The Basque Language and the Graphical Law

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Abstract

We study the Basque to English etymological dictionary of late R. L. Trask. We draw the natural logarithm of the number of words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised as well as unnormalised. We find that the words underlie a magnetisation curve of a Spin-Glass in presence of little external magnetic field.

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I. INTRODUCTION

"Gekhane dekhibe chhai,
uraiya dekho tai,
paile paite paro,
parosho ratan."
——Bengali proverb.

When the author was a student in class eight, he got a class-mate by the nickname "buru". Since then, he was wondering where from the word orginated. The road ended when he stumbled on the "Etymological Dictionary of Basque",[1], written by the late R. L. Trask. "Buru" is a two thousand years old Basque word, meaning headman. The dictionary is a treasure trove. Sirname of a colleague of the author is Askari. "Askari" in the Basque language means lunch or, tea. We often overhear the word "zaldibaji". In the Basque language "zaldi" means horse. Interestingly, in the Basque language, "Bilarri" means ear, "biki" means twin or, pair of twins, "bihar" means tomorrow, "biru" means thread or, fibre, "giro" is atmosphere, "jagon" is guard, "kabra" stands for a spiny red fish, "kopar" means "basin", "handi" is big, "lur" is earth, "patar" is hard liquor. Surprisingly, "judu" refers to Jew.

Basque people, apparently, originated from Aquitanian tribes populating the coast of South of France and North of Spain. Luis Michelena chronicled the Basque language,[1]. Developing on Luis Michelena, the late R. L. Trask constructed the dictionary of the Basque language.

Basque is a subject-Object-Verb language. It does not have grammatical gender or, noun classification. According to L. Michelena, there are nine dialects of Basque: Bizkaian, Gipuzkoan, High Navarese, Aezkoan, Salazarese, Ronclaese, Lapurdian, Low Navarese and Zuberoan. The language has five vowels: i, e, a, o,u. The consonants are split in fortis: (p), t, k, tz, ts, N, L, P and lenis: b, d, g, z, s, n, l, r. There are five diphthongs: ai, ei, oi, au, eu[1]. The Basque alphabet is composed of twenty two letters.

Name of a railway station in the Bardhaman district of West Bengal is Guskara. Guskara is place predominantly inhabited by farming people. If we remove the "G" from the word, we come across Uskara or, Euskara, which happens to be another name of the Basque language.

"Mihidana" is a famous sweet from Bardhaman. Mihi is as well a Basque language word meaning tongue. A block in Sundarban goes by the name Gosaba. If we remove G from the front, we obtain "osaba" which is a Basque word meaning stepfather. Tamal is a bengali name, which in the Basque language means misfortune. Makal is a deregatory epithet in the bengali language, in the Basque language it stands for weak. Obi is a bengali name, in the Basque language it means cavity. "durduri" in both the bengali and Basque languages means resteless or, nervous. "esan" is a bengali word representing south-west, in the Basque language it means south. "faltsu" in the Basque language means false, in the bengali language "faltu" means also false. "garai" is a surname in Bardhaman. In the Basque language, though it is rare in use means high. Road to Orissa used to be through Bardhaman. The Oriya language has lot of similarities with the bengali language hundred and fifty years back. Two common words in Oriya are "toki" meaning girl and "baina" meaning brother. In the Basque language "toki" means place, "baina" means but. A prominent network of seafaring businessmen mostly hailing from Orissa coast, in the medieval era was "Sadhavas".

Hence, we ponder on whether the Basque language has something to do with the bengali language. Both are Subject-Object-Verb languages. Speakers of both the languages are divided into two countries, united by sea. Bengalis and basques are passionate about football, food,art. Both have the history of seafaring. Both the languages bear the influence of non-native speakers, willy-nilly.

As we go along this article, we will find that the same kind of magnetisation curve(s) underlie both the languages to some extent.

In preliminary study, [2], the present author took a trip to probe into the word (and verb,adverb,adjective) contents along the letters in a language. The letters were arranged in ascending order of their ranks from the rank one. The letter with the highest number of words starting with, was taken as of rank one. For a natural language, a dictionary from it to English, was a natural choice for that type of study. The author has found that behind each language which was subjected to investigation, there is a curve of magnetisation. From that the author has conjectured that behind any written natural language there are curves of magnetisation, for words, verbs, adverbs and adjectives respectively. The graphical law was found also to exist in the contemporary chinese usages, [2]. Moreover, the curve drawn for the Arabian language was found to be very close to Onsager exact solution i.e. reduced magnetisation vs. reduced temperature, of two dimensional Ising model,[3]. We have not studied Hebrew or, Persian. We hope, Hebrew or, Persian like Arabian will also be close to Onsager solution. It happened so that Ising, was a Jew, [4], who investigated the model by his name but due to his advisor Lenz.

Moreover, we looked into, [5], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of graphical law behind bengali language,[6]. This was pursued by finding of graphical law behind Romanian, [7], five more disciplines of knowledge, [8], Onsager core of Abor-Miri, Mising languages,[9] and Onsager Core of Romanised Bengali language,[10].

We describe how a graphical law is hidden within in the Basque language, in this article. We organise the paper as follows. We explain our method of study in the section IV after giving an introduction to magnetisation and the the standard curves of magnetisation of Ising model in the sections II and III respectively. In the ensuing section, section V, we narrate our graphical results. We describe how natural logarithm of number of words arranged in descending order, normalised by different normalisers when plotted against the respective rank are fit with lines of magnetisations. Then we conclude about the existence of the graphical law. The section VI is Discussion. In that section we try to find out relationship of the Basque language with other languages on the basis of underlying magnetisation curves. We end up through acknowledgement section VII and bibliography.

II. MAGNETISATION

The two dimensional Ising model,[3], in absence of external magnetic field, is prototype of an Ising model. In case of square lattice of planar spins, one spin interacts with four other nearest neighbour spins i.e. on an average to another one spin. Below a certain ambient temperature, denoted as T_c, the two dimensional array of spins reduces to a planar magnet with magnetic moment per site varying as a function of $\frac{T}{T_c}$. This function was inferred, [4], by Lars Onsager way back in 1948, [11] and thoroughly deduced thereafter by C.N.Yang[12]. This function we are referring to as Onsager solution. Moreover, systems, [13], showing behaviour like Onsager solution is rare to come across. Graphically, the Onsager solution appears as in fig.1. In the Bragg-Williams and Bethe-Peierls approximations for an Ising model in any dimension, in (absence)presence of external magnetic fields, reduced magnetisation as a function of reduced temperature, below the phase transition temperature, T_c, vary as in the figures 2-4. The Bragg-Williams and Bethe-Peierls approximations are motivated below.

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like paramagnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins.

$L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[3], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [14], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_BT})$, [15]. In the Bragg-Williams approximation,[16], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_BT} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L+c}{\frac{T}{T_c}} \tag{1}$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [17]. $\frac{T}{T_c}$ is referred to as reduced temperature.
Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [14]. W. L. Bragg was a professor of Hans Bethe. Rudlof Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudlof Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [3],[14],[15],[16],[17], due to Bethe-Peierls, [18], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe datas generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those datas. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.2. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$				$\frac{M}{M_{max}}$,
BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE I. Datas for Reduced temperature[for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$ respectively and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature data set(say, data set BW(c=0)) is drawn along the x-axis and the corresponding Reduced magnetisation data set is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.[For example, for drawing BW(c=0), ".dat" file, say denoted as "0.dat", contains BW(c=0) data set in first column and reduced magnetisation data set in second column. Moreover, after (0.944,0.400), next pair of points will be (0.969,0.300), then (0.987,0.200), ...and so on in the "0.dat" file.]

C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [18], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c};factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula ala [18] is given in the appendix of [8].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c};factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe datas in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(4, $\beta H = 0.06$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4)' BP(4, $\beta H = 0.05$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(4), BP(4, $\beta H = 0.04$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4), BP(4, $\beta H = 0.02$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4), BP(4, $\beta H = 0.01$) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4), The data set is used to plot fig.3 and fig.4. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$							$\frac{M}{M_{max}}$,
BP(4,$\beta H = 0.1$)	BP(4,$\beta H = 0.08$)	BP(4,$\beta H = 0.06$)	BP(4,$\beta H = 0.05$)	BP(4,$\beta H = 0.04$)	BP(4,$\beta H = 0.02$)	BP(4,$\beta H = 0.01$)	reduced magnetisation
0	0	0	0	0	0	0	1
0.597	0.589	0.583	0.580	0.577	0.572	0.569	0.978
0.603	0.593	0.587	0.584	0.581	0.575	0.572	0.977
0.660	0.655	0.647	0.643	0.639	0.632	0.628	0.961
0.673	0.665	0.657	0.653	0.649	0.641	0.637	0.957
0.688	0.679	0.671	0.667		0.654	0.650	0.952
			0.716			0.696	0.931
0.745	0.734	0.723	0.718	0.713	0.702	0.697	0.927
0.766	0.754	0.743	0.737	0.731	0.720	0.714	0.917
0.787	0.775	0.762	0.756	0.749	0.737	0.731	0.907
0.796	0.783	0.770	0.764	0.757	0.745	0.738	0.903
0.848	0.832	0.816	0.808	0.800	0.785	0.778	0.869
0.854	0.837	0.821	0.813	0.805	0.789	0.782	0.865
0.866	0.849	0.832	0.823	0.815	0.799	0.791	0.856
0.878	0.859	0.841	0.833	0.824	0.807	0.799	0.847
0.902	0.882	0.863	0.853	0.844	0.826	0.817	0.828
0.931	0.908	0.887	0.876	0.866	0.846	0.836	0.805
0.940	0.917	0.895	0.884	0.873	0.852	0.842	0.796
0.966	0.941	0.916	0.904	0.892	0.869	0.858	0.772
0.996	0.968	0.940	0.926	0.914	0.888	0.876	0.740
1			0.929			0.877	0.735
	0.977		0.936			0.883	0.730
	0.989		0.944			0.889	0.720
	0.990		0.945				0.710
	1.00		0.955			0.897	0.700
			0.963			0.903	0.690
			0.973			0.910	0.680
						0.909	0.670
			0.993			0.925	0.650
				0.976	0.942		0.651
			1.00				0.640
				0.983	0.946	0.928	0.628
				1.00	0.963	0.943	0.592
					0.972	0.951	0.564
					0.990	0.967	0.527
						0.964	0.513
					1.00		0.500
						1.00	0.400
							0.300
							0.200
							0.100
							0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

