

The metaphysics of physics

Jean Louis Van Belle, *Drs, MAEc, BAEd, BPhil*

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Email: jeanlouisvanbelle@outlook.com

Abstract: This wavefunction is a didactic exploration of the basic assumptions and concepts of the *Zitterbewegung* interpretation of quantum mechanics. Its novelty is in applying the concepts to photons.

Keywords: *Zitterbewegung*, mass-energy equivalence, wavefunction interpretations.

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The metaphysics of physics

1. The wavefunction of an electron

The wavefunction is a wonderful mathematical object which can be used in various ways. One way is to describe the *Zitterbewegung* (*zbw*) of an electron – or of any charged particle, as depicted below.

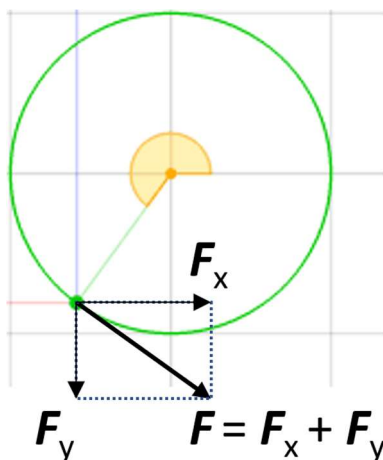


Figure 1: The force and position vector

The *zbw* model of an electron implies a dual view of the reality of the real and imaginary part of the wavefunction. On the one hand, they will describe the *physical* position (i.e. the *x*- and *y*-coordinates) of the pointlike *charge*. This is the green dot in the illustration. The elementary wavefunction describes its motion as follows:

$$a \cdot e^{j\theta} = x + j \cdot y \text{ with } x = a \cdot \cos(\omega t) \text{ and } y = a \cdot \sin(\omega t)$$

As such, the (elementary) wavefunction is viewed as an implicit function: it is equivalent to the $x^2 + y^2 = a^2$ equation, which describes the same circle. On the other hand, the $z\bar{w}$ model implies the circular motion of the pointlike charge is driven by a tangential force, which we write as:

$$\mathbf{F} = F_x \cdot \cos(\omega t + \pi/2) + i \cdot F_x \cdot \sin(\omega t + \pi/2)$$

The line of action of the force is the orbit, because a force needs something to grab onto, and the only thing it can grab onto in this model is the oscillating (or rotating) charge. The formula above suggests we should think of the composite force \mathbf{F} as the resultant force of two perpendicular oscillations. The *zbw* model – which is derived from Dirac’s wave equation for free electrons – tell us the velocity of the pointlike charge is equal to c . If the *zbw* frequency is given by Planck’s energy-frequency relation ($\omega = E/\hbar$), then we

can combine Einstein's $E = mc^2$ formula with the radial velocity formula ($c = a \cdot \omega$) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$a = \frac{\hbar}{mc} = \frac{\lambda_e}{2\pi} \approx 0.386 \times 10^{-12} \text{m}$$

This (two-dimensional) oscillator model also allows us to calculate the magnitude of the force. Indeed, the energy in the oscillator must be equal to the magnitude of the force times the length of the loop:

$$E = F\lambda_e \Leftrightarrow F = \frac{E}{\lambda_e} \approx \frac{8.187 \times 10^{-14} \text{ J}}{2.246 \times 10^{-1} \text{ m}} \approx 3.3743 \times 10^{-2} \text{ N}$$

Considering the sub-atomic scale, this is a significant force. The current is equally significant:

$$I = q_e f = q_e \frac{E}{h} \approx (1.6 \times 10^{-19} \text{ C}) \frac{8.187 \times 10^{-14} \text{ J}}{6.626 \times 10^{-34} \text{ Js}} \approx 1.98 \text{ A}$$

The A is not *ångström* but *ampere*. Hence, we have a household-level current here at the sub-atomic scale. Does that make sense? Maybe. Maybe not. But the result is consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which – using the results above – we can now calculate as:

$$\mu = I \cdot \pi a^2 = q_e \frac{mc^2}{h} \cdot \pi a^2 = q_e c \frac{\pi a^2}{2\pi a} = \frac{q_e c}{2} \frac{\hbar}{mc} = \frac{q_e}{2m} \hbar$$

All that is left is to check whether this is consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. The oscillator model implies the effective mass of the electron will be spread over the disk, which gives the form factor for the moment of inertia (I): 1/2. We write:

$$L = I \cdot \omega = \frac{ma^2}{2} \frac{c}{a} = \frac{mc}{2} \frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -g \left(\frac{q_e}{2m} \right) \mathbf{L} \Leftrightarrow \frac{q_e}{2m} \hbar = g \frac{q_e}{2m} \frac{\hbar}{2} \Leftrightarrow g = 2$$

The *vector* notation for $\boldsymbol{\mu}$ and \mathbf{L} (boldface) should make us think about the plane of oscillation. This question is related to the question of how we should analyze all of this is a moving reference frame. This is a complicated question. The Stern-Gerlach experiment suggests we may want to think of an oscillation plane that is perpendicular to the direction of motion, as illustrated below.

