Classical Unified Field Theory of Gravitation and Electromagnetism

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We present a five-dimensional unified framework that consistently incorporates gravitation and electromagnetism. Using an ADM-type $(4+1)$ decomposition, we construct a rotating and charged vacuum solution by extending the Kerr metric. Unlike the Kaluza–Klein scenario, our formalism eliminates the dilaton field through projection, leading directly to the Lorentz force law in four dimensions. A dimensional reduction of the five-dimensional Einstein–Hilbert action fixes the normalization of the electromagnetic sector and establishes the timelike nature of the extra dimension. Finally, by exploiting the underlying Killing symmetries, we derive the conserved mass, charge, and angular momentum, showing consistency with Komar and Tolman definitions. This framework provides a coherent unification of gravitational and electromagnetic dynamics while preserving the geometric interpretation of conserved quantities.

I. INTRODUCTION

The unification of gravitation and electromagnetism has been a longstanding goal of theoretical physics, dating back to the early attempts of Kaluza and Klein [1, 2]. While the Kaluza–Klein (KK) framework elegantly introduces electromagnetism through an extra spatial dimension, it also leads to the appearance of an additional scalar field (the dilaton) that lacks direct experimental support [3]. Moreover, the compactification procedure complicates the analysis of conserved quantities in higher-dimensional settings.

In this work, we propose an alternative approach based on the Arnowitt–Deser–Misner (ADM) formalism [4], where the five-dimensional metric is expressed in a $(4+1)$ decomposition. This formulation is particularly well-suited for addressing the role of symmetries and their associated conserved quantities. Within this setting, we construct a rotating and charged vacuum solution that extends the four-dimensional Kerr metric [5], and we demonstrate how the Lorentz force law naturally arises without introducing a dilaton field.

By analyzing the five-dimensional Einstein–Hilbert action [6–8] and performing dimensional reduction, we determine the physical normalization of the electromagnetic sector and clarify the relationship between four- and five-dimensional gravitational constants. Finally, through the application of Noether's theorem [9] and Komar integrals [10, 11], we show how mass, charge, and angular momentum emerge as conserved quantities associated with the timelike, axial, and extra-dimensional symmetries. This unified framework thus provides both mathematical consistency and physical interpretability.

II. METRIC ANSATZ AND FORMALISM

Unlike the conventional Kaluza–Klein (KK) ansatz, which introduces extra dimensions through compactification [1–3], we employ the Arnowitt–Deser–Misner (ADM) formalism [4, 12] to express the five-dimensional metric in a $(4+1)$ decomposition. This approach is more suitable for analyzing conserved quantities in higher-dimensional spacetimes [7, 11]. This representation, distinct from the KK ansatz, will serve as the foundation for constructing rotating and charged five-dimensional solutions. In particular, we allow both timelike and spacelike possibilities for the extra dimension.

The five-dimensional line element in ADM form is given by

$$ds^2 = \pm N^2(dx^5)^2 + g_{\mu\nu}\left(dx^\mu + \beta^\mu dx^5\right)\left(dx^\nu + \beta^\nu dx^5\right), \quad (1)$$

with $\mu, \nu = (ct, r, \theta, \phi)$.

From Eq. (1), the metric components can be read off as

$${}^{(5)}g_{ab} = \begin{bmatrix} \pm N^2 + \beta_\lambda \beta^\lambda & \beta_\nu \\ \beta_\mu & {}^{(4)}g_{\mu\nu} \end{bmatrix}, \quad (2)$$

where $a, b = (\omega, ct, r, \theta, \phi)$.

In order to evaluate Christoffel symbols and curvature quantities, the inverse metric is also required. Its explicit form is

$${}^{(5)}g^{ab} = \begin{bmatrix} \pm \frac{1}{N^2} & \mp \frac{\beta^\nu}{N^2} \\ \mp \frac{\beta^\mu}{N^2} & {}^{(4)}g^{\mu\nu} \pm \frac{\beta^\mu \beta^\nu}{N^2} \end{bmatrix}. \quad (3)$$

III. VACUUM SOLUTIONS IN FIVE DIMENSIONS

We now proceed to construct vacuum solutions under the symmetry assumptions $\partial_\omega g_{ab} = 0$ and $\partial_t g_{ab} = 0$, where ω denotes the fifth coordinate. Under these conditions, we evaluate the relevant Christoffel symbols and analyze the geodesic equation [7, 11].