D. Spin-Glass

In the case coupling between(among) the spins, not necessarily n.n, for the Ising model is(are) random, we get Spin-Glass. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like $\frac{1}{T-T_c}$ i.e. like the branch of rectangular hyperbola, upto the the phase transition temperature, followed by very little increase,[19–21], in magnetisation, as the ambient temperature continues to drop.

Theoretical study of Spin Glass started with the paper by Edwards, Anderson,[22]. They were trying to explain two experimental results concerning continuous disordered freezing(phase transition) and sharp cusp in static magnetic susceptibility. This was followed by a paper by Sherrington, Kickpatrick, [23], who dealt with Ising model with interactions being present among all neighbours. The interaction is random, follows Gaussian distribution and does not distinguish one pair of neighbours from another pair of neighbours, irrespective of the distance between two neighbours. In presence of external magnetic field, they predicted in their next paper, [24], below spin-glass transition temperature a spin-glass phase with non-zero magnetisation. Almeida etal, [25], Gray and Moore, [26],finally Parisi, [27], [28] improved and gave final touch, [29], to their line of work. Parisi and collaborators, [30]-[34], wrote a series of papers in postscript, all revolving around a consistent assumption of constant magnetisation in the spin-glass phase in presence of little constant external magnetic field.

In another sequence of theoretical work, by Fisher etal,[35–37], concluded that for Ising model with nearest neighbour or, short range interaction of random type spin-glass phase does not exist in presence of external magnetic field.

For recent series of experiments on spin-glass, the references, [38, 39], are the places to look into.

For an indepth account, accessible to a commonner, the series of articles by late P. W. Anderson in Physics Today, [40]-[46], is probably the best place to look into. For a book to enter into the subject of spin-glass, one may start at [47].

Here, in our work to follow, spin-glass refers to spin-glass phase of a system with infinite range random interactions.

III. CURVES OF MAGNETISATION

The Ising Hamiltonian,[3],[18],for a lattice of spins is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs, σ_i is i-th spin, H is external magnetic field and ϵ is coupling between two nearest neighbour spins. σ_i is binary i.e. can take values ±1. At a temperature T, below a certain temperature called phase transition temperature, T_c, for the two dimensional Ising model in absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [12], [18],

$$\frac{M}{M_{max}} = [1 - (sinh\frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

Graphically, the Onsager solution appears as in fig.1. In the Bragg-Williams and Bethe-

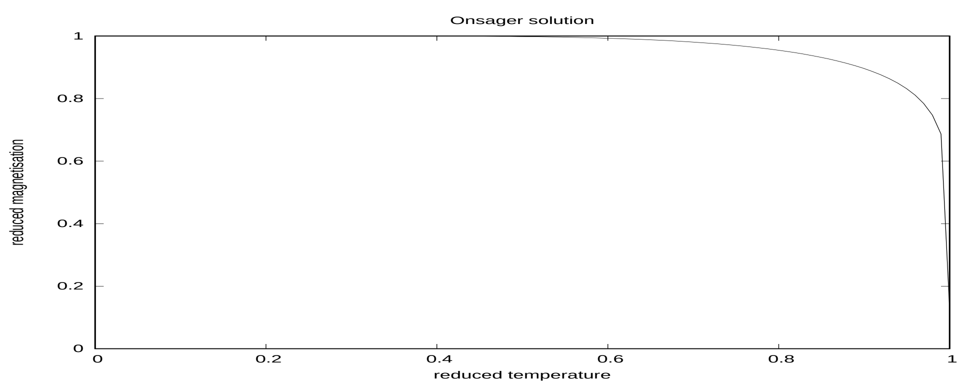

FIG. 1. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

Peierls approximations for an Ising model in any dimension, in presence of external magnetic fields, reduced magnetisation as a function of reduced temperature, below the phase transition temperature, T_c, vary as in the figures 3-5.

The graphs in the figures,1-4, are used in the sections to follow as reference curves.

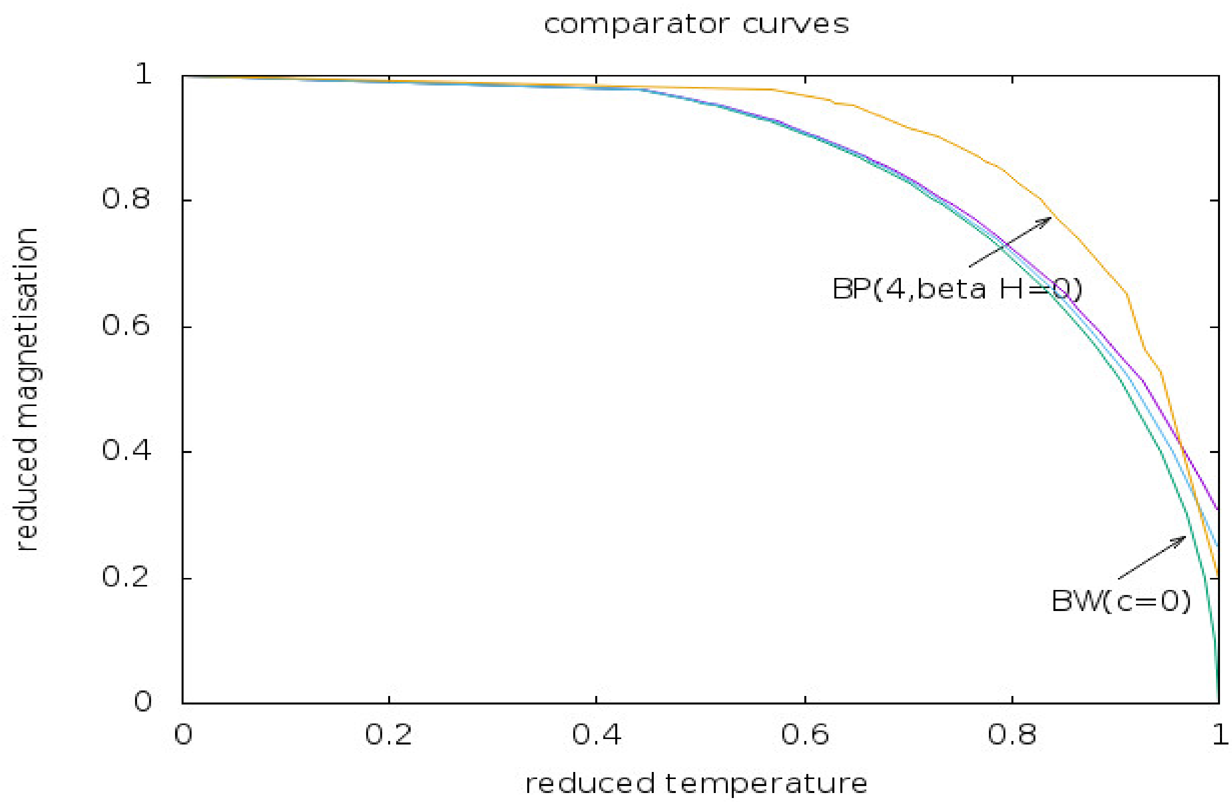

FIG. 2. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$, outwards; and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours (outer in the top).

