

Title :A COMPLETE COSMOLOGICAL MODEL OF DARK MATTER AND DARK ENERGY(v2)  
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15 Mai 26

## Abstract

This article introduces a novel model of dark matter, conceptualized as a new physical element — termed *dark substance* — which is not composed of classical particles. This dark substance is proposed to pervade the universe, constituting what is commonly regarded as the vacuum. By assigning simple physical properties to this dark substance, we provide a theoretical justification for the flat rotation curves observed in galaxies and the baryonic Tully-Fisher relation. Subsequently, we examine the potential distributions of dark matter within galaxies and galaxy clusters, along with the associated galaxy velocities in clusters, within the framework of our proposed dark matter theory. Furthermore, leveraging this new dark matter model, we propose a New Cosmological Model (NCM) of the Universe, which is finite and flat. The NCM comprises two distinct mathematical formulations. The first closely resembles the Standard Cosmological Model ( $\Lambda$ CDM), while clarifying the nature of dark matter and dark energy, and offering an interpretation of the CMB rest frame and cosmological time. The second mathematical model is significantly simpler than the SCM, yet yields theoretical predictions consistent with astronomical observations at sufficiently low redshifts. In the concluding sections, we demonstrate that both the proposed dark matter model and the NCM can interpret observations related to the primordial universe and the CMB power spectrum. Finally, we show that these two mathematical models can be utilized to address the Hubble tension, leading to the definition of a third mathematical framework, referred to as the  $\Delta$  model. An earlier version of this work, lacking recent astronomical data and covering fewer topics (particularly concerning the  $\Lambda$ CDM model, primordial universe, CMB power spectrum, and Hubble tension), was published in an applied physics journal (DELORT 2018).

**Keywords:** Tully-Fisher relation, dark matter, dark halo, cosmic microwave background, galaxy clusters, gravitational lensing, galaxy rotation curves, galaxy velocities, dark energy, structure formation,  $\Lambda$ CDM model.

## 1.INTRODUCTION

The primary objective of this study is to advance a comprehensive theory of dark matter and dark energy. In the first part of this paper, we propose a new model of dark matter in which a physical entity — termed *dark substance* — constitutes both dark matter and the apparent vacuum. According to this framework, the dark substance fills the universe and exhibits physical properties analogous to those of an ideal gas. Utilizing these properties, we provide a theoretical justification for the observed flat rotation curves of galaxies, adopting a dark substance density profile proportional to  $1/r^2$ . While similar density profiles have been suggested previously, no theoretical dark matter model has yet been proposed to explain this profile from first principles.

Additionally, we hypothesize simple thermal properties of the dark substance within this dark matter theory, which allow a theoretical justification of the baryonic Tully-Fisher

relation. Alternative theories, such as MOND (MILGROM 1983), also attempt to account for flat rotation curves; however, these contradict Newtonian gravity and are challenged by certain astronomical observations (PINA et al. 2021).

In this study, we develop a theory of dark matter encompassing various potential distributions within galaxies. We demonstrate that the proposed model predicts galaxy velocities within clusters and cluster masses in agreement with astronomical observations, and allows a theoretical prediction of galaxy dark radii consistent with empirical data. Moreover, this framework leads to a simple spherical geometric description of the Universe.

In the second part of the paper, we extend this theory to propose a new cosmological model, based on the geometric considerations introduced previously and on a new interpretation of the cosmic microwave background rest frame (CRF), here referred to as the *local cosmological frame* due to its significance in the new cosmological theory. This framework redefines cosmological distances analogously to those in the SCM. It remains compatible with Special and General Relativity locally, since the CRF cannot be detected through laboratory experiments but only via CMB observations.

We propose two possible mathematical models of cosmic expansion. The first model, like the SCM, is based on the equations of General Relativity ( $\Lambda$ CDM), but specifies the physical nature of dark matter and dark energy required by the SCM, while providing an interpretation of the CMB rest frame and cosmological time. This model reproduces the cosmological distances, redshifts, and Hubble constant values predicted by the SCM. Notably, some of its key equations may also be derived from classical Newtonian mechanics, which is discussed later in the paper.

The second mathematical model, in contrast, does not rely on General Relativity and is mathematically much simpler. Nevertheless, its predictions — particularly regarding the Hubble constant and cosmological distances — match astronomical observations to a good approximation for sufficiently low redshifts. However, the observed anisotropies of the CMB contradict this second model, for example regarding the cosmological time of CMB formation (400 000 years) and the comoving distance to the last scattering surface (43 billion light-years), which are in agreement with the first model but inconsistent with the second. Despite this, the second model provides the most accessible framework for understanding the physics of the NCM.

According to this new cosmological model, the Universe is flat, justifying the choice of  $\Omega_c=0$  in the Friedman equations of the  $\Lambda$ CDM model. Furthermore, the dark matter properties assumed in the NCM are consistent with those in  $\Lambda$ CDM (cold, dissipationless, collisionless).

In the final part of this work, we investigate the evolution of the dark substance temperature throughout cosmic history.

It is worth recalling that many astrophysicists and physicists have highlighted the challenges of the SCM, especially those related to dark matter and dark energy, arguing for a new paradigm (KROUPA, PAWLOWSKI & MILGROM 2012). This paper aims to offer such an evolutionary paradigm.

We also note that the dark matter and dark energy theory developed here remains compatible with the SCM (RAINE & THOMAS 2001; LIDDLE 2003; DODELSON & SCOTT 2008) within the first mathematical model, enabling interpretation of most astronomical observations not directly related to dark matter or dark energy — such as primordial element abundances, baryon formation at the same cosmological redshift as in the SCM, structure formation, the appearance of the CMB, and its temperature evolution (in  $1/(1+z)$ ), along with CMB anisotropies. Nevertheless, as discussed later, the dark matter model must be modified in the early Universe to account for the CMB power spectrum.

Importantly, the first part of this article describing the new dark matter theory is formulated within Newtonian mechanics, as are the two proposed mathematical models of the NCM. While some equations in the SCM and MSC also adopt Newtonian mechanics, others (not addressed here) require the mathematics of General Relativity, particularly for interpreting super-horizon modes ( $\ell < 100$ ) of the CMB power spectrum.

Finally, it is possible to define, in agreement with the NCM, a cosmological model termed  $\Lambda$ CDM-NCM, which relies on the  $\Lambda$ CDM equations but interpreted using the NCM's new physical concepts (e.g., Universal Cosmological Frame, Local Cosmological Frame, Universal Sphere, CMB rest frame, redefinition of cosmological time, etc.). This represents the weakest form of the NCM.

A preliminary version of this study was published in an applied physics journal (DELORT 2018). However, the present work substantially extends and refines the theory, particularly in its treatment of the  $\Lambda$ CDM model, the primordial Universe, and the CMB power spectrum. In the concluding discussion, we will demonstrate that the two mathematical models presented can together help address the so-called Hubble tension, motivating the introduction of a third and final mathematical model, the  $\Delta$  model.

## 2. THEORY OF DARK MATTER

As introduced in Section 1, we formulate the following **Postulate 1**, which defines the physical properties of the dark substance:

### **Postulate 1**

- a) A substance, termed *dark substance*, permeates the entire Universe.
- b) This substance does not interact with photons traversing it.
- c) The dark substance possesses mass and obeys Boyle's law, Charles's law, and their combined expression. For an element of dark substance with mass  $m$ , volume  $V$ , pressure  $P$ , and temperature  $T$ , the following relation holds (where  $k_0$  is a constant):

$$PV = k_0 mT$$

This relation is formally identical to the equation of state for an ideal gas, replacing  $k_0$  with a constant characteristic of a particular ideal gas  $G_0$  (denoted  $k(G_0)$ ). This equivalence follows directly from the universal gas law, which itself arises from Boyle's and Charles's laws.

From Postulate 1, we draw two immediate remarks:

- First, although the substance is labeled *dark*, it is more accurately described as translucent, since, per Postulate 1b, it does not interact with photons propagating through it.
- Second, by virtue of Postulate 1a, what is typically referred to as “emptiness” is, in reality, filled with this dark substance.

## 2.2 Flat Rotation Curves of Galaxies

By treating the dark substance as an ideal gas (Postulate 1c), we now demonstrate how a spherically symmetric concentration of dark substance in gravitational equilibrium can account for the dark matter component of a galaxy, thereby explaining the observed flat rotation curves.

According to Postulate 1c, for an element of dark substance with mass  $m$ , volume  $V$ , pressure  $P$ , and temperature  $T$ , we have:

$$PV = k_0 m T \quad (1)$$

Defining  $k_1 = k_0 T$ , this can be rewritten as:

$$PV = k_1 m \quad (2)$$

Introducing the mass density  $\rho$  of the element, we obtain:

$$P = k_1 \rho \quad (3a)$$

We hypothesize that a galaxy can be modeled as a concentration of dark substance exhibiting spherical symmetry, maintained at a homogeneous temperature  $T$ , and in a state of gravitational equilibrium.

Let us define the spherical surface  $S(r)$  as the sphere of radius  $r$  centered at the galaxy's center  $O$ . Similarly,  $S(r+dr)$  denotes the spherical surface of radius  $r+dr$ , also centered at  $O$ . We denote by  $S(O,r)$  the spherical volume of dark substance contained within radius  $r$  about the center  $O$ .

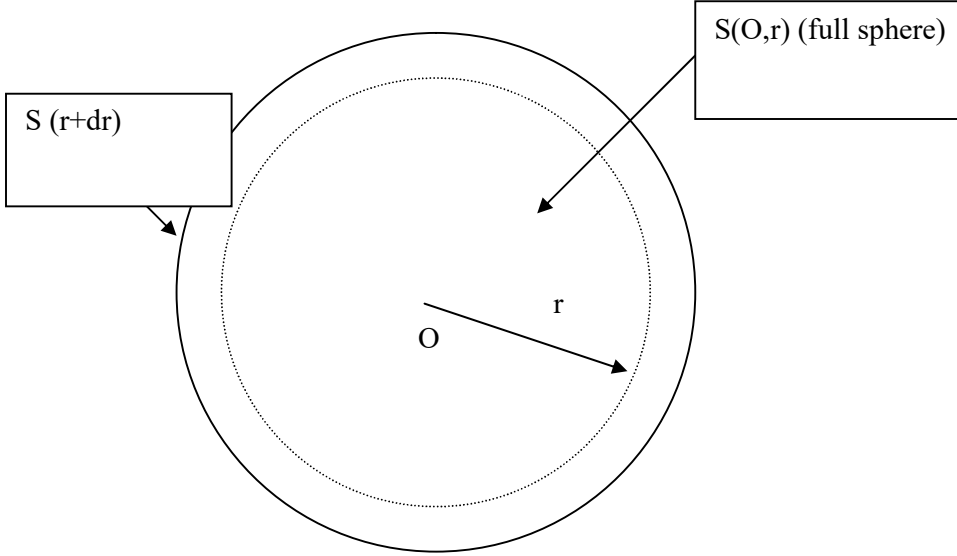


Figure 1: The spherical concentration of dark substance

The mass  $M(r)$  of the sphere  $S(O,r)$  is given by:

$$M(r) = \int_0^r \rho(x) 4\pi x^2 dx \quad (3b)$$

Assuming a spherical symmetry for the density of dark substance, using Newton's law ( $\Sigma \mathbf{F} = \mathbf{0}$  for a material element in equilibrium with a mass  $m$ ,  $\mathbf{F}_G(r) = m\mathbf{G}(r)$ ,  $\mathbf{F}_G(r)$  gravitational force acting on the element,  $\mathbf{G}(r)$  gravitational field defined by Newton's universal law of gravitation) and Gauss theorem in order to obtain  $\mathbf{G}(r)$ , we obtain the following equation (4) of equilibrium of forces on an element dark substance with a surface  $dS$ , a width  $dr$ , situated between  $S(O,r)$  and  $S(r+dr)$ :

$$dSP(r+dr) + \frac{G}{r^2} (\rho(r) dS dr) \left( \int_0^r \rho(x) 4\pi x^2 dx \right) - dSP(r) = 0 \quad (4)$$

Eliminating  $dS$ , we obtain the equation:

$$\frac{dP}{dr} = -\frac{G}{r^2} (\rho(r)) \left( \int_0^r \rho(x) 4\pi x^2 dx \right) \quad (5)$$

And using the equation (3) obtained using the Boyle-Charles' law assumed in the Postulate 1, we obtain the equation:

$$k_1 \frac{d\rho}{dr} = -\frac{G}{r^2} (\rho(r)) \left( \int_0^r \rho(x) 4\pi x^2 dx \right) \quad (6)$$

We then verify that the density profile of the dark substance,  $\rho(r)$ , satisfying the above conditions of gravitational equilibrium and the equation of state, is given by the following straightforward solution:

$$\rho(r) = \frac{k_2}{4\pi r^2} \quad (7)$$

A mass density profile of dark matter following an inverse-square law, as expressed in Equation (7), has already been proposed in the literature to account for the flat rotation curves of spiral galaxies. However, no prior model of dark matter has provided a theoretical foundation justifying this  $1/r^2$  density profile or deriving the constant  $k_2$ . In this work, we establish a theoretical justification for the  $1/r^2$  density profile and explicitly derive the value of the constant  $k_2$  (see Equation (8)). This result emerges naturally from our model of dark substance treated as an ideal gas, in accordance with Postulate 1.

To determine  $k_2$ , we substitute the expression of  $\rho(r)$  given by Equation (7) into Equation (6). This substitution immediately yields the following expression for  $k_2$ , for which Equation (6) is satisfied:

$$k_2 = \frac{2k_1}{G} = \frac{2k_0 T}{G} \quad (8)$$

By applying the density profile provided in Equation (7), we obtain that the mass  $M(r)$  enclosed within the spherical volume  $S(O,r)$  is expressed as:

$$M(r) = \int_0^r 4\pi x^2 \rho(x) dx = k_2 r \quad (9)$$

Neglecting the contribution of stellar mass within the galaxy, we then find that the velocity  $v(r)$  of a star located at a distance  $r$  from the galactic center  $O$  satisfies the relation  $v(r)^2/r = GM(r)/r^2$ , which leads directly to:

$$v(r)^2 = Gk_2 = 2k_1 = 2k_0 T \quad (10)$$

Equation (10) demonstrates that the orbital velocity of a star within the galaxy is independent of its distance  $r$  from the galactic center  $O$ .

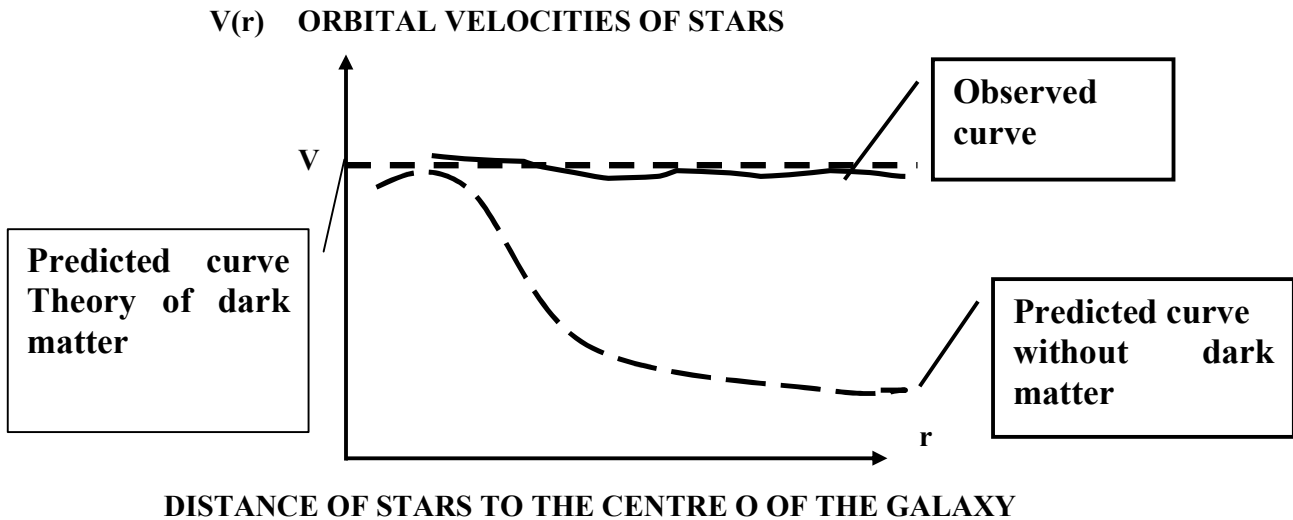


Figure 2 :Rotation curve of galaxies

### 2.3 Baryonic Tully-Fisher's law.

#### 2.3.1 Recall

Tully and Fisher performed observations (TULLY&FISHER 1977) of spiral galaxies exhibiting flat rotation curves. They established that the luminosity  $L$  of such spiral galaxies is proportional to the fourth power of the stellar rotation velocity  $v$ , leading to the well-known Tully-Fisher relation, where  $K_1$  is a constant:

$$L=K_1v^4 \quad (11)$$

Furthermore, in the galaxies examined by Tully and Fisher, the baryonic mass  $M$  was generally proportional to their luminosity  $L$ . This implies the relation, with  $K_2$ ,  $K_2$  a constant:

$$M=K_2v^4 \quad (12)$$

This second formulation is commonly referred to as the baryonic Tully-Fisher law.

More recent observations by McGAUGH have shown that the baryonic Tully-Fisher relation (Equation 12) appears to hold across all galaxies with flat rotation curves, including those whose luminosity is not directly proportional to their baryonic mass (McGAUGH 2011).

In what follows, we will demonstrate that adopting Postulate 1 along with a second postulate describing simple thermal properties of the dark substance (particularly its thermal interaction with baryonic matter) allows us to theoretically justify the baryonic Tully-Fisher relation.

#### 2.3.2 Theory of Quantified Calorific Energy Loss (by Nuclei)

From Equation (10), our model of dark substance implies that the square of the stellar rotation velocity in a galaxy with a flat rotation curve is proportional to the temperature  $T$  of the spherical concentration of dark substance forming the galaxy. Determining this temperature  $T$  is thus crucial.

A first, intuitive idea might be to link  $T$  to the CMB temperature. However, this cannot be the case, since it would imply that all stars in all flat-rotation-curve galaxies would move with the same velocity, which is not observed.

A second, more plausible possibility is that in a given galaxy, each baryon interacts thermally with the dark substance in which it is immersed, transferring a portion of its thermal energy. While this transferred energy is expected to be extremely small (otherwise it would have already been detected), the very low density of the dark substance, combined with cosmological timescales (noting that the baryonic diameters of galaxies can reach 100,000 light-years), could still result in appreciable temperatures for the dark substance.

In principle, one could expect this energy loss per baryon to depend on both the temperature of the baryon and the temperature  $T$  of the surrounding dark substance. However, if this were the case, calculating the total energy transferred by all baryons would be extremely difficult, and it would be unlikely that one could recover the very simple baryonic Tully-Fisher law.

