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The reflection symmetry is a physical property for inertial reference frames. It shows that the elapsed time in one inertial reference frame is identical to the elapsed time in another inertial reference frame. The same symmetry also leads to the conservation of the wavelength across inertial reference frames. The velocity of a wave is proportional to its frequency. Doppler effect shows that both the velocity and the frequency depend on the reference frame. The higher the detected frequency is, the faster the wave travels toward the detector. One example is the radar gun used by the traffic police. The reflected radio wave travels faster than the emitted radio wave. This results in frequency difference between two waves. This difference is used to calculate the velocity of a vehicle.

I. INTRODUCTION

The concept of symmetry is critical to physics because it leads to conservation law and conserved quantity. The reflection symmetry for inertial reference frame is particularly important for its conserved quantity across inertial reference frames. The conserved quantity keeps the same value in any inertial reference frame. One example is the wavelength of a wave. Another example is the elapsed time in an inertial reference frame.

The reflection symmetry for inertial reference frame allows both the velocity and the distance to be determined accurately. From the definition of the velocity, the elapsed time in an inertial reference frame can be calculated precisely. From the elapsed time and the velocity for a wave, the wavelength can also be calculated.

With the velocity of the wave determined in every inertial reference frame, the Doppler effect demonstrates precisely the dependency of frequency on velocity. The greater the velocity, the higher the frequency.

II. PROOF

Consider one dimensional motion.

A. Time From Reflection Symmetry

The reflection symmetry exists for an isolated system of two persons.

Let a person P_1 be stationary at the origin of a reference frame F_1 . Let another person P_2 be at a position x in F_1 .

Let the rest frame of P_2 be F_2 . P_2 is stationary at the origin of F_2 . From the relative reflection symmetry, P_1 is at the position of -x in F_2 .

Let F_2 move at the speed of v relative to F_1 . From the relative reflection symmetry, F_1 is moving at the speed of -v relative to F_2 .

Let t_1 be the time of F_1 . P_2 moves at the speed of v

in F_1 . This motion can be described as,

$$\frac{dx}{dt_1} = v \tag{1}$$

Let t_2 be the time of F_2 . P_1 moves at the speed of -v in F_2 . This motion can be described as,

$$\frac{d(-x)}{dt_2} = -v \tag{2}$$

From equations (1,2),

$$dt_1 = dt_2 \tag{3}$$

The elapsed time is conserved in both F_1 and F_2 .

$$t_1 = t_2 + A \tag{4}$$

The time of F_1 differs from the time of F_2 by a constant A which can be set to zero or any value by the initial condition.

From equation (3), if dt_1 is zero then dt_2 is also zero. Two simultaneous events in one inertial reference frame are also simultaneous in another inertial reference frame.

B. Wavelength From Reflection Symmetry

A stationary wave has zero frequency to a stationary observer because the wave signal to the observer remains constant. The frequency increases if the observer moves. This is the apparent frequency of a stationary wave detected by a moving observer.

Let a wave W_1 be stationary relative to a reference frame F_1 . Let an observer P_1 move at a speed of v relative to F_1 . W_1 can be represented by

$$W_1 = \sin(k_1 x_1) \tag{5}$$

The apparent frequency of W_1 detected by P_1 in F_1 is f_1 .

$$f_1 = \frac{v}{\frac{2\pi}{k_1}} = \frac{v * k_1}{2\pi} \tag{6}$$

Let the rest frame of P_1 be F_2 . W_1 is represented by a traveling wave W_2 in F_2 . W_2 travels at the speed of -v relative to P_1 .

$$W_2 = \sin(k_2 x_2 + w_2 t_2) \tag{7}$$

The frequency of W_2 detected by P_1 in F_2 is f_2 .

$$v = \frac{w_2}{k_2} = \frac{2\pi * f_2}{k_2} \tag{8}$$

From equation (3), both period and frequency are independent of inertial reference frame.

$$f_1 = f_2 \tag{9}$$

From equation (6,8,9),

$$k_1 = k_2 \tag{10}$$

The wavelength is independent of inertial reference frame.

C. Doppler Effect

In 1842, Christian Doppler discovered that the frequency of a wave depends on the motion of the source and the motion of the detector[1].

Let W_1 be a wave traveling at the speed of c relative to F_1 . Let P_1 move at a speed of v relative to F_1 .

$$W_1 = \sin(k_1 x_1 - w_1 t_1) \tag{11}$$

$$c = \frac{w_1}{k_1} \tag{12}$$

Let F_2 be the rest frame of P_1 . W_1 is represented by W_2 in F_2 .

$$W_2 = \sin(k_2 x_2 + w_2 t_2) \tag{13}$$

From equation (10,13),

$$W_2 = \sin(k_1 x_2 + w_2 t_2) \tag{14}$$

The apparent frequency of W_1 detected by P_1 in F_1 is f_1 . f_1 can be calculated from the motion of two adjacent crests. Let the first crest be at a distance of L from P_1 . The distance between the second crest and P_1 is $L+\lambda$. λ is the wavelength of W_1 .

