\begin{aligned}\sigma_p\sigma_t &\geq \frac{1}{2}|\int \Psi^*[\hat{p},\hat{t}]\Psi dt| \\ &\geq \frac{1}{2}|\int \Psi^*(i\frac{\hbar}{c})\Psi dt| \\ &\geq \frac{1}{2}|i\frac{\hbar}{c}\int \Psi^*\Psi dt| \qquad (10)\end{aligned}$$

and since $\int \Psi^*\Psi dt$ must sum to 1, we are left with

$$\begin{aligned}\sigma_p\sigma_t &\geq \frac{1}{2}|i\frac{\hbar}{c}| \\ \sigma_p\sigma_t &\geq \frac{\hbar}{2}\frac{1}{c} \qquad (11)\end{aligned}$$

That is, we have a new momentum time uncertainty principle that possibly leads to further directions in the study of time and physics. Establishing a consistent time operator could be important to making progress in quantum gravity [22], for example. Clearly, more work is needed on this area of quantum physics in all its forms.

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