We begin, as in GFR with differentiation of the relativistic relative momentum of two moving masses, with μ_0 being the reduced mass

$$F_T = \frac{d}{dt}\frac{(\mu_0 v)}{\sqrt{1-\frac{v^2}{c^2}}} = \mu_0\frac{c^3}{\left(\sqrt{c^2-v^2}\right)^3}\left(\frac{dv}{dt}\right)$$

This can be broken into two parts F_m and F_c with F_c being negligible at low speeds.

$$F_T = F_m + F_c = c\mu_0\frac{1}{\left(\sqrt{c^2-v^2}\right)}\left(\frac{dv}{dt}\right) + c\mu_0\frac{v^2}{\left(\sqrt{c^2-v^2}\right)^3}\left(\frac{dv}{dt}\right)$$

It is instructive and necessary to point out that F_m is inherently sinusoidal. In fact F_m is an elliptic integral of the first kind:

$$\int F_m\,dt = \int c\mu_0\frac{1}{\left(\sqrt{c^2-v^2}\right)}\left(\frac{dv}{dt}\right)dt = c\mu_0\int \frac{1}{\left(\sqrt{c^2-v^2}\right)}\,dv$$

$$= c\mu_0\arcsin(v/c) + constant$$

, so that, assuming the constant $=0$, we have, for some θ,

$$v/c = \sin\theta = \sin\left(\int F_m/(c\mu_0)\,dt\right)$$

, and setting $\int F_m/(c\mu_0)\,dt = Q/(c\mu_0)$ so

$$v = c\sin\theta \;;\; v = c\sin(Q/(c\mu_0))$$

, and if we choose $Q = kr$, where k is a constant

and r is the radial distance travelled from the centre of radial motion, along the focal line.

$$v = c\sin(kr/(c\mu_0))$$

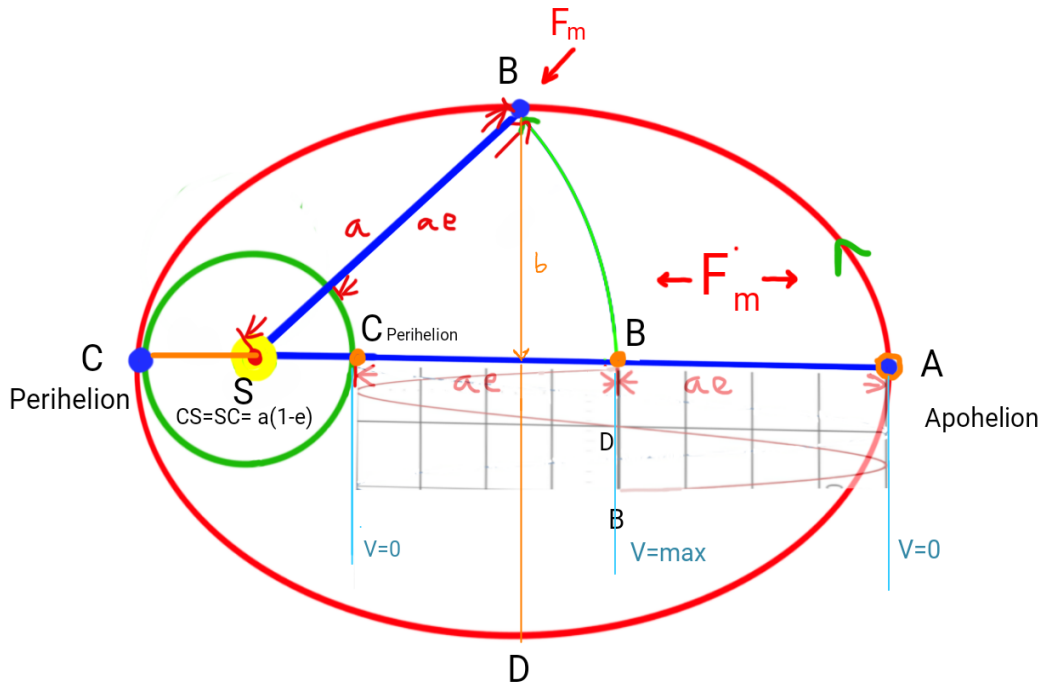, which, for θ being a small angle, makes $v = c(kr/(c\mu_0)) = kr/\mu_0$.