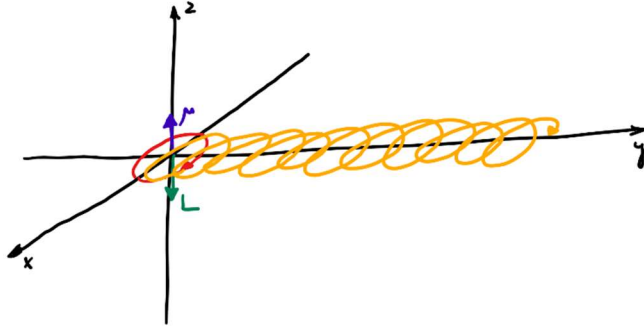


Figure 2: The *zbw* electron traveling through a Stern-Gerlach apparatus?

Of course, the Stern-Gerlach experiment assumes the application of a (non-homogenous) magnetic field. In the absence of such field, we may want to think of the plane of oscillation as something that is rotating in space itself. The idea, then, is that it sort of snaps into place when an external magnetic field is applied.

As for the question of how we should look at the motion in a moving reference frame – and, in particular, when the electron would move at a relativistic speed – we will come back to this later. Let us first see how we can use the wavefunction concept to interpret a photon. Before we do so, however, we should say something about the nature of the force.

The assumption is that the force grabs onto a pointlike charge. Hence, the force must be electric. We write:

$$\mathbf{F} = q_e \mathbf{E}.$$

The \mathbf{E} in this formula is an electric field vector (as opposed to the energy E). The field must be humongous because the force is humongous (a force of 0.0375 N is equivalent to a force that gives a mass of 37.5 *gram* ($1 \text{ g} = 10^{-3} \text{ kg}$) an acceleration of 1 m/s per second). Hence, what makes the charge go round and round? There is no wire to confine its motion. The $E = ma^2\omega^2 = mc^2$ is intuitive: the energy of any oscillation will be proportional to the square of (i) the (maximum) amplitude of the oscillation and (ii) the frequency of the oscillation, with the mass as the proportionality coefficient. But what does it mean?

This question is difficult to answer. This is why we insist that the *Zitterbewegung* idea must be complemented with the idea of a two-dimensional oscillation. We explore this

elsewhere¹ and, hence, will not dwell on this here. We will only make one or two remarks below which may or may not help the reader to develop his or her own interpretation.

When everything is said and done, we should admit that the bold $c^2 = a^2 \cdot \omega^2$ assumption interprets spacetime as a relativistic aether – a term that is taboo but that is advocated by Nobel Prize Laureate Robert Laughlin². It is inspired by the most obvious implication of Einstein's $E = mc^2$ equation, and that is that the ratio between the energy and the mass of *any* particle is always equal to c^2 :

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photo}}{m_{photo}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

This reminds us of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ of harmonic oscillators – with one key difference, however: the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more) degrees of freedom.³ In contrast, $c^2 = E/m$ for *any* particle, *always*. This is the point: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light c emerges here as *the* defining property of spacetime. It is, in fact, tempting to think of it as some kind of *resonant frequency* but the $c^2 = a^2 \cdot \omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we will now refer to as *the rest energy oscillation*.

It is now time to look at how we can use the very same wavefunction to describe the conceptual opposite of matter: the photon.

¹ See: Jean Louis Van Belle, 24 November 2018, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung* (<http://vixra.org/abs/1811.0364>).

² Robert Laughlin (2005), as quoted in the Wikipedia article on aether theories (https://en.wikipedia.org/wiki/Aether_theories).

³ The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. We will usually also include a resistance in an electric circuit to introduce a damping factor or, when analyzing a mechanical spring, a drag coefficient. Both are usually defined as a fraction of the *inertia*, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as $R = \gamma L$ respectively. This is a third degree of freedom in classical oscillators.