Hence, the study makes the following simplifying assumption describing the thermal transfer between dark substance and baryons, expressed in Postulate 2a:

**Postulate 2a**

- Each atomic nucleus in a galaxy loses thermal energy to the surrounding dark substance in which it is immersed.
- This thermal transfer depends only on the number  $n$  of nucleons constituting the nucleus (and is therefore independent of the nucleus temperature). If  $p$  is the thermal power dissipated by a nucleus, there exists a constant  $p_0$  (the thermal power dissipated per nucleon) such that

$$p=np_0 \quad (13)$$

From Equation (13), the total thermal power transmitted by all atoms in a galaxy to the spherical concentration of dark substance is proportional to the total number of nucleons in the galaxy, and therefore proportional to its baryonic mass. Denoting  $m_0$  the mass of a single nucleon, and  $M$  the baryonic mass of the galaxy, we find that the total thermal power  $P_r$  received by the concentration of dark substance is

$$P_r=(M/m_0)p_0=K_3M \quad (14)$$

with  $K_3=p_0/m_0$

Regarding Postulate 2a, it is possible (though not strictly required) that it only applies to atoms whose temperature exceeds the temperature  $T$  of the dark substance concentration.

This postulate is key because it leads to the simple expression (14), which will be essential to recover the baryonic Tully-Fisher law.

### 2.3.3 Derivation of the Baryonic Tully-Fisher Law

In accordance with the galaxy model proposed in Section 2.2, we describe a galaxy with a flat rotation curve as a spherical concentration of dark substance at temperature  $T$ , itself embedded within a medium composed of “intergalactic dark substance” characterized by temperature  $T_0$  and density  $\rho_0$ .

It is natural to impose the continuity of the density  $\rho(r)$ . Denoting by  $R$  the radius where the density of the concentration of dark substance equals the ambient density  $\rho_0$ , we identify  $R$  as the dark radius of the galaxy. Therefore, the following relation holds:

$$\rho(R) = \rho_0 \quad (15)$$

Then according to (7) and (8):

$$\frac{k_2}{4\pi R^2} = \rho_0 \quad (16)$$

in which  $k_2$  was determined previously in Equation (8), and  $\rho_0$  denotes the ambient density of the intergalactic dark substance.

$$\frac{2k_0 T}{G} \times \frac{1}{4\pi R^2} = \rho_0 \quad (17)$$

Hence, we obtain that the radius  $R$  of the spherical concentration of dark substance making up the galaxy is approximately determined by the condition of continuity at its boundary. This yields the relation:

$$R = \left( \frac{2k_0 T}{4\pi G \rho_0} \right)^{1/2} = K_4 T^{1/2} \quad (18)$$

where the constant  $K_4$  is given by:

$$K_4 = \left( \frac{2k_0}{4\pi G \rho_0} \right)^{1/2} \quad (19)$$

In other words, the dark radius  $R$  of the galaxy is essentially set by the equilibrium between the central dark substance density profile (falling as  $1/r^2$ ) and the surrounding intergalactic dark substance with its constant density  $\rho_0$ . This matching condition defines a finite spatial extent for the effective dark matter halo associated with the galaxy.

We then consider that the sphere of dark substance with radius  $R$  and temperature  $T$  is in thermal interaction with the surrounding intergalactic dark substance at temperature  $T_0$ . The simplest and most natural mode of thermal transfer in such a configuration is convective heat transfer. We formulate this in the following:

**Postulate 2b**

- The thermal interaction between the spherical concentration of dark substance making up the galaxy (with a density profile falling as  $1/r^2$  and a homogeneous temperature  $T$ ) and the intergalactic dark substance surrounding it (at temperature  $T_0$ ) can be described as a convective thermal transfer.

Let  $\phi$  denote the local thermal flux (heat flow per unit surface area) on the boundary of the spherical concentration of dark substance of radius  $R$ . Then the total power  $P_l$  lost by the spherical concentration of dark substance to its surroundings is given by the surface integral of this flux, that is:

$$P_l = 4\pi R^2 \phi \quad (20)$$

where  $\phi$  can be expressed, under standard convective heat transfer laws, as proportional to the temperature difference between the surface of the concentration and the surrounding intergalactic medium:

$$\phi = h(T - T_0) \quad (21)$$

with  $h$  denoting an effective convective heat transfer coefficient (which depends on the nature of the interface and the properties of the dark substance). Therefore, we can write the total power loss:

$$P_l = 4\pi R^2 h(T - T_0) \quad (22)$$

This power loss must balance, in steady state, the total thermal power  $P_r$  received from the baryonic component of the galaxy as previously given in Equation (14). This will permit us to determine the stationary temperature  $T$  of the dark substance concentration, crucial for connecting the model to the baryonic Tully-Fisher law.

According to the equations (14) and (22), ( $M$  being the baryonic mass of the galaxy), we have:

$$K_3 M = 4\pi R^2 h(T - T_0) \quad (23)$$

Using then the equation (18) :

$$K_3 M = 4\pi K_4^2 h T(T - T_0) \quad (24)$$

Making the approximation  $T_0 \ll T$  :

$$M = 4\pi \frac{K_4^2}{K_3} h T^2 \quad (25)$$

Consequently we obtain the expression of T, defining the constant  $K_5$  :

$$T = \left( \frac{K_3}{4\pi K_4^2 h} \right)^{1/2} M^{1/2} = K_5 M^{1/2} \quad (26)$$

And then according to the equation (10) :

$$v^2 = 2k_0 T = 2k_0 K_5 M^{1/2} \quad (27)$$

So :

$$M = \left( \frac{1}{2k_0 K_5} \right)^2 v^4 \quad (28)$$

So we finally obtain :

$$M = K_6 v^4 \quad (29)$$

The constant  $K_6$  being defined by:

$$K_6 = \left( \frac{1}{2k_0 K_5} \right)^2 = \frac{4\pi K_4^2 h}{4k_0^2 K_3} \quad (30)$$

$$K_6 = \frac{4\pi h}{4k_0^2 K_3} \times \frac{2k_0}{4\pi G \rho_0} \quad (31)$$

$$K_6 = \frac{m_0 h}{2k_0 G \rho_0 p_0} \quad (32)$$

We recover the baryonic Tully-Fisher relation (12), identifying  $K_2 = K_6$ . It is natural to propose that the convective coefficient  $h$  depends on the intergalactic dark substance density  $\rho_0$ . The simplest expression is to set  $h = C_1 \rho_0$  where  $C_1$  is a constant. With this form, the coefficient  $K_6$  becomes independent of  $\rho_0$ , thus allowing the baryonic Tully-Fisher relation to define reliable standard candles for estimating cosmological distances.

## 2.4 Temperature of the intergalactic dark substance

We have introduced the temperature  $T_0$  of the intergalactic dark substance. One might hypothesize that  $T_0$  equals the temperature of the cosmic microwave background (CMB). However, it is crucial to recall that in order to recover the baryonic Tully-Fisher relation, we assumed  $T_0 \ll T$ , with  $T$  representing the temperature of the spherical concentration of dark substance within a galaxy. If  $T_0$  were equal to the CMB temperature, this would imply excessively high temperatures for these spherical concentrations (tens of thousands of Kelvin degrees), which appears unrealistic.

Consequently, within the framework of the present dark matter theory, we consider that the temperature  $T_0$  of the intergalactic dark substance generally differs from the temperature of the CMB, except potentially in the primordial Universe.

We may distinguish the following scenarios:

- a) The temperature  $T_0$  of the intergalactic dark substance at the current cosmological epoch (cf. equation (21)) is far below the CMB temperature.
- b) Baryons can transfer thermal energy to the dark substance, as postulated in Postulate 2a), even if their temperature is lower than that of the dark substance.

It should be emphasized, consistent with Postulate 1b), that the dark substance does not interact with photons, and in particular does not couple to the photons of the CMB. As such, the dark substance does not absorb radiative energy.

## 2.5 Geometry of the Universe

The core elements of the new dark matter theory have been outlined previously. As a result, the emergence of flat galactic rotation curves and the baryonic Tully-Fisher relation are both compatible with the standard cosmological model (SCM). The full dark matter theory as presented also remains consistent with the range of topological models permitted within the SCM framework.

Nevertheless, the proposed dark matter model naturally allows for a novel and particularly simple geometric model of the Universe: a sphere filled with dark substance (termed the **Universal Sphere**) surrounded by a medium referred to as “nothingness,” representing the state of the medium prior to the Big Bang.

Let  $R_U(t)$  denote the radius of the Universal Sphere at cosmological time  $t$ . Defining  $1+z$  as the expansion factor of the Universe between cosmological times  $t_1$  and  $t_2$ , the expansion of this Universal Sphere can be expressed by the relation:

$$R_U(t_2) = (1+z) R_U(t_1) \quad (33)$$

## 2.6 Superposed Sphere

Let us consider a spherical concentration of dark substance, characterized by a density varying as  $1/r^2$  (as defined previously), moving through space. One might expect its velocity or mass to be altered by its motion, due, for instance, to Archimedean-type forces or to absorption or loss of dark substance during its motion. Although such effects could, in principle, be negligible, the proposed theory provides a much stronger justification for them being strictly zero.

According to this new theory, the dark substance exhibits two possible behaviors: it can behave as an ordinary substance possessing mass, or alternatively as absolute emptiness. For baryonic particles immersed within the dark substance, it systematically behaves as absolute emptiness; consequently, the velocity of baryonic particles is never affected by any Archimedean force arising from their motion through the dark substance.

In addition, the intergalactic dark substance in which the spherical concentration of dark substance is embedded likewise behaves as absolute emptiness with respect to the motion of the spherical concentration itself. Therefore, neither the velocity nor the mass of the spherical concentration of dark substance is altered by its motion through the intergalactic dark substance. To describe this property, we say that the spherical concentration of dark substance is **superposed** upon the surrounding intergalactic dark substance.

In classical Newtonian gravitation, it is assumed that only baryonic matter contributes to the mass density, and furthermore that the Universe is static. Neither of these assumptions holds within the present dark matter theory, which — like the standard cosmological model (SCM) — allows for an expanding Universe. As a result, the equations of Newtonian dynamics must be adapted to remain consistent with this new dark matter theory. We now present three elementary examples of such adaptations.

In Section 2.2, we assumed spherical symmetry about the galactic center  $O_{GA}$  to establish our model of a superposed sphere with density varying as  $1/r^2$ . The present study therefore proposes the following first rule for adapting Newton's laws to account for the fact that dark matter can behave as absolute emptiness:

### **Adaptation Rule 1:**

Consider a galaxy  $G_A$  formed by a superposed sphere of concentrated dark matter ( $S_{CDM}$ ) centered at  $O_{GA}$  with radius  $R_{GA}$ . Then  $O_{GA}$  experiences an acceleration  $\mathbf{G}(O_{GA})$ ,  $\mathbf{G}(O_{GA})$  defined by:

$$\mathbf{F}_G(S_{CDM}) = m(S_{CDM}) \mathbf{G}(O_{GA})$$

where  $\mathbf{F}_G(S_{CDM})$  is the gravitational force exerted on  $S_{CDM}$  by the surrounding intergalactic dark substance and by baryonic matter, and  $m(S_{CDM})$  is the mass of  $S_{CDM}$ . The combined effect of the intergalactic dark substance and baryonic matter acts upon  $S_{CDM}$  as though the  $S_{CDM}$  were a rigid solid, involving that baryonic matter exerts no influence on the internal density distribution of dark matter within  $S_{CDM}$ . This implies that the previous superposed sphere model with a density profile of  $1/r^2$  remains valid, provided spherical symmetry is assumed.

Thus, this constitutes a first concrete example of adapting Newtonian dynamics to the proposed dark matter theory.

Earlier, in Section 2.3, we introduced a convective thermal transfer model between a superposed sphere at temperature  $T$  and the intergalactic dark substance at temperature  $T_0$ , with a thermal flux given by:

$$\phi = h(T - T_0) \quad (34)$$

However, it is possible that the dark substance in which the superposed sphere is immersed behaves as absolute emptiness not only in gravitational terms, but also in thermal terms. This leads us to propose an alternative, second model of thermal transfer between the superposed sphere and the surrounding dark substance, with the thermal flux expressed as

$$\phi = hT \quad (35)$$

This flux remains formally analogous to a convective heat transfer, and can be interpreted as the limiting case of convective transfer between a medium at temperature  $T$  and another medium at  $T_0=0$ .

The second model of thermal transfer is particularly interesting because it implies that the baryonic Tully-Fisher law, as established in Section 2.3, remains valid regardless of the temperature  $T_0$  of the dark substance in which the superposed sphere is immersed. In contrast, the validity of the first thermal transfer model requires the condition  $T_0 \ll T$ . However, this first model is highly attractive, as it implies that the temperature of dark substance within a superposed sphere is always higher than that of intergalactic dark matter.

We have seen that dark substance possesses the remarkable property of sometimes behaving as absolute emptiness, devoid of mass, and at other times as an ordinary material substance with mass. A second fundamental property of dark substance, which we shall also accept, is that it can sometimes tend toward homogeneity, with its density not obeying Newton's law of gravitation, while at other times its density distribution does follow Newtonian laws.

This second property is essential because, if one assumes that on the scale of a star or a black hole the homogeneity tendency of dark substance dominates, then there would be no significant concentration of dark substance around individual stars within a galaxy. Consequently, there exist two principal types of dark matter distributions within galaxies:

- Galaxies immersed in a dark substance whose density varies as  $1/r^2$ .
- Galaxies immersed in intergalactic dark substance with a constant density.

## 2.7 Baryonic and Dark Radius of a Galaxy

In Section 2.1, we observed that if  $r$  denotes the distance from the center  $O$  of a spherical concentration of dark substance forming a galaxy, then the density profile of the dark substance  $\rho(r)$  is expressed as:

$$\rho(r) = \frac{k_3}{r^2} \quad (36)$$

where  $k_3$  is a constant (see Section 2.2, equation (7), with  $k_3 = k_2/4\pi k$ ).

So we obtain,  $M(r)$  being the mass of the sphere having its center in  $O$  and a radius  $r$  (See equation (9)):

$$M(r) = 4\pi k_3 r \quad (37)$$

Consequently,  $v$  being the velocity of a star at a distance  $r$  of  $O$  (see equation (10)):

$$v^2 = \frac{GM}{r} = 4\pi k_3 G \quad (38)$$

Consequently:

$$k_3 = \frac{v^2}{4\pi G} \quad (39)$$

We also know that if  $\rho_0$  represents the local density of the intergalactic dark substance surrounding the spherical concentration of dark substance forming the galaxy, then the radius  $R$  of this concentration is given by the expression (see equation (15)):

$$\rho(R) = \frac{k_3}{R^2} = \rho_0 \quad (40)$$

Consequently:

$$R = \sqrt{\frac{k_3}{\rho_0}} = v \sqrt{\frac{1}{4\pi G \rho_0}} \quad (41)$$

In a previous section, we referred to  $\mathbf{R}$  as the *dark radius* of the considered galaxy. Hence, in a galaxy for which there exists a spherical concentration of dark substance with a density varying as  $1/r^2$ , two distinct types of radii must be distinguished:

- The first, termed the *dark radius*, refers to the radius of the spherical concentration of dark substance itself.
- The second corresponds to the radius of the smallest sphere enclosing all the stars in the galaxy; we shall designate this as the *baryonic radius*.

## 2.8 Other Models of Dark Matter Distribution in the Universe

The present analysis concludes that dark substance is not ordinary matter, and, a priori, does not necessarily share the physical properties of ordinary matter. For example, according to the proposed model, dark substance may behave as absolute emptiness. In this section, and in the subsequent discussion on the dynamics of galaxy clusters, the study proposes new, simple but unconventional physical properties of dark substance, allowing for an interpretation of astronomical observations traditionally attributed to dark matter.

### 2.8.1 The Dual Behavior of Dark Substance

In addition to the first distribution model presented in Section 2.2, describing a dark substance density varying as  $1/r^2$  in galaxies with flat rotation curves, a second model of distribution must be considered, involving a constant dark substance density  $\rho(r)=\rho_0$ , where  $\rho_0$  is the density of the intergalactic dark substance, assumed in Section 2.2 to be homogeneous both in temperature and in density.

This second model emerges as a consequence of a possible behavior of dark substance: its ability to remain homogeneous in density, in violation of the usual force equilibrium equations.

Therefore, dark substance may display two distinct behaviors:

- It may be homogeneous in density within a given volume, thereby violating the equilibrium equations (as in the case of intergalactic dark substance), or
- It may have its density obey the equations of force equilibrium (as in the model of galaxies with flat rotation curves).

The study specifies, within the proposed framework, the conditions under which dark substance behaves according to each of these two modes. Furthermore, it was shown that the dark halo of a galaxy with a flat rotation curve consists of a *superposed sphere* of dark substance. This leads to the following hypothesis:

**Hypothesis a):**

Dark substance maintains a constant density everywhere in the Universe outside the superposed spheres.

To extend the model, we propose a second hypothesis:

**Hypothesis b):**

A local concentration of dark substance within a differential volume  $dV$  belonging to a superposed sphere  $S$  (with  $dV$  small relative to the volume of  $S$ ) can only exist if  $dV$  lies within a second sphere of dark substance  $S'$ , itself superposed to  $S$ .

The preceding hypotheses a) and b) lead to a very simple description of dark matter density at any point in the Universe.

One might question whether multiple levels of superposed spheres are possible — i.e., whether a sphere filled with dark substance  $S'$  can itself be superposed on another sphere  $S$ , as hypothesized in b). The simplest assumption is that this is not possible, a notion supported by current observations. Therefore, the following hypothesis is also proposed:

**Hypothesis c):**

Multiple levels of superposed spheres cannot exist.

Hypothesis a) implies that if a star lies outside any superposed sphere, there will be no local concentration of dark substance around it. Hypotheses b) and c) further imply that within a superposed sphere  $S$  constituting the dark halo of a galaxy with a flat rotation curve, there are no local concentrations of dark substance — neither around stars nor around dwarf galaxies.

If both hypotheses b) and c) are correct, then there are no dark matter concentrations locally around the Magellanic Clouds. However, if future astronomical observations establish that the Magellanic Clouds are galaxies with flat rotation curves consistent with the baryonic Tully-Fisher law, this would contradict Hypothesis c) (while maintaining Hypothesis b)). Nevertheless, Hypothesis c) is not essential to the broader dark matter theory proposed here. The justification of the baryonic Tully-Fisher law could, in principle, be applied to a sphere  $S'$  superposed on another superposed sphere  $S$ . Still, according to current observations, neither the Large Magellanic Cloud nor the Small Magellanic Cloud exhibit flat rotation curves consistent with the baryonic Tully-Fisher law.

Finally, we state a last fundamental hypothesis regarding dark substance that helps explain several observations:

**Hypothesis d):**

- Baryonic matter does not influence the density of dark substance; therefore, the baryonic mass contribution must be taken as null when determining the density of dark substance.
- Neither the intergalactic dark substance nor a superposed sphere  $S_A$  has any influence on the density of another superposed sphere  $S_B$  distinct from  $S_A$ .