The nonvanishing five-dimensional Christoffel symbols

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take the form

$$^{(5)}\Gamma^5_{\mu\nu} = \pm\tfrac{1}{2N^2}\left(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu\right), \tag{4}$$

$$^{(5)}\Gamma^5_{\mu 5} = \tfrac{1}{N}\partial_\mu N \pm \tfrac{1}{N^2}\beta^\lambda\nabla_\mu\beta_\lambda \mp \tfrac{\beta^\lambda}{2N^2}\left(\nabla_\mu\beta_\lambda - \nabla_\lambda\beta_\mu\right), \tag{5}$$

$$^{(5)}\Gamma^5_{55} = \pm\tfrac{\beta^\sigma}{2N^2}\,\partial_\sigma\left(\pm N^2 + \beta^\lambda\beta_\lambda\right). \tag{6}$$

At this stage we assume that $\frac{dx^5}{ds}$ remains constant along the geodesic. This quantity is not yet identified with a physical charge-to-mass ratio, but the assumption allows us to simplify the connection coefficients consistently.

The fifth component of the geodesic equation is

$$\frac{d}{ds}\left(\frac{dx^5}{ds}\right) + {}^{(5)}\Gamma^5_{ab}\frac{dx^a}{ds}\frac{dx^b}{ds} = 0. \tag{7}$$

Under the constancy assumption, ${}^{(5)}\Gamma^5_{ab}$ can be set to zero, which implies from Eq. (4) that

$$\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu = 0, \tag{8}$$

namely the Killing equation in four dimensions. Since $\partial_t{}^{(4)}g_{\mu\nu} = 0$, β is proportional to the timelike Killing vector:

$$\beta_\mu = \alpha\,{}^{(4)}g_{t\mu}, \tag{9}$$

with α a constant. In addition, Eq. (5) reduces to

$$^{(5)}\Gamma^5_{\mu 5} = \tfrac{1}{N}\partial_\mu N, \tag{10}$$

which requires N to be constant.

With these conditions, the remaining five-dimensional Christoffel symbols become

$$^{(5)}\Gamma^\mu_{\lambda\rho} = {}^{(4)}\Gamma^\mu_{\lambda\rho}, \tag{11}$$

$$^{(5)}\Gamma^\mu_{\lambda 5} = {}^{(4)}g^{\mu\nu}F_{\lambda\nu}, \tag{12}$$

$$^{(5)}\Gamma^\mu_{55} = -\,{}^{(4)}g^{\mu\nu}\partial_\nu\phi, \tag{13}$$

with

$$F_{\mu\nu} = \partial_\mu\left(\tfrac{\beta_\nu}{2}\right) - \partial_\nu\left(\tfrac{\beta_\mu}{2}\right), \qquad \phi = \tfrac{1}{2}\,{}^{(5)}g_{55}. \tag{14}$$

From these expressions, the five-dimensional Ricci tensor can be written as

$$^{(5)}R_{ab} = \begin{bmatrix} -\Box\phi + F_{\lambda\rho}F^{\lambda\rho} & \nabla^\rho F_{\nu\rho} \\ \nabla^\rho F_{\mu\rho} & {}^{(4)}R_{\mu\nu} \end{bmatrix}. \tag{15}$$

Further manipulation shows that the condition for a vacuum solution reduces to

$$^{(4)}R_{\mu\nu} = 0, \tag{16}$$

so that the four-dimensional metric must itself be Ricci flat.

Choosing the four-dimensional Kerr solution [5], the full five-dimensional vacuum metric takes the form

$$^{(5)}g_{ab} = \begin{bmatrix} \pm N^2 + \alpha^2\, g^{\rm Kerr}_{tt} & \alpha\, g^{\rm Kerr}_{t\nu} \\ \alpha\, g^{\rm Kerr}_{t\mu} & g^{\rm Kerr}_{\mu\nu} \end{bmatrix}, \tag{17}$$

with indices $\mu,\nu = (ct, r, \theta, \phi)$ and $a, b = (\omega, ct, r, \theta, \phi)$. A direct computation confirms that the Ricci tensor vanishes identically for constant N and α.