2. The wavefunction of a photon

Photons may or may not have a wavefunction but, if they do, we would probably want to visualize it as a circularly polarized wave, as illustrated below: a rotating electric field vector (\mathbf{E}) which can be analyzed as the sum of two orthogonal components: $\mathbf{E} = \mathbf{E}_x + \mathbf{E}_y$.⁴

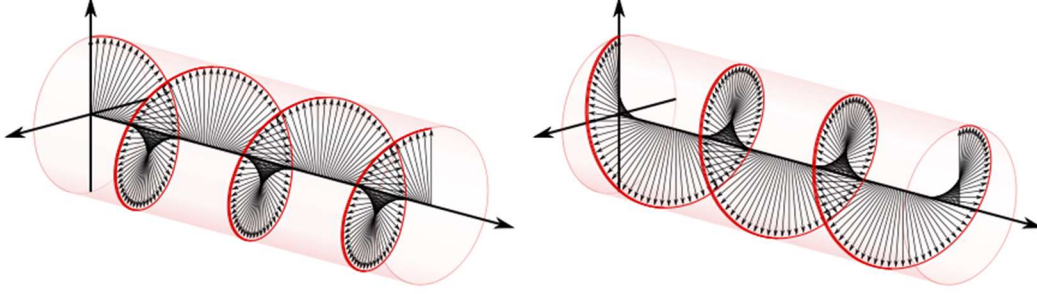


Figure 3: LHC- and RHC-polarized light

This is a very different view of the (elementary) wavefunction. It is *not* an implicit function anymore. It is a proper function now. To be precise, we think of $a \cdot e^{i\theta}$ as a function from some *domain* ($\Delta \mathbf{x}, \Delta t$) to an associated *range* of values $a \cdot e^{i\theta}$. We may write this as:

$$(\mathbf{x}, t) \rightarrow a \cdot e^{i\theta} = a \cdot e^{i(\omega \cdot t - \mathbf{k} \cdot \mathbf{x})} = a \cdot \cos(\omega \cdot t - \mathbf{k} \cdot \mathbf{x}) + i \cdot a \cdot \sin(\omega \cdot t - \mathbf{k} \cdot \mathbf{x})$$

Hence, while the domain of this wavefunction has to be *limited* in space and in time, the wave itself will, effectively, occupy some space at any point in time and, conversely, will only have non-zero values over a limited time *interval* at any point in space. Of course, the amplitude is not necessarily uniform. If you have ever recorded someone playing the guitar (yourself, perhaps), then you are probably aware of how an actual wave packet looks like: it is a transient oscillation, as shown below. Note that its shape reverses depending on whether we take the horizontal axis to be time (t) or spatial position (x).

⁴ One should not confuse the electric field vector \mathbf{E} with the energy E . Boldface is used to denote a vector. Of course, there is more scope for confusion when we will use E to denote the magnitude of the electric field, which we will do shortly. We could have introduced new symbols but the context should make clear what we are talking about.

