Chen's Formulas of the Fine-structure Constant (viXra:2002.0203vH)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper gives two series of formulas of the fine-structure constant α which are reasonable, precise, smart and elegant. It also demonstrates there are two values of α, i.e., α_1=1/137.035999037435 and α_2=1/137.035999111818, which are consistent with but much more accurate than those experiment measured values. The formulas consist of 2π-e formulas and some factors related to nucleon numbers of nuclides. A brief explanation of the fine-structure constant shows 1/α≈137.036 is the equal ratio factor between 112 and 168 (more precisely 168-1/3). Based on these, all 119th to 170th ideal extended elements were predicted, the speed of light in atomic units was mathematically calculated by $c_{au}=1/(\alpha_1\alpha_2)^{1/2}$=137.035999074627, Schrödinger equation of hydrogen atom was simplified and correlated with α_1/α_2, classical electron radius was calculated to be 2.81794032658(43) fm and proton charge radius was hypothetically calculated to be 0.833027202999(13) fm. In the end, it was found that the approximate rational numbers of 2π marvelously related to nuclides, a mathematic shell model of nuclides was established and a picture of elements and ideal extended elements was depicted.

Keywords: formulas; the fine-structure constant; the ideal extended elements; the speed of light; Schrödinger equation of hydrogen atom; the proton charge radius; 2π.

1. Introduction

The fine-structure constant (Sommerfeld's constant) is a critical dimensionless constant in physics, it is a century mystery of physics, it has been one of the biggest enigmas in physics since it was introduced by Arnold Sommerfeld in 1916. Its definition, some interpretations and the latest measured values are as follows[1, 2]:

$$\alpha=\frac{\lambda_e}{2\pi a_0},\ \alpha=\frac{2\pi r_e}{\lambda_e},\ \frac{a_0}{r_e}=\frac{1}{\alpha^2};\ \alpha=\frac{e^2}{4\pi\varepsilon_0\hbar c}=\frac{v_e}{c},\ \frac{c}{v_e}=\frac{1}{\alpha}$$

in atomic units, the speed of light $c_{au}=\frac{1}{\alpha}$

the 2014 CODADA recomended value: $\alpha=1/137.035999139(31)$

the 2018 CODADA recomended value: $\alpha=1/137.035999084(21)$

Science 13 April 2018 reported value: $\alpha=1/137.035999046(27)$

The ratio of Bohr radius of hydrogen atom a_0 to the classical electron radius r_e is $1/\alpha^2$. The ratio of the speed of light c to the line velocity of ground state electron in hydrogen atom v_e is $1/\alpha$, this means in atomic units $c=1/\alpha$ and $E=mc^2=m/\alpha^2$ or $\alpha^2=m/E$. In quantum electrodynamics it substantially characterizes the strength of electromagnetic interaction between elementary charged particles such as electron and proton, so it is the coupling constant of electric charges. It is one of the 25 fundamental constants (could not be calculated theoretically, could only be determined by experiments) in Standard Model of physics and should be the most important one. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. However, to our knowledge, up to now (except this work), no one knows how it comes from, no one could give reasonable explanations to it or formulas of it since it was introduced.

In 2016 Paul Davis gave the following comment[3]: "Physicists have long wondered where this number, 1/137.035999, comes from. Is there a deep reason why α has to be precisely this number for the world to function as it does? There is a long history of attempts to derive α from physical theory or to concoct a mathematical formula that has this value. For a brief time in the 1920s, when it looked as if α might be exactly 1/137, astronomer Arthur Eddington searched for a theory that would throw up the numbers naturally, but his ideas ultimately led nowhere. Then in 1969 a young Swiss mathematician, Armand Wyler, pointed out that $(9/16\pi^3)(\pi/5!)^{1/4}$ comes close to 1/137.036, which matched the value of α to the precision known at the time. However, his formula was not accompanied by any credible theory and was regarded as little more than a numerical curiosity. Several other attempts at α numerology have been made since, none of which have gained traction in the physics community."

As for the fascination of the fine-structure constant, in the middle of 1980s, Richard Feynman stated[4]: "It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the hand of God wrote that number, and we don't know how He pushed his pencil."

This paper shows how God pushed his pencil to write the fine-structure constant and how God used it to coordinate elements.

2. 2π-e formula(s)

2π-e formula, its related formulas and their preliminary applications were deduced independently by us from April to December of 2013.

Fig. 1. Diagram of y=1/x.

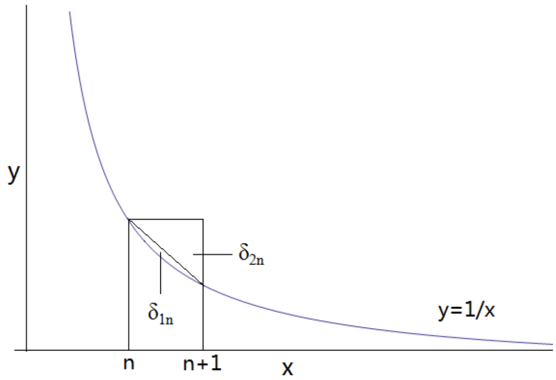
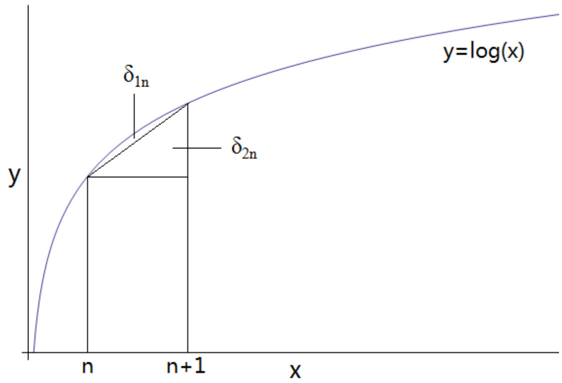


Fig. 2. Diagram of y=log(x).



Euler-Mascheroni constant γ : $\sum_{n=1}^{\infty}\frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{\infty}=\ln\infty+\gamma$

As for $y=1/x$ (Fig. 1), $\gamma=\sum_{n=1}^{\infty-1}\delta_{2n}+\sum_{n=1}^{\infty-1}\delta_{1n}=\frac{1}{2}+\gamma_1=0.5+0.077215\cdots=0.577215\cdots$

$\gamma_1=\sum_{n=1}^{\infty-1}\delta_{1n}=\lim_{N\to\infty}(\sum_{n=1}^{N\text{-}1}\frac{1}{n}-\int_1^N\frac{1}{x}\mathrm{dx})-\frac{1}{2}$, Generally $\gamma_s=\lim_{N\to\infty}(\sum_{n=1}^{N\text{-}1}\frac{1}{n^s}-\int_1^N\frac{1}{x^s}\mathrm{dx})-\frac{1}{2}$, $\mathrm{s}\in\mathrm{N}$

As for $y=\log(x)$ (Fig. 2), $\delta_{1,n}=\int_n^{n+1}\ln x dx-\frac{1}{2}\ln\frac{n+1}{n}-\ln n=(x\ln x-x)\Big|_n^{n+1}-\frac{1}{2}\ln(n+1)n$

$=(n+1)\ln(n+1)-n\ln n-1-\frac{1}{2}\ln(n+1)-\frac{1}{2}\ln n=(n+\frac{1}{2})\ln(1+\frac{1}{n})-1$

$$\gamma_{c,N}=\sum_{n=1}^{N}\delta_{1,n}=\sum_{n=1}^{N}[(n+\frac{1}{2})\ln(1+\frac{1}{n})-1]=\sum_{n=1}^{N}\ln\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}=\ln\prod_{n=1}^{N}\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}$$

$$\gamma_c=\gamma_{c,\infty}=\sum_{n=1}^{\infty}\delta_{1,n}=\lim_{N\to\infty}(\int_1^{N+1}\log(\mathrm{x})\,\mathrm{dx}-\sum_{n=1}^{N}\log(n)-\frac{\log(\mathrm{N+1})}{2})$$

$$=\sum_{n=1}^{\infty}[(n+\frac{1}{2})\ln(1+\frac{1}{n})-1]=\sum_{n=1}^{\infty}\ln\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}=\ln\prod_{n=1}^{\infty}\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}$$

$$\ln N!=\sum_{n=1}^{N}\ln n=\int_1^{N+1}\ln x dx-\sum_{n=1}^{N}\delta_{1,n}-\sum_{n=1}^{N}\delta_{2,n}=(x\ln x-x)\Big|_1^{N+1}-\ln e^{\gamma_{c,N}}-\sum_{n=1}^{N}\frac{\ln(n+1)-\ln n}{2}$$

$$=(N+1)\ln\frac{(N+1)}{e}+\ln\frac{e}{e^{\gamma_{c,N}}}-\frac{1}{2}\ln(N+1)=\ln[\frac{e^{1-\gamma_{c,N}}}{\sqrt{N+1}}(\frac{N+1}{e})^{(N+1)}]$$

$N!=\frac{e^{1-\gamma_{c,N}}}{\sqrt{N+1}}(\frac{N+1}{e})^{(N+1)}$, compared to Stirling formula : $N!\sim\sqrt{2\pi N}(\frac{N}{e})^N$

$(N+1)!=(N+1)N!\sim\sqrt{2\pi(N+1)}(\frac{N+1}{e})^{N+1}$, $N!\sim\frac{\sqrt{2\pi}}{\sqrt{N+1}}(\frac{N+1}{e})^{N+1}$

Compared to previous formula, gives $\sqrt{2\pi}\sim e^{1-\gamma_{c,N}}$ or $2\pi=(\frac{e}{e^{\gamma_c}})^2$

$2\pi-e$ formula(s): $2\pi=(\frac{e}{e^{\gamma_c}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$, $(2\pi)_k=(\frac{e}{e^{\gamma_{c/k}}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}$

$\gamma_c=0.0810614668\cdots$, $e^{\gamma_c}=1.0844375\cdots$

2π-e formula is an expanding form of Stirling formula. To our knowledge, it was first deduced by us. If it was new, it could be named Chen's 2π-e formula.

3. Some Formulas Related to 2π-e Formula

The following formulas which correlate each other and has similar form could be called Chen's natural group formulas, and the form is called natural group.

$$1=4\gamma_c+\frac{4\gamma_1}{1(1+1)}+\frac{4\gamma_2}{2(2+1)}+\frac{4\gamma_3}{3(3+1)}+\cdots$$

$$=|B|\frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{|B_{2n}|(\pi/2)^{2n}}{(2n)!}=\sum_{n=1}^{\infty}\frac{|B_{2n}|\pi^{2n}}{(2n)!}=-|B|\frac{3\pi}{2}+\sum_{n=1}^{\infty}\frac{|B_{2n}|(3\pi/2)^{2n}}{(2n)!}$$

$$N\sim-\frac{3}{2}|B|+\sum_{n=1}^{\mathrm{N}}\frac{|B_{2n}|(2\pi)^{2n}}{2(2n)!}$$

$$e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots$$

$$2\pi=(\frac{e}{e^{\gamma_c}})^2=e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots$$

B, B_{2n}: the Bernoulli numbers such as $-\frac{1}{2},\ -\frac{1}{6},\ -\frac{1}{30},\ \frac{1}{42},\ -\frac{1}{30},\ \cdots$

$$\gamma_c=\lim_{N\to\infty}(\int_1^{N+1}\log(\mathrm{x})\,\mathrm{dx}-\sum_{n=1}^{N}\log(n)-\frac{\log(\mathrm{N}+1)}{2})=0.0810614668\cdots$$

$$\gamma_s=\lim_{N\to\infty}(\sum_{n=1}^{N\text{-}1}\frac{1}{n^s}-\int_1^N\frac{1}{x^s}\mathrm{dx})-\frac{1}{2},\quad \mathrm{s}\in\mathrm{N}$$

$$\gamma_1=0.077215\cdots,\ \gamma_2=0.144934\cdots,\ \gamma_4=0.24899\cdots,\ \gamma_8=0.36122\cdots,\ \gamma_{16}=0.433349\cdots,\ \cdots,\ \gamma_\infty=0.5$$

$\gamma_c\ \gamma_1\ \gamma_2\ \gamma_3\cdots$ are called Chen's natural group constants (analogue to Bernoulli numbers).

The following are some other formulas related to 2π-e Formula.

$$\sqrt{2\pi}=e^{1-\gamma_c},\quad e=\sqrt{2\pi}e^{\gamma_c}=\sqrt{2\pi}(1+\sum_{n=1}^{\infty}\frac{\gamma_c{}^n}{n!})$$

$$\gamma_c=\sum_{n=1}^{\infty}[(n+\frac{1}{2})\ln(1+\frac{1}{n})-1]=\sum_{n=1}^{\infty}\frac{(2^{2n}-1)|B_{2n}|\pi^{2n}-2(2n)!}{2(2n+1)!}=\frac{1}{4}-\sum_{s=1}^{\infty}\frac{\gamma_s}{s(s+1)}$$

$$\gamma_g=\sum_{n=1}^{\infty}(n+\frac{1}{2})\ln(1+\frac{1}{n})-\int_1^{\infty}(x+\frac{1}{2})\ln(1+\frac{1}{x})dx$$

$$\gamma_{cg}=\frac{1}{2}\lim_{N\to\infty}[\sum_{n=1}^{N}\frac{(2^{2n}-1)|B_{2n}|\pi^{2n}}{(2n+1)!}-\ln N]$$

$$\frac{\pi}{2}=(\frac{e}{e^{\gamma_g}})^2,\ e=\sqrt{\frac{\pi}{2}}e^{\gamma_g}=\sqrt{\frac{\pi}{2}}(1+\sum_{n=1}^{\infty}\frac{\gamma_g{}^n}{n!});\quad \frac{\pi}{2}=(\frac{e^{\gamma/2}}{e^{\gamma_{cg}}})^2,\ \gamma=\ln\frac{\pi}{2}+2\gamma_{cg}$$

$$\gamma_c=\gamma_g-\ln 2=1-\frac{\gamma}{2}+\gamma_{cg}-\ln 2,\ \gamma_{cg}=\frac{1}{2}+\sum_{s=2}^{\infty}\frac{\gamma_s}{s(s+1)}-\ln 2$$

$$\gamma_c=0.0810614668,\quad \gamma_g=0.7742086474,\quad \gamma_{cg}=0.0628164798$$

$$\frac{\pi}{2}=\sum_{n=1}^{\infty}\frac{|B_{2n}|\pi^{2n}}{2n(2n)!};\quad \sum_{n=1}^{\infty}[\zeta(2n)-1]=\frac{3}{4},\quad \zeta(2n)=\sum_{k=1}^{\infty}\frac{1}{k^{2n}}$$

$$\sum_{n=1}^{\infty}\frac{1}{n}=\sum_{n=1}^{\infty/2}\frac{|B_{2n}|(2\pi)^{2n}}{2n(2n)!}=\sum_{n=1}^{\infty}\frac{|B_{2n}|(2n2^{2n}+1)\pi^{2n}}{2n(2n+1)!}$$

4. Some Applications of 2π-e Formula and its Related Formulas

(1). 2π-e formula is basically an algebraic expanding of Stirling formula, but it is more meaningful, it exhibits the relationship between 2π and e. In 2π-e formula, γ_c is a real constant with geometric definition like Euler-Mascheroni constant γ. With 2π-e formula and its related formulas, 2π can be calculated from e and vice versa. So it is the real 2π-e relationship formula.