Thus we obtain a rotating and charged five-dimensional vacuum solution. At this stage the parameters N and α are purely mathematical constants ensuring self-consistency. In the next chapter we will determine their physical interpretation: α by matching to the Lorentz force law, and N by comparing the five-dimensional Einstein–Hilbert action with the four-dimensional gravitational and electromagnetic sectors.

IV. LORENTZ FORCE AND CONSISTENCY

In this section we derive the Lorentz force within the five-dimensional framework and verify the validity of the assumption made in the previous section that dx^5/ds is a constant of motion. Unlike in the conventional Kaluza–Klein theory [1–3], our formulation shows that the scalar degree of freedom g_{55} (the dilaton) does not appear once the geodesic equation is projected onto the four-dimensional hypersurface. As a result, the effective dynamics reduces directly to the Lorentz force law.

Time-translation symmetry and the fifth-dimensional symmetry imply the conserved quantities

$$^{(5)}u_5 = \Big(\pm N^2 + \alpha^2\,{}^{(4)}g_{tt}\Big)\frac{dx^5}{ds} + \alpha\,{}^{(4)}g_{t\mu}\frac{dx^\mu}{ds}, \tag{18}$$

$$^{(5)}u_0 = \alpha\,{}^{(4)}g_{tt}\frac{dx^5}{ds} + {}^{(4)}g_{t\mu}\frac{dx^\mu}{ds}. \tag{19}$$

These relations show that dx^5/ds is itself conserved.

The relation between the full five-dimensional derivative and its four-dimensional projection is

$$\frac{d}{ds} = \frac{dx^a}{ds}\,\partial_a, \qquad P\frac{d}{ds} = \frac{dx^\mu}{ds}\,\partial_\mu, \tag{20}$$

where the first acts in five dimensions, while the second corresponds to the projected operator on the four-dimensional hypersurface.

Applying this projection eliminates the scalar component g_{55}. Thus, unlike in the Kaluza–Klein framework [3], no dilaton field arises. The projected geodesic equation takes the form

$$\frac{d^2x^\mu}{ds^2} + {}^{(4)}\Gamma^\mu_{\lambda\rho}\frac{dx^\lambda}{ds}\frac{dx^\rho}{ds} = \frac{dx^5}{ds}\,F^\mu{}_\nu\frac{dx^\nu}{ds}, \tag{21}$$

where we used ${}^{(5)}\Gamma^\mu_{\lambda\rho} = {}^{(4)}\Gamma^\mu_{\lambda\rho}$ [7, 11].

Equation (21) has exactly the structure of the Lorentz force law in four dimensions, with dx^5/ds playing the role of a constant coupling. Identifying

$$\frac{dx^5}{ds} = \frac{q}{m}, \tag{22}$$

the geodesic equation reproduces the standard Lorentz force. Under this identification, $F_{\mu\nu}$ is interpreted as the electromagnetic field strength tensor, and the metric component $g_{5\mu} = \alpha\, {}^{(4)}g_{t\mu}$ fixes the value of the constant α, in agreement with the result stated at the end of the previous section.

Thus the key difference from the Kaluza–Klein theory lies in the projection: in KK theory the dilaton inevitably appears [3], while in our approach the projection onto the four-dimensional hypersurface eliminates it and yields the Lorentz force in its physical form.

V. EINSTEIN–HILBERT ACTION IN FIVE DIMENSIONS

In this section we analyze the five-dimensional Einstein–Hilbert action in order to determine the explicit value of N^2. This also explains why N must be constant not only from a mathematical point of view, as in the previous section, but also from physical consistency. By reducing the five-dimensional action to four dimensions, we show that the relative normalization of the electromagnetic term fixes N^2 directly.

The five-dimensional action is [6–8]

$$ {}^{(5)}S_{\mathrm{EH}} = \frac{1}{16\pi\, {}^{(5)}G} \int {}^{(5)}R \sqrt{|{}^{(5)}g|}\, d^5x, \tag{23} $$

where ${}^{(5)}R$ is the five-dimensional Ricci scalar, $|{}^{(5)}g|$ the determinant of the metric, and $G^{(5)}$ the Newton constant in five dimensions.