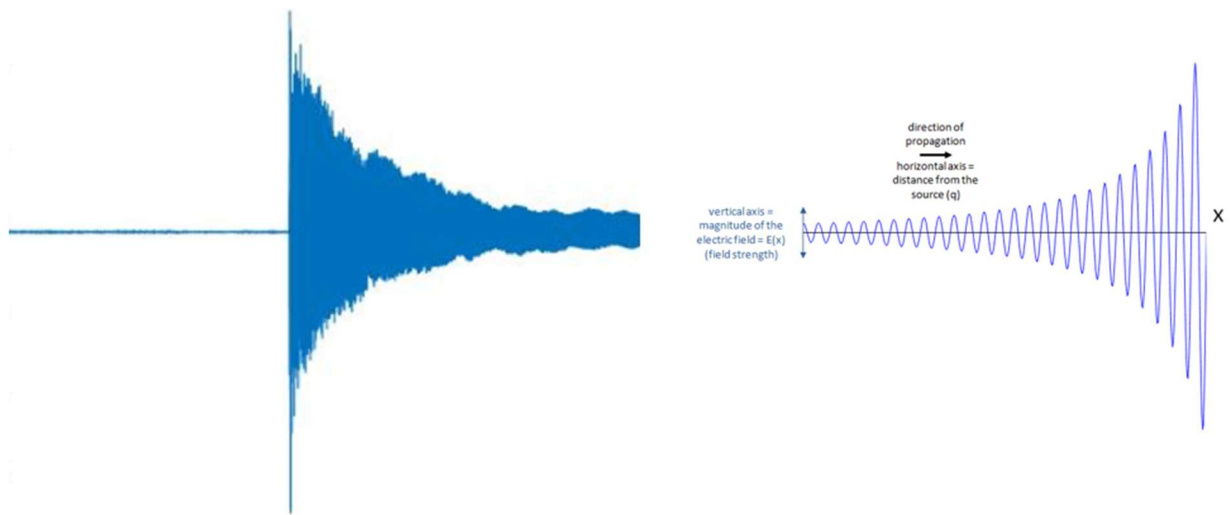


Figure 4: An actual wave is usually a transient

This may look outlandish but it makes sense if we think photons are emitted – and absorbed – by an atomic transition from one energy state to another. We think of these atoms as atomic oscillators, and we can calculate their Q : it's of the order of 10^8 (see, for example, Feynman's *Lectures*, I-33-3), which means that, after about as many oscillations, the amplitude will have died by a factor $1/e \approx 0.37$. Let us give an example, because it gives rise to interesting questions. For sodium light – which has a frequency of 500 THz (500×10^{12} oscillations per second) and a wavelength of 600 nm (600×10^{-9} meter) – the *decay time* of the radiation will be some 3.2×10^{-8} seconds. That makes for about 16 million oscillations. Now, the wavelength is small but the speed of light is huge. The length of the wave train is, therefore, still quite considerable: about 9.6 meter.

This sounds lunatic: a photon with a length of 9.6 meter? Yes. Fortunately, we are saved by relativity theory: as this wave train zips by at the speed of light, relativistic length contraction reduces its *length* to zero. What about the field strength? Because the electric field is perpendicular to the direction of propagation, we like to think the amplitude remains what it is. However, that requires, perhaps, a more careful consideration.

At this point in the argument, we have no choice but to think about relativistic transformations of the wavefunction or, to be precise, relativistic transformations of its argument. Before we do so, we need to make one more note. It should, intuitively, be obvious that the energy of a photon – the energy of the wave train, really – is packed over many oscillations. Zillions, literally. Each of these oscillations will, therefore, pack an exceedingly small (but real) amount of energy. As any oscillation, each oscillation takes some time (the cycle time) and, in the case of the photon, some space (the wavelength). In contrast, the electron picture was different: *one* oscillation – one cycle, really – packs

all of the energy $E = F\lambda_e = m_e c^2$. Hence, the magnitude of the associated electric field is humongous as compared to the amplitude of the oscillations of our photon.

Let relativity enter the picture now.

3. The relativistic invariance of the wavefunction

Let us consider the idea of a particle traveling in the positive x -direction at constant speed v . This idea implies a pointlike concept of position and time: we think the particle will be somewhere at some point in time. The *somewhere* in this expression does not necessarily mean that we think the particle itself will be dimensionless or pointlike. It just implies that we can associate some *center* with it. Think of the *zbw* model here, for example: we have an oscillation around some center, but the oscillation has a *physical* radius, which we refer to as the Compton radius of the electron.⁵ Of course, two extreme situations may be envisaged: $v = 0$ or $v = c$. However, let us not consider these right now (we will do so later, of course).

The point is: in our reference frame, we have a position – a mathematical *point* in space, that is – which is a function of time: $x(t) = v \cdot t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t' . Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t' , and the position and time in the two reference frames will be related as follows:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows:

$$\theta = \frac{1}{\hbar}(E_v t - p x) = \frac{1}{\hbar} \left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} t - \frac{E_0 v}{c^2 \sqrt{1 - \frac{v^2}{c^2}}} x \right) = \frac{1}{\hbar} E_0 \left(\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = \frac{E_0}{\hbar} t'$$

⁵ The language is quite subtle: the Compton *radius* is the reduced Compton wavelength: $a = r_c = \lambda_e/2\pi$.