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \prod_{n=1}^{\infty} \frac{e^2}{(1+\frac{1}{n})^{2n+1}} = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots$$

$$e = \sqrt{2\pi} e^{\gamma_c} = \sqrt{2\pi}\left(1+\sum_{n=1}^{\infty} \frac{\gamma_c^{\ n}}{n!}\right), \quad \gamma_c = \sum_{n=1}^{\infty} \frac{(2^{2n}-1)\left|B_{2n}\right|\pi^{2n} - 2(2n)!}{2(2n+1)!}$$

(2). 2π-e formula demonstrates 2π is a natural constant rather than π. π/2 is somewhat fundamental but not as complete as 2π. π is neither fundamental nor complete. In 2001 mathematician Bob Palais said "π is wrong"[5]. 2π-e formula and the Taylor expansion of e have similar form (natural group form), this should give a conclusive proof that 2π is a real natural constant and π is not.

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \Rightarrow 2\pi \text{ or } \sqrt{2\pi} \text{ is a natural constant}$$

$$\frac{\pi}{2} = \left(\frac{e}{e^{\gamma_g}}\right)^2 = \left(\frac{e^{\gamma/2}}{e^{\gamma_{cg}}}\right)^2 \Rightarrow \frac{\pi}{2} \text{ or } \sqrt{\frac{\pi}{2}} \text{ is almost a natural constant}$$

$$\pi = \left(\frac{e}{e^{\gamma_c}\sqrt{2}}\right)^2 = \left(\frac{e\sqrt{2}}{e^{\gamma_g}}\right)^2 \Rightarrow \pi \text{ or } \sqrt{\pi} \text{ is not a natural constant}$$

Table 1 lists some points of view of Piist who support π is a natural constant, Tauist who support 2π is a natural constant and this work which supports the later.

Table 1. Comparison of points of view of Piist, Tauist and this work.

	Piist	Tauist	This work
Circumference of a circle	πd	2πR	2πR
Area of a circle	πR^2		(1/2)(2πR)R
Volume of sphere	$(4/3)\ \pi R^3$	$(2/3)(2\pi)R^3$	$(2\pi R^2/3)2R$
Volume of n-dimension sphere	$\frac{\pi^{n/2}}{\Gamma(n/2+1)} R^n$	$\frac{(2\pi)^{n/2}}{2^{n/2}\Gamma(n/2+1)} R^n$	$\frac{2\pi R^2}{n} V_{n-2}$
Euler's identity	$e^{i\pi}+1=0$	$e^{2\pi i}=1$	$e^{2\pi i}=1$
Gauss integral	$\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$		$\int_{-\infty}^{+\infty} e^{-x^2} dx = \frac{e}{e^{\gamma_c}} \frac{1}{\sqrt{2}}$

(3). As 2π is a square number, the frequent appearing of its square root in some important equations such as Gaussian distribution (normal distribution) and Maxwell –Boltzmann distribution becomes reasonable and understandable. And the distributions can be transformed as follows.

$$\text{Standard Normal Distribution: } f(x,0,1)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}=e^{-\frac{x^2+2(1-\gamma_c)}{2}}$$

$$\text{Maxwell – Boltzmann Distribution: } f(v)=\frac{2}{\sqrt{2\pi}}v^2(\frac{m}{kT})^{\frac{3}{2}}e^{-\frac{mv^2}{2kT}}=2(\frac{m}{kT})^{\frac{3}{2}}v^2e^{-(\frac{m}{kT}\frac{v^2}{2}+1-\gamma_c)}$$

(4). Euler's identity (Euler's equation) $e^{i\pi}+1=0$ is called God formula and the most beautiful formula in mathematics. However, as 2π is the real natural constant and π is not, $e^{2\pi i}=1$ should be more beautiful.

(5). $\gamma=\ln(2\pi)+\gamma_{cg}$ may help to prove γ is an irrational number or even a transcendental number.

(6). The natural group formulas help us to establish "Chen's Periodic Table of Elements and Natural Group Theory"[6] (2014-2017).

(7). The mathematic expression of chirality is $\pm 2\pi$. This concept is helpful for us to establish "Chirality and Poetry Model of Atomic Nuclei"[7] (2017/12-2018/3).

(8). Based on the above theories, Chen's formulas of the fine-structure constant were deduced (2018/4-6)[8], then modified and extended (2018/7-2020/6).

5. Original Inspiration for Formulas of the Fine-structure Constant

1. According to $\alpha=\frac{e^2}{4\pi\varepsilon_0\hbar c}=\frac{\lambda_e}{2\pi a_0}=\frac{2\pi r_e}{\lambda_e}\approx\frac{1}{137.036}$, the formulas of α should relate to 2π.

2. $\frac{137.036}{2\pi}=\frac{137.036}{6.28318}=21.81$, $137.036=21.81\times 2\pi$

3. According to 2π-e formula: $2\pi=(\frac{e}{e^{\gamma_c}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$

   2π is a square number, suppose $21.81=x^2$, $x=4.670\approx 14/3$

   so: $\frac{1}{\alpha}\approx(\frac{14}{3})^2 2\pi$ or $\alpha\approx(\frac{3}{14})^2\frac{1}{2\pi}$ (Discover: about 2 am on 2018/4/12)

4. Apply with 2π-e formula (in the afternoon of 2018/4/12, a meeting in the morning)

$$\alpha=(\frac{3}{14})^2\frac{1}{(2\pi)_{112}}=(\frac{3}{14})^2\frac{1}{e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{113}{112})^{225}}}=137.035781520\text{, closest to the real value.}$$

As 112 is one of the most important stable numbers and the 112th element ${}^{285}_{112}Cn^{*}_{173}$ is the natural end of elements according to our Chen's Chirality and Poetry Model of Atomic Nuclei[6].

So: **Eureka!** Subsequently transformed to: $\alpha=\frac{6^2}{7(2\pi)_{112}}\frac{1}{112}=137.035781520$,

Finally modified to: $\alpha=\frac{6^2}{7(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}=137.035999037435$

6. Logical Deduction of Chen's Formulas of the Fine-structure Constant

Physicist Richard Feynman noticed a hydrogen-like atom with Z protons and only one electron, according to Bohr model, the line velocity of the nth rank electron $v_{e/Z/n}$ satisfies:

$$\frac{v_{e/z/n}}{c}=\frac{Ze^2}{n4\pi\varepsilon_0\hbar c}=\frac{Z}{n}\alpha,\ \text{as } v_{e/z/n}\le c,\ \ \alpha=\frac{v_{e/z/n}}{c}\frac{n}{Z}\approx\frac{1}{Z_{\max-\text{ideal}}}=\frac{1}{Fy}=\frac{1}{137}$$

The 137th hydrogen-like element Fy (Feynmanium) is an ideal (imaginative) element,

in reality, the above formula should be modified to: $\alpha = f(\mathrm{Z}_{real})\dfrac{1}{Z_{\max-\text{real}}}$

According to Chen's Chirality and Peotry Model of Atomic Nuclei[6],

$Z_{\max-\text{real}}=112=2\cdot 56,$ so $\alpha = f(\mathrm{Z}_{real})\dfrac{1}{Z_{\max-\text{real}}}=f(\mathrm{Z}_{real})\dfrac{1}{112}$

Compared to $\alpha=\dfrac{\lambda_e}{2\pi a_0}$, the formula should have a 2π factor:

$$\alpha = f(\mathrm{Z}_{real})\frac{1}{Z_{\max-\text{real}}}=\frac{n}{\mathrm{m}(2\pi)}\frac{1}{Z_{\max-\text{real}}}=\frac{6^2}{7\cdot(2\pi)}\frac{1}{112}=1/136.8$$

Apply with 2π-e formula: $2\pi = e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots$

the formula is transformed to:

$$\alpha=\frac{n}{\mathrm{m}(2\pi)_k}\frac{1}{Z_{\max-\text{real}}}=\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112}=\frac{6^2}{7\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{113}{112})^{225}}}\frac{1}{112}=1/137.035782$$

Above deduction on 2018/4/12, only $(2\pi)_{112}$ gives the closest value to α,
this coincidence of one part per infinity proves the formula itself is correct.

Added an calibration factor (δ=1/75^2) on 2018/4/20, the accurate formula is:

$$\alpha_1=\frac{\lambda_e}{2\pi a_0}=\frac{6^2}{7\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{113}{112})^{225}}}\frac{1}{112+\dfrac{1}{75^2}}=1/137.035999037435$$

Discover: 2018/4/12; Revise: 2018/4/20 (add $1/75^2$ factor)

By the same procedure but compared to $\alpha=\dfrac{2\pi r_e}{\lambda_e}$, the other formula is:

$$\alpha_2=\frac{2\pi r_e}{\lambda_e}=\frac{13\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{279}{278})^{557}}}{10^2}\frac{1}{112-\dfrac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

Discover: 2018/4/24; Revise: 2018/9/18-20 ($280\to 278$, $-\dfrac{1}{39^2}+\dfrac{1}{780^2}\to-\dfrac{1}{3\cdot 29\cdot 64}$)

Another amazing coincidence is 6^2 and 10^2 are square numbers in accordance with $2\pi=(\dfrac{e}{e^{\gamma_c}})^2$

This also demonstrates that α has two values with two kinds of formulas.

As $f(\mathrm{Z}_{real})=\dfrac{n}{\mathrm{m}(2\pi)_k}$ or $f(\mathrm{Z}_{real})=\dfrac{\mathrm{m}(2\pi)_k}{n}$, m n k δ should relate to nucleon numbers of nuclides.

7. The Two Most Important Formulas

The above two formulas for α_1 and α_2 were our first gained formulas and are the most important formulas among their serial formulas which will be given followed in this paper. Calculation to give the values of α_1 and α_2 is shown in **Fig. 3** and **Table 2**.

Fig. 3. Calculation diagram of α_1 and α_2 (2018/4-6).

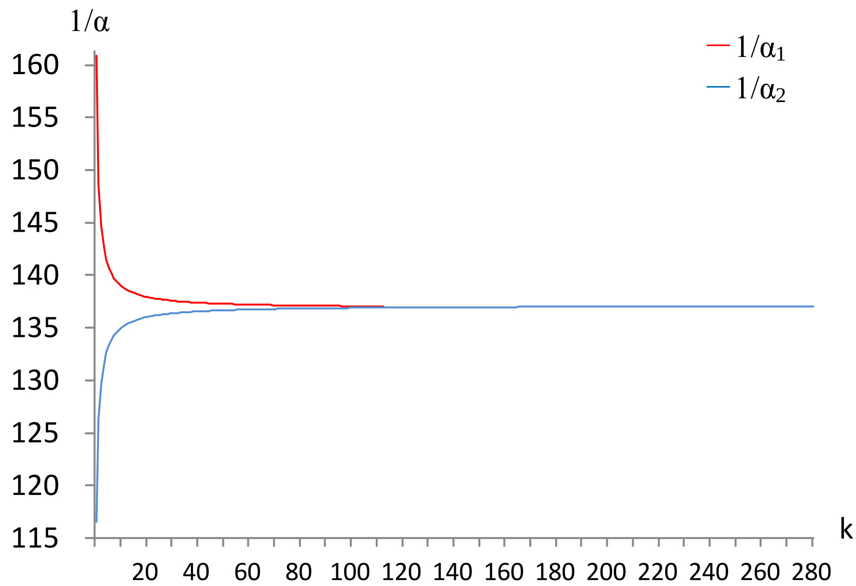


Table 2. Calculation of α_1 and α_2 (2018/4-6).

k	$(2\pi)_k$	$1/\alpha_1$	k	$(2\pi)_k$	$1/\alpha_2$
	7.389056099	160.917477134		7.389056099	116.596364743
1	6.824768754	148.628533230	1	6.824768754	126.236816375
2	6.640803185	144.622165589	2	6.640803185	129.733867427
3	6.549956514	142.643723845	3	6.549956514	131.533251879
4	6.49586908	141.465817857	4	6.49586908	132.628454999
5	6.46000004	140.684668634	5	6.46000004	133.364872233
6	6.434476503	140.128821836	6	6.434476503	133.893888578
7	6.415388754	139.713132398	7	6.415388754	134.292263980
8	6.400576029	139.390543654	8	6.400576029	134.603053878
9	6.388747203	139.132937708	9	6.388747203	134.852272701
10	6.379083388	138.922480953	10	6.379083388	135.056563407
14	6.353377324	138.362659116	14	6.353377324	135.603008624
28	6.319398093	137.622665802	28	6.319398093	136.332142298
56	6.301583891	137.234711452	56	6.301583891	136.717545138
110	6.29262658	137.039640822	112	6.292459356	136.915795771
111	6.292542221	137.037803660	224	6.28784124	137.016353814
112	**6.292459356**	**137.035999037435**	276	6.286966940	137.035408057
113	6.292377945	137.034226098	277	6.286953333	137.035704647
114	6.292297952	137.032484014	**278**	**6.286939823**	**137.035999111818**
			279	6.286926410	137.036291474
			280	6.286913093	137.036581756

In these two formulas (deduced from the modification of Z_{max}), there are some factors which are essentially related to nucleon numbers of some nuclides especially some important stable numbers (stipulated by Chen's Chirality and Poetry Model of Atomic Nuclei[7]) such as 28, 42, 56, 83, 84, 112, 126, 166, 167, 168 *et al.* And these numbers correlate with each others. This kind of relationship is shown in the follows.

