The Collatz Conjecture - simplified

Richard L. Hudson 8-14-2025

Abstract

Originated by Lothar Collatz in 1937 [1], the conjecture states: given the recursive function, y=3x+1 if x is odd, or y=x/2 if x is even, for any positive integer x, y will equal 1 after a finite number of steps. This analysis examines why the Collatz process is true.

1. examples

An example for a random selection of 9, using the original method:

1. S9=(9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1)

An example for a random selection of 12, using the original method:

2. S12= (12, 6, 3, 10, 5, 16, 8, 4, 2, 1)

2. functions

The recursive function is replaced with function d for odd values (2n-1), with

$$d(x) = 3x+1 = u = 2^k y \qquad (2.0)$$

and function e for even values, which removes factors of 2,

$$e(u) = u/2 \qquad (2.1)$$

Since d(x) cannot produce (0 mod 3) terms, those can only begin a descending sequence.

3. conversion to binary

X	U	binary set	Y
1	4	4	1
5	16	16	1
7	22	16+4+2	11
9	28	16+8+4	7
11	34	32+2	17
13	40	32+8	5
17	52	32+16+4	13

fig.1

Using example 1, each odd integer is expressed in terms of a binary set of elements. Y denotes the binary set divided by the smallest of its members. This forms a different ordering of elements for the sequence.

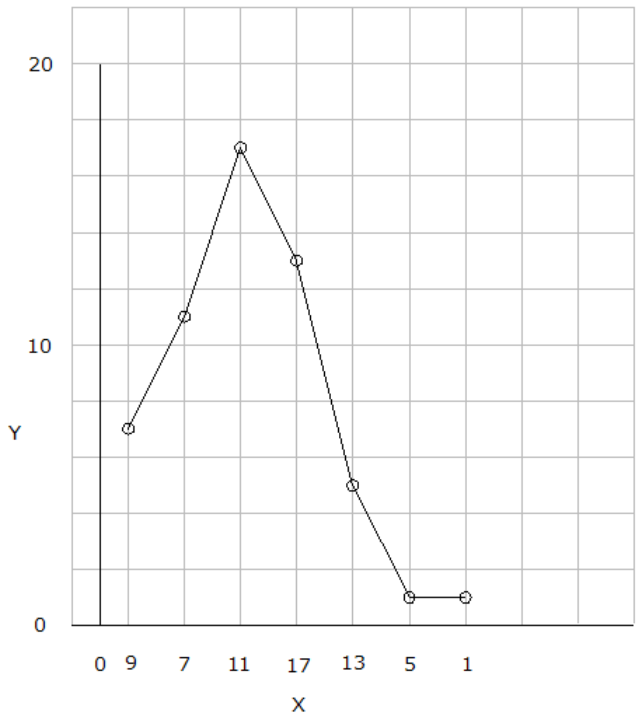

fig.2

In fig.2, the x integers are in an a different order resulting from the binary reduction.

Termination with 1

X	diff	-	+
9			
7	(7-9)	2	
11	(11-7)		4
17	(17-11)		6
13	(13-17)	4	
5	(5-13)	8	
1	(1-5)	4	
	total	18	10

fig.3

The difference between an integer and its successor is shown in columns 3 or 4 as plus or minus. For the example 9+10-18=1.
Column 2 shows each intermediate term from 7 to 5 has a + and - sign, and cancel.
The initial term of a sequence cancels leaving 1.
For S9, 9-9+1=1.
For S7, 7-7+1 =1.

reference

1. Wikipedia.org/Collatz Conjecture, Mar 2018