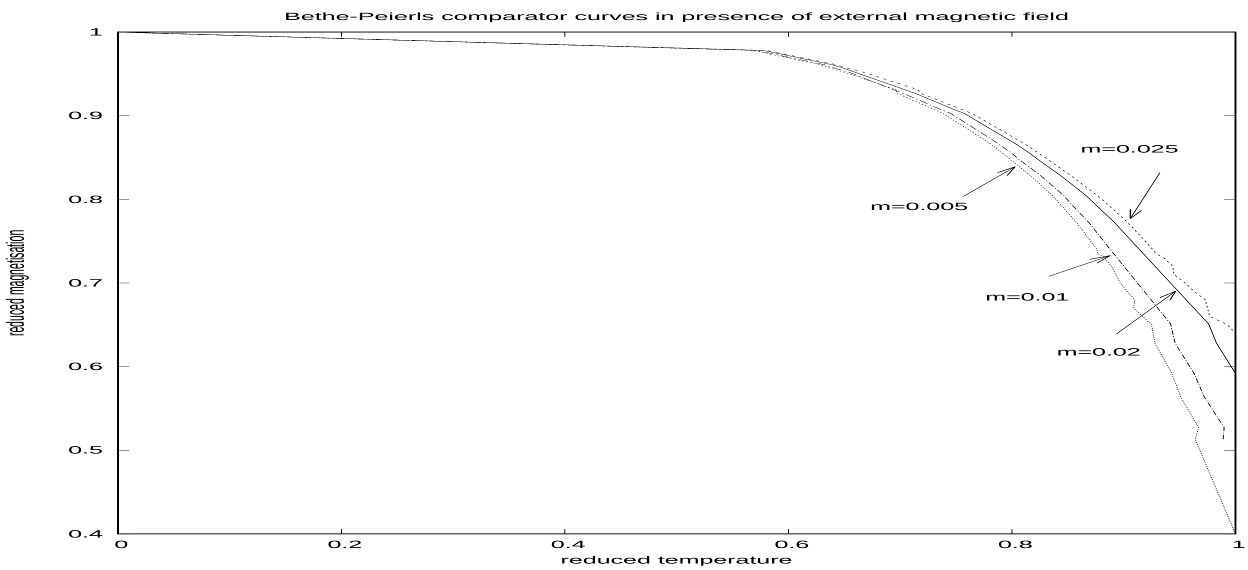

FIG. 3. Reduced magnetisation vs reduced temperature curves, BP(4,βH), for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

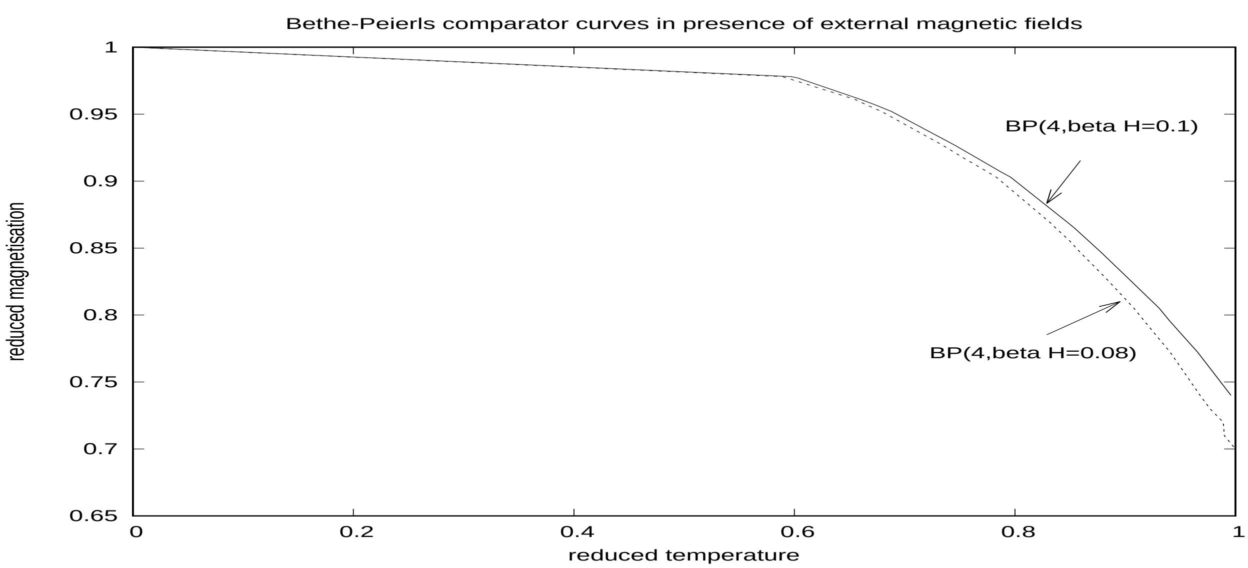

FIG. 4. Reduced magnetisation vs reduced temperature curves, BP(4,βH=0.1) and BP(4,βH=0.08).

letter	A	B	D	E	F	G	H	I	J	K	L	M	N	O	P	R	S	T	U	X	Y	Z
number	1506	885	1009	956	46	566	397	679	194	376	438	627	290	624	435	6	269	306	381	100	7	507
splitting	1445+61	824+61	979+30	804+152	26+20	509+57	6+391	562+117	167+27	260+116	409+29	566+61	270+20	587+37	357+78	1+5	239+30	193+113	259+122	17+83	1+6	458+49

TABLE III. basque words: the first row represents letters of the basque alphabet in the serial order, the second row is the respective number of words, the third row describes the splitting of words.

IV. METHOD OF STUDY

The Basque language alphabet is composed of twenty two letters. We take the Basque-English etymological dictionary,[1]. Then we count all the head-words, written in boldface in the dictionary, [1], one by one from the beginning to the end, starting with different letters. This has been done in two steps for the dictionary. First, we have counted all head-words in bold initiating with A form the section for the letter A. The number is one thousand four hundred forty five, Second, we have enlisted all head-words in bold initiating with A form the sections for the letters B, D,..,Z. Then we have removed from the list words already appearing in the section belonging to A. Then we have counted the number of the words in that list. The number is sixty one. As a result total number of words beginning with A is one thousand five hundred and six. This exercise was then followed for B,C,..Z. The result is the table, III. The largest number of words, one thousand five hundred and six to be specific, start with the letter "A". The next block of words numbering one thousand nine with the letter "D" as the initial, followed by block of words starting with "E" totalling nine hundred fifty six in number and so on. To visualise the pattern of change of number of words along the the letters initiating with, we draw the number of words of the blocks vs. sequence number of the respective letters in the fig.5.

To explore for the occurance of graphical law, we assort the letters according to the number of words, in the descending order, denoted by f and the respective rank, denoted by k. k is a positive integer starting from one. Moreover, we attach a limiting rank, k_{lim}, and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty three and the limiting number of words is one. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$. We then ignore the letters with the

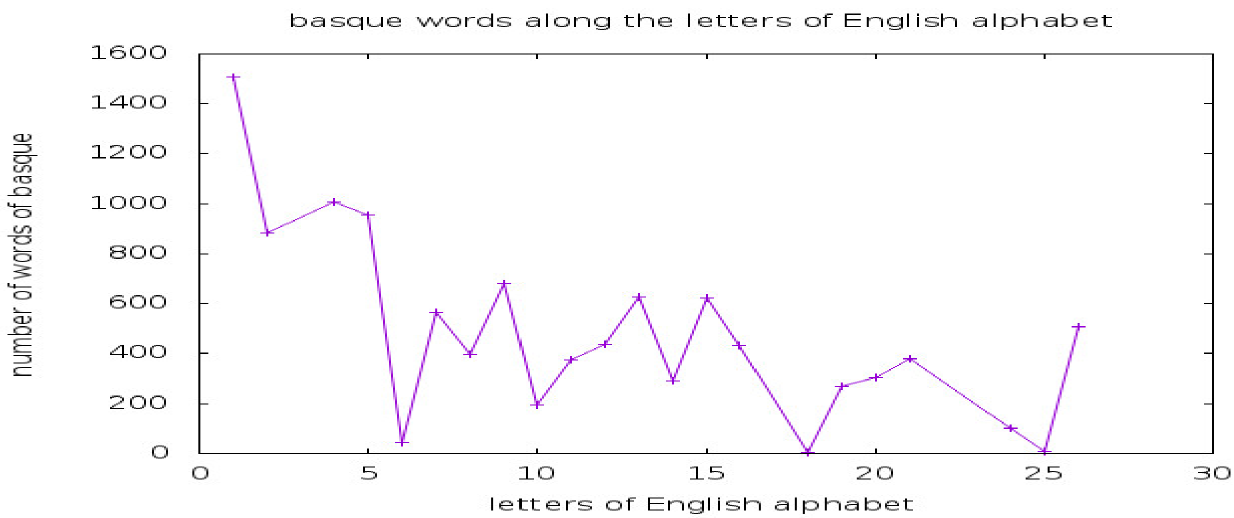

FIG. 5. Vertical axis is number of words of basque and horizontal axis is the respective letters of the English alphabet. Letters are represented by the number in the alphabet or, English dictionary sequence.

highest, then next highest, then next next highest and so on number of words and redo the plot, normalising the lnfs with next-to-maximum $lnf_{nextmax}$, and starting from $k = 2$; next-to-next-to-maximum $lnf_{nextnextmax}$, and starting from $k = 3$; next-to-next-to-next-to-maximum $lnf_{nextnextnextmax}$, and starting from $k = 4$, nnnnmax $lnf_{nnnnmax}$, and starting from $k = 5$, nnnnnmax $lnf_{nnnnnmax}$, and starting from $k = 6$, nnnnnnmax $lnf_{nnnnnnmax}$, and starting from $k = 7$, nnnnnnnnnnmax $lnf_{nnnnnnnnnnmax}$, and starting from $k = 11$. The results are the table IV and the figures (fig.5-fig.13) of the next section.

