AN ALGEBRAIC WAY OF SIMULTANEOUSLY ANALYZING BOTH EINSTEIN, PODOLSKY AND ROSEN PAPER AND BOHR'S REPLY TO IT

N. GURAPPA

Velammal College, Chennai-600066,
Tamil Nadu, India.
dr.n.gurappa@gmail.com

In their celebrated paper titled "Can quantum mechanical description of physical reality be considered complete?", Einstein, Podolsky and Rosen (EPR) showed for the first time the existence of 'Spooky action-at-a-distance'. Though the result of their paper is unquestionable, but the conclusion of the same became sensational because of its challenge to quantum mechanical formalism whether it's complete or not in describing the physical reality of Nature. Bohr's physical and philosophical reply to that conclusion justified the completeness of quantum mechanics. Here, a simple algebraic way is presented for the results of these two classic papers in such a way that the actual reason behind why quantum world necessarily exhibits the action-at-a-distance and how Bohr defended against the incompleteness of the quantum formalism will become clear.This approach naturally reveals what physical assumption of EPR went wrong while considering the entangled quantum system and also provides the missing mathematical argument in Bohr's reply.

Keywords: EPR Paradox; Quantum Entanglement; Spooky action-at-distance.

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1. Introduction

Quantum entanglement is a natural consequence of the quantum formalism whose existence in Nature is experimentally confirmed [1,2,3,4,5,6,7,8], thanks to Bell's inequalities [9]. Nature is indeed quantum mechanically spooky. Any measurement on a remote quantum particle has an instantaneous influence on its entangled partner which can, in principle, be separated to a distance as much as the known dimensions of the Universe. This present paper is an algebraic effort to harmoniously mix and analyze the essence of both Einstein, Podolsky and Rosen (EPR) paper [10,11] and Bohr's reply [12,13,11] to it and hence to pin-down the necessity for the Nature to maintain such a quantum entanglement or spooky action-at-a-distance as one of Her fundamental requirements.

Consider the crucial extracts from page:140 of EPR paper [10], "*We see therefore that, as a consequence of two different measurements performed upon the first sys-*

tem, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems. Thus, it is possible to assign two different wave functions to the same reality (the second system after the interaction with the first)" and from Bohr's short note [12], "*It is true that in the measurements under consideration any direct mechanical interaction of the system and the measuring agencies is excluded, but a closer examination reveals that the procedure of measurements has an essential influence on the conditions on which the very definition of the physical quantities in question rests*". Aim of the present paper is to distill out the physically relevant essence of these two extracts algebraically.

In quantum mechanics, Heisenberg's uncertainty relation states that the position and momentum of a quantum particle can not be precisely and simultaneously measured which is a consequence of the canonical commutation relation $[\hat{x}, \hat{p}] = i\hbar$; where, $\hat{x}$ is the position operator, $\hat{p}$ is the momentum operator, $i = \sqrt{-1}$ and $\hbar$ is the reduced Plank's constant, respectively. In fact, this commutation relation is the very basis for the entire quantum formalism. The main purpose of EPR argument was to show that in the case of two entangled particles, it becomes possible to simultaneously measure both the position and momentum of both the particles which invalidates the correctness of uncertainty relation and hence the corresponding basic commutation relation itself. Hence, they concluded by questioning the completeness of quantum mechanics in describing the reality of Nature. The present analysis exposes what classical physics assumption of EPR will not hold to be true in the case of quantum mechanics as pointed out by Bohr. Also, the missing mathematical argument for Bohr's reply is provided. A simple proof is given to show that the conserved quantities are responsible for the existence of the spooky action in the quantum world.

2. Conserved quantities and the Spooky action-at-a-distance

Consider the EPR case of two entangled quantum particles in one-dimension. They had interacted initially at some position, x_0, for a very brief time (like the case of elastic collision) and became entangled. When they are separated to a large distance, then it is assumed that there are no more physically known interactions acting between them.