A brief illustration of the relationships between the fine-structure constant and nulides:

${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{209}_{83}Po^*_{126}$ ${}^{209}_{84}Po^*_{125}$ ${}^{222}_{86}Rn^*_{136}$ ${}^{223}_{87}Fr^*_{136}$ ${}^{226}_{88}Ra^*_{138}$ ${}^{227}_{89}Ac^*_{138}$ ${}^{285}_{112}Cn^*_{173}$

Above nuclides indicate that 136 – 138, which can be called the fine-structure constant numbers, definitely relate to 112 and 166 – 168 (double of 56 and 83 – 84, the most stable numbers in nuclides).

$$\alpha_1=\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}=\frac{6^2}{7\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{113}{112})^{225}}}\frac{1}{112+\frac{1}{75^2}}=1/137.035999037435$$

Relations to nuclides $(7\cdot(2\pi))_{112}\approx 44;\ {}^{nucleon}_{proton}X_{nutron})$: ${}^{7}_{3}Li_4$ ${}^{9}_{4}Be_5$ ${}^{11}_{5}B_6$ ${}^{12}_{6}C_6$ ${}^{14}_{7}N_7$ ${}^{19}_{9}F_{10}$ ${}^{24}_{12}Mg_{12}$ ${}^{28}_{14}Si_{14}$ ${}^{52}_{24}Cr_{28}$
${}^{55}_{25}Mg_{30}$ ${}^{64}_{28}Ni_{36}$ ${}^{75}_{33}As_{42}$ ${}^{84}_{36}Kr_{48}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{100}_{44}Ru_{56}$ ${}^{88}_{38}Sr_{50}$ ${}^{112}_{48}Cd_{64}$ ${}^{122}_{50}Sn_{72}$ ${}^{125}_{52}Te_{73}$ ${}^{129}_{54}Xe_{75}$ ${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{144}_{60}Nd_{84}$
${}^{150}_{62}Sm_{88}$ ${}^{180}_{72}Hf_{108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^*_{126}$ ${}^{209}_{84}Po^*_{125}$ ${}^{210}_{85}At^*_{125}$ ${}^{222}_{86}Rn^*_{136}$ ${}^{223}_{87}Fr^*_{136}$ ${}^{226}_{88}Ra^*_{138}$ ${}^{227}_{89}Ac^*_{138}$ ${}^{285}_{112}Cn^*_{173}$

$$\alpha_2=\frac{13\cdot(2\pi)_{278}}{10^2}\frac{1}{112-\frac{1}{3\cdot29\cdot64}}=\frac{13\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{9\cdot31}{278})^{557}}}{10^2}\frac{1}{112-\frac{1}{3\cdot29\cdot64}}=1/137.035999111818$$

Relations to nuclides $(13\cdot(2\pi))_{278}\approx 82;\ {}^{nucleon}_{proton}\mathrm{X}_{nutron})$: ${}^{20}_{10}Ne_{10}$ ${}^{27}_{13}\mathrm{Al}_{14}$ ${}^{29}_{14}\mathrm{Si}_{15}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}\mathrm{Fe}_{28,30,31,32}$
${}^{58}_{28}Ni_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{93}_{41}Nb_{52}$ ${}^{100}_{44}Ru_{56}$ ${}^{112,116}_{48}Cd_{64,68}$ ${}^{116}_{50}Sn_{66}$ ${}^{128}_{52}Te_{76}$ ${}^{136,137,38}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$
${}^{149}_{62}Sm_{87}$ ${}^{156,157,160}_{64}Gd_{92,93,96}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{192}_{76}Os_{116}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^*_{126}$ ${}^{223}_{87}Fr^*_{136}$ ${}^{237}_{93}Np^*_{144}$ ${}^{257}_{100}Fm^*_{157}$ ${}^{285}_{112}Cn^*_{173}$

The value of the front part of each above formula is almost equal to $1/(3/2)^{1/2}$ (because 112 is the element natural proton end and 168 is the element natural neutron end as shown in ${}_{112}\mathrm{Cn}_{168+5}$), so the formulas can be transformed to the follows.

$$\alpha_1=\alpha_{1-(3/2)}=\frac{1}{(\frac{3}{2}-\frac{1}{3\cdot112+1}+\frac{1}{2^2\cdot3\cdot5^3\cdot13\cdot23-\frac{30}{64}})^{\frac{1}{2}}}\frac{1}{112+\frac{1}{75^2}}=1/137.035999037435$$

2019 / 4 / 25 Relations to nuclides：

${}^{27}_{13}Al_{14}$ ${}^{27}_{15}P_{16}$ ${}^{40-48}_{20}Ca_{20-28}$ ${}^{51}_{23}V_{28}$ ${}^{55}_{25}Mn_{30}$ ${}^{56}_{26}Fe_{30}$ ${}^{64}_{30}Zn_{34}$ ${}^{72,74,76}_{32}Ge_{40,42,44}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{106,110}_{46}Pd_{60,64}$
${}^{112}_{48}Cd_{64}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{124-130}_{52}Te_{72-78}$ ${}^{155-160}_{64}Gd_{91-96}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{207}_{82}Pb_{125}$ ${}^{209}_{84}Po^*_{125}$

$$\alpha_2=\alpha_{2-(3/2)}=\frac{1}{(\frac{3}{2}-\frac{1}{3\cdot112+1}+\frac{1}{2\cdot7\cdot11\cdot19\cdot29+\frac{36}{75^2}})^{\frac{1}{2}}}\frac{1}{112-\frac{1}{3\cdot29\cdot64}}=1/137.035999111818$$

2019 / 4 / 25 Relations to nuclides:

${}^{28}_{14}Si_{14}$ ${}^{50}_{22}Ti_{28}$ ${}^{58}_{28}Ni_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{88}_{38}Ni_{50}$ ${}^{100,102}_{44}Ru_{56,58}$ ${}^{112,116}_{48}Cd_{64,68}$ ${}^{116,118,119}_{50}Sn_{66,68,69}$
${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{180}_{72}Hf_{108}$ ${}^{187}_{75}\mathrm{Re}_{112}$ ${}^{188,192}_{76}Os_{112,116}$ ${}^{193}_{77}\mathrm{Ir}_{116}$ ${}^{194}_{78}\mathrm{Pt}_{116}$ ${}^{209}_{83}Bi^*_{126}$ ${}^{209}_{84}Po^*_{125}$ ${}^{223}_{87}Fr^*_{136}$

8. The Integrated Fine-structure Constant

Multiplication of α_1 and α_2 should almost divide out the 2π factors and give 3/2 and 112×112 factors, this means $\alpha_1\alpha_2$ is almost equal to 112×168, so we define $\alpha_c=(\alpha_1\alpha_2)^{1/2}$ as the integrated fine-structure constant or Chen's fine-structure constant.

$$\frac{1}{\alpha_c{}^2}=\frac{1}{\alpha_1\alpha_2}=\frac{2\pi a_0}{\lambda_e}\frac{\lambda_e}{2\pi r_e}=\frac{a_0}{r_e}=(\frac{c}{v_e})^2$$

$$=112\times(168-\frac{1}{3}+\frac{1}{12\cdot 47}-\frac{1}{6\cdot 29\cdot 53\cdot 59-79/47})\quad 2018/6/8\text{-}9,\ 9/18\text{-}19,\ 2019/4/19$$

$$=136(138+\frac{1}{2}-\frac{1}{10\cdot 29}+\frac{1}{12\cdot 53\cdot(6\cdot 53-1)-27/47})\qquad 2019/4/17\text{-}19$$

$$=137(137+\frac{1}{13}-\frac{1}{7\cdot 29}+\frac{1}{32\cdot 33\cdot 89+16/49})\qquad 2019/4/17\text{-}19$$

$$=112\cdot 167.668437878408=18778.865042381$$

${}^{27}_{13}\mathrm{Al}_{14}$ ${}^{29}_{14}Si_{15}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{53}_{24}\mathrm{Cr}_{29}$ ${}^{54,56,58}_{26}\mathrm{Fe}_{28,30,32}$ ${}^{59}_{27}\mathrm{Co}_{32}$ ${}^{58,60,61}_{28}\mathrm{Ni}_{30,32,33}$ ${}^{63,65}_{29}\mathrm{Cu}_{34,36}$ ${}^{79}_{35}\mathrm{Br}_{44}$ ${}^{87}_{38}\mathrm{Sr}_{49}$
${}^{100,102}_{44}\mathrm{Ru}_{56,58}$ ${}^{112}_{48}Cd_{64}$ ${}^{113}_{49}In_{64}$ ${}^{135-138}_{56}\mathrm{Ba}_{79-82}$ ${}^{136,138}_{58}Ce_{78,80}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{158,160}_{64}\mathrm{Gd}_{94,96}$ ${}^{159}_{65}Tb_{94}$ ${}^{166,168}_{68}Er_{98,100}$
${}^{174}_{70}Yb_{104}$ ${}^{188}_{76}Os_{112}$ ${}^{197}_{79}Au_{118}$ ${}^{203}_{81}\mathrm{Tl}_{122}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ar^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{293}_{116}Lv^{ie}_{177}$

$$\alpha_c{}^2=\alpha_1\alpha_2=[\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}][\frac{13\cdot(2\pi)_{278}}{10^2}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}]$$

$$=\frac{13\cdot 3^2}{7\cdot 5^2}\frac{e^2}{(\frac{2\cdot 3\cdot 19}{113})^{227}}\frac{e^2}{(\frac{115}{114})^{229}}\cdots\frac{e^2}{(\frac{9\cdot 31}{2\cdot 139})^{557}}\frac{1}{112^2-\frac{1}{30^2\cdot 5}+\frac{1}{60^2\cdot 15}-\frac{1}{120^2\cdot 15\cdot 29}}$$

$$=1/18778.865042381\qquad 2019/12/14$$

${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{39}_{19}K_{20}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}\mathrm{Cu}_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{87}_{38}Sm_{49}$
${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{112-120-124}_{50}Sn_{62-70-74}$ ${}^{135-138}_{56}Ba_{79-82}$ ${}^{139}_{57}La_{82}$ ${}^{136,138}_{58}Ce_{78,80}$ ${}^{144,145}_{60}Nd_{83,84}$ ${}^{157}_{64}Gd_{93}$
${}^{200}_{80}Hg_{120}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{278+7}_{112}Cn^{*}_{166+7}$ ${}^{284}_{113}Nh^{ie}_{9\cdot 19}$

$$\alpha_c{}^2=\alpha_1\alpha_2=\frac{1}{(\frac{3}{2}-\frac{1}{3\cdot 112+1}+\frac{1}{7\cdot 19\cdot 29\cdot 37-\frac{25}{44}})}\frac{1}{112+\frac{1}{75^2}}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}$$

$$=1/18778.865042381\qquad 2019/12/14$$

${}^{39}_{19}K_{20}$ ${}^{47,50}_{22}Ti_{25,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}\mathrm{Cu}_{34,36}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{87,88}_{38}Sr_{49,50}$ ${}^{99,100,102,104}_{44}Ru_{55,56,58,60}$ ${}^{112}_{48}Cd_{64}$
${}^{112,114,115,116,120,124}_{50}Sn_{62,64,65,66,70,74}$ ${}^{5\cdot 37,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$

9. A Brief Explanation of the Fine-structure Constant

According to Chen's Chirality and Poetry Model of Atomic Nuclei[7], the ratio of neutron number N to proton number Z in nuclides increases from 1/1 to 3/2 (eventually slightly above 3/2) along with the increasing of atomic number, for example, from ${}_{14}\mathrm{Si}_{14}$, ${}_{26}\mathrm{Fe}_{30}$, ${}_{29}\mathrm{Cu}_{34,36}$, ${}_{56}\mathrm{Ba}_{82}$, ${}_{84}\mathrm{Po}_{125}$ to ${}_{112}\mathrm{Cn}^{*}{}_{168+5}$. In this process, $(3/2)^{1/2}$ will act as a transition foothold. As for nuclide ${}_{112}\mathrm{Cn}_{168+5}$ with Z=112, N=168+5 and 168/112=3/2, 137 is just right their $(3/2)^{1/2}$ times intermediate stage. This should be why 137 exists and what's the real meaning of 137.

$$\frac{112}{1/\alpha_1} \approx \frac{1/\alpha_2}{168-1/3} \quad \text{or} \quad \frac{112}{137.036} \approx \frac{137.036}{168-1/3}$$

$$137.036^2 \approx 112 \cdot (168-1/3)$$

$$112 \cdot (\frac{3}{2} - \frac{1}{336+1})^{\frac{1}{2}} \approx 137.036, \; 137.036 \cdot (\frac{3}{2} - \frac{1}{336+1})^{\frac{1}{2}} \approx 168-1/3$$

$$\frac{112}{1/\alpha_1} \approx \frac{1/\alpha_2}{168} \quad \text{or} \quad \frac{112}{137} \approx \frac{137}{168}$$

$$137^2 \approx 112 \cdot 168, \; 112(\frac{3}{2})^{\frac{1}{2}} \approx 137, \; 137(\frac{3}{2})^{\frac{1}{2}} \approx 168$$

The relationships between the fine-structure constant and elements are mainly reflected by correlation of nucleon numbers of 56, 68-69, 82, 82-84, 112, 136-138, and 166-168 which are derived from the three key numbers 112, 137 and 168, and by correlation of nucleon numbers of the other factors in Chen's formulas of the fine-structure constant. The former type is illustrated as follows.