V. RESULTS

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{nmax}	lnf/lnf_{nnmax}	lnf/lnf_{nnnmax}	lnf/$lnf_{nnnnmax}$	lnf/$lnf_{nnnnnmax}$	lnf/$lnf_{nnnnnnmax}$	lnf/$lnf_{nnnnnnnnnnmax}$
1	0	0	1506	7.317	1	Blank	Blank	Blank	Blank	Blank	Blank	Blank
2	0.69	0.220	1009	6.917	0.945	1	Blank	Blank	Blank	Blank	Blank	Blank
3	1.10	0.350	956	6.863	0.938	0.992	1	Blank	Blank	Blank	Blank	Blank
4	1.39	0.443	885	6.786	0.927	0.981	0.989	1	Blank	Blank	Blank	Blank
5	1.61	0.513	679	6.521	0.891	0.943	0.950	0.961	1	Blank	Blank	Blank
6	1.79	0.570	627	6.441	0.880	0.931	0.939	0.949	0.988	1	Blank	Blank
7	1.95	0.621	624	6.436	0.880	0.930	0.938	0.948	0.987	0.999	1	Blank
8	2.08	0.662	566	6.339	0.866	0.916	0.924	0.934	0.972	0.984	0.985	Blank
9	2.20	0.701	507	6.229	0.851	0.901	0.908	0.918	0.955	0.967	0.968	Blank
10	2.30	0.732	438	6.082	0.831	0.879	0.886	0.896	0.933	0.944	0.945	Blank
11	2.40	0.764	435	6.075	0.830	0.878	0.885	0.895	0.932	0.943	0.944	1
12	2.48	0.790	397	5.984	0.818	0.865	0.872	0.882	0.918	0.929	0.930	0.985
13	2.56	0.815	381	5.943	0.812	0.859	0.866	0.876	0.911	0.923	0.923	0.978
14	2.64	0.841	376	5.930	0.810	0.857	0.864	0.874	0.909	0.921	0.921	0.976
15	2.71	0.863	306	5.724	0.782	0.828	0.834	0.844	0.878	0.889	0.889	0.942
16	2.77	0.882	290	5.670	0.775	0.820	0.826	0.836	0.869	0.880	0.881	0.933
17	2.83	0.901	269	5.595	0.765	0.809	0.815	0.824	0.858	0.869	0.869	0.921
18	2.89	0.920	194	5.268	0.720	0.762	0.768	0.776	0.808	0.818	0.819	0.867
19	2.94	0.936	100	4.605	0.629	0.666	0.671	0.679	0.706	0.715	0.716	0.758
20	3.00	0.955	46	3.829	0.523	0.554	0.558	0.564	0.587	0.594	0.595	0.630
21	3.04	0.968	7	1.946	0.266	0.281	0.284	0.287	0.298	0.302	0.302	0.320
22	3.09	0.984	6	1.792	0.245	0.259	0.261	0.264	0.275	0.278	0.278	0.295
23	3.14	1	1	0	0	0	0	0	0	0	0	0

TABLE IV. basque words: ranking,natural logarithm, normalisations

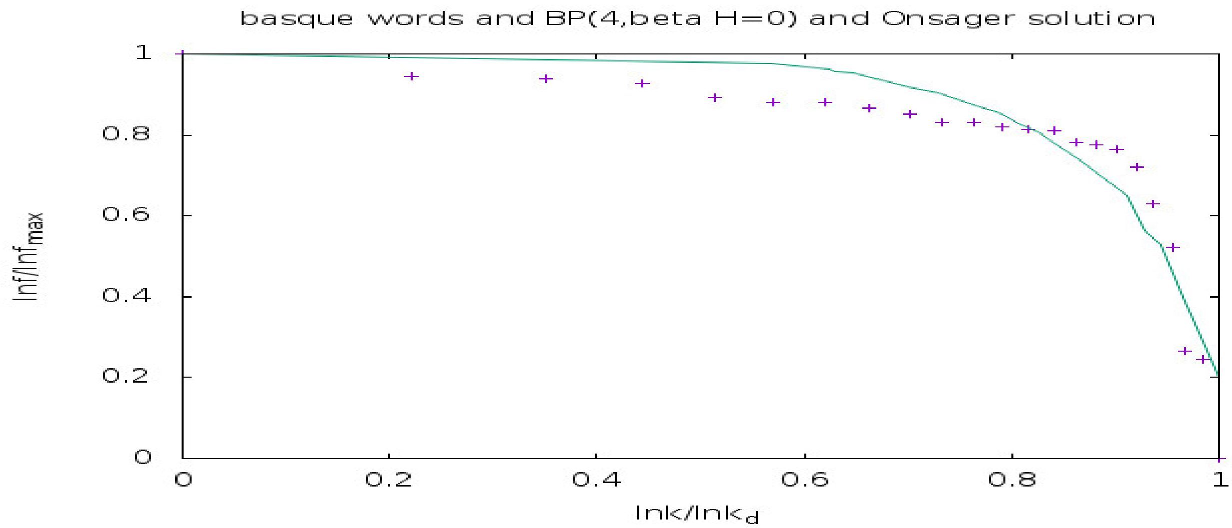

FIG. 6. Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and in absence of external magnetic field, $m = 0$ or $\beta H = 0$. The uppermost curve is the Onsager solution.

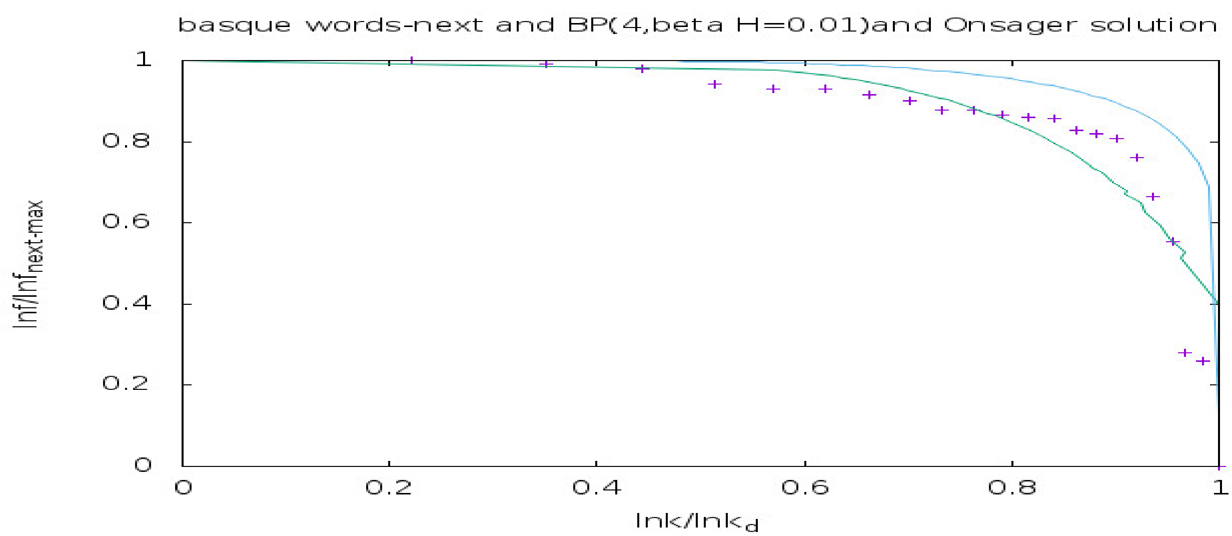

FIG. 7. Vertical axis is $\frac{lnf}{lnf_{next-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.005$ or, $\beta H = 0.01$. The uppermost curve is the Onsager solution.

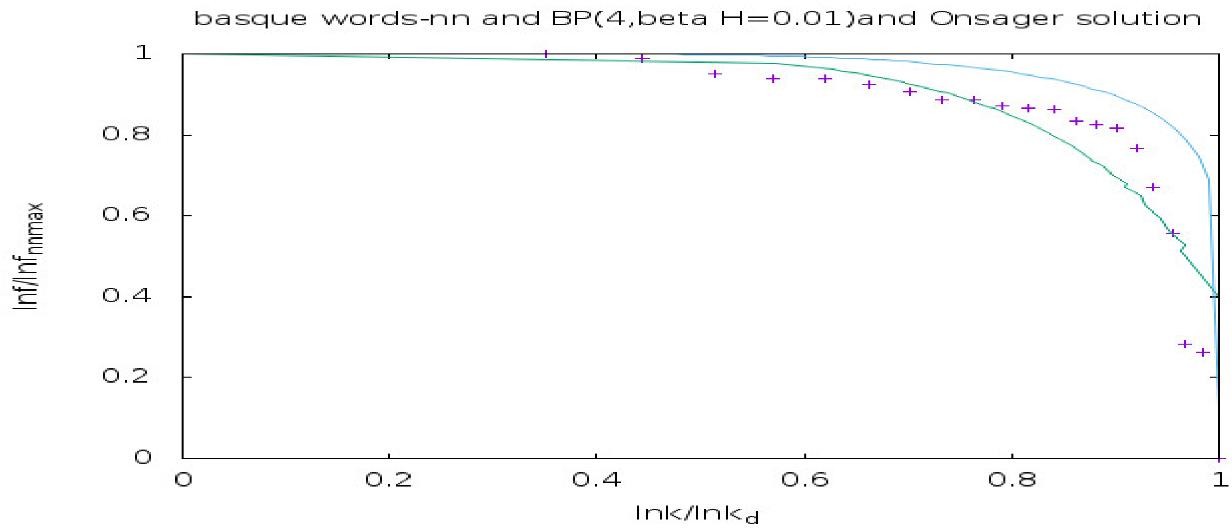

FIG. 8. Vertical axis is $\frac{lnf}{lnf_{nn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.005$ or, $\beta H = 0.01$. The uppermost curve is the Onsager solution.