Now, make the position measurement of particle-1 accurately. Hence, the outcome of its momentum measurement becomes uncertain. On the particle-2, perform the momentum measurement and hence its position gains a huge uncertainty. This is in perfect agreement with the position-momentum commutation relations as given below:

$$[\hat{x}_1, \hat{p}_1] = [\hat{x}_2, \hat{p}_2] = i\hbar \tag{1}$$

where, $\hat{x}_1$ and $\hat{x}_2$ are position operators and $\hat{p}_1$ and $\hat{p}_2$ are momentum operators of particle-1 and particle-2, respectively.

Now, according to the EPR assumption, the two particles were treated as independent systems like in the case of classical physics because at the time of measurement they were no longer interacting with each other by any known physical interactions. It implies the following commutation relations:

$$[\hat{x}_1, \hat{p}_2] = [\hat{x}_2, \hat{p}_1] = 0 \tag{2}$$

It can be easily seen that the relative position operator, $\hat{x}_0 = \hat{x}_1 - \hat{x}_2$, corresponding to the distance of separation between the two particles and the total momentum operator, $\hat{P}_{cm} = \hat{p}_1 + \hat{p}_2$, corresponding to the center-of-mass motion of the combined two particle system commute with each other,

$$[\hat{x}_0, \hat{P}_{cm}] = [\hat{x}_1 - \hat{x}_2, \hat{p}_1 + \hat{p}_2] = [\hat{x}_1, \hat{p}_1] - [\hat{x}_2, \hat{p}_2] = 0 \tag{3}$$

and therefore, they can be measured accurately and simultaneously.

So, from the accurate knowledge of x_1, p_2, x_0 and P_{cm}, one can know x_2 and p_1 as accurately as desired, which is a violation of the uncertainty principle. Even in this case, the validity of uncertainty relation requires that somehow the measurement of x_1 should not allow for the accurate measurement of p_2. It means that the measurement of x_1 must have an influence on the other particle's outcome. But we have $[\hat{x}_1, \hat{p}_2] = 0$ implying that it's possible to simultaneously and accurately measure both x_1 and p_2 since their corresponding operators commute with each other. Therefore, $x_1(x_2)$ and $p_2(p_1)$ can be measured accurately which, in turn, is the violation of the original commutation relations $[\hat{x}_1, \hat{p}_1] = i\hbar$ and $[\hat{x}_2, \hat{p}_2] = i\hbar$. This violation can not be acceptable as mentioned earlier. To circumvent this situation, it becomes necessary to accept that x_1 measurement necessarily influences the distant out come, p_2, even though the particles are no more interacting. Einstein called this kind of influence as spooky because such an action-at-a-distance is not at all visible at a mere glance at the quantum formalism.

The above equations (1), (2) and (3) were considered by Bohr in the footnote of his reply to the EPR paper [13]. But actually he missed out the following observation needed to explain the EPR correlations:

In the case of entangled particles, $\hat{x}_1$ and $\hat{p}_2$ and $\hat{x}_2$ and $\hat{p}_1$ should not commute with each other respectively, in order to maintain the original commutation relations, $[\hat{x}_1, \hat{p}_1] = [\hat{x}_2, \hat{p}_2] = i\hbar$ and also the law of total momentum conservation. Since, the center-of-mass and relative degrees of freedom are always independent of each other, one has

$$\begin{aligned}
[\hat{x}_0, \hat{P}_{cm}] &= 0 \\
&= [\hat{x}_1 - \hat{x}_2, \hat{p}_1 + \hat{p}_2] \\
&= [\hat{x}_1, \hat{p}_1] - [\hat{x}_2, \hat{p}_2] + [\hat{x}_1, \hat{p}_2] - [\hat{x}_2, \hat{p}_1] \\
&= [\hat{x}_1, \hat{p}_2] - [\hat{x}_2, \hat{p}_1]
\end{aligned} \tag{4}$$

Therefore, the commutators $[\hat{x}_1, \hat{p}_2]$ and $[\hat{x}_2, \hat{p}_1]$ need not be equal to zero independently, but they should always be equal to each other i.e.,$[\hat{x}_1, \hat{p}_2] = [\hat{x}_2, \hat{p}_1]$. Now, it's easy to see that the EPR assumption about the well-separated entangled particles as two independent and mutually non-influencing systems will not hold to be true within the quantum formalism and that was precisely the whole point Bohr explained with suitable physical examples.