Let the time for the first crest to reach P_1 be t_a . The distance for the first crest to travel to P_1 is,

$$c * t_a = L + v * t_a \tag{15}$$

Let the time for the second crest to reach P_1 be t_b . The distance for the second crest to travel to P_1 is,

$$c * t_b = L + \lambda + v * t_b \tag{16}$$

The apparent period of W_1 detected by P_1 in F_1 is T.

$$T = t_b - t_a \tag{17}$$

From equation (15, 16, 17),

$$T = \frac{\lambda}{c - v} \tag{18}$$

The detected frequency in F_1 is

$$f_1 = \frac{1}{T} = \frac{c - v}{\lambda} = k_1 \frac{c - v}{2\pi} = \frac{w_1}{2\pi} \frac{c - v}{c}$$
(19)

This is the Doppler effect. The detected frequency (f_1) depends on the motion of the observer (v).

From equation (9), the frequency is independent of inertial reference frame. P_1 detects the same frequency f_2 in both F_1 and F_2 . Therefore, the speed of W_2 in F_2 can be determined from equations (9,10,12,19).

$$f_2 * \lambda = f_1 * \lambda = f_1 * \frac{2\pi}{k_1} = c - v$$
 (20)

The speed of a wave depends on the motion of the observer. As v changes with time, the speed of a wave in the rest frame of the observer also changes with time.

D. Velocity and Reference Frame

Let $\vec{C_1}$ be the velocity of a radio wave in a reference frame F_1 . Let F_2 move at velocity \vec{v} relative to F_1 . Let $\vec{C_2}$ be the velocity of the same radio wave in F_2 .

From equation (20),

$$\vec{C}_2 = \vec{C}_1 - \vec{v} \tag{21}$$

Let a reflective surface within the y-z plane be stationary relative to F_1 . The incident wave travels along the x-axis toward the reflective plane and is reflected back along the x-axis.

Let \vec{C}_1^i be the velocity of the incident wave in F_1 . Let \vec{C}_1^r be the velocity of the reflected wave in F_1 . From Fresnel's equations[2],

$$\vec{C_1^i} = -\vec{C_1^r}$$
 (22)

$$C_1^i = C_1^r \tag{23}$$

Let $\vec{C_2^i}$ be the velocity of the incident wave in F_2 . Let $\vec{C_2^r}$ be the velocity of the reflected wave in F_2 . From equation (21),

$$\vec{C}_2^i = \vec{C}_1^i - \vec{v} \tag{24}$$

$$\vec{C}_2^r = \vec{C}_1^r - \vec{v} \tag{25}$$

From equations (22,24,25),

$$-\vec{C}_2^r = \vec{C}_2^i + 2\vec{v} \tag{26}$$

The speeds of both waves can be determined from equation (26) based on the direction of \vec{v} .

If both \vec{C}_2^i and \vec{v} point to the same direction,

$$C_2^r = C_2^i + 2v (27)$$

If both $\vec{C_2^r}$ and \vec{v} point to the same direction,

$$C_2^i = C_2^r + 2v (28)$$

The reflected wave travels at a different speed from the incident wave if the reflective plane is in motion.

E. Doppler Radar

Radar gun is used by the traffic police to measure the speed of an approaching car. It demonstrates how the detected frequency depends on the reference frame.

The equation from Doppler effect to calculate the frequency of the radar wave in radar gun[3,4] is

$$f_r - f_i = 2v \frac{f_i}{c} \tag{29}$$

 f_r is the frequency of the reflected radar wave. f_i is the frequency of the incident radar wave. v is the speed of the car.

From equation (27),

$$\frac{C_2^r}{\lambda_r} = \frac{C_2^i + 2v}{\lambda_i} \tag{30}$$

$$f_2^r = f_2^i + \frac{2v}{\lambda_i} \tag{31}$$

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$$f_2^r - f_2^i = 2v \frac{1}{\lambda_i} \tag{32}$$

This is exactly the formula used by radar gun in equation (29). Therefore Doppler radar provides an excellent experimental verification that the radio wave accelerates upon reflection by an approaching car.

III. CONCLUSION

The reflection symmetry leads to the invariant wavelength across inertial reference frame. The wavelength of a wave is independent of inertial reference frames.

The symmetry also leads to the conservation of the elapsed time. As a result, the time in an inertial reference frame differs from the time in another inertial reference frame by a constant. This constant can be initialized by any preset condition.

The velocity of a wave depends on the reference frame because the wavelength is invariant. The higher the detected frequency is, the faster the wave travels. This property applies to radio wave, visible light and mechanical wave. Consequently, any wave can be accelerated by a non-inertial reference frame.

One good example is the radar gun. The emitted radio wave is accelerated by the moving vehecle. As a result, the frequency of the return signal increases. This increase in frequency is used to calculate the velocity of that moving vehecle precisely.

The invariant elapsed time across the inertial reference frames also indicates that two simultaneous events are always simultaneous in all inertial reference frames.

This indicates that the time dilation from Lorentz transformation[5] is impossible for the real world because it violates the reflection symmetry in physics.

Lorentz transformation is the foundation of the theory of Special Relativity[6]. As a result, all predictions from this theory are incorrect in physics because of Lorentz transformation.

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