We have just shown that the argument of the wavefunction is relativistically invariant.⁶ It makes us think that of the argument of the wavefunction and – therefore – the wavefunction itself – might be more real – in a *physical* sense, that is – than the various wave equations (Schrödinger, Dirac, Klein-Gordon) for which it is some solution. Let us, therefore, further explore this. We have been interpreting the wavefunction as an implicit function again: for each x , we have a t , and vice versa. There is, in other words, no uncertainty here: we think of our particle as being *somewhere* at any point in time, and the relation between the two is given by $x(t) = v \cdot t$. We will get some linear motion. If we look at the $\psi = a \cdot \cos(p \cdot x/\hbar - E \cdot t/\hbar) + i \cdot a \cdot \sin(p \cdot x/\hbar - E \cdot t/\hbar)$ once more, we can write $p \cdot x/\hbar$ as Δ and think of it as a phase factor. We will, of course, be interested to know for what x this phase factor $\Delta = p \cdot x/\hbar$ will be equal to 2π . Hence, we write:

$$\Delta = p \cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi \cdot \hbar/p = h/p = \lambda$$

We now get a meaningful interpretation of the *de Broglie* wavelength. It is the distance between the crests (or the troughs) of the wave, so to speak, as illustrated below.

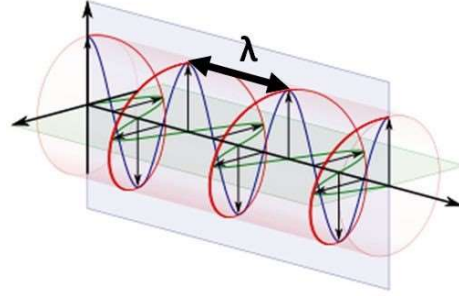


Figure 5: An interpretation of the *de Broglie* wavelength

Of course, we should probably think of the plane of oscillation as being *perpendicular* to the plane of motion – or as oscillating in space itself – but that doesn't matter. Let us explore some more. We can, obviously, re-write the argument of the wavefunction as a function of *time* only:

$$\theta = \frac{1}{\hbar}(E_v t - p x) = \frac{1}{\hbar} \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} \left(t - \frac{v}{c^2} v t \right) = \frac{1}{\hbar} \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 - \frac{v^2}{c^2} \right) t = \sqrt{1 - \frac{v^2}{c^2}} \cdot \frac{E_0}{\hbar} t$$

We recognize the *inverse* Lorentz factor here, which goes from 1 to 0 as v goes from 0 to c , as shown below.

⁶ E_0 is, obviously, the rest energy and, because $p' = 0$ in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t'/\hbar$ in the reference frame of the electron itself.

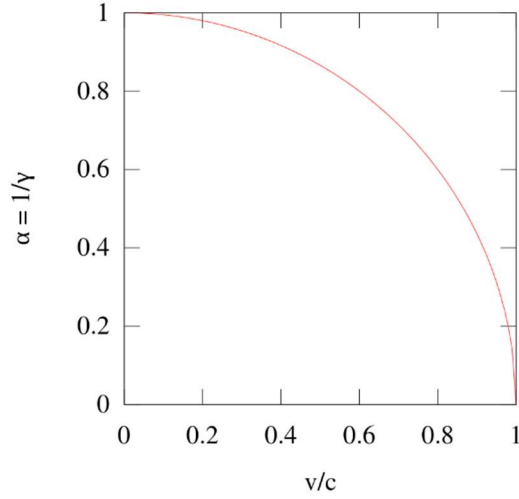


Figure 6: The inverse Lorentz factor as a function of (relative) velocity (v/c)

Note the shape of the function: it is a simple circular arc. This result should not surprise us, of course, as we also get it from the Lorentz formula:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{v^2}{c^2} t}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{1 - \frac{v^2}{c^2}} \cdot t$$

What does it all mean? We can go through a simple numerical example to think this through. Let us assume that, for example, that we are able to speed up an electron to, say, about one tenth of the speed of light. Hence, the Lorentz factor will then be equal to $\gamma = 1.005$. This means we added 0.5% (about 2,500 eV) – to the rest energy E_0 : $E_v = \gamma E_0 \approx 1.005 \cdot 0.511 \text{ MeV} \approx 0.5135 \text{ MeV}$. The relativistic momentum will then be equal to $m_v v = (0.5135 \text{ eV}/c^2) \cdot (0.1 \cdot c) = 5.135 \text{ eV}/c$. We get:

$$\theta = \frac{E_0}{\hbar} t' = \frac{1}{\hbar} (E_v t - p x) = \frac{1}{\hbar} \left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} t - \frac{E_0 v}{c^2 \sqrt{1 - \frac{v^2}{c^2}}} x \right) = 0.955 \frac{E_0}{\hbar} t$$