$${}^{56}_{26}Fe_{30} \; {}^{63,65}_{29}Cu_{34,36} \; {}^{68}_{30}Zn_{38} \; {}^{69}_{31}Ga_{38} \; {}^{75}_{33}As_{42} \; {}^{82,83,84}_{36}Kr_{46,47,48} \; {}^{98}_{42}Mo_{56} \; {}^{110-112-114,116}_{48}Gd_{62-66,68} \; {}^{118,119}_{50}Sn_{68,69} \; {}^{125,126}_{52}Te_{73,74}$$

$${}^{136,137,138}_{56}Ba_{80,81,82} \; {}^{142,143,144}_{60}Nd_{82,83,84} \; {}^{148-150,152,154}_{62}Sm_{66-68,90,92} \; {}^{151,153}_{63}Eu_{88,90} \; {}^{166,167,168}_{68}Er_{98,99,100} \; {}^{169}_{69}Tm_{100} \; {}^{185,187}_{75}Re_{110,112}$$

$${}^{186-188-190}_{76}Os_{110-112-114} \; {}^{208}_{82}Pb_{126} \; {}^{209}_{83}Bi^{*}_{126} \; {}^{209}_{84}Po^{*}_{125} \; {}^{210}_{85}At^{*}_{125} \; {}^{222}_{86}Rn^{*}_{136} \; {}^{223}_{87}Fr^{*}_{136} \; {}^{226}_{88}Ra^{*}_{138} \; {}^{227}_{89}Ac^{*}_{138} \; {}^{232}_{90}Th^{*}_{142} \; {}^{231}_{91}Pa^{*}_{140} \; {}^{238}_{92}U^{*}_{146} \; {}^{237}_{93}Np^{*}_{144}$$

$${}^{244}_{94}Pu^{*}_{150} \; {}^{243}_{95}Am^{*}_{148} \; {}^{247}_{96}Cm^{*}_{151} \; {}^{251}_{98}Cf^{*}_{153} \; {}^{252}_{99}Es^{*}_{153} \; {}^{257}_{100}Fm^{*}_{157} \; {}^{258}_{101}Md^{*}_{157} \; {}^{259}_{102}No^{*}_{157} \; {}^{268}_{105}Db^{*}_{163} \; {}^{269}_{106}Sg^{*}_{163} \; {}^{270}_{107}Bh^{*}_{163} \; {}^{277}_{108}Hs^{*}_{169} \; {}^{285}_{112}Cn^{*}_{173} \; {}^{294}_{118}Og^{ie}_{176}$$

$${}^{312,314}_{125,126}Ch^{ie}_{187,188} \; {}^{344-348}_{136-138}Fy^{ie}_{208-210} \; {}^{366-376}_{144-149}Ch^{ie}_{222-227} \; {}^{384,386,392,394}_{153,154,155,156}Ch^{ie}_{231,232,237,238} \; {}^{400,402}_{157,158}Ch^{ie}_{243,244} \; {}^{410}_{163}Ch^{ie}_{247} \; {}^{416-420}_{166,168}Ch^{ie}_{250/252} \; {}^{426}_{169}Ch^{ie}_{257}$$

Several clusters of ideal extended elements (*ie*) Fy and Ch are hence predicted.

10. Comparison to Experiment Determined Values

The above two calculated values of the fine-structure constant, i.e., α_1=1/137.035999037435 and α_2=1/137.035999111818 are consistent with those experiment measured values[2], but much more accurate with several more digits.

The above theoretical analysis and formulas also demonstrate there are two different values of the fine-structure constant, i.e. α_1 and α_2. Accordingly, we have found that up to now the experiment determinations of α have almost proved this because the α ranges measured by two different but accurate methods couldn't overlap each other [2]. It seems that the time comes to a critical point to prove there are two values of the fine-structure constant theoretically and experimentally.

11. Theoretical Calculation of the Speed of Light

In atomic units, the line velocity of the ground state electron in hydrogen atom can be assigned as the natural unit of speed ($v_{e/au}$=1), then the speed of light becomes the reciprocal of the fine-structure constant, i.e., c_{au}=1/α=137.035999. However, we have demonstrated that there are two values of α, but the speed of light shouldn't have two values, so by referring to Maxwell's formula of calculating the speed of electromagnetic wave or light, it should be reasonable to suppose the speed of light to be the integrated fine-structure constant, i.e., $c_{au}=1/\alpha_c= 1/(\alpha_1\alpha_2)^{1/2}=137.035999074627$. It means we've theoretically/mathematically calculated the speed of light, the formula is intrinsically consistent with Maxwell's formula, and the value is much accurate.

In atomic units ($e=m_e=\hbar=1$ and $\varepsilon_0=\frac{1}{4\pi}$), $v_{e/au}=\alpha c_{au}=\frac{e^2}{4\pi\varepsilon_0\hbar}=1$, so $c_{au}=\frac{1}{\alpha}$

There are two α (α_1 and α_2), but there shouldn't be two c or c_{au},

so it should be: $c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$ (*au*: atomic units)

Compared to Maxwell Formula $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$ should be reasonable.

$$\mathrm{c}_{au}=\frac{1}{\sqrt{\mu_{0/au}\varepsilon_{0/au}}},\ \mu_{0/au}\varepsilon_{0/au}=\alpha_1\alpha_2,\ \mu_{0/au}=4\pi\alpha_1\alpha_2 \qquad (2019/11/30)$$

So the theroretical formula of the speed of light in atomic units is as follows:

$$\mathrm{c}_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}=\frac{1}{\sqrt{\left(\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}\right)\left(\frac{13\cdot(2\pi)_{278}}{10^2}\frac{1}{112-\frac{1}{3\cdot29\cdot64}}\right)}}$$

$$=\frac{5}{3}\sqrt{\frac{7}{13}\frac{(2\pi)_{112}}{(2\pi)_{278}}\left(112^2-\frac{1}{30^2\cdot5}+\frac{1}{60^2\cdot15}-\frac{1}{120^2\cdot15\cdot29}\right)}$$

$$=\sqrt{\frac{\frac{5\cdot17}{11}-\frac{10}{11\cdot23}}{\frac{36}{7}-\frac{1}{7\cdot19}}\frac{(2\pi)_{12389}}{(2\pi)_{28186}}\left(112^2-\frac{23\cdot(12\cdot41-1)}{10^{10}}\right)}$$

$$=\frac{5}{3}\sqrt{\left(\frac{2^3\cdot17}{11\cdot23}+\frac{2\cdot17}{11\cdot23\cdot97}\right)\frac{(2\pi)_{34450}}{(2\pi)_{28186}}\left(112^2-\frac{18\cdot97+1}{2\cdot10^9}\right)}$$

$$=\sqrt{\left(\frac{3}{2}-\frac{1}{3\cdot112+1}+\frac{1}{7\cdot19\cdot29\cdot37-\frac{25}{44}}\right)\left(112^2-\frac{1}{30^2\cdot5}+\frac{1}{60^2\cdot15}-\frac{1}{120^2\cdot15\cdot29}\right)}$$

$$=\sqrt{\frac{3}{2}\left(112-\frac{1}{3^2}+\frac{1}{12^2\cdot13-\frac{30\cdot19}{100}-\frac{1}{125\cdot100}}\right)}=\sqrt{\frac{3}{2}-\frac{1}{3\cdot112+1}+\frac{1}{14\cdot53\cdot193-\frac{33}{2\cdot47}}}\times112$$

$$=\sqrt{112\times\left(168-\frac{1}{3}+\frac{1}{12\cdot47}-\frac{1}{6\cdot29\cdot53\cdot59-79/47}\right)}$$

$$=\sqrt{137.035999037435\times137.035999111818}=137.035999074627$$

Note: $112/278\approx27/67$, $12389/28186\approx11/25$, $34450/28186\approx11/9\approx66/29$

Discover: 2019/12/16; Revise and Supplement: 2020/1/5–8, 2/24, 3/28-29

12. The Special 29 and 75 Factors

In the above formulas some factors especially 29 and 75 appear several times. This feature should be analyzed and explained. Accompanying N/Z ratio from 1/1 to slightly above 3/2 along with the increasing of atomic number, $_{29}Cu_{34,36}$ is the critical point of N/Z ratio approaching $(3/2)^{1/2}$ and $_{75}Re_{110,112}$ is the critical point of N/Z ratio approaching 3/2 (**Table 3**, **Fig. 4** and **Fig. 5**), so 29 and 75 are important factors and hence frequently appear in the formulas.

Table 3. N/Z ratios of the Elements (2019/4/23).

	Z	N	N/Z		Z	N	N/Z		Z	N	N/Z		Z	N	N/Z
H	1	0	0	Ga	31	38.80	1.25	Pm	61	84	1.38	Pa*	91	140	1.54
He	2	2.00	1.00	Ge	32	40.71	1.27	Sm	62	88.45	1.43	U*	92	146	1.59
Li	3	3.92	1.31	As	33	42	1.27	Eu	63	89.04	1.41	Np*	93	144	1.55
Be	4	5	1.25	Se	34	45.05	1.33	Gd	64	93.33	1.46	Pu*	94	150	1.60
B	5	5.80	1.16	Br	35	44.98	1.29	Tb	65	94	1.45	Am*	95	148	1.56
C	**6**	**6.01**	**1.00**	Kr	36	47.89	1.33	Dy	66	96.57	1.46	Cm*	96	151	1.57
N	7	7.00	1.00	Rb	37	48.56	1.31	Ho	67	98	1.46	Bk*	97	150	1.55
O	8	8.00	1.00	Sr	38	49.71	1.31	Er	68	99.33	1.46	Cf*	98	153	1.56
F	9	10	1.11	Y	39	50	1.28	Tm	69	100	1.45	Es*	99	153	1.55
Ne	10	10.19	1.02	Zr	40	51.32	1.28	Yb	70	103.11	1.47	Fm*	100	157	1.57
Na	11	12	1.09	Nb	41	52	1.27	Lu	71	104.03	1.47	Md*	101	157	1.55
Mg	12	12.32	1.03	Mo	42	54.04	1.29	Hf	72	106.54	1.48	No*	102	157	1.54
Al	13	14	1.08	Td	43	55	1.28	Ta	73	108	1.48	Lr*	103	159	1.54
Si	14	14.11	1.01	Ru	44	57.16	1.30	W	74	109.89	1.49	Rf*	104	159	1.55
P	15	16	1.07	Rh	45	58	1.29	**Re**	**75**	**111.25**	**1.48**	Db*	105	163	1.55
S	16	16.09	1.01	Pd	46	60.51	1.32	Os	76	114.27	1.50	Sg*	106	165	1.56
Cl	17	18.48	1.09	Ag	47	60.96	1.30	Ir	77	115.25	1.50	Bh*	107	163	1.52
Ar	18	21.99	1.22	Cd	48	64.52	1.34	Pt	78	117.12	1.50	Hs*	108	162	1.56
K	19	20.13	1.06	In	49	65.91	1.35	Au	79	118	1.49	Mt*	109	169	1.53
Ca	20	20.12	1.01	Sn	50	68.81	1.38	Hg	80	120.62	1.51	Ds*	110	171	1.55
Sc	21	24	1.14	Sb	51	70.86	1.39	Tl	81	123.41	1.52	Rg*	111	170	1.52
Ti	22	25.92	1.18	Te	52	75.70	1.46	Pb	82	125.24	1.53	**Cn***	**112**	**173**	**1.54**
V	23	28	1.22	I	53	74	1.40	Bi*	83	126	1.52	Nh*	113	173	1.51
Cr	24	28.06	1.17	Xe	54	77.39	1.43	Po*	84	125	1.49	Fl*	114	175	1.54
Mn	25	30	1.20	Cs	55	78	1.42	At*	85	125	1.47	Mc*	115	174	1.50
Fe	26	29.91	1.15	Ba	56	81.42	1.45	Rn*	86	136	1.58	Lv*	116	177	1.53
Co	27	32	1.19	La	57	82	1.44	Fr*	87	136	1.56	Ts*	117	177	1.51
Ni	28	30.76	1.10	Ce	58	82.21	1.42	Ra*	88	138	1.57	Og*	118	176	1.49
Cu	**29**	**34.62**	**1.19**	Pr	59	82	1.39	Ac*	89	138	1.55				
Zn	30	35.45	1.18	Nd	60	84.41	1.41	Th*	90	142	1.58				

Z: atomic number, N: average neutron number or neutron number of the most stable isotope.

1. N/Z from 1/1 ($_{6}$C) to slightly above 3/2 (such as $_{112}$Cn which is the natural end of elements demonstrated by Chen's Chirality and Poetry Model of Atomic Nuclei[7]).
2. For $_{29}$Cu, N/Z ratio 1.19 is near to $(3/2)^{1/2}$=1.22，slightly less is because of stability effect.
3. For $_{75}$Re, N/Z ratio 1.48 is near to 3/2=1.50，slightly less is because of stability effect.
4. From $_{6}$C to $_{112}$Cn, the middle of N/Z 1.5 range is at (76.5-5)/(112-5)=0.668≈2/3 position.

Fig. 4 and **Fig. 5** shows that stability effect of nucleon number 64 makes the neutron numbers of $_{29}$Cu's isotopes are relatively less (34 and 36) than normal so that its N/Z ratio is a little less than $(3/2)^{1/2}$ which is otherwise it should be. Also the stability effect of nucleon numbers 110 and 112 make the neutron numbers of $_{75}$Re's nuclides are relatively less (110 and 112) than normal so that its N/Z ratio is a little less than 3/2 which otherwise it should be.

Fig. 4. Complete Graph of N/Z Ratios of Elements (2019/4/23-24).