We predict that ordinary baryonic matter and the superposed sphere  $S_A$  exert a global gravitational influence on the superposed sphere  $S_B$ . This means that, although neither ordinary matter nor  $S_A$  affect the *density* of the dark substance constituting  $S_B$ , their gravitational force on  $S_B$  is effectively applied at the center of  $S_B$ . An alternative to Hypothesis d) would be to assume that, on the scale of stars, the intrinsic tendency of dark substance toward homogeneity dominates.

Our theory of dark matter enables an estimation of the mass of the Milky Way that is consistent with values determined through astronomical observations.

Specifically, further observational data — interpreted through the dynamical model of galaxy clusters within our dark matter theory — allows us to estimate the density of the intergalactic dark substance  $\rho_0$ . From this, applying equation (41), one can estimate the radius of the Milky Way's dark matter halo as  $R_H=550,000$  light-years. Subsequently, an estimate of the Milky Way's mass  $M_{MW}$  can be obtained, with  $v$  being the orbital velocity at a distance  $R_H$  from the Milky Way's center, using the relation:

$$GM_{MW}/R_H=v^2 \quad (41a)$$

Taking  $v\approx 205$  km/s yields  $M_{MW}\approx 1.54\times 10^{12}$  S.M, which is in excellent agreement with recent measurements by NASA and ESA teams (WATKINS et al., 2019).

### 2.8.2 Generation of the Superposed Spheres

An interesting research question concerns the mechanism by which the superposed spheres of dark substance arise in the Universe. Observationally, there are no significant concentrations of dark matter found locally around stars or around low-mass black holes. According to our hypotheses a) and b), this indicates that there are no superposed spheres forming around such objects. We therefore introduce the following:

#### **Hypothesis e):**

No planets, stars, or black holes with small masses generate superposed spheres.

Nevertheless, it is possible that superposed spheres could be generated by supermassive black holes. If so, there should exist a supermassive black hole at the center of every galaxy with a flat rotation curve, and conversely, any galaxy whose central object is a supermassive black hole should exhibit a flat rotation curve. Another possibility is that superposed spheres were generated by primordial black holes formed in the very dense early Universe, but have since disappeared.

Accordingly, there are two main possibilities for the formation of superposed spheres: either they are generated by particular celestial objects (e.g., supermassive black holes), or they originate from processes in the primordial Universe.

### 2.8.3 Rotation Curves Near the Galactic Center

In our model for galaxies with flat rotation curves, the density varies as  $1/r^2$  (with  $r$  being the distance from the galactic center). However, astronomical observations show that near the center of galaxies, the rotation curve is not flat; rather,  $v(r)=0$  at  $r=0$ .

We propose a simple explanation for this feature. As previously discussed, dark substance can exhibit two behaviors: either a homogeneous density violating equilibrium force equations, or a density obeying those equations. We now state the following hypothesis to account for this central behavior:

#### **Hypothesis f):**

For any temperature  $T$ , there exists a maximal density  $\rho_M(T)$  beyond which dark substance reverts to behaving as a homogeneous medium, violating the equilibrium of forces.

From Hypothesis f), it follows that in a galaxy with a flat rotation curve, there is a characteristic distance  $d_0$  such that for  $0 < r < d_0$ , the dark substance density is constant and equal to  $\rho_M(T)$ , while for  $r > d_0$   $\rho(r)$  decreases asymptotically toward the intergalactic density  $\rho_0$ . At large  $r$ , the density profile approaches the  $1/r^2$  behavior of our initial model without Hypothesis f). This is fully consistent with observations.

To determine  $\rho(r)$  under Hypothesis f), we proceed as follows. Let  $a > 0$ , then we define  $\rho_{Sa}(r)$  by:

- (i) For  $0 \leq r \leq a$ :  $\rho_{Sa}(r) = \rho_M(T)$ .
- (ii) For  $a < r$   $\rho_{Sa}(r)$  is the solution of the equilibrium equations, asymptotically approaching the  $1/r^2$  profile.

Then, define  $\rho_{Sam}(r)$  as the unique function among the  $\rho_{Sa}(r)$  satisfying:

- (i)  $\rho_{Sam}(r) \leq \rho_M(T)$  for all  $r$ .
- (ii)  $a$  is minimal (equal to  $a_m$ ).

This function  $\rho_{Sam}$  represents the dark matter density profile in a galaxy with a flat rotation curve under Hypothesis f), and  $d_0 = a_m$ .

Consider a galaxy **Gal** with a flat rotation curve. Let  $\rho_N(r)$  be the density obtained as the solution to the equilibrium equation used in Section 2.2, and thus without the  $\rho_M(T)$  limit. Consequently,  $\rho_N(r)$  varies as  $1/r^2$ .

One can then obtain an approximation of  $d_0$  using the approximations  $\rho_N(d_0) \approx \rho_M(T)$  and, for  $r > d_0$ ,  $\rho_{Sam}(r) \approx \rho_N(r)$ .

If  $\rho_0$  denotes the present density of the intergalactic dark substance, taking  $\rho_M(T) = 10^6 \rho_0$  and assuming **Gal** is the Milky Way, we obtain results consistent with

observation. (If the radius of the superposed sphere is approximately 500,000 light-years, as will be shown later, then  $d_0 \approx 500$  light-years.)

#### *2.8.4 The Intracluster Medium and the Baryonic Tully-Fisher Law*

Astronomical observations have established the existence of a plasma of baryonic matter within galaxy clusters — known as the intracluster medium — whose mass generally exceeds that of all galaxies within the cluster.

However, in deriving the baryonic Tully-Fisher relation within our theory of dark matter, we considered that all baryonic particles inside the halo of a given galaxy transfer thermal energy to the dark substance constituting the dark halo. If we included the plasma, we would fail to recover the Tully-Fisher relation based solely on the stellar and visible gas mass of the galaxy.

We therefore propose the following explanation: the plasma is composed of ionized particles, typically hydrogen or helium. We obtain the baryonic Tully-Fisher law by counting only the mass of stars and visible gas if we stipulate that a charged baryonic particle (such as an ion) does not transfer thermal energy to the surrounding dark substance in which it is immersed.

This is consistent with astronomical observations showing that the plasma particles do not cool down appreciably. They also support S. McGaugh's approximation of neglecting ionized gas when empirically establishing the baryonic Tully-Fisher relation. Since hydrogen and helium account for nearly all of the baryonic mass of galaxies (e.g., ~99% in the Milky Way), our model remains valid under the assumption that only helium and hydrogen transfer thermal energy to the dark substance.

#### *2.8.5 Collisions Between Dark Matter and Baryonic Matter*

No astronomical observations to date have confirmed any collisions between dark matter and baryonic matter. This is naturally explained in our dark matter theory. Since dark substance fills all of space and can behave as absolute emptiness, collisions between absolute vacuum and baryonic matter are impossible. Accordingly, no Archimedean buoyancy forces act on a particle moving through dark substance. Therefore, the theory does not predict any possible collisions between baryonic matter and dark substance.

### **2.9 Other Observations of Dark Matter**

There are, a priori, two main models regarding the distribution of dark substance within galaxy clusters. In the first model — analyzed in detail — the observed mass of dark substance in a galaxy cluster is much greater than the total mass of the dark halos of the galaxies contained within it. In the second model, the observed dark substance mass in a galaxy cluster equals the sum of the masses of the superposed sphere halos of its constituent galaxies.

In the first model, one must explicitly consider the mass of intergalactic dark substance, i.e., dark substance outside the dark halos, assumed to be homogeneous in density. Consequently, it is necessary to postulate a dual gravitational behavior for intergalactic dark substance depending on whether it is inside or outside a concentration of baryonic matter.

Hence the first model of dark matter in galaxy clusters adopts the following fundamental principle:

If a point P belongs to a concentration of baryonic matter (such as a galaxy cluster or a baryonic anisotropy in the early Universe), then its true dark matter density must be used in Newton's equations. Conversely, if P does not belong to any concentration of baryonic matter nor to any dark halo, then a zero density should be considered at P in Newton's equations.

It is worth recalling that models of galaxy formation — i.e., structure formation — generally require the presence of dark matter. The above-stated property may be the origin of the apparent dark matter effects observed during cosmic structure formation.

We now turn to interpreting experimental data on the velocities of galaxies within galaxy clusters under the framework of our new dark matter theory, focusing exclusively on the first distribution model.

In this first model of dark matter distribution, we account for the total mass of dark substance present within the galaxy cluster. Therefore, the velocity of a galaxy inside a cluster is determined by three contributions:

- the baryonic mass contained in the cluster (e.g., stars, gas, etc.),
- the masses of the dark halos of the constituent galaxies,
- and the mass of the intergalactic dark substance.

According to the previous section, we admit that a galaxy cluster contains only either:

- galaxies whose dark substance density follows a  $1/r^2$  profile as defined in Section 2.1 (the first model of galactic dark matter distribution), or
- galaxies with a homogeneous dark matter density equal to  $\rho_0$ , the intergalactic dark substance density (the second model of galactic dark matter distribution).

A very interesting result arises when considering the mean density of galaxies in the first model (with dark substance density in  $1/r^2$ ). From equation (18), the dark radius of these galaxies is given by

$$R_S = (2k_0T/4\pi G\rho_0)^{1/2} \quad (42)$$

Further, using equation (8) :

$$k_2 = 2k_0T/G \quad (43)$$

Consequently :

$$R_S = (k_2/4\pi\rho_0)^{1/2} \quad (44)$$

So according to equation (9) the total mass of the dark halo is:

$$M_S(R_S) = \frac{k_2^{3/2}}{(4\pi\rho_0)^{1/2}} \quad (45)$$

Let us now calculate the mass of a sphere with radius  $R_S$  and a uniform density equal to the intergalactic dark substance density  $\rho_0$ .

$$M_I(R_S) = \rho_0 \frac{4}{3} \pi \left( \frac{k_2}{4\pi\rho_0} \right)^{3/2} = \frac{1}{3} \frac{k_2^{3/2}}{(4\pi\rho_0)^{1/2}} \quad (46)$$

Consequently :

$$M_I(R_S) = M_S(R_S)/3 \quad (47)$$

So the mean density of the halos of galaxies belonging to the first model of dark matter distribution is equal to  $3\rho_0$ , regardless of the radius and the temperature of the considered halo, and consequently regardless of the orbital velocity of stars in the considered galaxy.

The study also assumes that the dark mass of clusters is much greater than the baryonic mass of the galaxies belonging to those clusters.

Therefore, for a cluster A with a mean density  $\rho_{mA}$ , and if we neglect the baryonic density, we have:

$$\rho_0 < \rho_{mA} < 3\rho_0 \quad (48)$$

The mean densities of galaxy clusters make it possible to estimate the density  $\rho_0$  of the intergalactic dark substance.

We will see that these theoretical predictions agree with astronomical observations.

It is interesting to introduce the mean volume of the dark halo corresponding to the first model of dark substance distribution per galaxy, which we denote by  $Vol_{SG}$ . Then, if clusters contain the same kind of galaxies in the same proportions (which is not always the case), one can express the mean density of dark substance  $\rho_{mA}$  as a function of  $N_A$ , the number of galaxies inside the cluster A, and  $Vol_{SG}$ . Indeed, using the fact that the mean density of the dark halos corresponding to the first model of distribution is equal to  $3\rho_0$ , and that elsewhere the density of dark substance is equal to  $\rho_0$ , one can then express the mean density of the cluster from its total volume  $Vol_A$  as follows:

One considers that the contribution to the dark mass comes:

- from the dark halos of the galaxies (with a mean density  $3\rho_0$ ,
- and from the intergalactic medium contained in the remainder of the cluster (with density  $\rho_0$ ).

$$\rho_{mA} = \frac{1}{Vol_A} [3\rho_0 N_A Vol_{SG} + \rho_0 (Vol_A - N_A Vol_{SG})] \quad (50)$$

So we obtain,  $\rho_{\text{mAG}}$  being the mean density of the number of galaxies in the cluster,  $\rho_{\text{mAG}}=N_A/\text{Vol}_A$ :

$$\rho_{\text{mA}}=\rho_{\text{mAG}}(2\rho_0\text{Vol}_{\text{SG}})+\rho_0 \quad (51)$$

Moreover, denoting by  $\text{Vol}_A(H)$  the total volume of the dark halos of the galaxies belonging to the first model within the cluster A, we have, still using the fact that the mean density of the dark halos corresponding to the first model of dark substance distribution is equal to  $3\rho_0$  (according to Equation (47)), and that elsewhere the density of dark substance is equal to  $\rho_0$ :

$$\rho_{\text{mA}} = \frac{1}{\text{Vol}_A} [3\rho_0\text{Vol}_A(H) + \rho_0(\text{Vol}_A - \text{Vol}_A(H))] \quad (52)$$

$$\rho_{\text{mA}} = 2\rho_0 \frac{\text{Vol}_A(H)}{\text{Vol}_A} + \rho_0 \quad (53)$$

An important case is that in which we have  $\text{Vol}_A(H)/\text{Vol}_A \ll 1$  for all clusters. In this case, the mean density  $\rho_{\text{mA}}$  will be very close to  $\rho_0$  for all clusters. This implies, since  $\rho_0$  depends on the cosmological redshift  $z$ , that clusters corresponding to the same  $z$  will have approximately the same mean density  $\rho_{\text{mA}}$ , very close to  $\rho_0(z)$ .

We recall that we assumed the contribution of baryonic matter to the mean density of the cluster  $\rho_{\text{mA}}$  could be neglected. In what follows, still according to the first model of dark substance distribution, we will assume that generally for clusters  $\text{Vol}_A(H)/\text{Vol}_A \ll 1$ , and consequently  $\rho_{\text{mA}} \approx \rho_0$ . We remind that  $\rho_0$  depends on  $t$ , the age of the Universe. We will see further on that this previous assumption is in agreement with astronomical observations.

In the second model of dark substance distribution in galaxy clusters, the density of dark substance interacting gravitationally is that associated with the mass of dark halos:

$$\rho_{\text{mA}}=3\rho_0\text{Vol}_A(H) \quad (54)$$

Two dynamical models of clusters, which allow one to derive relations between the mass of clusters and the velocities of galaxies belonging to these clusters, have been studied. Only the second model is new, while the first model is generally accepted within the Standard Cosmological Model (SCM), but without a specific model of dark matter. We will observe that these two models yield theoretical predictions that are quite similar regarding the relations between the mass of a given cluster A, its radius, and the dispersion velocity of its galaxies or the maximal recession velocity of the galaxies in that cluster. However, we will see that the second dynamical model is based on our dark matter model and furthermore makes it possible to interpret some astronomical observations which are not explained by the first dynamical model. In what follows, we will study the first model of dark substance distribution in clusters, and we will see that its theoretical predictions are in good agreement with astronomical observations.

A first possible dynamical model of clusters is the model generally employed in the Standard Cosmological Model (SCM) (NARLIKAR 2002), which is based on the Virial theorem. We will therefore refer to this model as the Virial dynamical model of clusters.

According to this model, if  $\sigma_A$  is the velocity dispersion within a cluster A, with  $M_A$  the mass of the cluster and  $R_A$  its radius:

$$\frac{GM_A}{R_A} \approx \alpha_A \sigma_A^2 \quad (56)$$

In the previous expression,  $\alpha_A$  is of order unity and depends on cluster A. Very often, it is taken to be 1 or 2. One can also replace  $R_A$  in the preceding expression with the Abel radius (RAINE & THOMAS 2001). We recall that equation (56) obtained from the Virial model appears to be approximately consistent with astronomical observations. We will see that this will also be the case for the second dynamical model of clusters.

We now propose a second dynamical model of clusters, based on our dark matter model. In this model, with  $G_A$  being a galaxy of cluster A located at a point P within the cluster, we consider only the gravitational potential generated at P by the dark substance. We will thus refer to this second model as the *dynamical model of the dark potential of clusters*.

In order to obtain, within this second model, the gravitational potential generated by the dark substance at any point of the cluster, it is necessary to present the elements of our dark matter theory allowing the calculation of the gravitational field  $\mathbf{G}$  and the gravitational potential  $U$  at any point in the Universe. We have already examined two examples of adaptations of Newtonian mechanics to our dark matter theory (Sections 2.6 and 2.8). We have seen that these adaptations are necessary because in Newtonian gravitational theory, only baryonic matter exists, and there is no expansion, which is not the case in our dark matter theory. To obtain  $\mathbf{G}(Q)$  and  $U(Q)$  at a point Q in the Universe using Newtonian mechanics, in order to take into account the dark substance density at a point P, we recall that it is necessary to distinguish the cases in which P is inside a baryonic matter concentration, or not. Indeed, we have established the fundamental property:

a) Let us suppose that P is a point in the Universe belonging to no concentration of baryonic matter nor of dark substance, but belonging to the intergalactic dark substance. We know that the density of dark substance at P is equal to  $\rho_0$  (Sections 2.3 and 2.8). Due to the expansion of the Universe and the properties of dark substance, we will admit in our dark matter theory that there exists a symmetry for all points P satisfying these properties, implying that we must take  $\rho(P)=0$  in the Newtonian equations in order to obtain  $\mathbf{G}(Q)$  and  $U(Q)$ . This means that dark substance behaves as if it were absolute emptiness at P, exactly as in Section 2.8.

Thus, rule (a) justifies that between clusters, dark matter behaves as absolute emptiness, consistent with astronomical observations.

b) If P belongs to a significant concentration of baryonic matter (cluster, galaxy, star, a concentration due to baryonic matter anisotropies in the early Universe, etc.), then the symmetry at P is broken: we must take  $\rho(P)=\rho_0$  equal to the density of dark substance at P in the Newtonian equations in order to obtain  $\mathbf{G}(Q)$  and  $U(Q)$ .

Thus, we have a third example of adaptation of Newtonian equations to our dark matter theory that is due to expansion of the Universe, that did not exist in the Newton's theory of gravitation.

In this third dynamical model of clusters, we model a cluster as an ideal system with the following properties:

a) The cluster is a sphere of radius  $R_A$ , containing galaxies and dark substance, and possessing spherical symmetry.

b) In order to determine  $\mathbf{G}$  and  $U$  within the cluster, allowing the calculation of the velocities, accelerations, and energies of the galaxies, the galaxies being modeled as point masses located at their centers of mass, we can consider that within the cluster, the density is homogeneous and equal to  $\rho_{mA}$  (because of equation (53), assuming  $\text{Vol}_A(H)/\text{Vol}_A \ll 1$  and neglecting the baryonic matter contribution).

Concerning the galaxies of the cluster, the velocities and energies are calculated in the reference frame with origin  $O_A$ , the center of the cluster. The galaxies are modeled in the following way:

c) For a galaxy  $G_A$ , we define the ratio  $r(G_A)$  as  $r(G_A) = E_T(G_A)/m(G_A)$  where  $E_T(G_A)$  is the total energy of the galaxy  $G_A$  and  $m(G_A)$  its mass. We also define  $r_{AMax}$  as the maximum value of this ratio. Then, according to our cluster model:

(i) The radius  $R_A$  of the cluster is the maximum possible distance between a galaxy  $G_A$  in the cluster and  $O_A$ , the cluster center, subject to the condition  $r(G_A) \leq r_{AMax}$ .