This is interesting, and then it is not. A more interesting question is what happens to the *radius* of the oscillation. Does it change? It must, but how should we interpret this? In the moving reference frame, we measure higher mass and, therefore, higher energy – as it includes the kinetic energy. The $c^2 = a^2 \cdot \omega^2$ identity must now be written as $c'^2 = a'^2 \cdot \omega'^2$. Instead of the rest mass m_0 and rest energy E_0 , we must now use $m_v = \gamma m_0$ and $E_v = \gamma E_0$ in the formulas for the Compton radius and the Einstein-Planck frequency, which we just write as m and E in the formula below:

$$m\alpha'^2\omega'^2 = m \frac{\hbar^2}{m^2c^2} \frac{m^2c^4}{\hbar^2} = mc^2$$

This is easy to understand intuitively: we have the mass factor in the denominator of the formula for the Compton radius, so it must increase as the mass of our particle increases with speed. Conversely, the mass factor is present in the numerator of the *zbw* frequency, and this frequency must, therefore, increase with velocity. It is interesting to note that we have a simple (inverse) proportionality relation here.

The idea is visualized in the illustration below (for which credit goes to the modern *zbw* theorists Celani et al.), which depicts an *accelerating* electron: the *radius* of the circulatory motion must effectively diminish as the electron gains speed. Once again, however, we should warn the reader that he or she should also imagine the plane of oscillation to be possibly parallel to the direction of propagation, in which case the circular motion becomes elliptical.

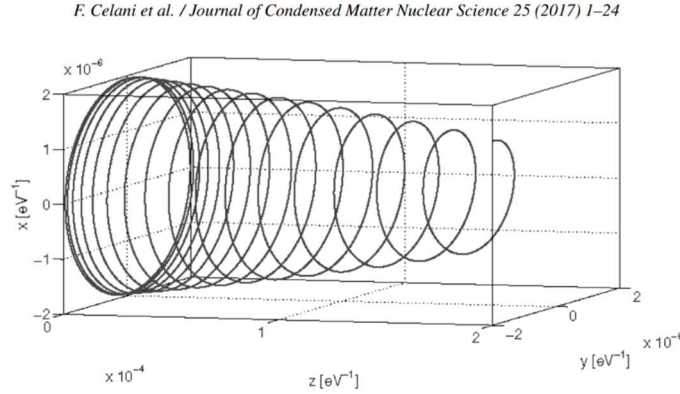


Figure 7: The Compton radius must decrease with increasing velocity

Can the velocity go to c ? This is where the analysis for an electron – or any other matter-particle – and for a photon part ways. In the *zbw* model, we have a rest energy which is explained by the *Zitterbewegung* of a pointlike electric charge. Hence, relativity tells us we can never accelerate it to the speed of light, because its mass – a measure for inertia to movement – becomes infinite.

In contrast, we have no such constraint for a photon. In fact, it does not have any rest energy (or rest mass). All of its energy is in its motion. What happens to the argument of the wavefunction? If we still think of the photon just like we would think of a particle – i.e. in terms of it being at some specific point in space at some specific point in time – then the argument vanishes:

$$\theta = \frac{1}{\hbar} (\mathbf{E} \cdot \mathbf{t} - \mathbf{p} \cdot \mathbf{x}) = \frac{1}{\hbar} (\mathbf{pc} \cdot \mathbf{t} - \mathbf{p} \cdot \mathbf{ct}) = 0$$

What does this tell us? In our humble view, this tells us that we should *not* think of a photon as we think of a particle. If we want to associate a wavefunction with a photon, then we should not think of it as an implicit function, but as a proper function, i.e. a function from some *domain* $(\Delta \mathbf{x}, \Delta t)$ to an associated *range* of values $a \cdot e^{i\theta}$. We can then use the superposition principle to shape it anyway we would want to shape it, and we should probably think of some transient here, rather than a nice symmetrical wave packet.

Jean Louis Van Belle, 25 November 2018

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to

1. Feynman's *Lectures on Physics* (<http://www.feynmanlectures.caltech.edu>). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Celani et al., because they single-handedly provide most of the relevant material to work on here:

2. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.
3. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, <https://arxiv.org/pdf/0802.2728.pdf>.
4. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

In addition, it is always useful to read an original:

5. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, <https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf>

The illustrations in this paper are open source or – in one or two case – have been created by the author. References and credits – including credits for open-source Wikipedia authors – have been added in the text.

One reference that has not been mentioned in the text is:

6. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

The latter work is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: “Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research.” (p. 1-10)

Last but not least, we pre-published a series of more speculative arguments on the viXra.org site. See: http://vixra.org/author/jean_louis_van_belle. The author intends to further develop these in future articles.