The Ricci scalar reads

$$ \begin{aligned} {}^{(5)}R = \; & \pm \frac{1}{N^2}\big(-\Box\phi + F_{\lambda\rho}F^{\lambda\rho}\big) \\ & \mp \frac{2}{N^2}\, \beta^\mu \nabla^\rho F_{\mu\rho} \\ & + \Big({}^{(4)}g^{\mu\nu} \pm \frac{\beta^\mu\beta^\nu}{N^2}\Big)\, {}^{(4)}R_{\mu\nu}. \end{aligned} \tag{24} $$

With the identification $\beta^\mu = 2A^\mu$ and $\nabla_\rho F^{\mu\rho} = \mu_0 J^\mu$, this reduces to

$$ {}^{(5)}R = {}^{(4)}R \pm \frac{1}{N^2} F_{\lambda\rho}F^{\lambda\rho} \mp \frac{2\mu_0}{N^2} J_\mu A^\mu \mp \frac{1}{N^2}\Box\phi. \tag{25} $$

Comparing with the four-dimensional action [1–3]

$$ S = \int d^4x \sqrt{-g} \left(\frac{c^4}{16\pi G}\, {}^{(4)}R - \frac{1}{4\mu_0} F_{\lambda\rho}F^{\lambda\rho} + J_\mu A^\mu \right), \tag{26} $$

we obtain

$$ \frac{1}{N^2} = \frac{\kappa}{2\mu_0}, \qquad \kappa \equiv \frac{8\pi G}{c^4}. \tag{27} $$

The minus sign for the $F_{\mu\nu}F^{\mu\nu}$ term must be chosen, implying that the fifth dimension is timelike.

Substituting back, the action becomes

$$ \frac{c^4}{16\pi G}\, {}^{(5)}R = \frac{c^4}{16\pi G}\, {}^{(4)}R - \frac{1}{4\mu_0} F_{\lambda\rho}F^{\lambda\rho} + \tfrac{1}{2} J_\mu A^\mu + \frac{1}{4\mu_0}\Box\phi. \tag{28} $$

The prefactor 1/2 in front of $J_\mu A^\mu$ does not change the field equations, since J_μ already contains A_μ. Thus the standard Maxwell equation

$$ \nabla_\nu F^{\mu\nu} = \mu_0 J^\mu \tag{29} $$

is recovered.

Finally, because the metric determinant factorizes as

$$ \sqrt{|{}^{(5)}g|} = N \sqrt{-{}^{(4)}g}, $$

with constant N, the four- and five-dimensional Newton constants are related by

$$ {}^{(4)}G \sim \frac{{}^{(5)}G}{N}. $$

Hence, although the metric effectively contains two timelike directions, the gravitational constant remains fixed rather than dynamical, in contrast with the situation in two-time physics.

VI. CONSERVED CHARGES

Our five-dimensional metric corresponds to a rotating, charged solution, and it admits three continuous symmetries: translation along the fifth dimension, time translation, and axial rotation about the ϕ-axis. According to Noether's theorem [9], each Killing symmetry is associated with a conserved charge:

$$ \partial_\omega \longrightarrow Q, \qquad \partial_t \longrightarrow M, \qquad \partial_\phi \longrightarrow J. \tag{30} $$

That is, the conserved quantities are the electric charge Q, the total mass M, and the angular momentum J. We now compute these charges explicitly.

Conserved currents and 3+1 foliation

The conserved quantity $\mathcal{C}$ is defined as

$$ \mathcal{C} = \int_{\Sigma_t} \rho\, d\Sigma. \tag{31} $$

Here, Σ_t denotes a three-dimensional hypersurface at fixed t. Introducing a current J^μ, the density can be written using the unit normal vector n_μ to the hypersurface as

$$ \rho = J^\mu n_\mu, \tag{32} $$

so that

$$ \mathcal{C} = \int_{\Sigma_t} J^\mu n_\mu\, d\Sigma, \tag{33} $$

where $d\Sigma = \sqrt{h}\, d^3x$ and h is the determinant of the induced metric.