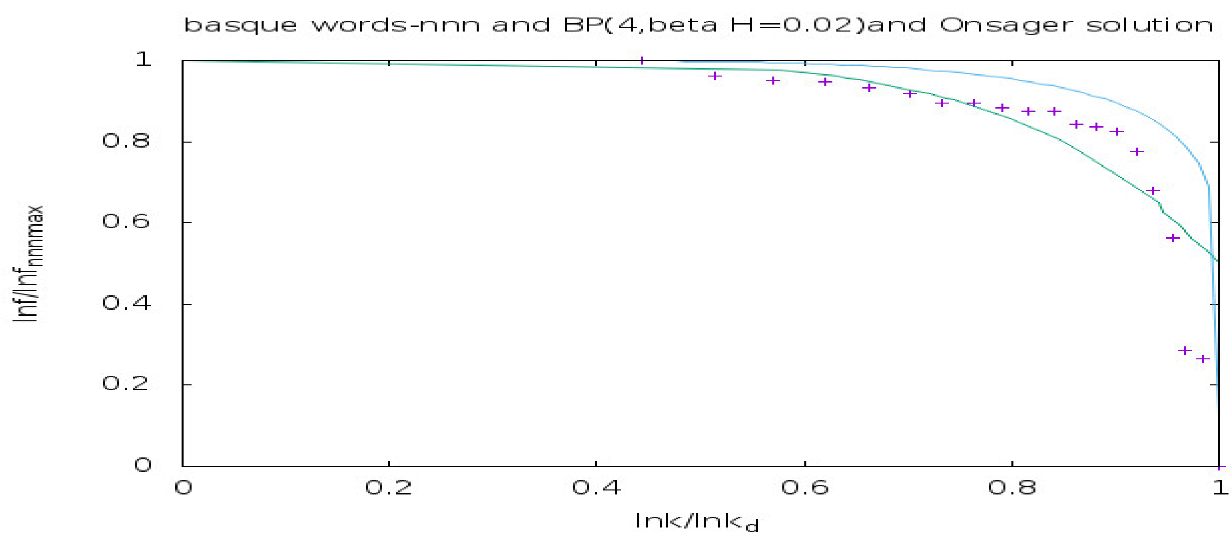

FIG. 9. Vertical axis is $\frac{lnf}{lnf_{nnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.01$ or, $\beta H = 0.02$. The uppermost curve is the Onsager solution.

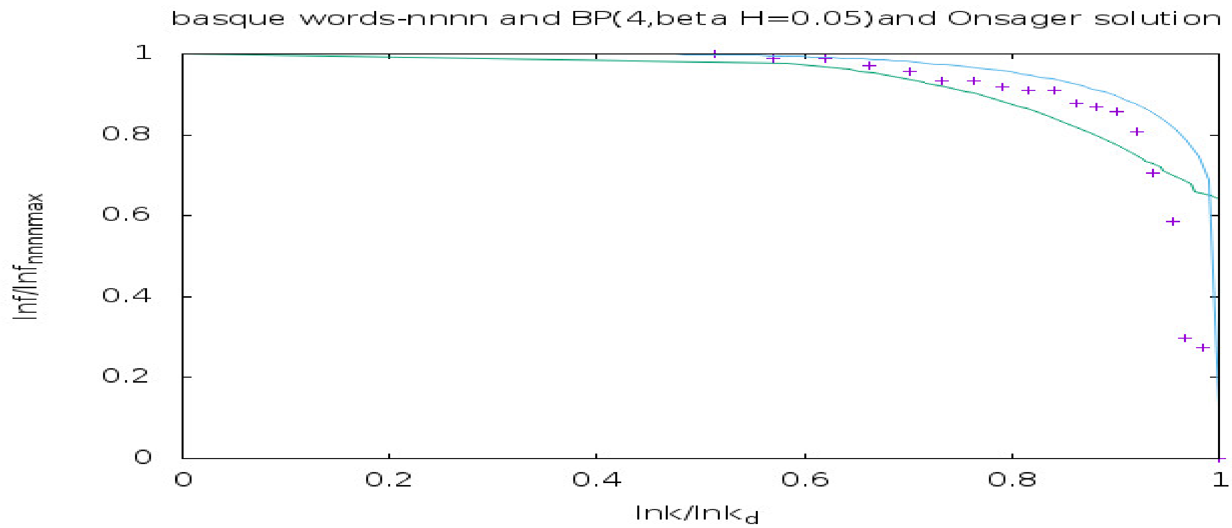

FIG. 10. Vertical axis is $\frac{lnf}{lnf_{nnnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.025$ or, $\beta H = 0.05$. The uppermost curve is the Onsager solution.

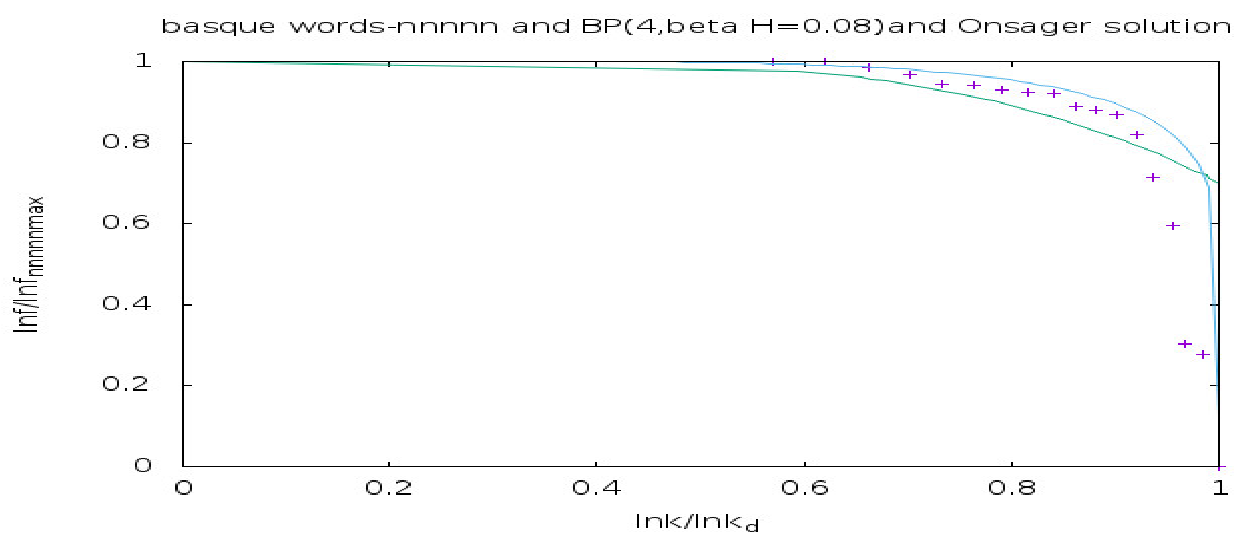

FIG. 11. Vertical axis is $\frac{lnf}{lnf_{nnnnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.04$ or, $\beta H = 0.08$. The uppermost curve is the Onsager solution.

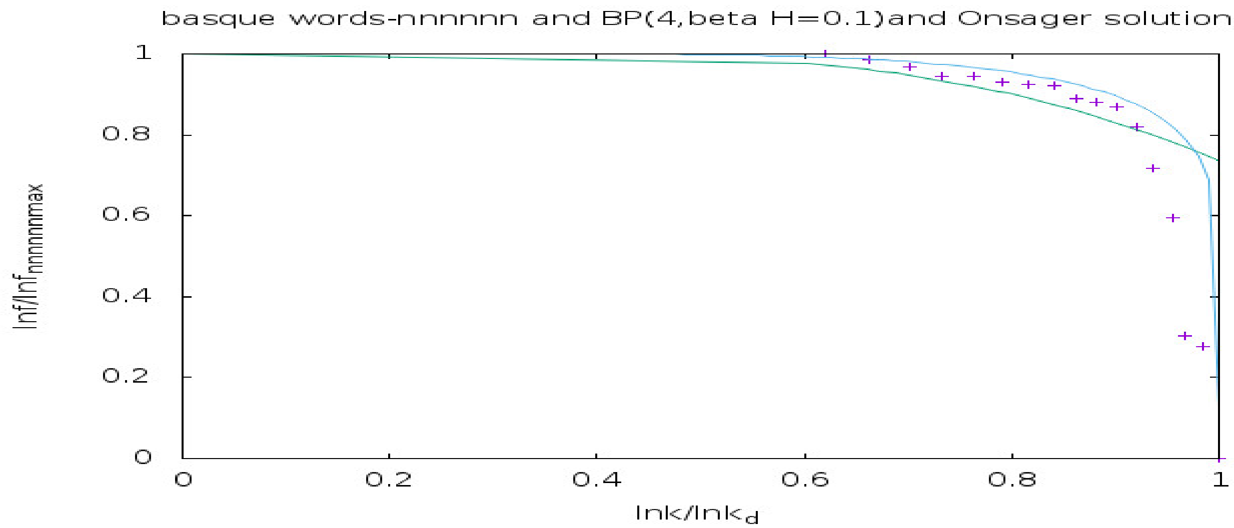

FIG. 12. Vertical axis is $\frac{lnf}{lnf_{nnnnnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m = 0.05$ or, $\beta H = 0.1$. The uppermost curve is the Onsager solution.