The quantity v/r can be determined from average radial velocity and distance, and has dimensions of frequency ω. This can be illustrated from elliptical planetary orbits, see Fig. 1.

At peak excursion at the centre of radial motion, where peak radial velocity is reached, at the minor axis point of the orbit $r = ae$, where a is the semi-major axis and e is the eccentricity so $v = k\,ae/\mu_0$. k can be determined from peak radial velocity which is average v divided by 0.637.

Then if the angular frequency is $\omega = k/\mu_0$, which, as it turns out from data from astronomy, it is, then the frequency, $f = \omega/2\pi$ and the orbital period (a year for Earth) is $T = 2\pi/\omega$ and $T = 2\pi\mu_0/k$.

This can be seen from the Basic Program (Fig. 2) results (Fig. 3) below, for a number of planets, that is that the results of this equation corresponds very well with the actual known periods of the planets.

Figure 1

The radial path, ABC, is shown, starting at B, where radial velocity is greatest.
The corresponding points on the elliptical orbit are shown by corresponding letter.
The corresponding sinewave of the radial position is labelled by the same letters.

Figure 2

```
REM Start of BASIC! Program

print "For Earth"
m1= 5.97219e24        rem mass of Earth
a1=1.49598262e11      rem semi-major axis
ec=0.01671123         rem eccentricity
orbper=365.26         rem known orbital period in Earth days
gosub calculation


print "Mars "
m1=0.641693e24
a1=  2.27943824e11
ec= 0.0933941
orbper=686.98         rem known orbital period in Earth days
gosub calculation


print "Venus"
m1=4.867320e24
a1=  1.08209475e11
ec= 0.00677672
orbper=224.70
gosub calculation


print "Mercury"
m1=0.330104e24
a1= 5.7909227e10
ec= 0.20563593
orbper=88
gosub calculation


print "Jupiter"
m1=1898.130e24
a1= 7.78340821e11
ec=0.04838624
orbper=4332.82
gosub calculation

print "Saturn"
m1= 568.319e24
a1= 1.426666422e12
ec= 0.05386179
orbper=10755.7
gosub calculation

print "Neptune"
m1=102.41e24
a1= 4.498396441e12
```

```
ec= 0.00859048
orbper=60190.03
gosub calculation

print "Uranus"
m1=86.8103e24
a1= 2.870658186e12
ec= 0.04725744
orbper=30687.15
gosub calculation

 end

Calculation:

M2=1.9891e30       rem mass of the Sun
Gn=0.667384e-10   rem Newton's constant
c=2.99792458e8
pi=2*asin(1)

mu=m1*m2/(m1+m2) rem reduced mass virtually = m1
rap=a1*(1-ec)          rem perihelion radius
raa=a1*(1+ec)          rem aphelion radius

distra=(raa-rap)*2      rem distance travelled
timtra=orbper*24*60*60  rem time travelled
vav=distra/timtra     rem average radial velocity  "m/s"

sinetheta=vav/c
theta=asin(vav/c)

vav=c*sinetheta       rem sinusoidal equation

c=2.99792458e8
pi=2*asin(1)

r=a1*ec            rem the approximate radius at max radial vel.
vp=vav/0.637       rem the peak radial velocity

rem vp=c*sin(k*r) peak velocity

k=asin(vp/c)/r


rem vp=c*sin(r*k/(c*m1))
rem vp= r*k/m1
k=vp* m1/r
w=(k/m1) rem angular frequency
orbper2=(2*pi* m1/k)/(60*60*24) rem derived orbital period in days
print orbper, "orbper"
print orbper2, "orbper2"

RETURN
```

Figure 3

For Earth

365.26, orbper

365.4781552490908, orbper2

Mars

686.98, orbper

687.3903057904519, orbper2

Venus

224.7, orbper

224.83420435982893, orbper2

Mercury

88.0, orbper

88.05255889481472, orbper2

Jupiter

4332.82, orbper

4335.407820802631, orbper2

Saturn

10755.7, orbper

10762.12395119271, orbper2

Neptune

60190.03, orbper

60225.97910745036, orbper2

Uranus

30687.15, orbper

30705.47821237511, orbper2

END