If the current is conserved,

$$\nabla_\mu J^\mu = 0, \tag{34}$$

then $\mathcal{C}$ is independent of t. For a Killing vector ζ^μ, the current

$$J^\mu = {}^{(4)}T^{\mu\nu}\zeta_\nu, \qquad \nabla_{(\mu}\zeta_{\nu)} = 0, \tag{35}$$

is conserved. By Einstein's equation,

$${}^{(4)}R_{\mu\nu} - \tfrac{1}{2}{}^{(4)}g_{\mu\nu}\,{}^{(4)}R = \kappa\,{}^{(4)}T_{\mu\nu}, \tag{36}$$

this becomes

$$J^\mu = \frac{1}{\kappa}\Big({}^{(4)}R^{\mu\nu} - \tfrac{1}{2}{}^{(4)}g^{\mu\nu}\,{}^{(4)}R\Big)\zeta_\nu. \tag{37}$$

Ricci tensor contracted with a Killing vector

Using the Ricci identity and the Killing equation [7, 11],

$$(\nabla_\lambda\nabla_\mu - \nabla_\mu\nabla_\lambda)\zeta^\lambda = {}^{(4)}R_{\nu\mu}\zeta^\nu, \qquad \nabla_\lambda\zeta^\lambda = 0, \tag{38}$$

so that

$${}^{(4)}R_{\mu\nu}\zeta^\nu = \nabla_\lambda\nabla_\mu\zeta^\lambda, \qquad {}^{(4)}R^{\mu\nu}\zeta_\nu = \nabla_\lambda\nabla^\mu\zeta^\lambda. \tag{39}$$

This shows explicitly why the Ricci tensor contracted with a Killing vector can be written as a divergence term, allowing volume integrals to be recast as surface integrals.

Observable mass

The conserved quantity is

$$M_{\rm tot} = \int_{\Sigma_t} T^{\mu\nu}\zeta_\nu\delta^0_\mu\sqrt{-{}^{(4)}g}\, d^3x. \tag{40}$$

Using Einstein's equation this decomposes as

$$\begin{aligned} M_{\rm tot} &= \frac{1}{\kappa}\int_{\Sigma_t}\nabla_\lambda\nabla^0\zeta^\lambda\sqrt{-{}^{(4)}g}\, d^3x \\ &\quad - \frac{1}{2\kappa}\int_{\Sigma_t}{}^{(4)}R\,\zeta^0\sqrt{-{}^{(4)}g}\, d^3x \\ &\equiv M_{\rm grav} + M_{\rm curv}. \end{aligned} \tag{41}$$

The first term $M_{\rm grav}$ is a divergence and hence convertible into a surface integral, meaning that only $M_{\rm grav}$ can be computed from the exterior geometry, while the second term $M_{\rm curv}$ depends on the interior geometry and cannot be obtained from exterior data.

For $\zeta = -2\partial_0$, one finds

$$M_{\rm grav} = \frac{1}{\kappa}\int -2R^0{}_0\sqrt{-{}^{(4)}g}\, d^3x \tag{42}$$

$$= \int(\rho c^2 + 3p)\sqrt{-{}^{(4)}g}\, d^3x \tag{43}$$

$$= M_{\rm Tolman}. \tag{44}$$

Thus $M_{\rm grav}$ coincides with both the Komar mass and the Tolman mass [10, 13]. Although the explicit values of ρ and p are not known, the volume integral can be converted into a surface integral, which makes the evaluation possible solely from the exterior geometry. This is directly analogous to electrostatics, where the electric field at infinity depends only on the total enclosed charge, irrespective of the details of the charge distribution.

In particular, $M_{\rm grav}$ is the same mass parameter M that appears in the exterior metric (e.g. via $g_{tt} = -(1 - 2GM/c^2r)$), as will be demonstrated explicitly below.

Surface integral representation

To evaluate $M_{\rm grav}$ explicitly, we choose the integration region on Σ_t as the ball $B_{r_0} = \{r \le r_0\}$. After attaching a formal dx^0 (to invoke the divergence theorem) and then dropping it, Gauss's theorem reduces the divergence piece to a surface integral over ∂B_{r_0}:

$$M_{\rm grav} = \frac{1}{\kappa}\int_{\Sigma_t}\nabla_\lambda\nabla^0\zeta^\lambda\sqrt{-{}^{(4)}g}\, d^3x \tag{45}$$

$$\to \frac{1}{\kappa}\int_{[t,t+\Delta t]\times\Sigma_t}\nabla_\lambda\nabla^0\zeta^\lambda\sqrt{-{}^{(4)}g}\, d^4x \tag{46}$$

$$\to \frac{1}{\kappa}\int_{[t,t+\Delta t]\times S_r}\nabla^0\zeta^r\sqrt{-{}^{(4)}g}\, d\theta\, d\phi\, dx^0 \tag{47}$$

$$\to \frac{1}{\kappa}\int_{S_r}\nabla^0\zeta^r\sqrt{-{}^{(4)}g}\, d\theta\, d\phi. \tag{48}$$