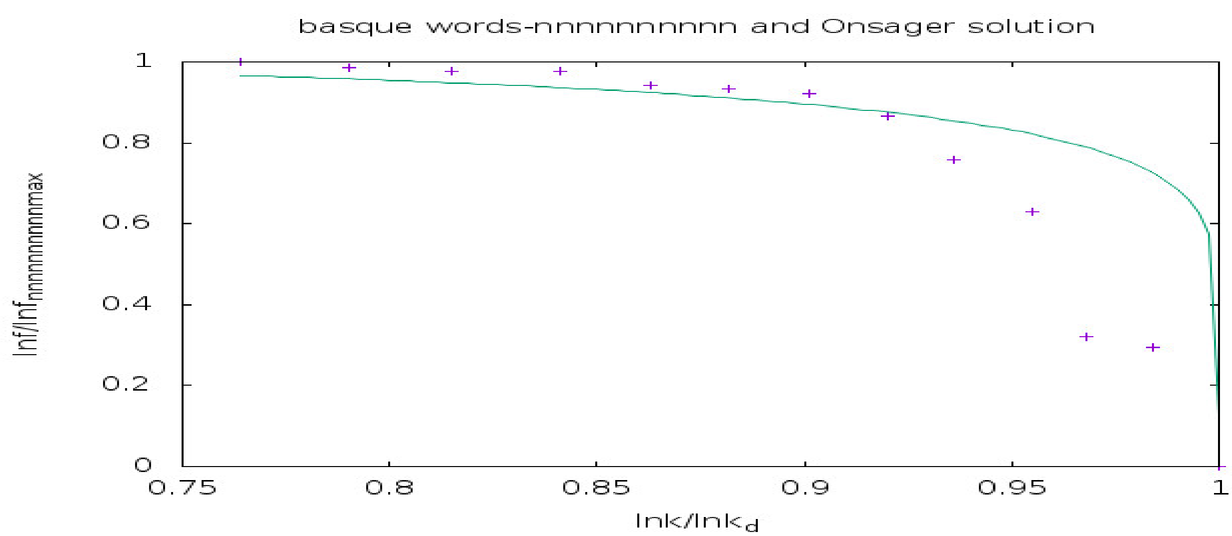

FIG. 13. Vertical axis is $\frac{lnf}{lnf_{nnnnnnnnnn-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the basque language. The reference curve is the Onsager solution. The Basque words not going over to the Onsager solution.

A. conclusion

From the figures (fig.5-fig.13), we observe that behind the words of the basque language etymological dictionary, [1], there is a magnetisation curve, BP(4,$\beta H = 0.01$), in the Bethe-Peierls approximation with four nearest neighbours, in presence of liitle magnetic field, $\beta H = 0.01$.

Moreover, the associated correspondance with the Ising model is,

$$\frac{lnf}{lnf_{next-to-maximum}} \longleftrightarrow \frac{M}{M_{max}},$$

and

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [48]. As temperature decreases, i.e. lnk decreases, f increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As basque language expands, the letters which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect as was first observed in [49] in another way.

On the top of it, on successive higher normalisations, words of basque language, do not go over to Onsager solution in the normalised lnf vs $\frac{lnk}{lnk_{lim}}$ graphs.

As matching of the plots in the figures fig.(5-13), with comparator curves i.e. the magnetisation curves of Bethe-Peierls approximations, is with large dispersions and dispersion does not reduce significantly over higher orders of normalisations, to explore for possible existence of spin-glass transition, in presence of little external magnetic field, $\frac{lnf}{lnf_{max}}$, $\frac{lnf}{lnf_{next-max}}$ and $\frac{lnf}{lnf_{nn-max}}$ are drawn against lnk in the figures fig.(14-16).

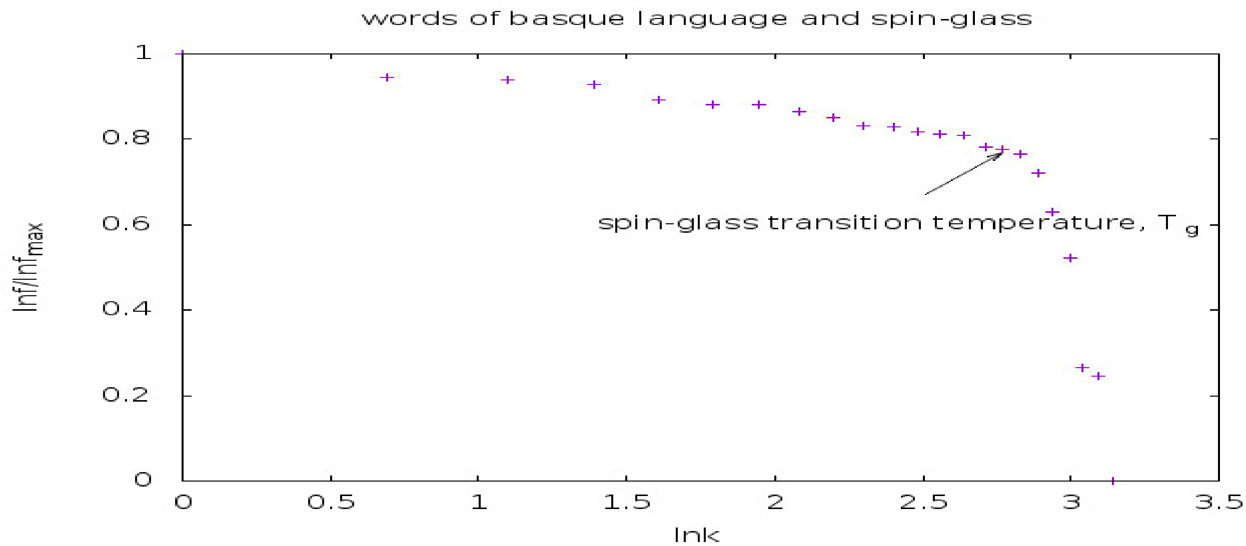

FIG. 14. Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is lnk. The + points represent the words of the basque language.

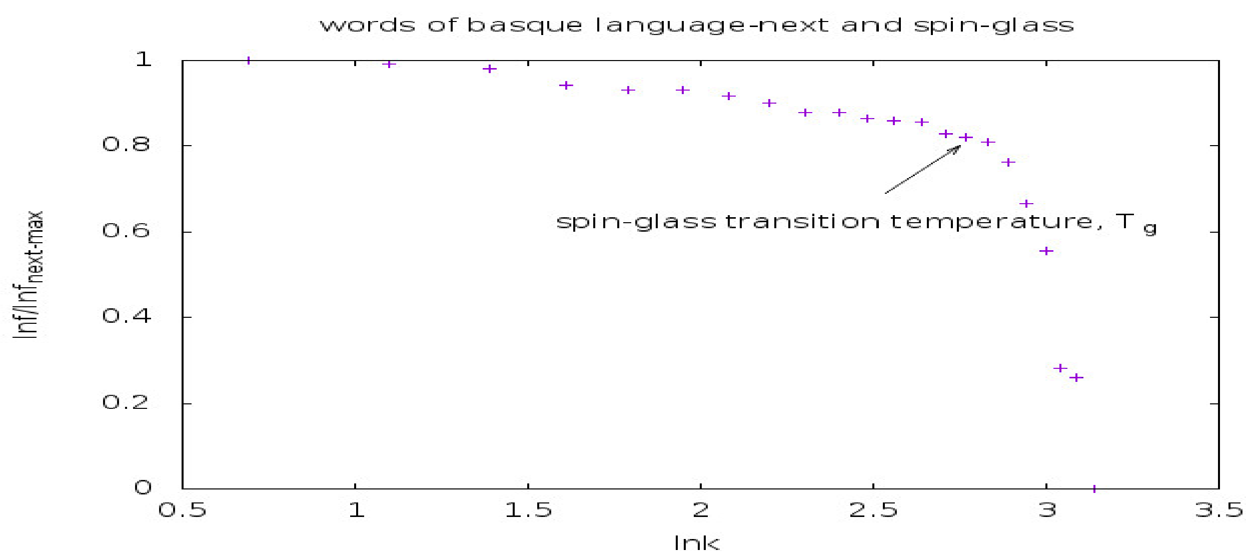

FIG. 15. Vertical axis is $\frac{lnf}{lnf_{next-max}}$ and horizontal axis is lnk. The + points represent the words of the basque language.