(ii) The galaxies  $G_A$  with  $r(G_A) = r_{AMax}$  have a high density within the cluster (not necessarily homogeneous). This means that at any point  $Q$  in the cluster, there exists a galaxy  $G_A$  close to  $Q$  such that  $r(G_A) = r_{AMax}$ . Furthermore, in the case where  $Q = O_A$ , the cluster center, due to spherical symmetry, if  $\mathbf{u}$  is any unit vector, there exists a galaxy  $G_{A0}$  near  $O_A$  with  $r(G_{A0}) = r_{AMax}$  such that, with  $\mathbf{V}(G_{A0})$  being its velocity vector:

$$\mathbf{V}(G_{A0}) \cdot \mathbf{u} \approx \|\mathbf{V}(G_{A0})\|,$$

meaning that the vector  $\mathbf{V}(G_{A0})$  is approximately collinear with  $\mathbf{u}$ .

d) The galaxies  $G_A$  such that  $r(G_A) = r_{AMax}$  maintain their energy and mass, and consequently  $r_{AMax}$  is constant.

Therefore, according to the preceding property (a) of our cluster model, and also our adaptation of the Newtonian equations (previous example):

$$U(R_A) = -GM_A/R_A \quad (57a)$$

$$\mathbf{G}(R_A) = -GM_A/R_A^2 \mathbf{u} \quad (57b)$$

Moreover, for a galaxy  $G_A$  located at a distance  $r$  from  $O_A$ , with mass  $m(G_A)$  and velocity  $\mathbf{V}(G_A)$ , its total energy  $E_T(G_A)$ , with  $U(r)$  the gravitational potential at a distance  $r$  from  $O_A$  is thus

$$E_T(G_A) = (1/2)m(G_A)V(G_A)^2 + m(G_A)U(r) \quad (58)$$

Using the spherical symmetry of our cluster model and applying Gauss's theorem, with  $M(r)$  being the mass of the sphere centered on  $O_A$  of radius  $r$ , the gravitational field  $G(r)$  is then:

$$\mathbf{G}(r) = -G \frac{M(r)}{r^2} \mathbf{u} \quad (59)$$

According to the property b) of our model of cluster,  $M(r)=(4/3)\pi r^3 \rho_{mA}$  and consequently :

$$\mathbf{G}(r) = -G \frac{4}{3} \pi r \rho_{mA} \mathbf{u} \quad (60)$$

By definition  $\mathbf{G}=-\mathbf{Grad}(U)$ , so we obtain,  $C_{AU}$  being a positive constant at a given age of the Universe:

$$U(r)=G(4/6)\pi r^2 \rho_{mA}-C_{AU} \quad (61)$$

This equation can also be written, in the approximation that the density of dark matter in the cluster is approximately constant an equal to  $\rho_{mA}$ ,  $M(r)$  being the mass of the sphere with the centre  $O_A$  and a radius  $r$  :

$$U(r)=GM(r)/2r-C_{AU} \quad (62)$$

Consequently we have,  $M_A=M(R_A)$  being the mass of the cluster, using the equation (57a) :

$$\frac{GM_A}{2R_A} - C_{AU} = -\frac{GM_A}{R_A} \quad (63)$$

So we finally obtain, with  $M_A$  and  $R_A$  depending a priori on  $t$ , age of the Universe:

$$C_{AU} = \frac{3}{2} \frac{GM_A(t)}{R_A(t)} \quad (64)$$

Therefore, using the equation (58), for a galaxy at a distance  $r$  from  $O_A$  :

$$\frac{1}{2} m(G_A) V(G_A)^2 + Gm(G_A) \frac{M(r)}{2r} = E_T(G_A) + m(G_A) C_{AU} \quad (65a)$$

Moreover we have defined, in the property c) of our model of cluster,  $r_{AMax}$  as being the maximal value of  $r(G_A)=E_T(G_A)/m(G_A)$ . So we have for any galaxy  $G_A$ :

$$\frac{1}{2} V(G_A) + G \frac{M(r)}{2r} \leq r_{AMax} + C_{AU} \quad (65b)$$

We are now going to consider a galaxy  $G_{A1}$  located at the outer boundary of the cluster (i.e., at  $r=R_A$ ), and a galaxy  $G_{A0}$  located at the cluster center  $O_A$  (i.e., at  $r=0$ ).

According to property c)(i) of our cluster model, the radius  $R_A$  of the cluster is the maximum possible distance between a galaxy  $G_A$  in the cluster and the cluster center  $O_A$ , under the condition  $r(G_A) \leq r_{AMax}$ . Taking into account the preceding inequality (65b), we therefore have, for a galaxy  $G_{A1}$  at the limit of the cluster,  $V(G_{A1})=0$  and consequently:

$$G \frac{M(R_A)}{2R_A} = r_{AMax} + C_{AU} \quad (66)$$

For a galaxy  $G_{A0}$  situated at the centre of the cluster ( $r=0$ ), such that  $r(G_{A0})=r_{AMax}$ , according to the equation (65a):

$$\frac{1}{2}V(G_{A0})^2 = r_{AMax} + C_{AU} \quad (67)$$

Therefore, because of equation (65b),  $V(G_{A0})$  is equal to the maximal velocity of the galaxies in the cluster  $V_{MA}$ . Consequently, using the equations (66) (67) we obtain:

$$V_{MA}^2 = \frac{GM_A}{R_A} \quad (68)$$

Moreover, according to property c) of our cluster model, for any unit vector  $\mathbf{u}$ , there exists a galaxy  $G_{A0}$  close to  $O_A$  such that  $r(G_{A0})=r_{AMax}$  and  $\mathbf{V}(G_{A0}) \cdot \mathbf{u} \approx \|\mathbf{V}(G_{A0})\|$  with  $\mathbf{V}(G_{A0})$  the velocity vector of  $G_{A0}$  and  $\|\mathbf{V}(G_{A0})\|$  its norm.

Therefore, this third model predicts that the velocity of galaxies is maximal for galaxies close to the center of the cluster, in agreement with astronomical observations (RAINE & THOMAS 2001), which is not the case for the second (Virial) model.

Moreover, if  $A_i$  and  $A_j$  are two clusters, we have:

$$M_{Ai} = (4/3)\pi\rho_{mAi}R_{Ai}^3 \quad (69)$$

We immediately obtain, by applying equation (68):

$$\frac{\rho_{mAj}}{\rho_{mAi}} = \left(\frac{V_{MAj}}{V_{MAi}}\right)^2 \left(\frac{R_{Ai}}{R_{Aj}}\right)^2 \quad (70)$$

But we have seen from equation (53) that if  $A_i$  and  $A_j$  are two galaxy clusters corresponding to the same cosmological redshift  $z$ , and if moreover  $\text{Vol}A_i(H)/\text{Vol}A_i \ll 1$  and  $\text{Vol}A_j(H)/\text{Vol}A_j \ll 1$ , then  $\rho_{mAj}/\rho_{mAi}$  should be close to unity.

We do not have sufficient data to rigorously validate or invalidate the previous dark matter model in galaxy clusters, nor the previous equation (70a). Moreover, real clusters can only be approximately modeled as ideal clusters. However, the limited data we have, concerning the Coma and Virgo clusters, is in agreement with this model. Since data varies depending on the source, we will consider the data given by Wikipedia as the most reliable.

For instance, consider the Virgo cluster  $A_2$  (with  $z_2 < 0.01$ ) and the Coma cluster  $A_4$  (with  $z_4 < 0.03$ ). According to astronomical observations, considering the galaxies NGC4388 and IC3258 as well as the galaxies with the greatest velocity relative to the center of the considered cluster, we can take  $V_{MA2}(u_2) = 1600$  km/s (SEDS MESSIER DATABASE 2006). Moreover we can take  $R_{A2} = 7.3$  million light-years (FOUQUE et al., 2001). These values of  $V_{MA2}$  and  $R_{A2}$  are also those given by Wikipedia (“Virgo Cluster”). For the Coma cluster, we can take  $V_{MA4} = 2300$  km/s (BIVIANO, 1997), and adopt the currently accepted value, given by Wikipedia (“Coma Cluster”),  $R_{A4} = 10$  million light-years = 3 Mpc. Then, using the previous experimental data and equation (70a), we find  $\rho_{mA4}/\rho_{mA2} = 1.1$ . The difference between this ratio and 1 could be explained by the approximate validity of our model. We did not account for the fact that the proportion of the mass of baryonic matter and of the dark halos of spiral galaxies is not necessarily identical in the two clusters. Moreover, these two clusters are not ideal clusters: only the Coma cluster is approximately spherical (a regular cluster), whereas the Virgo cluster is an irregular cluster, and neither is truly homogeneous because of the heterogeneity of baryonic matter and of the dark halos of spiral galaxies.

Taking into account the approximate nature of our model, we can expect that the ratio given by the previous equation (70a) is of the order of unity, which is indeed the case.

According to property d) of our cluster model,  $r_{AMax}$  remains constant in order to describe the evolution of the mass and radius of a galaxy cluster. According to equation (63) we obtain:

$$r_{AMax} = -G \frac{M_A(z)}{R_A(z)} \quad (71)$$

Therefore, because according to property d) of our model of galaxy clusters  $r_{AMax} \Gamma_{A, \setminus}$  is conserved, the ratio  $M_A(z)/R_A(z)$  is also conserved. Moreover, since  $M_A(z) = (4/3)\pi R_A(z)^3$ , and from equation (53), with  $Vol_A(H)/Vol_A \ll 1$ , it follows that  $\rho_{mA}(z) \approx \rho_0(z)$ , where  $\rho_0(z)$  is the density of the intergalactic dark substance for the Universe at cosmological redshift  $z$ . Therefore, according to the previous equation (70b), the evolution of  $M_A(z)$  and  $R_A(z)$  scales as  $1/\rho_0(z)^{1/2}$ .

But, as we will see further in this section,  $\rho_0(z) \approx \rho_0(0)(1+z)^3$ . Consequently, we obtain:

$$\begin{aligned} M_A(z) &\approx M_A(0)/(1+z)^{3/2} \\ R_A(z) &\approx R_A(0)/(1+z)^{3/2} \end{aligned} \quad (72)$$

For example,  $M_A(2) \approx M_A(0)/5$ ,  $M_A(1) \approx M_A(0)/3$ . This means, for instance, that the Coma cluster was approximately five times less massive for a Universe corresponding to a cosmological redshift  $z=2$ . Nevertheless, it is possible that  $r_{AMax}$  could depend on  $z$ , which would permit a constant  $M_A(z)$ , and therefore a constant mean density of dark matter in the Universe.

The apparent concentration of dark matter near the center of clusters could be explained by the presence of the most massive galaxies, whose rotation curves are flat, being preferentially located close to the center of the cluster.

The density of the intergalactic dark substance depends on the age of the Universe. As before, we will use the notation  $\rho_0(0)$  to represent the density of dark matter at the present epoch ( $z=0$ ), and  $\rho_0(z)$  to represent the density of the intergalactic dark substance at the age of the Universe corresponding to cosmological redshift  $z$ . The estimate of the intergalactic density  $\rho_0(0)$  obtained using this third dynamical model of clusters allows further theoretical predictions that support the validity of our dark matter model.

Indeed, according to equation (18), for a galaxy corresponding to the first model (with a dark matter density falling off as  $1/r^2$  immersed in the intergalactic dark substance, the dark radius  $R_S$  of this galaxy is given at the present age of the Universe by:

$$R_S = \left( \frac{2k_0 T}{4\pi G \rho_0(0)} \right)^{1/2} \quad (73)$$

Therefore,  $v$  being the orbital velocity of stars in this galaxy, according to equation (10):

$$R_S = \frac{v}{(4\pi G \rho_0(0))^{1/2}} \quad (74)$$

But the dynamical model of the dark potential exposed previously permits an estimation of  $\rho_0(0)$ . Let us for instance consider the case of the Milky Way. To get  $\rho_0(0)$ , we apply the dynamical model of the dark potential to the Virgo cluster A2 ( $z_{A2} < 0.01$ ). According to equation (68) we can write,  $\rho_{mA}$  being the mean density of the cluster A, and using  $M_A = \rho_{mA} (4/3) \pi R_A^3$ :

$$\rho_{mA} = \frac{1}{(4/3) \pi G} \frac{V_{MA}^2}{R_A^2} \quad (75)$$

If A is a cluster with  $z_A$  very close to 0, and assuming that  $\text{Vol}_A(H) \ll \text{Vol}_A$  in the equation (53), then we can consider that the mean density of galaxy cluster satisfies  $\rho_{mA} \approx \rho_0(0)$ . Therefore, by replacing  $\rho_0(0)$  in equation (72) with  $\rho_{mA}$  given by the equation (75), we obtain:

$$R_S = \frac{v}{\sqrt{3}} \frac{R_A}{V_{MA}} \quad (76)$$

Taking the Virgo cluster A<sub>2</sub> (with  $z_{A2} < 0.01$ ), using the experimental data  $R_2 = 7.3$  million light-years and  $V_{M2} \approx 1600$  km/s, and assuming for the Milky Way  $v \approx 205$  km/s, we find a dark radius of the Milky Way  $R_{SMW} \approx 540,000$  l.y Using the Coma cluster data similarly, we obtain  $R_{SMW} \approx 510,000$  l.y and  $M_{M.W} = v^2 R_{SM.W} / G = 1.5 \cdot 10^{12} M_\odot$ ,  $M_{M.W}$  mass of the Milky Way. These results are consistent and give a dark radius of the Milky Way larger than the distance to the Magellanic Clouds (approximately 250000 l.y (ALVES & NELSON 2000). This is a notable prediction of our dark matter model. The 5% discrepancy between these two estimates can be explained by the approximate validity of our models. Researchers can expect that different data sets yield  $R_S$  values of comparable order, which is indeed observed here. Since the Coma cluster is approximately spherical, we retain the value derived from Coma data. Moreover, if we take into account a possible difference between  $\rho_{mA}$  and  $\rho_0(0)$ , for

instance assuming  $\rho_{mA} \approx 1.2 \rho_0(0)$ , we get  $R_S \approx 550,000$  l.y, which has been used to estimate the Milky Way mass in reasonable agreement with modern estimates.

**On gravitational lensing:** Observations of gravitational lensing (predicted by General Relativity) involve light deflection due to cluster mass. According to the third adaptation of Newtonian mechanics in our model, the dark substance between clusters acts as if it were absolute vacuum within Newtonian equations. By analogy, generalizing this to General Relativity, the deflection of light rays by a cluster can be calculated as if the surrounding dark substance were absolute vacuum. It would be interesting to compare the cluster mass obtained from lensing with the mass obtained via the third dynamical model of clusters developed here.

**Mass estimates of clusters:** The mass of a cluster A is  $M_A = (4/3)\pi R_A^3 \rho_{mA}$ . As seen earlier, in the ideal cluster model,  $\rho_{mA} \approx \rho_0(z) = \rho_0(0)(1+z)^3$ . Inside a spherical concentration of dark substance, the mean density of dark substance is about  $3\rho_0(z)$ . Hence an estimate of  $\rho_0$  (in solar masses per cubic light-year) can be obtained with  $M_{M.W} \approx (4/3)\pi R_{M.W}^3 \times 3\rho_0$ , with  $M_{M.W} \approx 1.5 \times 10^{12} M_\odot$  and  $R_{SM.W} \approx 550,000$ ly from previous data.

Applying this to the Virgo cluster ( $A_2$ ) with  $R_{A2} \approx 7.3$  million ly, and to the Coma cluster ( $A_4$ ) with  $R_{A4} \approx 10$  million ly, we obtain  $M_{A2} \approx 1.1 \times 10^{15} M_\odot$ ,  $M_{A4} \approx 2.9 \times 10^{15} M_\odot$ . We should add 15% to account for baryonic matter and dark halos. These values agree with observed estimates (e.g.,  $M_{A4}$  sometimes cited around  $8 \times 10^{14} M_\odot$ ,  $M_{A2} \approx 1.2 \times 10^{15} M_\odot$  and  $M_{A2} \approx 1.2 \times 10^{15} M_\odot$ ). This agreement is remarkable given the approximations, keeping in mind that real clusters are not perfectly ideal and that a 5% error on  $R_A$  and  $\rho_{mA}$  can cause a 25% uncertainty in  $M_A$ .

## 2.10 Formation of large structures in the Universe

According to the standard cosmological model (SCM), the galaxies, stars, and large structures seen today developed from primordial density heterogeneities. However, the estimated baryonic-matter heterogeneities in the early Universe are far too small to account for these large structures. The SCM therefore suggests these heterogeneities are amplified by dark matter.

According to our theory of dark matter, the heterogeneities in the early Universe could be explained by generalizing our earlier hypotheses. Because of the properties of the dark substance, in the primordial Universe and due to expansion of the Universe, if a point P does not belong to a concentration of baryonic matter, then in the Newtonian gravitational equations we should set  $\rho_{SN}(P) = 0$  for the Newtonian “effective” densities, but  $\rho_{SN}(P) = \rho_0$ , if it belongs to a baryonic overdensity.

This hypothesis amplifies the gravitational effects of baryonic-matter heterogeneities, potentially explaining the origin of large-scale structures observed in the Universe today.

### 3. NEW COSMOLOGICAL MODEL

#### 3.1 Introduction

In the preceding Part 2, we presented a theory that interprets the entirety of astronomical observations associated with dark matter. This theory, built on the concept of a *dark substance* filling the entire Universe, naturally led us to propose a **spherical geometrical form** for the Universe.

In Part 3, the study proposes a **new cosmological model** based on this spherical form, together with a physical interpretation of the Cosmic Microwave Background (CMB) Rest Frame (CRF). Within this framework, the study defines cosmological distances fully analogous to those used in the Standard Cosmological Model (SCM) — namely, the angular distance, luminosity distance, comoving distance, and light-travel distance — as well as a Hubble constant analogous to that of the SCM.

This new cosmological model is physically simpler and arguably more transparent than the SCM. The study also proposes within this new framework two possible mathematical models of expansion, each capable of reproducing the cosmological scale factor  $(1+z)$  and the cosmological redshift  $z$ :

1. The **first mathematical model** of expansion is, like the SCM, based on General Relativity (as in the  $\Lambda$ CDM model). As in the  $\Lambda$ CDM model, it requires the existence of dark energy, and it predicts identical values for cosmological distances and the Hubble constant as in the SCM. However, it offers a novel interpretation of the nature of both dark matter and dark energy. We will see later that this first model can also be derived without using General Relativity, instead using a simpler Newtonian framework.
2. The **second mathematical model** of expansion is much simpler mathematically. Despite its simplicity, it predicts values of the Hubble constant and cosmological distances that match astronomical observations for sufficiently low redshift  $z$ . Remarkably, this second model does *not* require the existence of dark energy, unlike the first model and unlike the SCM.

