

# The Balan-Killing Manifolds

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## Abstract

We define here a differential equation over the metric of a Spin manifold, calling Balan-Killing metrics the solutions of this equation.

## 1 The Spin manifolds

A Spin manifold [F] admits a spinor bundle by reduction of the tangent bundle with the orthogonal group to the spinor bundle with the group spin. The spinor bundle admits a multiplication by the vectors called the Clifford multiplication.

## 2 The Balan-Killing metrics

We introduce here the following differential equation over the metric. Let  $R^\nabla$  be the spinor curvature.

**Definition 1** We call a Balan-Killing manifold, a manifold  $M$  with riemannian metric  $g$  such that  $\exists \mu, \forall X, Y$  vectors and  $\forall \psi$  spinor:

$$R^\nabla(X, Y)\psi = \mu \text{Ricc}^\nabla(X.Y - Y.X).\psi$$

. is the Clifford multiplication and  $\nabla$  is the Levi-Civita connection over the spinors,  $\text{Ricc}^\nabla$  is the Ricci curvature as an endomorphism of the spinors.

For example, a flat space is a Balan-Killing manifold, with  $\mu = 0$ .

## 3 The Balan-Killing spinors

**Definition 2** We call a Balan-Killing spinor, a spinor  $\psi$  which verifies the equation:

$$d(h(Z.\psi, \psi)(X, Y) = h((X.Y - Y.X).\psi, \psi)$$

$h$  is the hermitian metric over the spinor bundle,  $d$  is the differential operator over the forms.

It is a differential equation over the spinors.

## References

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