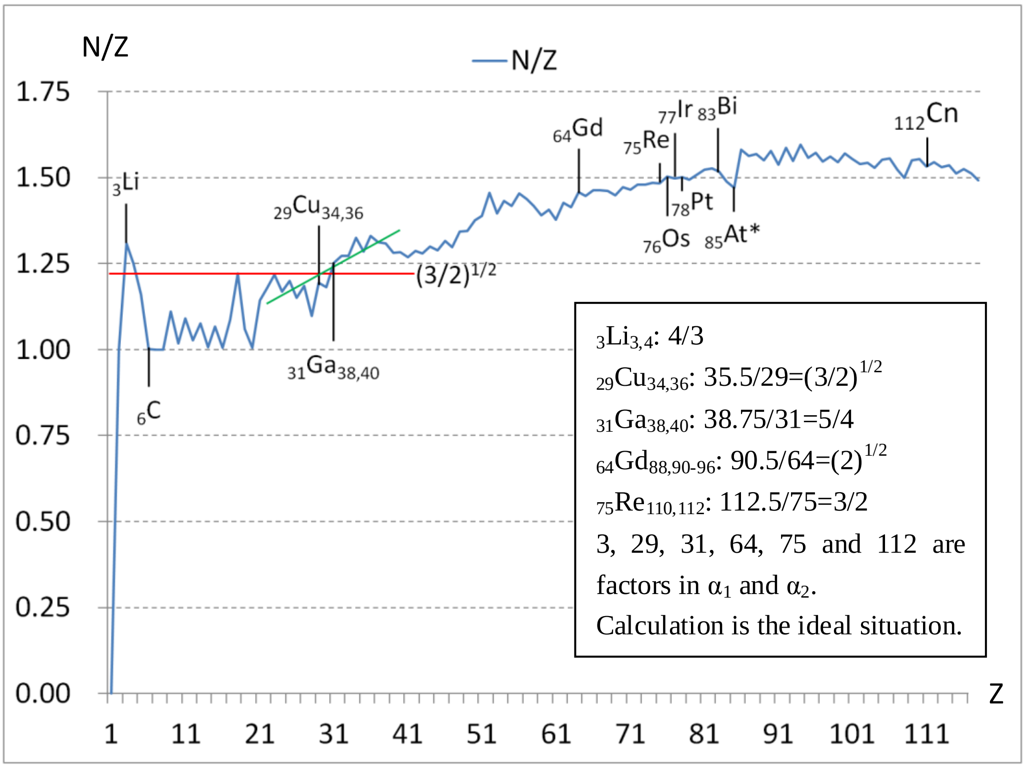
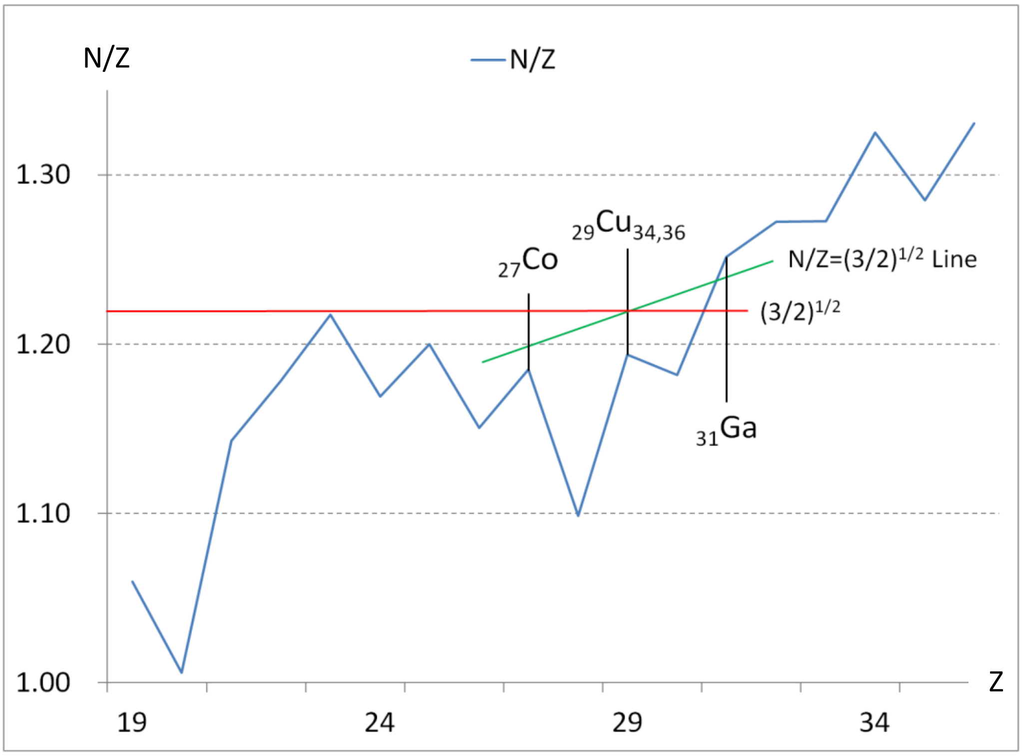


Fig. 5. Partially Amplified Graph of N/Z Ratios of Elements (2019/4/24).

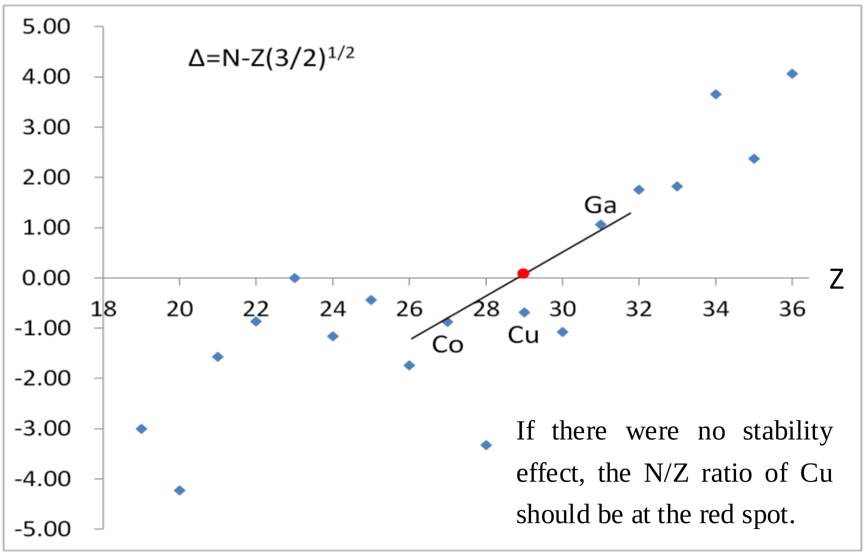


The general trend of N/Z ratio of elements is from 1/1 (${}_{6}C_{6}$) to slightly above 3/2 (${}_{112}Cn_{173}$) definitely. However, the increasing process is not smooth, the N/Z ratio rising fluctuates consecutively. According to Chen's Chirality and Poetry Model of Atomic Nuclei[7], there are some stable numbers (magic numbers) which can bring about this kind of fluctuation (**Table 4** and **Fig. 6**).

Table 4. Effect of Stable Numbers on N/Z ratio's fluctuation (2019/4/22).

Element	Z	N(Average)	$Z(3/2)^{1/2}$	$N\text{-}Z(3/2)^{1/2}$	Stable Number
K	19	20.13	23.27	-3.17	20
Ca	20	20.12	24.49	-4.41	20+20
Sc	21	24	25.72	-1.74	
Ti	22	25.92	26.94	-1.07	22+26=48
V	23	28.00	28.17	-0.23	28
Cr	24	28.06	29.39	-1.39	28
Mn	25	30	30.62	-0.68	
Fe	26	29.91	31.84	-1.99	26+30=56
Co	27	32.00	33.07	-1.14	
Ni	28	30.76	34.29	-3.60	28+30=58、28+32=60
Cu	**29**	**34.62**	**35.52**	**-0.97**	**64**
Zn	30	35.45	36.74	-1.36	30+34=64、30+36=66
Ga	31	38.80	37.97	0.75	
Ge	32	40.71	39.19	1.44	32+40=72
As	33	42.00	40.42	1.50	
Se	34	45.05	41.64	3.30	34+46=80
Br	35	44.98	42.87	2.03	
Kr	36	47.89	44.09	3.71	36+48=84

Fig. 6. Effect of Stable Numbers on N/Z ratio's fluctuation (2019/4/22-23)



13. α_1/α_2 in Schrödinger Equation of Hydrogen Atom

Stationary Schrodinger Equation $-\frac{\hbar^2}{2m}\nabla^2\psi + U\psi = E\psi$, applied to hydron atom:

$\nabla^2\psi + \frac{2m_e}{\hbar^2}(E + \frac{e^2}{4\pi\varepsilon_0 r})\psi = 0$, $E = -\frac{m_e e^4}{2n^2(4\pi\varepsilon_0)^2\hbar^2}$, do substitution and simlification:

$$\frac{2m_e}{\hbar^2}(\frac{m_e e^4}{2n^2(4\pi\varepsilon_0)^2\hbar^2} - \frac{e^2}{4\pi\varepsilon_0 r})\psi = \nabla^2\psi, \quad [\frac{1}{n^2}(\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2})^2 - \frac{2}{r}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2}]\psi = \nabla^2\psi,$$

$$[\frac{1}{n^2}(\frac{e^2}{4\pi\varepsilon_0\hbar c}\frac{m_e c}{\hbar})^2 - \frac{2}{r}\frac{e^2}{4\pi\varepsilon_0\hbar c}\frac{m_e c}{\hbar}]\psi = \nabla^2\psi,$$

As $\sqrt{\alpha_1\alpha_2} = \frac{v_e}{c} = \frac{e^2}{4\pi\varepsilon_0\hbar c}$, $\lambda_e = \frac{h}{m_e c}$ and $\alpha_1 = \frac{\lambda_e}{2\pi a_0}$:

$$[\frac{1}{n^2}(\sqrt{\alpha_1\alpha_2}\frac{2\pi}{\lambda_e})^2 - \frac{2}{r}\sqrt{\alpha_1\alpha_2}\frac{2\pi}{\lambda_e}]\psi = \nabla^2\psi,$$

$$[\frac{1}{n^2(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})^2} - \frac{2}{(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})r}]\psi = \nabla^2\psi,$$

$$[\frac{1}{n^2 a_0^2(\alpha_1/\alpha_2)} - \frac{2}{a_0 r\sqrt{\alpha_1/\alpha_2}}]\psi = \nabla^2\psi$$

As $\alpha_1/\alpha_2 \approx 1$, simplyfied to: $[\frac{1}{n^2 a_0^2} - \frac{2}{a_0 r}]\psi = \nabla^2\psi$ (factor 2 seems not beautiful)

In atomic units (au: $e = m_e = \hbar = 1$ and $\varepsilon_0 = \frac{1}{4\pi}$),

$$a_{0/au} = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = 1, \quad v_{e/au} = \frac{e^2}{4\pi\varepsilon_0\hbar} = 1, \quad c_{au} = \frac{v_{e/au}}{\alpha_c} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$[\frac{1}{n^2(\alpha_1/\alpha_2)} - \frac{2}{r_{au}\sqrt{\alpha_1/\alpha_2}}]\psi = \nabla_{au}^2\psi, \text{ or } (\frac{c_{au}^2}{\alpha_1^2 n^2} - \frac{2c_{au}}{\alpha_1 r_{au}})\psi = \nabla_{au}^2\psi$$

the above equation could be called Schrodinger-Chen eqution of hydarogen atom, the later form of the equation shows factor 2 is still reasonable and beautiful.

As $\alpha_1/\alpha_2 \approx 1$, simplyfied to: $[\frac{1}{n^2} - \frac{2}{r_{au}}]\psi = \nabla_{au}^2\psi$

Discover: 2018/4-6; Revise: 2019/12/13 (add au form)

$$\alpha_1/\alpha_2 = \frac{137.035999111818}{137.035999037435} = 1.0000000005428 = 1 + \frac{23\cdot 59}{25\cdot 10^{11}} = (1 + \frac{23\cdot 59}{50\cdot 10^{11}})^2$$

$$\sqrt{\alpha_1/\alpha_2} = 1 + \frac{23\cdot 59}{50\cdot 10^{11}} = 1.000000002714$$

Relations to nuclides: ${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{99,100}_{44}\mathrm{Ru}_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{137}_{56}Ba_{81}$ ${}^{118+1}_{50}Sn_{69}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{169}_{88}Ra^*_{137}$

2019/8/28–29

Solution of Schrödinger equation of hydrogen atom gives some quantum numbers such as n, l and m_l which determine the electron shell structure and the chemical properties of atoms. That means Schrödinger equation of hydrogen atom is the base of chemical periodicity of elements. On the other hand, from above analysis, we have already demonstrated the formulas of the fine-structure constant α are derived from Chen's Chirality and Poetry Model of Atomic Nuclei[7] and hence mainly connected to the stability of atomic nuclei. So, a question is whether and how α is connected to Schrödinger Equation of hydrogen atom. This question should reveal the connection of the theory of electron shell of atoms and the theory of nuclei of elements. The above deduction provides the answer. The fine-structure constant α relates to Schrödinger Equation of hydrogen atom in α_1/α_2 way which is subtle and negligible but could show the equation is really reasonable and beautiful.

14. The Two Kinds of General Formulas of the Fine-structure Constant

Based on the above two formulas of α_1 and α_2, it should be reasonable to assume there are two kinds of serial formulas of α_1 and α_2 which are listed in follows. Among these formulas, the above two first discovered formulas are the most fundamental and important. Some formulas both with a big m and an extra large k should be more important referring to the trend of the approximate values of α.

Approximate formulas:

$$\alpha_{1-m'}=\frac{n}{m\cdot(2\pi)_k}\frac{1}{112}=\frac{n}{m\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}\frac{1}{112}\approx 1/137.036$$

$$\alpha_{2-m'}=\frac{m\cdot(2\pi)_k}{n}\frac{1}{112}=\frac{m\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}{n}\frac{1}{112}\approx 1/137.036$$

Accurate Formulas:

$$\alpha_{1-m}=\frac{n}{m\cdot(2\pi)_k}\frac{1}{112+\delta_1}=\frac{n}{m\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}\frac{1}{112+\delta_1}$$

$=1/137.035999037435$

$$\alpha_{2-m}=\frac{m\cdot(2\pi)_k}{n}\frac{1}{112-\delta_2}=\frac{m\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}{n}\frac{1}{112-\delta_2}$$