In this Part 3, we will demonstrate that the new cosmological model is consistent with Special Relativity and General Relativity. According to the model, the CRF cannot be detected through conventional laboratory experiments but only through CMB observations. Therefore, the study maintains the validity of Special Relativity and General Relativity, even though alternative possibilities exist (cf. DELORT 2000, DELORT 2020).

From observations of CMB anisotropies (see Section 1 Introduction), it appears that these support the first mathematical model while contradicting the second. Nevertheless, in what follows, we will focus on the second model, which is easier to generalize into the first model if needed, though at the cost of more complex calculations.

According to this new cosmological framework, the Universe is flat, which will help justify why we should take  $\Omega_c=0$  in the Friedmann equations of the  $\Lambda$ CDM model.

Furthermore, the dark matter component of this new cosmological model is

compatible with the properties assumed for dark matter in  $\Lambda$ CDM (cold, dissipationless, collisionless).

### 3.2 Physical Interpretation of the CRF: Local and Universal Cosmological Frames

The CMB exhibits a Doppler effect that cancels in a particular frame, the so-called CMB Rest Frame (CRF). In the SCM, the CRF has no clear physical interpretation, but our theory provides one. This interpretation will allow us to define a new model of cosmological expansion, based on the spherical Universe geometry assumed in our theory. Within this framework, cosmological variables such as cosmological time, cosmological distances, and the Hubble constant can be defined in full analogy with their roles in the SCM.

To obtain the cosmological redshift  $z$  — fundamental to the new expansion model, just as in the SCM — our theory proposes two mathematical models of expansion:

- The **first model** uses General Relativity, like the SCM. It predicts identical formulas for the cosmological redshift  $z$  and other variables, but for a flat Universe as postulated by the new theory.
- The **second model** is much simpler. Despite this, it achieves predictions in excellent agreement with observations for sufficiently low  $z$ .

#### On the physical interpretation of the CRF:

- First, it is natural to define a CRF at each point in the Universe, not just on Earth. We can reasonably suppose that all these local CRFs have aligned corresponding axes.
- Second, the CRF provides a simple definition of cosmological time, identified with the age of the Universe. The most straightforward definition is that the time of the CRF (the time measured by clocks at rest in the CRF) coincides with the cosmological time. This hypothesis is consistent with astronomical observations. Indeed, it implies that cosmological time is approximately the same as the time experienced on Earth. We will refer to the CRF as the **local cosmological frame** and denote it  $R_{LC}$ .

Let  $H_S$  be a clock attached to the Sun, measuring time in the inertial frame  $R_S$  linked to the Sun, moving with velocity  $V_S$  relative to  $R_{LC}$ . According to Special Relativity, the transformations between  $R_S$  and  $R_{LC}$  are Lorentz transformations. If  $T_S$  is the time measured by  $H_S$  corresponding to a cosmological time  $T_C$ , then (dilatation of time):

$$T_S = T_C (1 - V_S^2/c^2)^{1/2} \quad (77)$$

Since  $V_S \ll c$  (with observations giving  $V_S \approx 300$  km/s), we have  $T_S \approx T_C$ . It is important to note that it is impossible for all local inertial frames (connected by Lorentz transformations) to define a *global* cosmological time (the age of the Universe). Therefore, it was not self-evident that the Sun's proper time would match cosmological time so closely.

- Third, Special Relativity tells us that the velocity of a photon relative to the local CRF stays constant in both magnitude and direction. We will refer to this as its **local velocity**  $c$ . A question arises about the evolution of this local velocity as the photon travels across the Universe. The simplest hypothesis is that the local velocity remains the same even as the photon transitions between different local CRFs. As we will see, this simple hypothesis yields theoretical predictions consistent with observations. It

provides a straightforward explanation of the expansion effects on photon wavelengths and on the distances between successive photons.

So we express the preceding hypothesis in the following **Postulate 3**:

**Postulate 3 (about the Cosmological Reference Frames, CRF):**

- a) At each point in the Universe, a CRF can be defined. We will assume that all CRFs have their corresponding axes parallel to one another.
- b) The cosmological time (identified with the age of the Universe) is the time measured by clocks at rest in any CRF; that is, it is the time coordinate common to all CRFs.
- c) The local velocity of a photon, as measured in the CRF in which it is located, remains constant as the photon propagates throughout the entire Universe.

Given its fundamental role in cosmology, and following our theory of dark matter and dark energy, we will also refer to the CRF as the *local cosmological frame*.

We recall that, due to Postulate 3b), and since we know that the inertial reference frame  $R_S$  associated with the Sun moves with a velocity  $v_S \ll c$  relative to the local CRF, the time measured in  $R_S$  is very close to the cosmological time. This is consistent with observation. Thus, Postulate 3b) justifies the identification of the time coordinate of  $R_S$  with the cosmological time, which is not an obvious result a priori. Moreover, according to astronomical observations, locally (i.e., near the Milky Way), all galaxies have a local velocity (relative to their own CRF) that is very small compared to  $c$ . Therefore, by Postulate 3b), the time coordinate of any star in a galaxy near the Milky Way is very close to the cosmological time.

It is reasonable to extend this property to the entire Universe, so that the time coordinate of any star (and thus any planet) across the Universe is approximately equal to the cosmological time.

We now need to fully define all CRFs. We have previously seen that, according to our theory of dark matter, the Universe is finite with boundaries, and we will assume it is spherical, with a center denoted  $O$ . It is worth noting that what follows could be generalized to other geometric models of finite universes with boundaries. We therefore model the Universe as an expanding sphere centered at  $O$ , with a radius  $R_E(t)$  (or  $R_U(t)$ ), where  $t$  is the cosmological time. We have seen in Section 2.5 that  $R_E(t_0) = R_E(t) (1+z)$  where  $t$  and  $t_0$  are any cosmological times with  $t < t_0$ , and where  $1+z$  is the expansion factor of the Universe between  $t$  and  $t_0$ . We will later see how to determine  $1+z$  from mathematical models of cosmic expansion.

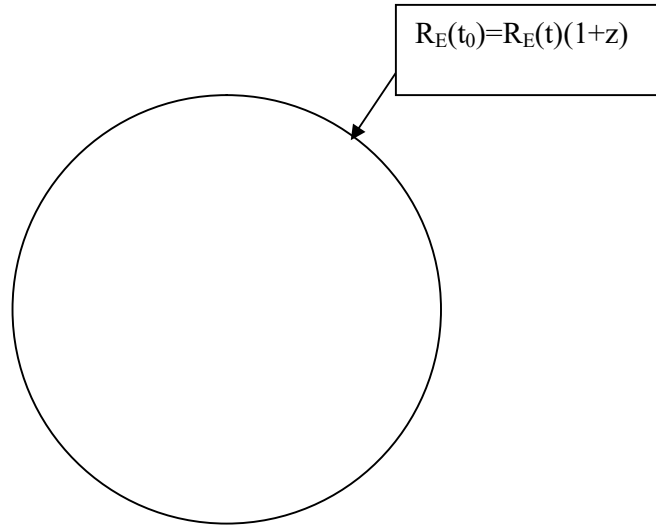


Figure 3: The spherical model of the Universe in expansion.

In order to fully define the CRF (or, equivalently, the local cosmological frames), we now introduce a new type of frame, denoted  $R_C$ , called the *Universal Cosmological Frame*. The origin of  $R_C$  is located at  $O$ , the center of the Universe. The time coordinate of the Universal Cosmological Frame  $R_C$  is defined as the same cosmological time used in the CRFs (see Postulate 3b)). Furthermore, the axes of  $R_C$  are defined to be parallel to the corresponding axes of each CRF (Postulate 3a)), and to yield locally the same measurements of distance as the CRFs.

The Universal Cosmological Frame  $R_C$  permits the definition of distances between any pair of points (A,B) in the Universe, unlike the local CRFs, which only provide local distances. We will see that all of the classical cosmological distance measures used in the standard cosmological model (SCM)—namely the luminosity distance, angular distance, comoving distance, and light-travel distance—can be expressed as functions of the distances measured in  $R_C$ , the cosmological time, and the cosmological redshift  $z$ .

This study introduces very important points of the Universal Cosmological Frame  $R_C$ , which we will call the *comoving points* of the expanding sphere.

Let us assume that  $P(t)$  is a point on the boundary of the expanding sphere at cosmological time  $t$ , with the vector  $\mathbf{OP}(t)$  (where  $O$  is the center of the expanding sphere) remaining along a fixed direction  $\mathbf{u}$ , a fixed vector of  $R_C$ .

A comoving point  $A(t)$  of the expanding sphere is then defined by the following conditions:

- $A(t)$  lies on the segment  $[O,P(t)]$ .
- Its distance from the center satisfies:

$$OA(t)=a OP(t) \quad (78)$$

with a constant in the interval  $[0,1]$ .

Hence both  $O$  and  $P(t)$  are particular cases of comoving points of the expanding sphere.

Moreover, if  $A(t)$  and  $B(t)$  are two comoving points on a given radius  $[O,P(t)]$ , and if  $t_1$  and  $t_2$  are two cosmological times with expansion factor  $1+z=OP(t_2)/OP(t_1)$ , then the following relations hold:

$$A(t_2)B(t_2)=(1+z) A(t_1)B(t_1) \quad (79)$$

and

$$[A(t_2),B(t_2)]/[A(t_1),B(t_1)] \quad (80)$$

(here, as usual, for points  $P,Q$ ,  $PQ$  denotes the distance between  $P$  and  $Q$  measured in  $R_C$ ;  $[P,Q]$  is the segment with endpoints  $P$  and  $Q$ ; and  $(P,Q)$  is the straight line passing through  $P$  and  $Q$  ).

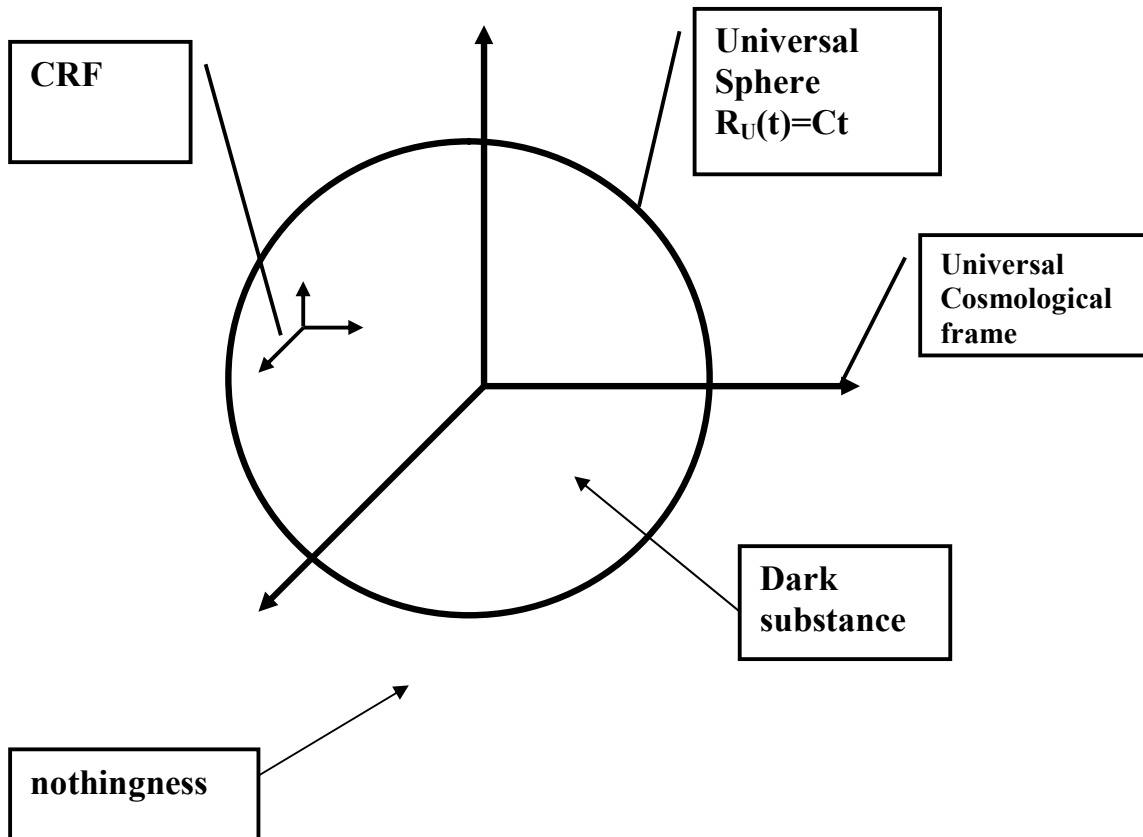


Figure 4 :New Cosmological model. In the 1<sup>st</sup> model,  $R_U(t)=Csinh^{2/3}(t/t_\Lambda)$ .

The study shows, using Thales' theorem, that the previous relations (79) and (80) remain valid for any pair of comoving points of the expanding sphere (as defined by relation (78)), even if they do not necessarily belong to the same segment  $[O,P(t)]$ .

Let us consider two comoving points (distinct from O)  $A(t_1)$  and  $B(t_1)$  at cosmological time  $t_1$ . We assume that  $A(t)$  belongs to the segment  $[O,P(t)]$ , with  $P(t)$  a point on the boundary of the expanding sphere, and similarly that  $B(t)$  belongs to the segment  $[O,Q(t)]$  with  $Q(t)$  a point on the boundary of the expanding sphere.

At a cosmological time  $t_2 > t_1$ , according to relation (71), the points  $O, B(t_1), B(t_2)$  lie on the same straight line, and the same applies to  $O, A(t_1), A(t_2)$ .

We then consider the triangle  $(O, A(t_2), B(t_2))$ . In this triangle, according to relation (78), with  $1+z$  the factor of expansion of the Universe between  $t_1$  and  $t_2$ , we have:

$$OA(t_2)/OA(t_1) = OP(t_2)/OP(t_1) = 1+z \quad (81)$$

And in the same way:

$$OB(t_2)/OB(t_1) = 1+z \quad (82)$$

Therefore:

$$OA(t_2)/OA(t_1) = OB(t_2)/OB(t_1) = 1+z \quad (83)$$

By applying Thales' theorem to the triangle  $(O, A(t_2), B(t_2))$ , we deduce that the segment connecting the comoving points is similarly expanded by the same factor  $1+z$ . i.e.

$$A(t_2)B(t_2) = (1+z)A(t_1)B(t_1) \quad (84)$$

Furthermore, the segment  $[A(t_2), B(t_2)]$  remains parallel to the segment  $[A(t_1), B(t_1)]$ :

$$[A(t_2), B(t_2)] // [A(t_1), B(t_1)] \quad (85)$$

Hence the homothetic character of the expansion is preserved for any pair of comoving points of the expanding sphere, not only those lying along a single fixed radius. This property will be fundamental in deriving the classical cosmological distances as functions of the distances measured in  $R_C$  and the redshift  $z$ .

We remark that if  $A(t)$  is a comoving point of a segment  $[O,P(t)]$ , according to relations (78), if  $V_P(t)$  and  $V_A(t)$  are respectively the velocities of  $P(t)$  and  $A(t)$  measured in the Universal Cosmological frame  $R_C$ , we obtain, with  $a$  being a constant:

$$V_A(t) = a V_P(t) \quad (86)$$

The previous definition of the comoving points of the sphere in expansion allows us to complete the definition of the local Cosmological frames (CRF) in the following Postulate 4:

#### Postulate 4:

- a) The Universe is a sphere in expansion.
- b) The origins of the local Cosmological frames (CRF) are the comoving points of this expanding sphere.

The study then stated the factor of expansion  $1+z$  in our new cosmological model of the expansion of the Universe. It proposes two possible mathematical models of expansion within our new cosmological framework, allowing the derivation of  $1+z$ . Both mathematical models are not equivalent and do not yield the same expression for  $1+z$ . Nevertheless, we shall see that both models lead to theoretical predictions in good agreement with astronomical observations for  $z < 9$ .

According to the **first mathematical model** of expansion,  $1+z$  is obtained as in the SCM with a flat Universe: we apply locally the equations of General Relativity, assuming the same values as in the SCM for the densities of dark substance, baryonic matter, and dark energy, and assuming the Universe is flat. Therefore in this first model, the expansion factor  $1+z$  can be expressed mathematically exactly as in the SCM for a flat Universe. A consequence of this is that the first mathematical model predicts cosmological distances and a Hubble constant with the same mathematical expressions as in the SCM, for an observer sufficiently far from the borders of the Universe. Hence the new cosmological model with this first mathematical expansion is very close to the SCM, while clarifying the nature of dark matter and dark energy, and also interpreting the CMB rest frame.

In our **second mathematical model** of expansion, we assume that the velocity of the borders of the spherical Universe measured in the Universal Cosmological frame  $R_C$  is a constant  $C$ . We will see further that it is possible to obtain a lower limit for this constant  $C$ . Despite its simplicity, the theoretical predictions of this second model are in good agreement with all astronomical observations for  $z < 9$ .

In this second mathematical model, if  $P(t)$  is a point on the border of the sphere, then:

$$OP(t) = Ct$$

and between two cosmological times  $t$  and  $t_0$  (with  $t_0 > t$ ), the factor of expansion is simply:

$$1+z = (C t_0) / (C t) = t_0 / t \quad (87a)$$

In the new cosmological model with the first mathematical model, we recall (see RAINE & THOMAS 2001, DODELSON & SCOTT 2008) that according to the  $\Lambda$ CDM model, with the usual notations, one has (after the radiation era):

$$(1+z)^{-1} = (\Omega_m / \Omega_\Lambda) \sinh^{2/3}(t/t_\Lambda) \quad (87b)$$

$$\text{With } t_\Lambda = 2 / (3H_0 \Omega_\Lambda^{1/2}) = 2 / (3\Lambda^{1/2}).$$

Consequently,

$$R_{UMI}(t) = C \sinh^{2/3}(t/t_\Lambda). \quad (87c)$$

We recall that this first mathematical model is supported by the observations of the anisotropies of the CMB.

Within our framework, we can demonstrate, that if two photons move along the same straight line towards the origin  $O$  of  $R_C$ , then between two cosmological times  $t_1$  and  $t_2$  (with  $t_2 > t_1$ ), the distance between the two photons and their wavelengths are both scaled by the expansion factor  $1+z$  between  $t_1$  and  $t_2$ . This holds true for both mathematical models of expansion.

Furthermore, it is possible to replace  $O$  by any comoving point  $O'$  of the sphere in expansion, thanks to the homothetic properties of the commoving segments.

Consider two photons,  $ph1$  and  $ph2$ . We use the following notations: at cosmological time  $t$ ,  $ph1$  is located at the point  $ph1(t)$  of  $R_C$ , and  $ph2$  is located at the point  $ph2(t)$  of  $R_C$ . Suppose that at a given cosmological time  $t_1$   $ph1(t_1)$  coincides with a comoving point  $A_1(t_1)$  and  $ph2(t_1)$  with a comoving point  $A_2(t_1)$ . We also assume there exists a unit vector  $\mathbf{u}$  of  $R_C$  such that  $A_1(t_1)$  and  $A_2(t_1)$  belong to the same segment  $[O, P(t_1)]$  with  $(O, P(t_1))$  parallel to  $\mathbf{u}$ , and that the local velocities of  $ph1$  and  $ph2$  are identical and equal to  $c\mathbf{u}$ . We recall that, according to Postulate 3, these local velocities remain constant.