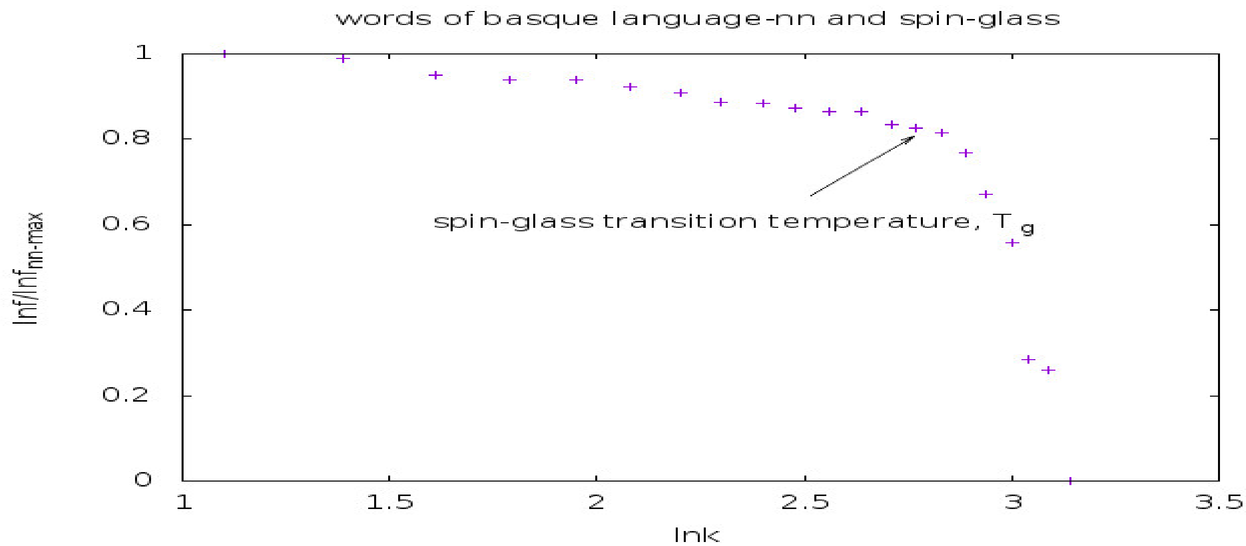

FIG. 16. Vertical axis is $\frac{lnf}{lnf_{nn-max}}$ and horizontal axis is lnk. The + points represent the words of the basque language.

In the figures Fig.14-Fig.16, the points has a clearcut transition at T_g. Above the transition point(s), T_g, the line is almost horizontal, increasing little and below the transition point(s), T_g, pointsline rises sharply like the branch of a rectangular hyperbola. Hence, the words of the basque language, better be described, to underlie a Spin-Glass magnetisation curve, [19], in the presence of little external magnetic field.

	basque	romanian
$\frac{lnf}{lnf_{max}}$ vs $\frac{lnk}{lnk_{lim}}$	BP(4, β H=0)	BW(c=0.01)
$\frac{lnf}{lnf_{next-max}}$ vs $\frac{lnk}{lnk_{lim}}$	BP(4, β H=0.01)	BP(4, β H=0)
$\frac{lnf}{lnf_{nnmax}}$ vs $\frac{lnk}{lnk_{lim}}$	BP(4, β H=0.01)	BP(4, β H=0)
$\frac{lnf}{lnf_{nnnmax}}$ vs $\frac{lnk}{lnk_{lim}}$	BP(4, β H=0.02)	BP(4, β H=0)
$\frac{lnf}{lnf_{nnnnmax}}$ vs $\frac{lnk}{lnk_{lim}}$	BP(4, β H=0.05)	
$\frac{lnf}{lnf_{nnnnnmax}}$ vs $\frac{lnk}{lnk_{lim}}$	BP(4, β H=0.08)	
$\frac{lnf}{lnf_{nnnnnnmax}}$ vs $\frac{lnk}{lnk_{lim}}$	BP(4, β H=0.1)	
$\frac{lnf}{lnf_{10nmax}}$ vs $\frac{lnk}{lnk_{lim}}$	Onsager:no	Onsager:no
Onsager core	NO	NO
spin-glass	transition	consideration
$\frac{lnf}{lnf_{max}}$ vs lnk	rectangular hyperbolic rise	rectangular hyperbolic rise
$\frac{lnf}{lnf_{next-max}}$ vs lnk	rectangular hyperbolic rise	rectangular hyperbolic rise
$\frac{lnf}{lnf_{nn-max}}$ vs lnk	rectangular hyperbolic rise	

TABLE V. comparison of words of the basque and the romanian languages

VI. DISCUSSION

Interesetingly, the Basque language is much closer to the Romanian language as far as the graphical law analysis reveals. We compare both the languages, [7], in the table, V.

A. vis a vis romanian

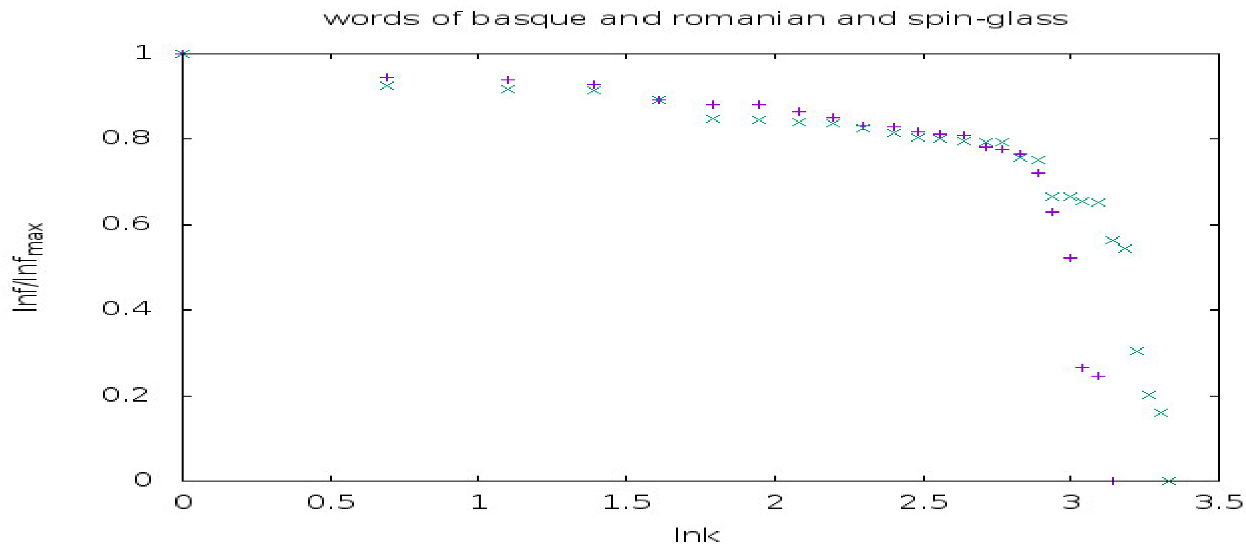

FIG. 17. Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is lnk. The + points represent the words of the basque language and the × points represent the words of the romanian language.

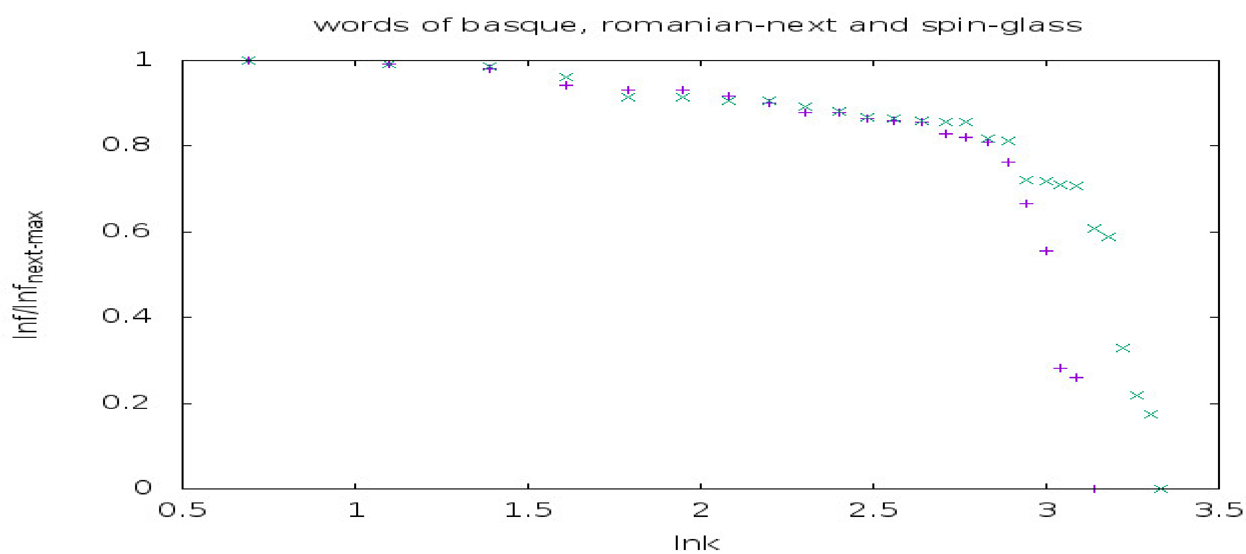

FIG. 18. Vertical axis is $\frac{lnf}{lnf_{next-max}}$ and horizontal axis is lnk. The + points represent the words of the basque language and the × points represent the words of the romanian language.

To make the comparison more explicit, we draw $\frac{lnf}{lnf_{max}}$ vs lnk as well as $\frac{lnf}{lnf_{next-max}}$ vs lnk simultaneously for both the langauges, to put forward their relative spin-glass natures. We find that the basque language is even better in manifesting spin-glass nature than the romanian,[7].