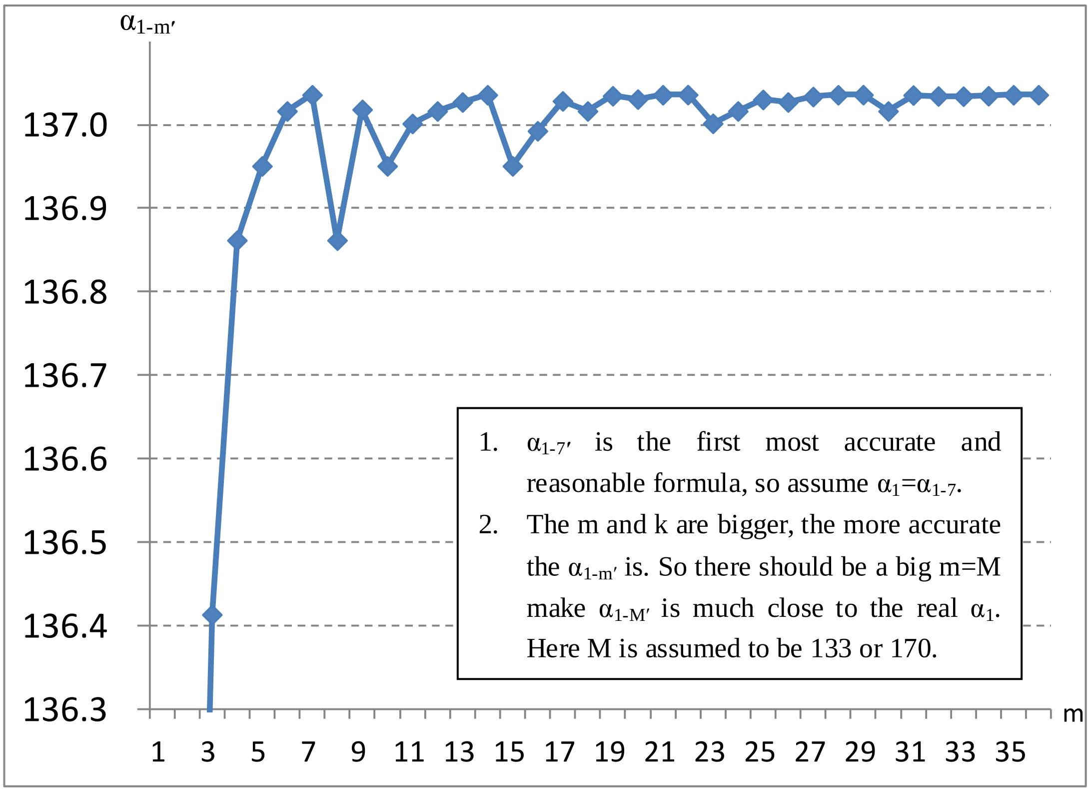
$=1/137.035999111818$

Discover: 2019/6/27; Revise: 2019/7/2-3

Table 5. Parameters and Results of Approximate Formulas of α_1 (2019/7/2).

m	n	k	$\alpha_{1\text{-}m'}$	m	n	k	$\alpha_{1\text{-}m'}$
1	6	1	122.265854937	24	124	27	137.016359405
2	11	2	135.230901223	**25**	129	34	137.030171763
3	**16**	4	136.413250690	26	134	46	137.027100696
4	21	7	136.861626741	27	139	66	137.033636049
5	26	13	136.950569252	28	144	112	137.035781520
6	31	27	137.016359405	29	149	**321**	137.035917078
7	**36**	**112**	**137.035781520**	30	155	27	137.016359405
8	42	7	136.861626741	31	**160**	32	137.035453560
9	47	9	137.018237882	**32**	165	40	137.034309209
10	52	13	136.950569252	33	170	52	137.034083409
11	57	18	137.001388822	34	**175**	72	137.034617877
12	62	27	137.016359405	35	180	112	137.035781520
13	67	46	137.027100696	**36**	185	**236**	137.035810961
14	72	112	137.035781520	43	221	**200**	137.035845637
15	78	13	136.950569252	50	257	**181**	137.035307038
16	83	16	136.992590996	59	303	**2645**	137.035986189
17	**88**	20	137.028423583	**81**	**416**	**1605**	137.035992406
18	93	27	137.016359405	**96**	493	**5806**	137.035998789
19	**98**	37	137.034579883	103	**529**	**1310**	137.035994308
20	103	58	137.030572071	**133**	**683**	**12389**	**137.035999034**
21	108	112	137.035781520	140	719	**1923**	137.035994882
22	**113**	**782**	**137.035967638**	155	**796**	**3988**	137.035997989
23	119	22	137.001596764	**170**	**873**	**34450**	**137.035999031**

Fig. 7. Results of Approximate Formulas of α_1 (2019/7/2).



$$\alpha_{1-1}=\frac{6}{1\cdot e^{2}(\frac{2}{1})^{2}}\frac{1}{112+\frac{17}{2}-\frac{1}{40}+\frac{1}{6\cdot 23\cdot 25-\frac{36}{55}}}=1/137.035999037434$$

${}^{12}_{6}C_{6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$
${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{74,78,80,82}_{34}Se_{40,44,46,48}$ ${}^{80,82,84,86}_{36}Kr_{44,46,48,50}$ ${}^{90,91,96}_{40}Zr_{50,51,56}$ ${}^{102,106}_{46}Pd_{56,60}$
${}^{116,118,119,120,122}_{50}Sn_{66,68,69,70,72}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{8\cdot 36}_{115=5\cdot 23}Mc^{ie}_{173}$

$$\alpha_{1-2}=\frac{11}{2\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}}\frac{1}{112+\frac{3}{2}-\frac{1}{200}+\frac{1}{5\cdot(3\cdot 42+1)\cdot(6\cdot 37-1)+\frac{2}{7}}}=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{79}_{35}Br_{44}$ ${}^{87}_{37}Rb_{50}$ ${}^{92,97,98}_{42}Mo_{50,55,56}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{90}_{40}Zr_{50}$ ${}^{200}_{80}Hg_{120}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{420}_{165}Ch^{ie}_{15\cdot 17}$

$$\alpha_{1-3}=\frac{4^{2}}{3\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\frac{e^{2}}{(\frac{4}{3})^{7}}\frac{e^{2}}{(\frac{5}{4})^{9}}}\frac{1}{112+1-\frac{1}{2}+\frac{1}{88}-\frac{1}{13\cdot(2\cdot 9\cdot 5\cdot 13+1)-\frac{2}{73}}}$$
$$=1/137.035999037435$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{40}_{18}Ar_{22}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$
${}^{73}_{32}Ge_{41}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{112}_{48}Cd_{64}$ ${}^{125}_{52}Te_{73}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\alpha_{1-4}=\frac{21}{2^{2}\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{8}{7})^{15}}}\frac{1}{112+\frac{1}{7}-\frac{1}{8\cdot 19\cdot 41-\frac{75}{98}}}=1/137.035999037435$$

${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{75}_{33}As_{42}$ ${}^{98}_{42}Mo_{56}$ ${}^{129}_{54}Ke_{75}$ ${}^{208}_{82}Pb_{126}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{5\cdot 37}$ ${}^{326,326/8\cdot 41}_{128,129}Ch^{ie}_{198,197/199}$
${}^{18\cdot 21,380,382}_{150,151,152=8\cdot 19}Ch^{ie}_{12\cdot 19,229,230}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{1-5}=\frac{26}{5\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{14}{13})^{27}}}\frac{1}{112+\frac{1}{14}-\frac{1}{9\cdot 71}+\frac{1}{67\cdot(75\cdot 100-1)+\frac{1}{10}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{100}_{44}Ru_{56}$ ${}^{117}_{50}Sn_{67}$ ${}^{126,128}_{52}Te_{74,76}$ ${}^{165}_{67}Ho_{98}$ ${}^{168}_{68}\mathrm{Re}_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$
${}^{185,186}_{75}\mathrm{Re}_{110,112}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{9\cdot 19}$ ${}^{20\cdot 17}_{134=2\cdot 67,135=5\cdot 27}Ch^{ie}_{206,5\cdot 41}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

$$\alpha_{1-6}=\frac{31}{6\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{28}{27})^{55}}}\frac{1}{112+\frac{1}{2\cdot 31}-\frac{1}{3\cdot 11\cdot 13\cdot 31-\frac{43}{4\cdot 27}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{75}_{33}As_{42}$ ${}^{89}_{39}Y_{50}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{108}_{46}Pd_{62}$ ${}^{148,150}_{62}Sm_{86,88}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{310,312}_{124,125}Ch^{ie}_{186,187}$
${}^{3\cdot 11^{2},28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{7\cdot 33,8\cdot 29}$ ${}^{392,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,238}$

$$\alpha_{1}=\alpha_{1-7}=\frac{6^{2}}{7\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{113}{112})^{225}}}\frac{1}{112+\frac{1}{75^{2}}}=1/137.035999037435$$

${}^{7}_{3}F_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{40}_{18}Ar_{22}$ ${}^{47}_{22}Ti_{25}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$
${}^{58,60,61,64}_{28}Ni_{30,32,33,36}$ ${}^{75}_{33}As_{42}$ ${}^{82-84}_{36}Kr_{46-48}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{88}_{38}Sr_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{125}_{52}Te_{73}$ ${}^{129}_{54}Xe_{75}$ ${}^{136-138}_{56}\mathrm{Ba}_{80-82}$
${}^{142-146,148,150}_{60}Nd_{82-86,88,90}$ ${}^{150}_{62}Sm_{88}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{207,208}_{82}Pt_{125,126}$ ${}^{209}_{84}Po^{*}_{125}$
${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171=9\cdot 19}$ ${}^{312,314}_{125,126}Ch^{ie}_{187,188}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{366,372}_{144,147}Ch^{ie}_{222,225}$

$$\alpha_{1-9}=\frac{47}{3^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{10}{9})^{19}}}\frac{1}{112+\dfrac{1}{4\cdot 17}-\dfrac{1}{2(8\cdot 9\cdot 37-1)+\dfrac{3\cdot 17}{157}}}=1/137.035999037436$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{106}_{51}Sb_{72}$ ${}^{152,168}_{68}Er_{94,100}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258=6\cdot 43}_{101}Md^{*}_{157}$ ${}^{7\cdot 37}_{102=6\cdot 17}Md^{*}_{157}$

${}^{24\cdot 13,2\cdot 157}_{125,126=9\cdot 14}Ch^{ie}_{11\cdot 17,4\cdot 47}$ ${}^{4\cdot 89}_{141=3\cdot 47}Ch^{ie}_{5\cdot 43}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$

$$\alpha_{1-11}=\frac{3\cdot 19}{11\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{19}{18})^{37}}}\frac{1}{112+\dfrac{1}{35}-\dfrac{1}{88\cdot 41-\dfrac{5\cdot 53}{22\cdot 13}}}=1/137.035999037435$$

${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{41}_{19}Ar_{22}$ ${}^{48}_{22}Ti_{26}$ ${}^{56,57}_{26}Fe_{30,31}$ ${}^{79}_{35}Br_{44}$ ${}^{93}_{41}Nb_{52}$ ${}^{99,100,101}_{44}Ru_{55,56,57}$ ${}^{127}_{53}I_{74}$ ${}^{139}_{57}La_{82}$ ${}^{180,186}_{74}W_{106,112}$

${}^{235}_{92}U^{*}_{143}$ ${}^{289}_{114}Fl^{ie}_{175}$ ${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$ ${}^{3\cdot 11^2,28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{404}_{159=3\cdot 53}Ch^{ie}_{7\cdot 35}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{1-13}=\frac{67}{13\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{47}{2\cdot 23})^{3\cdot 31}}}\frac{1}{112+\dfrac{1}{137}-\dfrac{1}{6(2\cdot 27\cdot 59+1)+\dfrac{9}{50}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{59}_{27}Co_{32}$ ${}^{89}_{39}Y_{50}$ ${}^{94}_{40}Zr_{54}$ ${}^{93}_{41}Nb_{52}$ ${}^{94,96}_{42}Mo_{52,54}$ ${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$

${}^{9\cdot 13,118,119}_{50}Sn_{67,68,69}$ ${}^{130-132}_{54}Xe_{76-78}$ ${}^{137}_{56}Ba_{81}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{157}_{64}Gd_{93}$ ${}^{165}_{67}Ho_{98}$ ${}^{169}_{69}Tm_{100}$ ${}^{196}_{78}Pt_{118}$ ${}^{198,200,201}_{80}Hg_{118,120,121}$

${}^{209}_{83}Bi^{*}_{126}$ ${}^{157}_{100}Fm^{*}_{157}$ ${}^{294}_{117=9\cdot 13}Ts^{ie}_{3\cdot 59}$ ${}^{310,24\cdot 13,314}_{124,125,126}Ch^{ie}_{186,187,4\cdot 47}$ ${}^{330,4\cdot 83}_{130,131}Ch^{ie}_{200,201}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-16}=\frac{83}{4^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{17}{16})^{33}}}\frac{1}{112+\dfrac{1}{28}-\dfrac{1}{6\cdot(18\cdot 41+1)+\dfrac{173}{2\cdot(2\cdot 75-1)}}}=1/137.035999037435$$

${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{60,61,62,64}_{28}Ni_{32,33,34,36}$ ${}^{73}_{32}Ge_{41}$ ${}^{75}_{33}As_{42}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{129}_{54}Xe_{75}$ ${}^{138}_{56}Ba_{82}$ ${}^{173}_{70}Yb_{103}$ ${}^{11\cdot 17}_{75}Re_{112}$

${}^{208}_{82}Pt_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Bi^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{32\cdot 13}_{164}Ch^{ie}_{36\cdot 7}$ ${}^{418}_{166,168}Ch^{ie}_{252,250}$

$$\alpha_{1-17}=\frac{2^2\cdot 22}{17\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{21}{20})^{41}}}\frac{1}{112+\dfrac{1}{137}-\dfrac{1}{2\cdot 19\cdot 23\cdot 59-\dfrac{30}{100}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{38,40}_{18}Ar_{20,22}$ ${}^{39,41}_{19}K_{20,22}$ ${}^{40,42,43,54}_{20}Ca_{20,22,23,34}$ ${}^{46,50}_{22}Ti_{24,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{64,68}_{30}Zn_{34,38}$

${}^{78,80,82}_{34}Se_{44,46,48}$ ${}^{80,82,84}_{36}Kr_{44,46,48}$ ${}^{84,88}_{38}Sr_{46,50}$ ${}^{100}_{44}Ru_{56}$ ${}^{105,106,108}_{46}Pd_{59,60,64}$ ${}^{118}_{50}Sn_{68}$ ${}^{137}_{56}Ba_{81}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{142,150}_{60}Nd_{82,88}$

${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{179,180}_{72}Hf_{107,108}$ ${}^{184,188}_{76}Hf_{108,112}$ ${}^{208}_{82}Pt_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{14\cdot 21}_{118}Og^{ie}_{16\cdot 11}$

${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$ ${}^{20\cdot 17}_{134,135=5\cdot 27}Ch^{ie}_{206,5\cdot 41}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{1-19}=\frac{2\cdot 7^2}{19\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{38}{37})^{75}}}\frac{1}{112+\dfrac{1}{2\cdot(8\cdot 54-1)+\dfrac{54}{19^2}}}=1/137.035999037440$$