Let  $1+dz$  be the expansion factor between  $t_1$  and  $t_1+dt$ . Then, applying the properties (84) of comoving points:

$$A_1(t_1+dt) A_2(t_1+dt) = (1+dz) A_1(t_1) A_2(t_1) = (1+dz) ph1(t_1) ph2(t_1) \quad (88)$$

Moreover, the local velocity of photons being equal to  $c$ :

$$A_1(t_1+dt) ph1(t_1+dt) = A_2(t_1+dt) ph2(t_1+dt) = c dt \quad (89)$$

According to the properties (relations (84)) of comoving points, and with the local velocities of  $ph1$  and  $ph2$  being parallel to  $\mathbf{u}$ , the points  $O$ ,  $A_1(t_1+dt)$ ,  $ph1(t_1+dt)$ ,  $A_2(t_1+dt)$  and  $ph2(t_1+dt)$  remain aligned on the same straight line as  $O$ ,  $A_1(t_1)$ , and  $A_2(t_1)$  (in the direction of  $\mathbf{u}$ ). Moreover, we assume that they are ranked in this order. Therefore, using an elementary diagram of a straight line on which  $A_1(t_1)$ ,  $A_1(t_1+dt)$ ,  $A_2(t_1)$ ,  $A_2(t_1+dt)$ ,  $ph1(t_1+dt)$ , and  $ph2(t_1+dt)$  are represented, we obtain:

$$ph1(t_1+dt) ph2(t_1+dt) = A_1(t_1+dt) A_2(t_1+dt) \quad (90)$$

$$ph1(t_1+dt) ph2(t_1+dt) = (1+dz) ph1(t_1) ph2(t_1) \quad (91)$$

Hence, between  $t_1$  and  $t_1+dt$ , the distance between  $ph1(t_1)$  and  $ph2(t_1)$  is increased by the factor of expansion of the Universe between  $t_1$  and  $t_1+dt$ , namely  $1+dz$ . Consequently, between  $t_1$  and  $t_2$ , the distance between  $ph1(t_1)$  and  $ph2(t_2)$  is increased by the factor of expansion of the Universe between  $t_1$  and  $t_2$ , which is  $1+z$ :

$$ph1(t_2) ph2(t_2) = (1+z) ph1(t_1) ph2(t_1) \quad (92)$$

To show the corresponding effect on the wavelengths of  $ph1$  and  $ph2$ , we proceed as follows: we model the photon  $ph1$  as a system whose endpoints are two mobile points  $a(t)$  and  $b(t)$ , with the length  $a(t) b(t)$  representing the wavelength of the photon. The position  $ph1(t)$

lies, as before, on a segment  $[O,P(t)]$ , with  $(O,P(t))$  parallel to the unit vector  $\mathbf{u}$ , and  $ph1(t)$  moving with local velocity  $c\mathbf{u}$ . We assume that for any photon  $ph1(t)$ , the points  $a(t)$  and  $b(t)$  move with the same local velocity  $c\mathbf{u}$ , and that  $a(t)$ ,  $b(t)$  also belong to  $[O,P(t)]$ . We then proceed with  $a(t)$  and  $b(t)$  in exactly the same way as we did with  $ph1(t)$  and  $ph2(t)$ . Therefore, in our new cosmological model, with  $\lambda(t)$  denoting the photon's wavelength, we obtain a relation analogous to (92):

$$\lambda(t_2)=\lambda(t_1)(1+z) \quad (93)$$

We have stated that the relations (92) and (93) are also valid in the expansion model of the SCM. Because relation (93) holds for any photon in accordance with our theory of dark matter and dark energy — just as in the SCM — we use the notation  $1+z$  to represent the expansion factor of the Universe. It should be recalled that in relation (80c),  $\lambda(t_1)$  and  $\lambda(t_2)$  must be measured in the local Cosmological Reference Frame (the CMB rest frame) in which the photon is located, and these wavelengths also correspond to distances as measured in the Universal Cosmological frame  $R_C$ , according to the definition of  $R_C$ .

We can show more generally, using an analogous reasoning, that if we only suppose that  $ph1$  and  $ph2$  have the same local velocity  $c\mathbf{u}$  (even if  $ph1(t)$  and  $ph2(t)$  do not necessarily belong to the same straight line containing  $O$ ), then between two cosmological times  $t_1$  and  $t_2$  the distance measured in  $R_C$  between  $ph1$  and  $ph2$  increases by the factor of expansion of the Universe between  $t_1$  and  $t_2$ , namely  $1+z$  (as in equation (80b)). Moreover,  $(ph1(t_2),ph2(t_2))$  and  $(ph1(t_1),ph2(t_1))$  remain parallel.

Alternatively, one could have obtained the evolution of  $\lambda(t_1)$ , and hence of  $E_{TOT}(ph1)$ , by assuming the Natural Postulate stating that, neglecting gravitational effects, for any particle  $p1$ , if  $\lambda_{ADB}(p1(t_1))$  denotes the de Broglie wavelength associated with  $p1$  at absolute time  $t1$ , measured in the Local Cosmological Reference Frame whose origin coincides with  $p1(t_1)$ , then  $\lambda_{ADB}(p1(t_1))$  evolves proportionally to  $(1+z)$ , for a cosmological expansion factor equal to  $(1+z)$ . (Recall that  $\lambda_{ADB}(p1(t_1)) = h / p_A(p1(t_1))$ , with  $p_A(p1(t_1))$  the momentum of  $p1$  measured at time  $t_1$  in the Local Cosmological Reference Frame whose origin coincides with  $p1(t_1)$ ).

One may also regard this Postulate as a generalization of the particular case where  $p1$  is a photon. We shall therefore admit this Postulate.

This Postulate allows one to obtain the evolution of the energy of a free particle  $p1$  while neglecting gravitational effects. Indeed, applying the above Postulate to  $p1$ ,  $\lambda_{ADB}(p1(t_1))$  evolves as  $(1+z)$ , and therefore  $p_A(p1(t_1))$  evolves as  $1/(1+z)$  for a cosmological expansion factor equal to  $(1+z)$ , as in the Standard Cosmological Model (SCM). Consequently, the energy of  $p1$  evolves as  $1/(1+z)$  if  $p1$  is relativistic, whereas its kinetic energy evolves as  $1/(1+z)^2$  if  $p1$  is non-relativistic.

This interpretation is fundamental, because it identifies the Reference Frame in which the momentum  $p_A(p1)$  of a particle  $p1$  evolves as  $1/(1+z)$ : namely, the Local Cosmological Reference Frame (local CMB rest frame) whose origin coincides with  $p1$  which is not the case in the SCM. It should be recalled that if  $p1$  has mass, it is impossible for this property to hold in all Lorentz frames.

It should be noted that, within the NCM, the validity of the preceding postulate may be restricted to photons and neutrinos, since no experimental observation has directly or indirectly confirmed the effect predicted by the SCM on the momentum of classical massive particles (such as protons, neutrons, or electrons).

We note that for any comoving point of the expanding sphere  $O'(t)$ , one can define a Cosmological Reference Frame  $R_C'$  whose origin is  $O'(t)$ , whose time coordinate is the cosmological time (identical to that of  $R_C$ ), and whose axes are parallel to the corresponding axes of  $R_C$ , defining the same distances between two points at a given cosmological time  $t$  as those defined in  $R_C$ . We will call this  $R_C'$  a **secondary Universal Cosmological frame**.

Then if  $A(t)$  is any comoving point of the expanding sphere defined earlier, and  $t_1$  and  $t_2$  are two cosmological times, according to the properties of comoving points (79), (80), if  $1+z$  is the expansion factor of the Universe between  $t_1$  and  $t_2$ , we have:

$$\frac{O'(t_2)A(t_2)}{(O'(t_2),A(t_2))} = (1+z) \frac{O'(t_1)A(t_1)}{(O'(t_1),A(t_1))} \quad (94)$$

Consequently,  $(O'(t_1),A(t_1))$  and  $(O'(t_2),A(t_2))$  lie in the same direction  $\mathbf{u}$  of  $R_C'$ .

The relations (78), (79), and (80) remain valid when replacing  $R_C$  with  $R_C'$  and  $O$  with  $O'$ . The point  $P(t)$  is still defined as a point belonging to the border of the expanding sphere, though we no longer have  $OP(t)=R_E(t)$ , where  $R_E(t)$  is the radius of the sphere at cosmological time  $t$ .

Therefore, it would be possible to define comoving points in  $R_C'$  in the same way we defined them in  $R_C$ . The expressions for cosmological distances and the Hubble constant derived in  $R_C$  are also valid in  $R_C'$ .

We will see that, generally, it is not possible to observe the entire Universe from any comoving point  $O'$  (which was also the case in the SCM: according to the SCM, it is not possible to observe the entire Universe from our own planet), but if  $O'$  is sufficiently far from the borders of the Universe, then the Universe observed from  $O'$  is approximately identical to the Universe observed from  $O$ .

A spherical form of the Universe could be confirmed if some celestial bodies did not have a homogeneous distribution but instead a distribution showing spherical symmetry with respect to a point  $O$ . According to our models, this point  $O$  would then be the center of the spherical Universe.

### 3.3 Hubble's law-Distances used in Cosmology.

We retain the notations of the previous section. Let  $R_C$  denote the Universal Cosmological Frame, with its origin  $O$  identified as the center of the Universe. (We recall that the following results can be generalized by replacing  $O$  with any comoving point  $O'$  sufficiently far from the boundaries of the Universe, and by replacing  $R_C$  with a secondary Universal Cosmological Frame  $R_C'$  whose origin is  $O'$ .)

Let us suppose that a photon is emitted from a star S located at a point  $Q(t_E)$  in  $R_C$  (where  $Q(t)$  is a comoving point on the expanding sphere) at cosmological time  $t_E$ , directed toward O. We assume that the photon reaches O at the current cosmological time  $t_0$ . Let us further assume that between  $t_E$  and  $t_0$ , the expansion factor of the Universe is  $1+z_0$ .

Between  $t$  and  $t+dt$ , we know that the photon travels a local distance of  $c dt$ . Consequently, between  $t_E$  and  $t_0$ , the total sum of the local distances traveled by the photon will be:

$$D_T = c(t_0 - t_E) \quad (95)$$

We shall refer to this distance, which is identical to the light-travel distance as defined in the SCM, by the same name. It may also be called the **lookback distance**, since it allows one to determine the cosmological time elapsed between the emission of the photon at  $Q(t_E)$  and its reception at O at cosmological time  $t_0$ .

In the second mathematical model of universal expansion, the Hubble constant can be very easily derived using the light-travel distance defined above. According to this second model and equation (87), with  $1+z_0$  being the expansion factor between  $t_E$  and  $t_0$ :

$$1+z_0 = (Ct_0)/(Ct_E) = t_0/(t_0 - D_T/c) \quad (96)$$

When  $D_T/ct_0 \ll 1$  we obtain  $z_0 \approx D_T/ct_0$

When  $D_T/ct_0 \ll 1$ , we obtain the approximation  $z_0 \approx D_T/ct_0$ , and consequently the Hubble constant is approximately equal to  $1/t_0$ . This simple relation (96) can easily be checked. For instance, taking  $t_0 = 15$  billion years, for  $z_0 = 0.5$ , we obtain  $D_T = 5$  billion light-years, and for  $z_0 = 9$  we obtain  $D_T = 13.5$  billion years. These predicted values agree with commonly accepted experimental determinations of the light-travel distance  $D_T$ .

At present, two principal models are used to estimate the Hubble constant  $H$  (also denoted  $H_0$ ):

- The first model, using standard candles such as supernovae, yields  $H_{SN} = 73$  km/s/Mpc<sup>-1</sup> (WONG et al., 2020).
- The second model, using CMB data, yields  $H_{CMB} = 67$  km/s/Mpc<sup>-1</sup> (AGHANIM et al., 2020).

The value of  $H_{CMB}$  implies  $t_0 = 13.8$  billion years in the 1<sup>st</sup> mathematical expansion model (as in  $\Lambda$ CDM model) and  $t_0 = 1/H_{CMB} \approx 14.4$  billion years in the 2<sup>nd</sup> mathematical expansion model. But we cannot use  $H_{CMB}$  in the 2<sup>nd</sup> mathematical model because  $H_{CMB}$  is obtained assuming mathematics of  $\Lambda$ CDM model.  $H_{SN}$  gives  $t_0 = 1/H_{SN} \approx 13.4$  billion years in the 2<sup>nd</sup> mathematical expansion model.

We continue to assume that a photon is emitted by a star S located at a comoving point  $Q(t_E)$ , where  $t_E$  is the age of the Universe at emission, and that it reaches the origin O of the Universal Cosmological Frame  $R_C$  at the present age of the Universe  $t_0$ . As discussed in Section 3.2, we may assume that the local velocity of S is small compared to  $c$ , similar to how

the local velocities of stars near our own Milky Way (measured in the local CMB rest frame) are small compared to  $c$ .

If the photon emitted by S at cosmological time  $t_E$  has a wavelength  $\lambda_0$  as measured in the inertial frame linked to S, and if it reaches, at time  $t_0$ , a planet T very close to O, with a local velocity much smaller than  $c$ , then if  $\lambda_T(t_0)$  is the photon's wavelength measured in the inertial frame linked to the planet T at  $t_0$ , according to equation (93), where  $1+z_0$  is the expansion factor between  $t_E$  and  $t_0$ , we have:

$$\lambda_T(t_0) \approx (1+z_0) \lambda_0 \quad (97)$$

In our model of an expanding spherical Universe, other types of cosmological distances can be defined in exactly the same way as in the SCM.

We have seen (equation (95)) that the light-travel distance can be expressed as:

$$D_T = \int_{t_E}^{t_0} c dt \quad (98)$$

The local distance covered by the photon between  $t$  and  $t+dt$  is, according to the Postulate 3 equal to  $c dt$ . This local distance, considered as a distance between 2 commoving points of the sphere in expansion, is then increased by the factor of expansion of the Universe  $1+z$  between  $t$  and  $t_0$  (See equation (87)).

In complete analogy with the SCM, we will define the *comoving distance* between O and S as the distance between  $Q(t_0)$  and  $O(t_0)$  measured in the Universal Cosmological frame  $R_C$ . This distance corresponds to the sum of all the local distances  $c dt$  traveled by the photon, scaled by the factor  $1+z$ . Denote this commoving distance by  $D_C$ :

$$D_C = \int_{t_E}^{t_0} c(1+z) dt \quad (99)$$

From this expression we define the luminosity-distance  $D_L$  between O and S (at the Cosmological time  $t_0$  and the angular-diameter distance  $D_A$  between O and S in complete analogy with their definitions in the SCM:

$$D_L = (1+z_0) D_C \quad (100)$$

$$D_A = D_C / (1+z_0) \quad (101)$$

The distance  $D_A$  appears to be the distance measured in  $R_C$  between  $Q(t_E)$  and O. In complete analogy with the SCM it permits to obtain some angles with a summit O in  $R_C$ .

The distance  $D_L$ , in complete analogy with its definition in the SCM, appears to be obtained measuring the luminous flow of a supernova considering the effect of the expansion of the Universe on the lengths of wave of the photons and on the distances between two photons (moving on the same axis). We saw in section 3.2 (Equations (92) and (93)) that this effect, predicted by the SCM, was also true in the model of expansion of the Universe proposed by our theory of dark matter and dark energy.

The mathematical expressions of the different kinds of distances used in Cosmology (99), (100), (101) are in agreement with their mathematical expression in the SCM, in which the comoving distance  $D_C$  is usually expressed as a function of the variable  $z$ , for a flat Universe.

In the 1st mathematical model of expansion, since  $1+z$  has the same mathematical expression as in the SCM ( $(1+z)^{-1}=(\Omega_m/\Omega_\Lambda)\sinh^{2/3}(t/t_\Lambda)$  with  $t_\Lambda=2/(3H_0\Omega_\Lambda^{1/2})$ ), the mathematical expression of those distances as a function of  $z_0$  is identical to their expression in the SCM. We also obtain an identical Hubble constant and an identical age of the Universe (13.8 billion years). Then we find that  $d_C(z)$  (comoving distance),  $d_A(z)$  (angular-diameter distance),  $d_L(z)$  (luminosity distance), and  $d_T(z)$  (light-travel distance) have the same expressions as in the  $\Lambda$ CDM model. But despite that the mathematics of the 1st model are widely identical to those of the  $\Lambda$ CDM model, the new Cosmological model is physically new, for instance concerning the geometry of the Universe, which is much simpler than in  $\Lambda$ CDM, the interpretation of Cosmological time, the new defined kinds of frames (local Cosmological frame and Universal Cosmological frame), the behavior of photons in those frames, and the physical interpretation itself of Cosmological distances.

In the 2nd model, the expressions of distances used in Cosmology are much simpler. Using  $1+z=t_0/t_E$  we obtain (Equation (87) and (99)):

$$D_C = \int_{t_E}^{t_0} c(1+z)dt = \int_{t_E}^{t_0} c(t_0/t)dt \quad (102)$$

So we obtain finally the mathematical expression of the comoving distance, using  $1+z_0=t_0/t_E$ :

$$D_C=ct_0\text{Log}(t_0/t_E)=ct_0\text{Log}(1+z_0) \quad (103)$$

Here also, this simple expression is in good agreement with the usual admitted experimental values for the comoving distance for  $z<9$ . We deduce very easily from this expression the luminosity distance and the angular distance (equations (100) (101)). We remark that in this 2nd model, in agreement with the previous equations, we have as in the SCM for  $z_0\ll 1$ :

$$D_T\approx D_C\approx D_A\approx D_L\approx ct_0z_0 \quad (104)$$

We know that according to the 2nd mathematical model, the velocity measured in  $R_C$  of any comoving point  $Q(t)$  is constant (according to equation (86), with  $V_P(t)=C$ ). Let  $V_Q$  be this velocity. Then the distance in  $R_C$  between  $O$  and  $Q(t_0)$ , which is the comoving distance  $D_C$ , is also equal to  $V_Q t_0$ . Therefore, from equation (103):

$$V_Q=c\text{Log}(1+z_0) \quad (105)$$

We can interpret in our new model of expansion of the Universe the observation of the explosion of a supernova in the same way as in the SCM, taking into account the effect of the expansion of the Universe on the wavelengths of photons and on the distances between photons moving on the same axis (equations (92) and (93)). So our new model of expansion of the Universe can interpret the astronomical observations concerning the explosion of a supernova (PERLMUTTER et al. 1998) the same way as the SCM.