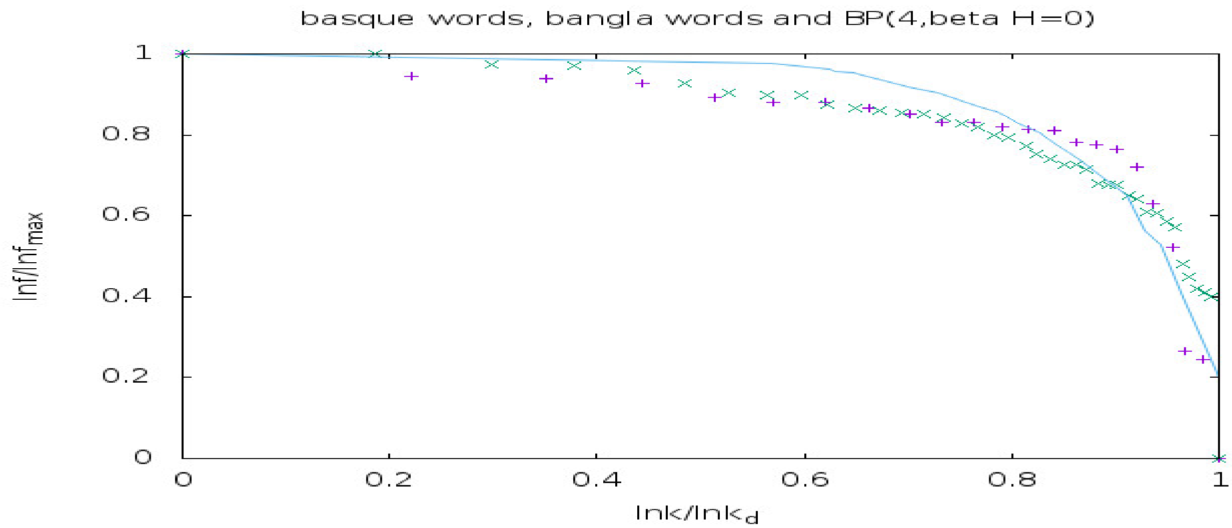

FIG. 19. Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points on dashed line represent the Basque language words and the × points represent the words of the bengali language.

Incidentally, Khasi does not have the letter c like that in the Basque. As far as proximity to the bengali language, [6], is concerned, we see from the figures, Fig.19- Fig.22, that two languages deviate from each other as we go away from $\frac{lnf}{lnf_{max}}$ vs $\frac{lnk}{lnk_{lim}}$ to higher order of normalisations. Bengali language is characterised by BP(4,βH=0.01), Basque also albeit with large dispersion.

B. vis a vis bengali

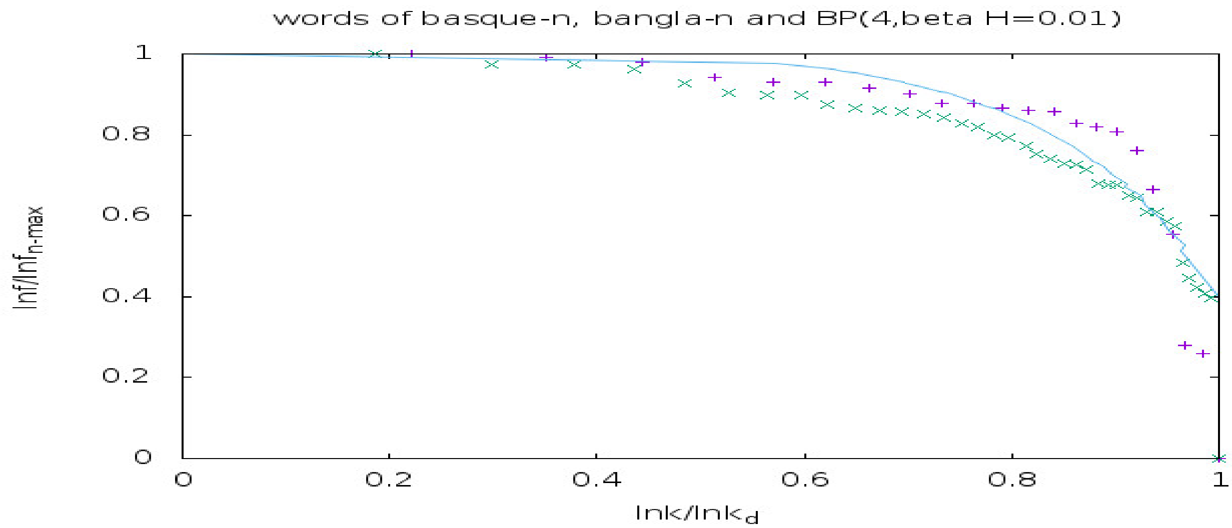

FIG. 20. Vertical axis is $\frac{lnf}{lnf_{next-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points on dashed line represent the Basque language words and the × points represent the words of the bengali language.

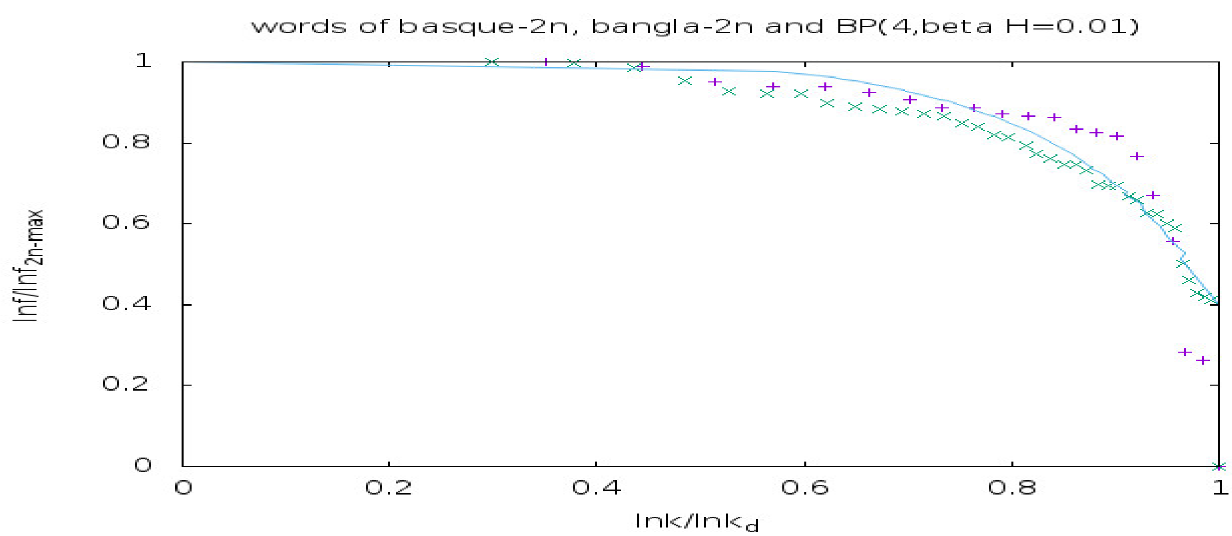

FIG. 21. Vertical axis is $\frac{lnf}{lnf_{nextnext-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points on dashed line represent the Basque language words and the × points represent the words of the bengali language.

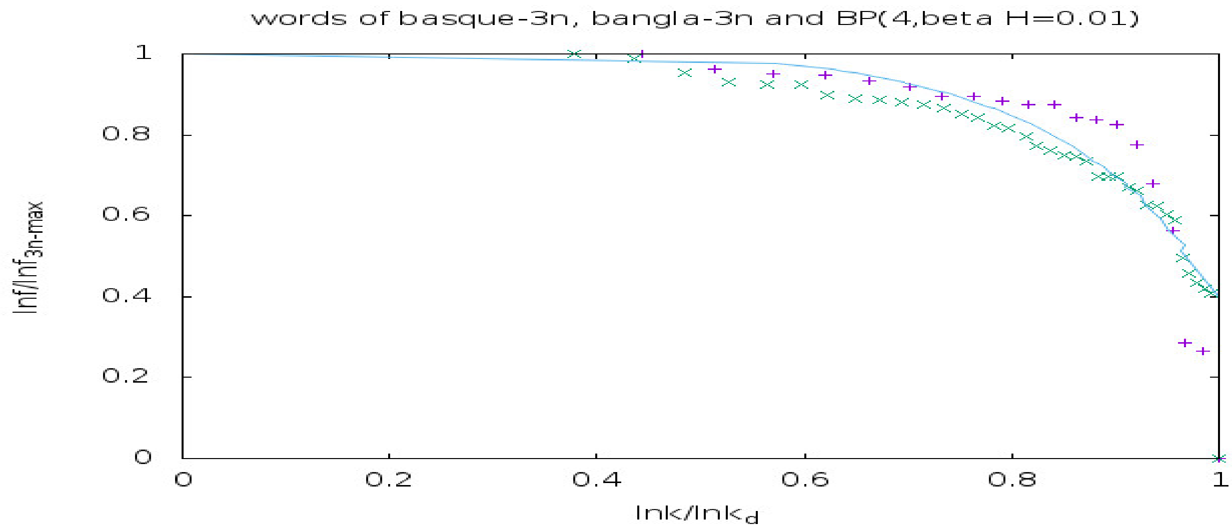

FIG. 22. Vertical axis is $\frac{lnf}{lnf_{nextnextnext-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points on dashed line represent the Basque language words and the × points represent the words of the bengali language.

VII. ACKNOWLEDGEMENT

The author came to know of the Basque language by reading a BBC news article. We have used gnuplot for drawing the figures.

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