Without loss of generality, by taking the total conserved momentum $\hat{P}_{cm}$ to be equal to zero,

$$\hat{p}_1 + \hat{p}_2 = 0 \tag{5}$$

and finding the commutator with $\hat{x}_1$ and $\hat{x}_2$ respectively, one has

$$[\hat{x}_1, \hat{p}_1 + \hat{p}_2] = [\hat{x}_1, \hat{p}_1] + [\hat{x}_1, \hat{p}_2] = 0 \tag{6}$$

and

$$[\hat{x}_2, \hat{p}_1 + \hat{p}_2] = [\hat{x}_2, \hat{p}_1] + [\hat{x}_2, \hat{p}_2] = 0 \tag{7}$$

which yield,

$$[\hat{p}_1, \hat{x}_2] = [\hat{p}_2, \hat{x}_1] = i\hbar \ . \tag{8}$$

In Eq. (8), if $[\hat{p}_1, \hat{x}_2] = [\hat{p}_2, \hat{x}_1] = 0$ as anticipated by EPR and accepted by Bohr in his reply, then it implies from Eq. (6) and Eq. (7) that either $[\hat{x}_1, \hat{p}_1] = [\hat{x}_2, \hat{p}_2] = 0$ or the law of conservation of total momentum is not valid in the quantum world. But, both the conclusions are physically unacceptable. Therefore, in the case of entangled particles, $\hat{x}_1$ will not commute with $\hat{p}_2$ and as well $\hat{x}_2$ with $\hat{p}_1$, like the case of unentangled free particles in such a way that the actual commutation relations, $[\hat{x}_1, \hat{p}_1] = [\hat{x}_2, \hat{p}_2] = i\hbar$ and the law of conservation of momentum were unaffected in the quantum mechanical description of the Nature, leaving no room to claim that quantum mechanics is incomplete. So, this simple explanation is sufficient to show that quantum mechanics is spooky. The entire Bohr's reply to EPR paper revolves around the explanation of the commutation relations $[\hat{p}_1, \hat{x}_2] = [\hat{p}_2, \hat{x}_1] = i\hbar$, which he did not explicitly consider, but only with some suitable experimental situations.

3. Summary

I algebraically studied the crucial results of both EPR paper and Bohr's reply to it. The EPR idea of treating two distant particles, which had interacted initially and no more interacting by any known physical mechanism, as two independent systems like in the case of classical physics can not hold to be true in the quantum world as pointed out by Bohr. This aspect was elegantly shown with the use of simple commutation relations and also, the missing mathematical argument for Bohr's reply is provided. In this approach, it becomes extremely transparent that even the quantum world, whose physical phenomena are based on the canonical quantum commutation relations, is bound to obey the conservation laws which naturally give

raise to the existence of Einstein's spooky action-at-a-distance among the entangled particles.

To visualize physically 'How entangled particles are able to communicate even though they are well-separated? (or) How actually the spooky-action-at-a-distance is becoming possible?, will necessarily depend on the physical reality of the Schrödinger wave function. In a recent paper, I gave a new non-dualistic interpretation for the quantum formalism where the Schrödinger wave function is interpreted as an 'instantaneous resonant spatial mode' [14]. This new interpretation unambiguously resolves the well-known paradoxes and puzzles in quantum mechanics at a single quantum level and provides a natural mechanism for the Einstein's spooky action-at-a-distance. Nature seems to be perfectly stubborn not to violate the conservation laws. Certainly, this instantaneous spooky action is not carried out by some physical carriers which require the exchange of energy and momentum between entangled particles because, we know that energy-momentum transport can't be superluminal and is bound to the Cosmic speed limit in accordance with the special theory of relativity. The main purpose for the existence of the spooky action in the quantum world seems to maintain strictly the conservation laws even in the absence of exchange interactions.

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