### 3.4 Cosmological limits of the observable Universe

In our model of a finite Universe in expansion, we cannot, as was also the case in the SCM, observe the Universe (through the observation of galaxies) before a given time  $t_{OU}$ . In this section, we are going to see how we can obtain this time  $t_{OU}$  according to our model of a finite Universe in expansion, and more precisely according to the 2nd mathematical model of expansion of the Universe, which is much simpler than the mathematical model of the SCM. We must proceed in the same way, just modifying mathematical expressions, to obtain  $t_{OU}$  according to the 1st mathematical model of expansion in our theory of dark matter and dark energy.

We keep in our theory the hypothesis, admitted in the SCM, of the existence of a dark age in the Universe during which light could not propagate. Let  $t_D$  be the end of this dark age. It is evident that  $t_{OU}$  must be greater than  $t_D$ . Moreover, galaxies cannot be observed before the Cosmological time  $t_G$ , the time of the apparition of the first galaxies.

There exists another limit according to our model of a spherical Universe in expansion, which is very clear in our 2nd model. According to equation (105), with  $V_Q$  necessarily smaller than  $C$ , we have:

$$C \geq c \text{Log}(1+z_0) \quad (106)$$

Consequently, with the notations of the previous section:

$$t_0/t_E = 1+z_0 \leq \exp(C/c) \quad (107)$$

Which implies that the Universe cannot be observed from  $O(t_0)$  (with  $t_0$  being the present age of the Universe) before the time  $t_1$  defined by:

$$t_1 = t_0 \exp(-C/c) \quad (108)$$

So, according to our theory of dark matter and dark energy,  $t_{OU}$ , the minimal Cosmological time for which the Universe can be observed, is the greatest time between  $t_1$ ,  $t_G$ , and  $t_D$ . Moreover, if  $t_{OU} > t_1$ , we cannot observe the borders of the Universe from  $O$ .

We remark that equation (106) allows us to give a lower bound to the constant  $C$  of the 2nd model: the fact that we have observed redshifts up to  $z=10$  implies  $C > 2.3 c$ . If we take  $C=10 c$ , we obtain  $t_1$  of the order of 1 million years.

We must use analogous methods if our galaxy is situated not at  $O$  but at another comoving point  $O'(t)$ . Then only  $t_1$  is modified, depending on the distance between  $O'(t_0)$  and the borders of the spherical Universe. Proceeding in the same way, replacing  $R_E(t)$  by its

value in the 1st mathematical model, we also obtain a minimal Cosmological observation time  $t_1$  in the 1st mathematical model.

### 3.5 The Cosmic Microwave Background.

As in the SCM, we admit the appearance of a CMB at a cosmological time very close to the Big Bang (we admit, as in the SCM, that the Big Bang occurs at a cosmological time equal to 0). Proceeding exactly as in the SCM, taking into account the effect of the expansion of the Universe on the wavelengths of photons and on photons moving along the same axis (effects obtained in section 3.2, equations (92) and (93)), we obtain in our theory of dark matter and dark energy that if the CMB appears at a cosmological time  $t_{\text{CMB}}$  corresponding to a temperature  $T_{\text{CMB}}$ , then at a cosmological time  $t$  greater than  $t_{\text{CMB}}$ , if the expansion factor between  $t_{\text{CMB}}$  and  $t$  is  $1+z$ , the CMB at time  $t$  corresponds to a temperature  $T_{\text{CMB}}(t)=T_{\text{CMB}}/(1+z)$ . (This is obtained exactly the same way as in the SCM, because in both cosmological models the density of photons is divided by  $(1+z)^3$  — since the radius of the Universe  $R_E(t)$  increases by a factor  $1+z$  — and the wavelengths of photons are increased by a factor  $1+z$ , according to equation (93)). Therefore, our new model of expansion of the Universe is in agreement with the observation of the CMB corresponding to a large redshift  $z_0$  (Raine & Thomas 2001).

However, we have now provided a very complete physical interpretation of the CMB rest frame that did not exist in the SCM, allowing a full definition of the CMB rest frame (Postulate 4) at any point of the Universe, and also giving fundamental physical properties of the CMB rest frame (Postulate 3). As noted in our Introduction, our theory of dark matter and dark energy remains compatible with the SCM when interpreting the anisotropies of the CMB.

It is important to know what happens to a photon reaching the borders of the spherical Universe. It could be absorbed, but this is not the only possible hypothesis. The simplest hypothesis would be that the photon is reflected, taking exactly as its new local velocity after reflection the opposite of its local velocity before reflection (as a vector). Under this hypothesis, one might expect to observe images of galaxies reflected on the borders of the Universe.

### 3.6 Dipole contribution of the CMB.

We know that according to the SCM we have the following fluctuations of temperature of the CMB <sup>(7)</sup>:

$$\left(\frac{\Delta T}{T}\right) = \frac{1}{4\pi} \sum_l l(2l+1)C_l \quad (109)$$

We will retain this expression in our theory of dark matter and dark energy. However, according to the preceding framework, the term with  $\ell=1$  represents the dipole contribution, corresponding—as in the SCM—to the motion of the Earth relative to the CRF (CMB Rest Frame). Thus, this dipole component is fully interpreted by our theory of dark matter and dark energy, whereas in the SCM, the CMB rest frame lacked a physical interpretation.

### 3.7 Evolution of the Temperature of the Intergalactic Dark Substance

We have seen that, according to our new cosmological model, the Universe is a sphere filled with dark substance, surrounded by a medium we refer to as “nothingness” (see Section 2.5). By analogy with the spherical concentrations of dark substance defined in Part 2, we may postulate a convective thermal transfer between the intergalactic dark substance and the surrounding nothingness. The resulting convective heat flux  $F$  would then be given by:  $F=h_n T_0(t)$ , where  $T_0(t)$  denotes the temperature of the intergalactic dark substance at cosmological time  $t$ . Generalizing this analogy, and using the constant  $K_3$  (as defined in Equation (14)),  $M_B$  the baryonic mass of the Universe, and  $R_E(t)$  the radius of the Universe at time  $t$ , we obtain the thermal equilibrium equation:

$$K_3 M_B = 4\pi R_E(t)^2 (h_n T_0(t)) \quad (110)$$

However, this equation is derived under the assumption of a convective heat exchange between the universal sphere and the nothingness (an exchange that may in fact be negligible or even non-existent), and it neglects other energetic factors potentially influencing the temperature of the intergalactic dark substance—an approximation that may not be valid. These factors will be examined in the next section.

We note that if (in analogy with the derivation of the baryonic Tully-Fisher law) we postulate a constant  $C_2$  such that  $h_n = C_2 \rho(t)$ , then Equation (94a) would imply that the temperature  $T_0(t)$  increases with  $t$ . Conversely, if we assume that  $h_n$  is constant, Equation (94a) implies that  $T_0(t)$  evolves as  $1/(1+z)^2$ , where  $1+z$  is the expansion factor of the Universe. In our theory, as in the SCM, we accept that the appearance of the CMB corresponds to a redshift  $z \approx 1100$ . We can adopt the standard assumption that at the time of recombination, the dark substance is significantly colder than the CMB. Nonetheless, the hypothesis of the equality of the temperature of dark substance and the temperature of the CMB for  $z=1100$  may result either from a thermal interaction for  $z < 1100$  or from an evolution of the dark substance temperature identical to that of the radiation temperature (i.e proportional to  $1/(1+z)$ , we will show later that such a scenario is theoretically possible, assuming an initial equality of the temperatures).

### 3.8 Dark Energy in the Universe

In the first part of our theory (2. THEORY OF DARK MATTER), we proposed that the Universe is filled with a dark substance that can be modeled as an ideal gas (see Section 2.1). It is thus natural to consider that, like an ideal gas, this dark substance possesses internal energy that could be identified with a form of dark energy—energy present throughout the entire Universe. However, as we will soon see, this assumption proves incorrect.

To obtain equation (110), we have assumed that the variation in the internal energy of the dark substance due to cosmic expansion is negligible. We call this the *first model* of the temperature evolution of the intergalactic dark substance. In this model, the dark substance remains at a constant temperature shortly after the Big Bang, provided the following inequality holds:

$$4\pi R_U(t)^2 h_n T_0(t) t \ll E_{INMSin} = K_{EIMS} M_{MS} T_0(0) \quad (111)$$

where  $E_{INMSin}$  denotes the initial internal energy of the dark substance.

Now let us consider a *second model* of temperature evolution, where we instead assume that the energy exchanged between baryons, photons, and the dark substance, as well as energy transferred through convection, is negligible compared to the internal energy variation of the dark substance. In this model, the dark substance is assumed to be homogeneously distributed throughout the Universe. As such, it follows the law of ideal gases (Postulate 1), and we also assume that it obeys Joule's law for ideal gases. That is, there exists a constant  $K_{ES}$  such that:

$$U(T(t))=K_{ES}M_S T(t) \quad (112)$$

where  $M_S$  is the total mass of the dark substance and  $U(T(t))$  its internal energy at time  $t$ .

Additionally, the energy lost as work done due to the expansion of volume  $dV$  under pressure  $P$  is:

$$W=-PdV \quad (113)$$

Assuming that the transformation is reversible and adiabatic, we apply Laplace's law, which states that there exists a constant  $\gamma$  such that, for a temperature  $T$  and volume  $V$  at time  $t$ , and temperature  $T_1$ :

$$TV^{\gamma-1}=T_1V_1^{\gamma-1} \text{ (constant)} \quad (114)$$

Since the expansion of the Universe implies  $V(t)=V(t_1)(1+z)^3$ , we finally obtain:

$$T(t)=T(t_1)(1+z)^{3(\gamma-1)} \quad (115)$$

In a 3<sup>rd</sup> *model of evolution of the temperature of the intergalactic dark substance* we consider every kind of energy received or lost by the dark substance. Nonetheless, we consider in this model that this was justified because the total volume of those dark halos was very small relative to the total volume of the Universe. We will take the following notations:

$dW(t,t+dt)$  is the energy received by the dark substance as a work (negative) due to the variation of volume of the dark substance between the ages of the Universe  $t$  and  $t+dt$ .

$dE_{TF}(t,t+dt)$  is the energy received by the dark substance (negative) due to the thermal transfer between the dark substance and the medium that we called "nothingness" between  $t$  and  $t+dt$ .  $R_U(t)$  being the radius of the Universe at the age of the Universe  $t$ , we have seen (equation (110)):

$$dE_{TF}(t,t+dt)=(-h_n T(t))(4\pi R_U(t)^2)dt \quad (116)$$

$dE_{TB}(t,t+dt)$  is the energy received by the dark substance (positive) received from the baryons, (Equation (14) and Equation (94b)) between  $t$  and  $t+dt$ .  $M_B(t)$  being the mass of the baryons at the age  $t$  of the Universe we have:

$$dE_{TB}(t,t+dt)=K_3 M_B(t)dt \quad (117)$$

Then the equation of equilibrium of the energy received and lost by the intergalactic dark substance between  $t$  and  $t+dt$  is:

$$dU(t,t+dt)=dW(t,t+dt) + dE_{TF}(t,t+dt) + dE_{TB}(t,t+dt) \quad (118)$$

We remind that according to the Boyle-Charles law,  $M_S$  being the total mass of the dark substance (assumed to be constant):

$$P(t)V(t)=k_0M_S T(t) \quad (119)$$

And,  $R_U(t)$  being the radius of the Universe,  $V(t)=(4/3)\pi R_U(t)^3$  and  $d(R_U(t))=dzR_U(t)$  ( $1+dz$  being the factor of expansion of the Universe between  $t$  and  $t+dt$ ),  $dV(t)=4\pi R_U(t)^2 dR_U(t)=4\pi R_U(t)^2 dz$  and consequently  $dV(t)/V(t)=3dz$ . So we have:

$$dW(t,t+dt)=-PdV(t)=-k_0M_S T(t)(dV(t)/V(t)) \quad (120)$$

$$dW(t,t+dt)=-3k_0M_S T(t)dz \quad (121)$$

So we obtain the following differential equation in  $T(t)$ , because  $dz$  and  $R_U(t)$  can be expressed as a function of  $t$ :

$$d(K_{ES}M_S T(t))=-3k_0T(t)dz-h_n T(t)(4\pi R_U(t)^2)dt+K_3M_B(t)dt \quad (122)$$

$$K_{ES}M_S(dT(t)/dt)=-3k_0M_S T(t)(dz/dt)-h_n(4\pi R_U(t)^2)T(t)+K_3M_B(t) \quad (123)$$

We can easily prove that with the previous notations, the parameter  $\gamma$  used in Laplace's equation (114) can be expressed by:

$$\gamma=1+k_0/K_{ES} \quad (124)$$

$k_0$  should be of the same order of magnitude as  $K_{ES}$ , by analogy with an existing gas modeled as an ideal gas. Using the previous equation (104b), we can then express the conditions of validity for the first model of the evolution of the temperature of the dark substance, in which we have neglected both the variation of its internal energy and the work received by the dark matter due to the variation of its volume. These conditions are:

$$-K_{ES}M_S(dT(t)/dt)\ll K_3M_B(t)$$

$$-K_{ES}M_S(dT(t)/dt)\ll h_n(4\pi R_U(t)^2)T(t)$$

$$3k_0M_S T(t)(dz/dt)\ll K_3M_B(t)$$

$$3k_0M_S T(t)(dz/dt)\ll h_n(4\pi R_U(t)^2)T(t) \quad (125)$$

The conditions for which the 2<sup>nd</sup> model of the evolution of the temperature of dark substance be valid are the inverse conditions (replacing “ $\ll$ ” by “ $\gg$ ”)

### 3.9 Evolution of the temperature of dark substance- 2<sup>nd</sup> model of expansion.

We consider now the application of the preceding Section 3.8 to the case of the second mathematical model of expansion of the Universe, in which  $R_U(t)=Ct$ , with  $C$  a constant (see Section 3.2). Consequently, between  $t$  and  $t+dt$ , one has  $1+dz=(t+dt)/t \Rightarrow dz=dt/t$ .

We recall that the mathematical function of expansion  $f(t)=Ct$  is not valid in a strict physical sense, but it is the simplest mathematical form allowing us to understand the general behavior of the temperature evolution of the dark substance. Furthermore, this analysis can be generalized to any other mathematical expansion function  $f(t)$  by proceeding analogously. In the last section, we will define a function of expansion compatible with all current observations, including those related to the primordial Universe and the CMB power spectrum, and consistent with an inflationary era as well as a radiation-dominated era.

We note that in the first model of the evolution of the temperature,  $T(t)$  evolves proportionally to  $1/(1+z)^2$ , and consequently in this second expansion model as  $1/t^2$ . In the second model of the evolution of the temperature,  $T(t)$  evolves as  $1/(1+z)^{3(\gamma-1)}$ , and consequently in this second expansion model as  $1/t^{3(\gamma-1)}$ . Thus, in both cases, the temperature  $T(t)$  evolves proportionally to  $1/t^p$  with  $p>0$ . For such a function  $T(t)$ , we see that as  $t \rightarrow \infty$ , both  $T(t)$  and the ratio  $(dT(t)/dt)/T(t)$  tend toward zero. Therefore, for sufficiently large  $t$ , the conditions expressed by relation (125) are satisfied, and the first model of the temperature evolution of the dark substance is valid.

On the contrary, for  $t \rightarrow 0$ , both  $(dT(t)/dt)/T(t)$  and  $T(t)$  tend toward infinity, and consequently for sufficiently small  $t$  (for example, just after the Big Bang), the inverse of the conditions expressed by relation (125) are satisfied, and thus the second model of the temperature evolution of the dark substance is valid.

These conclusions remain applicable in the first mathematical model of expansion, replacing  $R_E(t)$  by its corresponding expression in that model.

### 3.10 Estimation of Significant Internal Energies.

In *Postulate 1*, the Boyle–Charles law was defined for an element of dark substance at pressure  $P$ , in a volume  $V$ , at temperature  $T$ , and with mass  $m$ , where  $k_0$  is a constant:

$$PV=k_0 m T \quad (126)$$

Using the above law and Newton’s law of gravitation, Equation (10) was obtained, valid for galaxies exhibiting a flat rotation curve. In particular, for the Milky Way—where  $T_{MW}$  denotes the temperature of the Milky Way’s dark halo, and  $v_{MW}$  the orbital velocity of stars in the Milky Way—we have:

$$v_{MW}^2 \approx 2k_0 T_{MW} \quad (127)$$

Taking  $v_{MW} \approx 2 \times 10^5$  m/s, we then obtain:  $k_0 T_{MW} \approx 2 \times 10^{10}$  (SI units).

Let us compare Equation (126) with the analogous equation valid for hydrogen, modeled as an ideal gas. It is known that there exists a constant  $k_H$  such that, for an element of hydrogen with mass  $m_H$ , volume  $V$ , at temperature  $T$  and pressure  $P$ :

$$PV=k_H m_H T \quad (128)$$

For one mole of hydrogen at  $T=T_K=273$  K,  $V=20\times 10^{-3}$  m<sup>3</sup>,  $P=10^5$  Pa,  $m_H=10^{-3}$  kg, we have:

$$k_H T_K \approx PV/m_H = 10^5 \times 20 \times 10^{-3} \times 10^3 = 2 \times 10^6 \text{ (SI units)} \quad (129)$$

If we assume that dark substance and hydrogen both obey Joule's law, it follows that the internal energy of 1 kg of hydrogen at temperature  $T_K$  is of the order of  $k_H T_K$ , i.e.,  $2 \times 10^6$  J, whereas the internal energy of 1 kg of dark substance belonging to the Milky Way halo is of the order of  $k_0 T_{MW}$ , i.e.,  $2 \times 10^{10}$  J. The latter energy is therefore vastly greater than the former (Equation (124) is used here, with  $k_0/K_{ES}$  of order unity, as for ordinary ideal gases).

### 3.11 Experimental value of Hubble Constant.

It is possible that the center of mass of a galaxy cluster remains at rest in the **Local Cosmological Frame** (LCF). If this hypothesis holds true, it provides a method for determining the **Hubble constant**  $H_0$ , based on the dynamics of galaxy clusters.

### 3.12 Interpretation of the Friedmann-Robertson-Walker metric in the New Cosmological Model (NCM). Primordial Universe.

The Friedmann–Robertson–Walker (FRW) metric is fundamental in the Standard Cosmological Model (SCM). It is expressed, assuming  $k=0$  (zero spatial curvature, which is the case in the New Cosmological Model, NCM), as

$$ds^2 = -c^2 dt^2 + a(t)^2 (dr^2 + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2)) \quad (130)$$

We interpret this metric in the NCM as follows. Let us consider, at a given cosmic age  $t$ , a photon traveling from a point  $P_1(t)$  to a point  $P_2(t)$  within the Universal Cosmological Frame  $R_U(t)$  between times  $t$  and  $t+dt$ . We denote by  $d\mathbf{M}(t)$  the vector  $P_1(t)P_2(t)$ . At the present age of the Universe  $t_0$ , we know that in the NCM,  $d\mathbf{M}(t)$  becomes  $d\mathbf{M}(t_0)$ , with

$$d\mathbf{M}(t) = (1+z)^{-1}(t) d\mathbf{M}(t_0) = a(t) d\mathbf{M}(t_0)$$

Using the fact that the norm  $n(d\mathbf{M}(t))$  is equal to  $c dt$  in the NCM, we obtain, by setting  $ds^2=0$ , the equation (107) expressed in the spatial coordinates of  $d\mathbf{M}(t_0)$ .