For intuition one can write

$$\nabla^0\zeta^r = \nabla^0\Big(\tfrac{\zeta^r}{2}\Big) - \nabla^r\Big(\tfrac{\zeta^0}{2}\Big). \tag{49}$$

Evaluation for different Killing vectors

By applying Gauss's theorem, $M_{\rm grav}$ reduces to a flux integral over the boundary S_r. Depending on the choice of Killing vector, we obtain the corresponding conserved quantities:

For $\zeta = -c\partial_0 = -\partial_t$ (timelike Killing vector), the integral becomes

$$\mathcal{C}_t = \frac{r_s c}{2}\int_0^{2\pi}\int_0^{\pi}\frac{(r^2 - a^2\cos^2\theta)(r^2 + a^2)}{(r^2 + a^2\cos^2\theta)^2}\sin\theta\, d\theta\, d\phi = 4\pi\frac{GM}{c}. \tag{50}$$

For the extra-dimensional Killing vector $\zeta = \partial_\omega$, we obtain

$$\mathcal{C}_\omega = -\frac{\alpha r_s}{2}\int_0^{2\pi}\int_0^{\pi} \frac{(r^2 - a^2\cos^2\theta)(r^2 + a^2)}{(r^2 + a^2\cos^2\theta)^2}\sin\theta\, d\theta\, d\phi = 4\pi \frac{Q}{4\pi\epsilon_0 c}. \tag{51}$$

Finally, for the axial Killing vector $\zeta = \partial_\phi$, one finds

$$\mathcal{C}_\phi = \frac{r_s a}{2}\int_0^{2\pi}\int_0^{\pi} \frac{\sin^3\theta\,(-a^2 r^2\sin^2\theta - a^4\cos^2\theta + 3r^4 + 2a^2 r^2)}{(a^2\cos^2\theta + r^2)^2}\, d\theta\, d\phi = 4\pi\frac{2GJ}{c^3}, \qquad J = Mac. \tag{52}$$

Summary

We have checked the conserved quantities associated with the spacetime symmetries:

$$\zeta = -\partial_t \;\Rightarrow\; M, \qquad \zeta = \partial_\omega \;\Rightarrow\; Q, \qquad \zeta = \partial_\phi \;\Rightarrow\; J.$$

In particular, for $\zeta = -2\partial_0$ one finds

$$M_{\text{grav}} = Mc^2, \tag{53}$$

showing that the Komar mass [10], Tolman mass [13], and the metric mass parameter coincide.

VII. CONCLUSION

We presented a five-dimensional framework that unifies gravitation and electromagnetism through an ADM-type decomposition rather than compactification. Starting from symmetry assumptions, we constructed a rotating and charged vacuum solution that generalizes the Kerr metric while correctly reproducing the physical mass, charge, and angular momentum.

A central outcome of our analysis is the elimination of the dilaton degree of freedom. By projecting the geodesic equation onto the four-dimensional hypersurface, the Lorentz force law emerges directly, providing a physical identification of the extra-dimensional momentum with electric charge [1–3]. The reduction of the Einstein–Hilbert action further fixes the relative normalization between the gravitational and electromagnetic sectors, and crucially, it requires that the extra dimension be identified as timelike. This feature distinguishes our framework from conventional Kaluza–Klein models and ensures the correct sign for the electromagnetic action [7, 8].

Finally, the application of Noether's theorem and Komar-type surface integrals demonstrates the consistency of conserved quantities across different definitions, thereby reinforcing the geometric foundation of charge, mass, and angular momentum [9, 10, 13]. Taken together, these results establish a coherent and self-consistent classical unified field theory that preserves the geometric clarity of general relativity while incorporating electromagnetism in a natural way.

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