${}^{49}_{22}Ti_{27}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{87}_{38}Sr_{49}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{128,129,130}_{54}Xe_{74,75,76}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{18\cdot 21,380,382}_{150,151,152=8\cdot 19}Ch^{ie}_{12\cdot 19,229,230}$

$$\alpha_{1-20}=\frac{103}{2^2\cdot 5\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{59}{2\cdot 29})^{9\cdot 13}}}\frac{1}{112+\dfrac{1}{32\cdot 45\cdot 79+\dfrac{22}{3\cdot 17}}}=1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{70,72,76}_{32}Ge_{38,40,44}$ ${}^{89}_{39}Y_{50}$ ${}^{103}_{45}Rh_{58}$ ${}^{135,136}_{56}Ba_{79,80}$ ${}^{136,138}_{58}Ce_{78,80}$ ${}^{197}_{79}Au_{118}$ ${}^{173}_{70}Yb_{103}$

${}^{14\cdot 21}_{118}Og^{ie}_{16\cdot 11}$ ${}^{3\cdot 11^2,28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$ ${}^{394,394}_{155,156=12\cdot 13}Ch^{ie}_{3\cdot 79,14\cdot 17}$ ${}^{402}_{158}Ch^{ie}_{244}$

$$\alpha_{1-22}=\frac{113}{22\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{27\cdot 29}{2\cdot 17\cdot 23})^{5\cdot(2\cdot 157-1)}}}\,\frac{1}{112+\dfrac{1}{2\cdot[2\cdot 3\cdot 17\cdot(10\cdot 19+1)+1]+\dfrac{29}{49}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{39}_{19}K_{20}$ ${}^{49,50}_{22}Ti_{27,28}$ ${}^{50,51}_{23}Cd_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{77,78,80}_{34}\mathrm{Se}_{43,44,46}$ ${}^{98-102}_{44}Ru_{54-58}$ ${}^{104,106}_{46}Pd_{58,60}$ ${}^{113,115}_{49}In_{64,66}$

${}^{157}_{64}Gd_{93}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{6\cdot 43}_{101}Md^{*}_{157}$ ${}^{7\cdot 37}_{102=6\cdot 17}No^{*}_{157}$ ${}^{284}_{113}Nh^{ie}_{9\cdot 19}$ ${}^{24\cdot 13,2\cdot 157}_{125,126}Ch^{ie}_{11\cdot 17,188}$

${}^{22\cdot 17}_{148}Ch^{ie}_{226}$ ${}^{3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$ ${}^{400}_{157}Ch^{ie}_{9\cdot 27}$

$$\alpha_{1-23}=\frac{7\cdot 17}{23\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{23}{22})^{45}}}\,\frac{1}{112+\dfrac{1}{35}-\dfrac{1}{4\cdot 13\cdot 43-\dfrac{2\cdot 29}{16\cdot 17-1}}}=1/137.035999037435$$

${}^{32,33,34}_{16}S_{16,17,18}$ ${}^{35}_{17}Cl_{18}$ ${}^{51}_{23}Cd_{28}$ ${}^{56}_{26}Fe_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{238=14\cdot 17}_{92=4\cdot 23}U^{*}_{146}$ ${}^{288}_{115}Mc^{ie}_{173}$

${}^{298,300}_{119,120}Ch^{ie}_{179,4\cdot 45}$ ${}^{3\cdot 11^{2},28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$ ${}^{14\cdot 29}_{160,161=7\cdot 23}Ch^{ie}_{6\cdot 41,5\cdot 49}$ ${}^{420=12\cdot 35}_{165=11\cdot 15}Ch^{ie}_{15\cdot 17}$

$$\alpha_{1-25}=\frac{3\cdot 43}{5^{2}\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{35}{34})^{3\cdot 23}}}\,\frac{1}{112+\dfrac{1}{11\cdot 19}-\dfrac{1}{13^{2}(16\cdot 17-1)+\dfrac{11}{25}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{25}_{12}Mg_{13}$ ${}^{27}_{13}Al_{14}$ ${}^{34}_{16}S_{18}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,41}_{19}K_{20,22}$ ${}^{47,49}_{22}Ti_{25,26}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$

${}^{78,79,81}_{34}Se_{43,44,46}$ ${}^{79,81}_{35}Br_{44.46}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{100}_{44}Ru_{56}$ ${}^{169}_{69}Tm_{100}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$

${}^{24\cdot 13,314}_{125,126}Ch^{ie}_{11\cdot 17,188}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{3\cdot 11^{2},28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{418}_{168}Ch^{ie}_{250}$

$$\alpha_{1-27}=\frac{139}{27\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{67}{66})^{7\cdot 19}}}\,\frac{1}{112+\dfrac{1}{11\cdot 47+\dfrac{18}{23}+\dfrac{1}{138\cdot 137}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{40}_{18}Ar_{22}$ ${}^{41}_{19}K_{22}$ ${}^{47,49,50}_{22}Ti_{25,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{131,132,134}_{54}Xe_{77,78,80}$ ${}^{133}_{55}Cs_{78}$ ${}^{137}_{56}Ba_{81}$

${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{334,2\cdot 168}_{132,133=7\cdot 19}Ch^{ie}_{6\cdot 37,7\cdot 29}$ ${}^{340}_{134=2\cdot 67,135=5\cdot 27}Ch^{ie}_{206,205}$ ${}^{346,348}_{137,138}Fy^{ie}_{209,210}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{209}$

$$\alpha_{1-29}=\frac{149}{29\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{14\cdot 23}{3\cdot 107})^{643}}}\,\frac{1}{112+\dfrac{1}{6\cdot 8\cdot(12\cdot 26-1)+\dfrac{11}{18}}}=1/137.035999037434$$

${}^{12}_{6}C_{6}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29}_{14}Si_{14,15}$ ${}^{40}_{18}Ar_{22}$ ${}^{51}_{23}V_{28}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{100}_{44}Ru_{56}$ ${}^{106,108}_{46}Pd_{60,62}$ ${}^{107}_{47}Ag_{60}$

${}^{112,114}_{48}Cd_{64,66}$ ${}^{149}_{62}Sm_{87}$ ${}^{179}_{72}Hf_{107}$ ${}^{2\cdot 149,300}_{119=7\cdot 17,120}Ch^{ie}_{179,180}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$ ${}^{14\cdot 29}_{160,161=7\cdot 23}Ch^{ie}_{6\cdot 41,5\cdot 49}$

$$\alpha_{1-31}=\frac{4^{2}\cdot 10}{31\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{33}{32})^{5\cdot 13}}}\,\frac{1}{112+\dfrac{1}{12\cdot 11\cdot 17-\dfrac{4\cdot 49}{5\cdot 41}}}=1/137.035999037434$$

${}^{71}_{31}Ga_{40}$ ${}^{73,74,76}_{32}Ge_{41,42,44}$ ${}^{75}_{33}As_{42}$ ${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ ${}^{93}_{41}Nb_{52}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$

${}^{310,312}_{124,125}Ch^{ie}_{186,187}$ ${}^{3\cdot 11^{2},28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{12\cdot 31}_{147=3\cdot 49}Ch^{ie}_{225}$ ${}^{8\cdot 49,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,14\cdot 17}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$ ${}^{420}_{165=5\cdot 33}Ch^{ie}_{15\cdot 17}$

$$\alpha_{1-32}=\frac{15\cdot 11}{2\cdot 4^{2}\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{41}{40})^{81}}}\,\frac{1}{112+\dfrac{1}{25\cdot 29-\dfrac{5\cdot 83}{19\cdot 23}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{39}_{19}K_{20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{70,72,73,76}_{32}Ge_{38,40,41,44}$ ${}^{75}_{33}As_{42}$

${}^{7\cdot 19}_{55}Cs_{78}$ ${}^{11\cdot 13}_{60}Nd_{83}$ ${}^{208,11\cdot 19}_{83}Bi^{*}_{125,126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{24\cdot 17}_{162=2\cdot 81}Ch^{ie}_{6\cdot 41}$ ${}^{420}_{165=15\cdot 11}Ch^{ie}_{15\cdot 17}$ ${}^{32\cdot 13,22\cdot 19}_{166}Ch^{ie}_{250,252}$

$$\alpha_{1-33}=\frac{170}{33\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{53}{4\cdot 13})^{105}}}\,\frac{1}{112+\frac{1}{22\cdot 29+\frac{4\cdot 73}{5\cdot 83}}}=1/137.035999037436$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{47,48,50}_{22}Ti_{25,26,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{77,78}_{34}Se_{43,44}$ ${}^{120,122,125}_{52}Te_{68,70,73}$ ${}^{140,142}_{58}Ce_{82.84}$

${}^{180,181}_{73}Ta_{107,108}$ ${}^{140,142}_{58}Ce_{82.84}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{3\cdot 11^2,28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$ ${}^{404,14\cdot 29}_{159,161}Ch^{ie}_{5\cdot 49}$ ${}^{420}_{165}Ch^{ie}_{255}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{1-34}=\frac{7\cdot 5^2}{34\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{73}{72})^{5\cdot 29}}}\,\frac{1}{112+\frac{1}{15\cdot 59+\frac{13}{15}+\frac{1}{3\cdot(2\cdot 15\cdot 17-1)}}}=1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64}_{30}Zn_{34}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{79,80}_{35}Br_{44,46}$ ${}^{89}_{39}Y_{50}$ ${}^{124,125}_{52}Te_{72,73}$ ${}^{5\cdot 29}_{60}Nd_{85}$

$$\alpha_{1-36}=\frac{5\cdot 37}{6^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{3\cdot 79}{4\cdot 59})^{11\cdot 43}}}\,\frac{1}{112+\frac{1}{5\cdot(31\cdot 42-1)+\frac{3\cdot 31}{14\cdot 13}}}=1/137.035999037436$$

${}^{27}_{13}Al_{14}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{78,80,84,86}_{36}Kr_{42,44,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{310,311}_{124,125}Ch^{ie}_{186,187}$ ${}^{402}_{158}Ch^{ie}_{244}$

${}^{392,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{3\cdot 79,14\cdot 17}$

$$\alpha_{1-43}=\frac{13\cdot 17}{43\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{3\cdot 67}{200})^{401}}}\,\frac{1}{112+\frac{1}{8\cdot(12\cdot 83+1)+\frac{4}{3\cdot 13}}}=1/137.035999037436$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{77}_{34}Se_{43}$ ${}^{83}_{36}Kr_{47}$ ${}^{89}_{39}Y_{50}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{9\cdot 13}_{50}Sn_{67}$ ${}^{11\cdot 13}_{60}Nd_{83}$ ${}^{168}_{68}Er_{100}$

${}^{169}_{69}Tm_{100}$ ${}^{200}_{80}Hg_{120}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{330,4\cdot 83}_{130,131}Ch^{ie}_{200,201}$ ${}^{20\cdot 17}_{134=2\cdot 67,135}Ch^{ie}_{206,205}$ ${}^{6\cdot 67}_{158}Ch^{ie}_{244}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{1-50}=\frac{2\cdot 257}{100\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{14\cdot 13}{181})^{3\cdot 11^2}}}\,\frac{1}{112+\frac{1}{29\cdot 61-\frac{157}{16\cdot 11}}}=1/137.035999037436$$

${}^{27}_{13}Al_{14}$ ${}^{47,48,50}_{22}Ti_{25,26,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{116,120}_{50}Sn_{66,70}$ ${}^{5\cdot 29}_{61}\mathrm{Pm}^{*}_{84}$ ${}^{157}_{64}Gd_{93}$

${}^{226}_{88}Ra^{*}_{138}$ ${}^{257}_{100}\mathrm{Fm}^{*}_{157}$ ${}^{6\cdot 43}_{101}Md^{*}_{157}$ ${}^{7\cdot 37}_{102=6\cdot 17}No^{*}_{157}$ ${}^{14\cdot 21}_{118}Og^{ie}_{16\cdot 11}$ ${}^{302,16\cdot 19}_{121,122=2\cdot 61}Ch^{ie}_{181,182=14\cdot 13}$ ${}^{24\cdot 13,314}_{125,126=9\cdot 14}Ch^{ie}_{187=11\cdot 17,188}$

${}^{3\cdot 11^2,28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{400,402}_{157,158}Ch^{ie}_{243,4\cdot 61}$ ${}^{426}_{169}Ch^{ie}_{257}$

$$\alpha_{1-59}=\frac{3\cdot 101}{59\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot 27\cdot 49}{5\cdot 23^2})^{11\cdot 13\cdot 37}}}\,\frac{1}{112+\frac{1}{48\cdot 64\cdot 31-\frac{17}{81}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{37}_{17}Cl_{20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{105}_{46}Pd_{59}$ ${}^{112}_{48}Cd_{64}$ ${}^{113,115}_{49}In_{64,66}$

${}^{137}_{56}Ba_{81}$ ${}^{205}_{81}Tl_{124}$ ${}^{5\cdot 47,14\cdot 17}_{92=4\cdot 23}U^{*}_{11\cdot 13,146}$ ${}^{293}_{116}Lv^{ie}_{3\cdot 59}$ ${}^{6\cdot 49}_{117=9\cdot 13}Ts^{ie}_{3\cdot 59}$ ${}^{6\cdot 49}_{118=2\cdot 59}Og^{ie}_{16\cdot 11}$ ${}^{310,311}_{124,125}Ch^{ie}_{186,187}$

${}^{3\cdot 11^2,28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{8\cdot 49,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,14\cdot 17}$ ${}^{24\cdot 17}_{162=2\cdot 81}Ch^{ie}_{6\cdot 41}$

$$\alpha_{1-81}=\frac{4^2\cdot 26}{9^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{22\cdot 73}{15\cdot 107})^{13^2\cdot 19}}}\,\frac{1}{112+\frac{1}{2\cdot 81\cdot 17\cdot 67+\frac{35}{88}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35}_{17}Cl_{18}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{59}_{27}Co_{32}$ ${}^{78,80,82}_{34}Se_{44,46,48}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{137}_{56}Ba_{81}$ ${}^{165}_{67}Ho_{98}$ ${}^{180,181}_{73}Ta_{107,108}$

${}^{340}_{134,135}Ch^{ie}_{206,205}$ ${}^{402}_{158}Ch^{ie}_{244}$ ${}^{24\cdot 17}_{162=2\cdot 81}Ch^{ie}_{6\cdot 41}$

$$\alpha_{1-96}=\frac{17\cdot 29}{4^2\cdot 6\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{16\cdot 3\cdot 11^2-1}{27\cdot 5\cdot 43+1})^{79\cdot 147}}}\,\frac{1}{112+\frac{163\cdot(8\cdot 21\cdot 37+1)}{50\cdot 10^{11}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{45}_{21}Sc_{24}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{27}As_{42}$ ${}^{85,87}_{37}Rb_{48,50}$