Nevertheless, in the case of a perturbation caused by a nonzero fluctuation in the density of dark matter or baryonic matter, we must employ the mathematical framework of General Relativity to derive the perturbed FRW metric including these perturbations. We recall this metric:

$$ds^2 = -(1+2\Phi)c^2 dt^2 + a(t)^2 (1-2\Psi) (dr^2 + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2)) \quad (131)$$

where  $\Psi$  and  $\Phi$  are obtained in the same way as in the SCM.

The perturbed FRW metric is used to predict the power spectrum of the CMB for multipoles  $l < 100$  (super-horizon modes). The existence of density perturbations of dark matter in the primordial Universe, deduced from observation of the CMB power spectrum, implies that the hypothesis of strict homogeneity of dark substance is not valid in the early

Universe. However, we can assume that the dark substance remains locally homogeneous. Moreover, the value of the dark matter density obtained using the present model is incompatible with the value deduced from observations of the CMB. This difference can be justified by admitting the hypothesis that an element of dark substance, denoted  $E_{\text{ITS}}$ , does not have the same mass in the primordial Universe — let this mass be denoted  $m_{\text{SPR}}$ , associated with a primordial density  $\rho_{\text{SPR}}$  — as its mass in the Universe after star and galaxy formation, denoted  $m_{\text{SAS}}$ , associated with an astral density  $\rho_{\text{SAS}}$ . We will assume that there exists a cosmic age  $t_{\text{DMS}}$  such that for  $t < t_{\text{DMS}}$  (or  $\rho_{\text{SPR}} > \rho_{\text{LIMDS}}$ , the dark substance has apparently a primordial density, and for  $t > t_{\text{DMS}}$  (or  $\rho_{\text{SPR}} < \rho_{\text{LIMDS}}$ ), the dark substance has apparently an astral density. While this is obviously impossible for ordinary matter, dark substance, being an exotic matter, may possess this property, and we have already seen that it can exhibit zero gravitational mass under certain conditions. We will have  $m_{\text{SPR}} \ll m_{\text{SAS}}$ , and an interesting hypothesis, which we will adopt, is that the true mass of  $E_{\text{ITS}}$  is  $m_{\text{SAS}}$ , with  $E(E_{\text{ITS}}) = m_{\text{SAS}}c^2$ , and  $m_{\text{SPR}}$  its apparent mass in the primordial Universe,  $\rho_{\text{SPR}}$  being called the *effective density* of dark substance in the primordial Universe, while the difference  $m_{\text{SAS}} - m_{\text{SPR}}$  represents the *hidden mass* of  $E_{\text{ITS}}$  in the primordial Universe. In this case, the energy of  $E_{\text{ITS}}$  is preserved when  $m_{\text{SPR}}$  transitions to  $m_{\text{SAS}}$ . Nevertheless, to calculate the expansion of the Universe using the Friedmann equations, we will employ in the NCM the primordial density of dark substance, even for  $t > t_{\text{DMS}}$ ; consequently we will also call this primordial density the *effective density of expansion* of dark substance. We recall that in these equations, the mean baryonic density is used, even though the actual baryonic density is neither homogeneous nor equal to this mean density except in the primordial Universe. It is possible that the astral density  $\rho_{\text{SAS}}$  is also an effective density, lower than the true local density. The energy of dark substance linked to its real mass or its internal energy linked to its temperature or the energy of the radiation, could naturally be at the origin of particle creation (quarks, photons, etc.) in the primordial Universe.

The observation of the CMB power spectrum implies the existence of a phenomenon called inflation, a very rapid (essentially exponential) expansion just after the Big Bang by a factor of about  $10^{26}$  between  $t_{\text{PL}} = 10^{-36}$  s (Planck time) and  $t_{\text{EINF}} = 10^{-33}$  s. We will also adopt this phenomenon within the NCM, with  $R_U(t) = C_{\text{INF}} \exp(Ht)$ , where  $R_U(t)$  denotes the radius of the universal sphere. We set  $R_U(t_{\text{PL}}) = C_{\text{PL}} t_{\text{PL}}$ , with  $C_{\text{PL}}$  a constant speed or average speed of the expanding sphere's boundary between  $t=0$  and  $t=t_{\text{PL}}$ . We can verify the following approximate model, neglecting the radiation-dominated era and dark energy. In this model: For  $t_{\text{PL}} < t < t_{\text{EINF}}$ ,  $R_U(t) = C_{\text{INF}} \exp(Ht)$ , for  $t_{\text{EINF}} < t < t_0$ :  $R_U(t) = C' t^{2/3}$ .

Given the continuity of  $R_U(t)$  at  $t=t_{\text{EINF}}$ :

$$10^{26} C_{\text{PL}} t_{\text{PL}} = C' t_{\text{EINF}}^{2/3} \quad (132a)$$

Setting  $C_{\text{PL}} = xc$  (with  $x$  dimensionless) and inserting the numerical values of  $t_{\text{PL}}$  and  $t_{\text{EINF}}$ :

$$C' = 10^{26} t_{\text{PL}} xc / t_{\text{EINF}}^{2/3} = 10^{12} xc \text{ s}^{1/3} \quad (132b)$$

$t_0$  being the present age of the Universe, we assume  $R_U(t_0) \approx C' t_0^{2/3}$ ,  $t_0 \approx 15 \cdot 10^9$  years,  $R_U(t_0) \approx 45 \cdot 10^9$  l.y, and we set  $T_A = 1 \text{ year} \approx 30 \cdot 10^6$  s. Therefore  $R_U(t_0) \approx 45 \cdot 10^9 T_A c$  and  $C'$  corresponding to  $R_U(t_0) \approx 45 \cdot 10^9$  l.y is given by:

$$C' \approx R_U(t_0) / t_0^{2/3} \approx 7 \cdot 10^3 c T_A^{1/3} \approx 7 \cdot 10^3 c (30 \cdot 10^6)^{1/3} \approx 2 \cdot 10^6 \text{ s}^{1/3} \quad (133)$$

Therefore, if  $C_{PL}=xc=2\times 10^{-6}c$  ( $x=2\times 10^{-6}$ ), then  $R_U(t_0)\approx 45$  billion light-years that is the minimal size in order that an observer located at the center of the Universe could observe the CMB at its formation ( $z\approx 1100z$ ) in an isotropic manner (comoving distance). If  $C_{PL}=c$  ( $x=1$ ), then  $R_U(t_0)$  becomes enormous, on the order of millions of billions of light-years. Nevertheless, in both cases the Universe has boundaries and is finite — a model more conceivable than the infinite, unbounded Universe of the SCM.

We can also consider a more refined approximate model that takes into account the radiation-dominated expansion era, with the following modifications: For  $t_{EINF}<t<t_{ERAD}$ ,  $R_U(t)=C_{RAD}^{1/2}$  ( $t_{ERAD}=50000$  y.l, marks the end of the radiation-dominated era of expansion), for  $t_{ERAD}<t<t_0$ ,  $R_U(t)=C't^{2/3}$ .

Comparing the expansion rates between  $t_{EINF}$  and  $t_{ERAD}$  in both models, we find that for the same present-day size  $R_U(t_0)$   $C_{PL}$  is approximately  $2\times 10^7$  times larger in the second model than in the first one (hence, for  $R_U(t_0)=45\times 10^9$  ly  $C_{PL}\approx 40c$  in the second model).

The phenomenon of inflation is primarily invoked to address the horizon problem (the near-isotropy of the CMB), the flatness problem (why the Universe is nearly flat), and to explain the observed power spectrum of the CMB. However, the NCM does not require inflation to resolve the flatness or horizon problems, since it is inherently flat and began with a very small, perfectly homogeneous, isotropically evolving volume of dark substance, which implies it would remain homogeneous for a long time. Nevertheless, the NCM does require inflation to reproduce the observed CMB power spectrum.

It should be noted that the model treating dark substance as an ideal gas may not be valid in the primordial Universe.

In this way, all the equations of the SCM relating to the primordial Universe and to the CMB power spectrum (Boltzmann equations, etc.) can be interpreted within the framework of the NCM and the new model of dark matter (constituting “vacuum”).

#### 4.CONCLUSION

In the theory of dark matter presented in this article, we have modeled dark matter as a “dark substance” whose physical properties — in particular, its ability to be modeled as an ideal gas — allow us to interpret all the astronomical observations related to dark matter. For example, these physical properties provided a theoretical justification for the flat rotation curves of galaxies and for the baryonic Tully–Fisher relation. To achieve this, we interpreted galaxies with flat rotation curves as spherical concentrations of dark substance in gravitational equilibrium. We also showed that our concept of dark substance naturally led to proposing a new geometric shape of the Universe, namely flat, finite, and spherical.

We have studied, according to our theory of dark matter, the effects of the displacement of a concentration of dark substance on its mass and velocity, and we found that these effects are null. We also showed that this theory permits the definition, in agreement with astronomical observations, of two characteristic radii for galaxies: the baryonic radius and the dark radius. Subsequently, we presented, within this framework, different models for

the distribution of dark matter in galaxies. We then demonstrated that this theory predicts significant relationships between the masses of galaxy clusters and the velocities of galaxies within those clusters, as well as relationships between the mean densities of certain clusters corresponding to the same cosmological redshift. Furthermore, the theory provides a model for the role of dark matter in the formation of cosmic structures. Finally, we showed that our theory allows us to estimate the dark radius of galaxies, giving such an estimate for the Milky Way, and to derive the mean density of the Universe as well as the density of the intergalactic dark substance for any cosmological redshift  $z$ . We have seen that the new theory of dark matter is compatible with the mathematics of the  $\Lambda$ CDM model.

In the second part of this article (Section 3: **Dark Energy in the Universe**), we proposed a new cosmological model based on the geometric shape of the Universe established in the first part (spherical), and on the physical interpretation of the Cosmic Microwave Background (CMB) rest frame (CRF), which we also called the local cosmological frame. This new cosmological model allowed us to give a simple interpretation of cosmological time, consistent with all astronomical observations. It also led us to define a new and fundamental frame of reference, which we named the Universal Cosmological Frame. Within this new cosmological model, we defined a first mathematical model of cosmic expansion, based — like the SCM — on general relativity (the  $\Lambda$ CDM model), with most theoretical predictions identical to those of the SCM. We recall that in this model, the Universe is described as a swelling sphere with a radius  $R_U(t)$ , and in the first mathematical model,  $R_{UM1}(t) = C \sinh^{2/3}(t/t_\Lambda)$ . However, this first mathematical model retained the same notions of dark matter and dark energy as required in the SCM.

We also described a second mathematical model of expansion, much simpler than the first, with  $R_{UM2}(t) = Ct$ , which, despite its great simplicity, leads to theoretical predictions in good agreement with many astronomical observations for sufficiently low redshifts. Moreover, this second mathematical model does not require a dark energy component, contrary to the SCM and the first model of expansion, and thus, just like the first model, offers a solution to the enigma of dark energy. Finally, we studied — within the framework of our theory of dark matter and dark energy — the evolution of the temperature of the dark substance from the Big Bang up to the present age of the Universe, and showed the existence throughout the Universe of an energy corresponding to the internal energy of the dark substance, identified with that of an ideal gas.

We have seen that the observation of the anisotropies of the CMB supports the first mathematical model and contradicts the second. For example, the observed cosmological time of the CMB's appearance (approximately 400,000 years after the Big Bang) agrees with the prediction of the first model, which matches the SCM. Moreover, the observed comoving distance of the last scattering surface of approximately 43 billion light-years is also consistent with the predictions of the  $\Lambda$ CDM model. We remark that in this new cosmological model, dark matter possesses the same properties as assumed by the  $\Lambda$ CDM model: it is cold, dissipationless, and collisionless. However, the dark energy cannot be identified with the internal energy of the dark substance treated as a gas, since in that case the total dark energy would depend on the temperature and mass of the dark substance. Nevertheless, we may assume that the dark substance acts on cosmic expansion through an effective or “virtual” energy, termed the *effective expansion energy of dark substance*, with a constant density  $\rho_\Lambda(z) = \rho_\Lambda$  ( $\rho_\Lambda = \Lambda/8\pi G$  equal to that in the  $\Lambda$ CDM model). We note that the new cosmological model describes a flat Universe, which justifies setting  $\Omega_c = 0$  in the Friedmann equation of the  $\Lambda$ CDM model and adopting a cosmological constant consistent with flatness.

Regarding dark matter, we saw in the preceding section that an element of dark substance does not have the same mass in the primordial Universe (the “primordial mass”) as in the Universe after the formation of stars and galaxies (the “astral mass”). Therefore, the relation between the density of dark matter in the primordial Universe and in the present Universe is not the same as for baryonic matter. We have argued that the astral density can be regarded as the true density of dark substance, while the primordial density is the effective density of dark substance in the early Universe and the effective density of expansion of dark substance. The primordial density is obtained through observations of the CMB power spectrum, while the astral density is derived from the dynamics of galaxy clusters, as we did in this work.

According to recent observations, two different methods yield incompatible estimates of the Hubble constant  $H_0$ : the first,  $H_{0SN}=73 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , from supernova observations (RIESS et al. 2022), and the second,  $H_{0PL}=67 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , from observations of the CMB (Planck satellite). This problem, known as the “Hubble tension,” can be resolved within the new cosmological model. We admit the existence of a time  $t_\Delta$  such that for  $t < t_\Delta$ ,  $R_U(t)=f_1(t)$ , and for  $t > t_\Delta$ ,  $R_U(t)=f_2(t)$ , with the continuity condition  $f_1(t_\Delta)=f_2(t_\Delta)$ . We shall refer to this new mathematical model as the  **$\Delta$  model**. One may take  $f_1(t)=C\sinh^{2/3}(t/t_\Delta)$  and  $f_2(t)=C'(t-\beta)$  (with constant acceleration  $a''(t)=0$ , if  $a(t)=(1+z)^{-1}(t)$ ), or  $f_2(t)=C'(t-\beta)^\alpha$ . We remark that if  $t_\Delta=t_0-2$  billion years, the CMB observations at high redshift remain essentially identical to those of the  $\Lambda$ CDM model, and therefore the estimates of  $H_{0PL}$  and of all  $\Omega_x$  parameters remain nearly unchanged. For  $z=0,1$  one can show using the luminosity distance  $D_L(z)$  that the estimate of  $H_{0SN}$  differs by about 2.5% depending on whether the  $\Lambda$ CDM model or  $f_2(t)=t-\beta$  is adopted (denoted  $H_{0SN2}$ ). This justifies the difference between  $H_{0PL}$  and  $H_{0SN}$  within the  $\Delta$  model.

For instance, with  $f_2(t)=C'(t-\beta)$  taking  $\beta=0$ ,  $H_{0SN}=73 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $H_{0SN2}=(100-2,5)\%H_{0SN}\approx 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$  we obtain  $t_0\approx 13,8$  billion years, considering the uncertainty on  $H_{0SN}$ , and taking  $\beta=200$  million years we obtain  $t_0\approx 14$  billion years ((since  $H_{0SN2}=1/(t_0-\beta)$ ), both values of  $t_0$  being acceptable, whereas for the same  $H_{0SN}$  the  $\Lambda$ CDM model gives  $t_0=12.6$  billion years, which is completely unacceptable.

Nonetheless, if  $t_\Delta=t_0$  and  $f_1(t)=C\sinh^{2/3}(t/t_\Delta)$ , then the age of the Universe  $t_0$  can be determined using  $H_{0PL}$  and the  $\Lambda$ CDM model. Consequently, we may expect that if  $t_\Delta$  is close to  $t_0$ , for instance  $t_\Delta\approx t_0-2$  billion years, the age of the Universe derived from  $H_{0SN}$  in the  $\Delta$  model will be close to that obtained using  $H_{0PL}$  and the  $\Lambda$ CDM model. In that case, we find  $t_0\approx 13.8$  billion years, which is an acceptable value. (The closer  $t_\Delta$  is to  $t_0$  the closer the two estimated ages of the Universe become.) Thus the  $\Delta$  model resolves the HUBBLE-tension issue by predicting two distinct values of  $H_0$  which nevertheless remain mutually compatible, being both in agreement with the same value of the age of the Universe  $t_0$ , compatible with the age of the eldest stars. We may denote *free era* the interval corresponding to the age of the Universe  $t$  belonging to  $[t_\Delta, t_0]$ .

This model involving the functions  $f_2(t)$  is possible because, according to the New Cosmological Model (NCM), the expansion rate of the Universe can be entirely independent of the densities of baryonic and dark matter. For instance, we have shown that the actual value of the dark energy density may be zero, even if its effective expansion energy is non-zero. We also recall that an equation identical to the Friedman equation with zero curvature,  $((a'/a)^2=8\pi G/3)\rho$ , can be derived using Newtonian mechanics in a Universe model very similar to the one proposed in the NCM—considering a homogeneous expanding sphere of radius  $R(t)$ , and applying Newton’s law to a mass element located at the boundary. Although

Newtonian mechanics do not strictly apply within the NCM, the same holds true for General Relativity. We therefore accept this equation as a postulate in order to mathematically define the expansion of the Universe in the context of the NCM, which is, by its construction, compatible with any monotonically increasing expansion function. Moreover, we recall that many of the equations used to interpret the power spectrum of the Cosmic Microwave Background (CMB)—especially for sub-horizon modes ( $\ell > 100$ )—can also be derived from Newtonian theory. However, certain equations (not used in this article), particularly those relevant for interpreting super-horizon modes ( $\ell < 100$ ), do require the mathematical formalism of General Relativity.

It is possible to define, within the framework of the NCM, a cosmological model designated as  $\Lambda$ CDM–NCM. This model uses the equations of the  $\Lambda$ CDM model but reinterprets them according to the physics of the NCM, incorporating its new physical concepts (e.g., Universal Cosmological Frame, Local Cosmological Frame, Universal Sphere, interpretation of the CMB rest frame, redefinition of cosmological time). In this context, the concept of dark substance is replaced by that of the interstellar medium, which constitutes what is otherwise interpreted as vacuum—but without mass or density, and not forming dark matter.  $\Lambda$ CDM–NCM thus represents the minimal or weakest form of the New Cosmological Model.

In the final section, we acknowledged the inflationary epoch within the NCM. We note that, according to the NCM, the Universe could possess a relatively modest present radius,  $R_U(t_0) \approx 45$  billion light-years, which is approximately the minimum radius required for an observer situated at the center of the Universe to detect the CMB at the time of its emergence ( $z \approx 1100$ ).

Finally, we highlight that one of the most appealing features of the geometrical model of the Universe proposed by our theory of dark matter and dark energy is its conceptual clarity: a finite, spherical Universe with well-defined boundaries is far more accessible to human intuition than the models offered by the Standard Cosmological Model (SCM), which posit either an infinite Universe or a finite Universe without boundaries. It is our model of dark substance that has allowed us to naturally conceive such a Universe—both flat and finite.

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