${}^{100}_{44}Ru_{56}$ ${}^{112}_{48}Cd_{64}$ ${}^{116,118,120,124}_{50}Sn_{66,68,70,74}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$

${}^{2\cdot 163}_{128,129=3\cdot 43}Ch^{ie}_{198,197}$ ${}^{372}_{147}Ch^{ie}_{225}$ ${}^{3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{420}_{165=11\cdot 15}Ch^{ie}_{15\cdot 17}$

$$\alpha_{1-103}=\frac{23^2}{103\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{3\cdot 19\cdot 23}{7\cdot 11\cdot 17+1})^{2621}}}\,\frac{1}{112+\frac{1}{6\cdot(12\cdot(8\cdot(64\cdot 7+1)+1)+1)+\frac{3}{4}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{23}_{11}K_{12}$ ${}^{39}_{19}K_{20}$ ${}^{51}_{23}V_{28}$ ${}^{70,72,74}_{32}Ge_{38,40,42}$ ${}^{103}_{45}Rh_{58}$ ${}^{102,104,106,110}_{46}Pd_{56,58,60,64}$ ${}^{112}_{48}Cd_{64}$ ${}^{131}_{54}Xe_{77}$ ${}^{156}_{64}Gd_{92}$ ${}^{173}_{70}Yb_{103}$

$$\alpha_{1-133}=\frac{683}{133\cdot(2\pi)_{12389}}\,\frac{1}{112+\frac{14651}{50\cdot 10^{11}}}=\frac{6^2\cdot 19-1}{7\cdot 19\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{59\cdot 210}{13\cdot(8\cdot 7\cdot 17\cdot+1)})^{71\cdot(12\cdot 29+1)}}}\,\frac{1}{112+\frac{7^2\cdot 13\cdot 23}{50\cdot 10^{11}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{56}_{26}Fe_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{84}_{36}Kr_{48}$ ${}^{105}_{46}Pd_{59}$ ${}^{5\cdot 23}_{49}In_{66}$

${}^{116,118,119,120,122}_{50}Sn_{66,68,69,70,72}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{17^2}_{114=6\cdot 19}Fl^{ie}_{175}$ ${}^{14\cdot 21}_{118}Og^{ie}_{176}$ ${}^{334,2\cdot 168}_{132,133=7\cdot 19}Ch^{ie}_{202,7\cdot 29}$ ${}^{14\cdot 29}_{161=7\cdot 23}Ch^{ie}_{5\cdot 49}$

$$\alpha_{1-140}=\frac{6^2\cdot 20-1}{140\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{4\cdot 13\cdot 37}{3\cdot(64\cdot 10+1)})^{3847}}}\,\frac{1}{112+\frac{1}{4\cdot 9\cdot(2\cdot 3\cdot 29\cdot 47+1)+\frac{29}{54}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}So_{14,15.16}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{107,109}_{37}Rb_{48,50}$ ${}^{85,87}_{47}Ag_{60,62}$

${}^{112}_{48}Cd_{64}$ ${}^{24\cdot 13,314}_{125,126=18\cdot 7}Ch^{ie}_{187,4\cdot 47}$ ${}^{32\cdot 11,4\cdot 89}_{140,141=3\cdot 47}Ch^{ie}_{212,215}$

$$\alpha_{1-155}=\frac{2^2\cdot 199}{5\cdot 31\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{[\frac{19\cdot 210-1}{3^2\cdot(2\cdot 13\cdot 17+1)+1}]^{7977}}}\,\frac{1}{112+\frac{1}{5\cdot 17\cdot 31\cdot(2\cdot 13\cdot 17+1)-\frac{15}{43}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40}_{19}K_{20,21}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{64,68}_{30}Zn_{34,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77}_{34}Se_{43}$ ${}^{88}_{38}Sr_{50}$ ${}^{93}_{41}Nb_{52}$

${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{17^2}_{114=6\cdot 19}Fl^{ie}_{175}$ ${}^{8\cdot 41}_{129=3\cdot 43}Ch^{ie}_{199}$ ${}^{392,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,14\cdot 17}$ ${}^{420}_{170}Ch^{ie}_{250}$

$$\alpha_{1-170}=\frac{873}{170\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{34451}{34450})^{68901}}}\,\frac{1}{112+\frac{4171}{8\times 10^{11}}}$$

$$=\frac{3^2\cdot 97}{170\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{[\frac{47(12\cdot 61+1)}{2\cdot 25\cdot 13\cdot 53}]^{3\cdot 7\cdot 17\cdot 193}}}\,\frac{1}{112+\frac{43\cdot 97}{8\cdot 10^{11}}}=1/137.035999037435$$

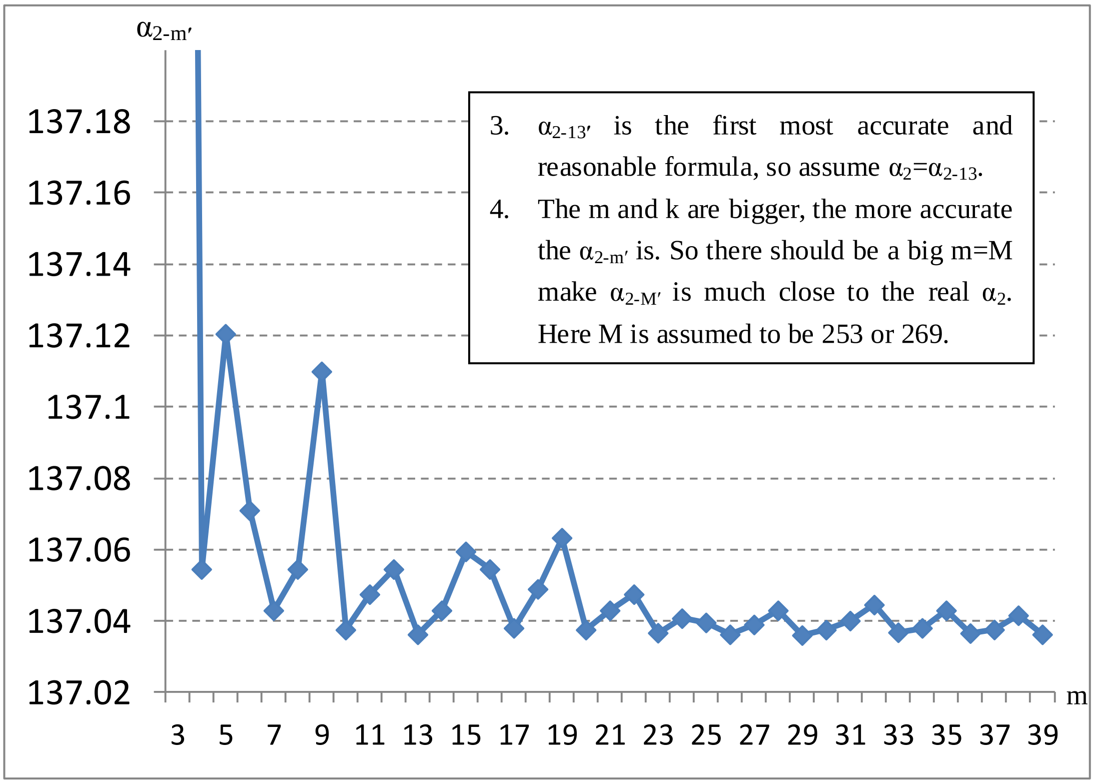
${}^{35,37}_{17}Cl_{18,20}$ ${}^{76,77}_{34}Se_{42,43}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{95,97}_{42}Mo_{53,55}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{127}_{53}I_{74}$ ${}^{145}_{61}Pm^{*}_{84}$

${}^{163}_{66}Dy_{97}$ ${}^{168}_{68}Er_{100}$ ${}^{193}_{77}Ir_{116}$ ${}^{202}_{80}Hg_{122}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{13\cdot 19}_{97}Bk^{*}_{150}$ ${}^{304}_{122}Ch^{ie}_{14\cdot 13}$ ${}^{6\cdot 53,320}_{127}Ch^{ie}_{191,193}$ ${}^{8\cdot 43}_{136}Fy^{ie}_{208}$ ${}^{404}_{159}Ch^{ie}_{245}$ ${}^{420}_{170}Ch^{ie}_{250}$

Table 6. Parameters and Results of Approximate Formulas of α_2 (2019/7/3).

m	n	k	$\alpha_{2\text{-}m'}$	m	n	k	$\alpha_{2\text{-}m'}$
1	**8**	4	137.933814383	22	170	32	137.047480404
2	16	4	137.933814383	23	177	**161**	137.036664793
3	24	4	137.933814383	**24**	185	62	137.040748949
4	31	20	137.054511358	**25**	193	39	137.039552569
5	39	11	137.120466691	26	200	278	137.036218856
6	47	8	137.070996332	27	**208**	80	137.038980680
7	**54**	48	137.042951195	28	216	48	137.042951195
8	62	20	137.054511358	**29**	**223**	**655**	**137.036002235**
9	70	14	137.109928583	30	231	104	137.037530964
10	77	**104**	137.037530964	31	239	58	137.040063944
11	85	32	137.047480404	**32**	247	41	137.044550585
12	93	20	137.054511358	33	254	**138**	137.036795730
13	**100**	**278**	**137.036218856**	34	262	70	137.038016730
14	108	48	137.042951195	35	270	48	137.042951195
15	**116**	28	137.059466839	**36**	277	**190**	137.036562950
16	124	20	137.054511358	37	285	85	137.037566566
17	131	70	137.038016730	38	293	56	137.041569603
18	139	37	137.048943854	39	300	278	137.036218856
19	147	26	137.063298933	**125**	**961**	**4293**	137.035999678
20	154	104	137.037530964	**253**	**1945**	**28186**	**137.035999128**
21	162	48	137.042951195	**269**	2068	**41654**	**137.035999118**

Fig. 8. Results of Approximate Formulas of α_2 (2019/7/3).



$$\alpha_{2-1}=\frac{e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\frac{e^2}{(\frac{5}{4})^9}}{2\cdot 2^2}\frac{1}{112-1+\frac{1}{3}-\frac{1}{16}+\frac{1}{41\cdot(12\cdot 13+1)+\frac{13}{41}}}=1/137.035999111816$$

${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{73}_{32}Ge_{41}$ ${}^{93}_{41}Nb_{52}$ ${}^{112}_{48}Cd_{64}$ ${}^{128}_{52}Te_{76}$ ${}^{138}_{56}Ba_{82}$ ${}^{157}_{64}Gd_{93}$ ${}^{208}_{82}Pb_{126}$ ${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$
${}^{24\cdot 17}_{162=2\cdot 81}Ch^{ie}_{6\cdot 41}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{2-4}=\frac{2^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{21}{20})^{41}}}{31}\frac{1}{112-\frac{1}{66}+\frac{1}{71\cdot(14\cdot 43-1)-\frac{56}{95}}}=1/137.035999111818$$

${}^{28}_{14}Si_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{161}_{66}Dy_{95}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{222}_{86}Rn^{*}_{136}$
${}^{284}_{113}Nh^{ie}_{171}$ ${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$ ${}^{310,312}_{124,125}Ch^{ie}_{186,187}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{2-5}=\frac{5\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{12}{11})^{23}}}{39}\frac{1}{112-\frac{1}{14}+\frac{1}{10\cdot 41}-\frac{1}{23\cdot(14\cdot 11\cdot 79+1)+\frac{11}{16}}}=1/137.035999111818$$

${}^{11}_{5}B_{6}$ ${}^{23}_{11}Na_{12}$ ${}^{39,41}_{19}K_{20,22}$ ${}^{51}_{23}V_{28}$ ${}^{78}_{34}Se_{44}$ ${}^{79}_{35}Br_{44}$ ${}^{89}_{39}Y_{50}$ ${}^{134,135,138}_{56}Ba_{78,79,82}$ ${}^{288}_{115=5\cdot 23}Mc^{ie}_{173}$ ${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$ ${}^{402}_{158}Ch^{ie}_{244}$
${}^{14\cdot 29}_{160,161=7\cdot 23}Ch^{ie}_{6\cdot 41,5\cdot 49}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{2-6}=\frac{6\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{9}{8})^{17}}}{47}\frac{1}{112-\frac{1}{2\cdot 17}+\frac{1}{2\cdot(36\cdot 17+1)-\frac{4}{47}}}=1/137.035999111818$$

${}^{34}_{16}S_{18}$ ${}^{35}_{17}Cl_{18}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{136}_{56}Ba_{80}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{312,314}_{125,126}Ch^{ie}_{187,188}$
${}^{4\cdot 89,358/360}_{141=3\cdot 47,142}Ch^{ie}_{215,6\cdot 36/218}$

$$\alpha_{2-7}=\frac{7\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{49}{48})^{97}}}{6\cdot 3^2}\frac{1}{112-\frac{1}{11\cdot 16+\frac{2\cdot 23}{3\cdot 25\cdot 137}}}=1/137.035999111819$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{59}_{27}Co_{32}$ ${}^{75}_{33}As_{42}$
${}^{92,95,96,97,98,100}_{42}Mo_{50,53,54,55,56,58}$ ${}^{96,98,99,100,104}_{44}Ru_{52,54,55,56,60}$ ${}^{102,106,108,110}_{46}Pd_{56,60,62,64}$ ${}^{110,111,112,113,114,116}_{48}Cd_{62,63,64,65,66,68}$
${}^{113,115}_{49}In_{64,66}$ ${}^{116-119,120,122}_{50}Sn_{66-69,70,72}$ ${}^{129}_{54}Xe_{75}$ ${}^{137,138}_{56}Ba_{81,82}$ ${}^{138}_{57}La_{81}$ ${}^{150}_{62}Sm_{88}$ ${}^{169}_{69}Tm_{100}$ ${}^{174,176}_{70}Yb_{104,106}$ ${}^{105,176}_{71}Lu_{104,105}$
${}^{180}_{72}Hf_{108}$ ${}^{185,187}_{75}Re_{110,112}$ ${}^{226}_{88}Ac^{*}_{138}$ ${}^{235,238}_{92}U^{*}_{143,146}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{277}_{108}Hs^{*}_{169}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{6\cdot 49}_{118}Og^{ie}_{176}$ ${}^{346,348}_{137,138}Fy^{ie}_{209,210}$ ${}^{7\cdot 54}_{150}Ch^{ie}_{228}$

$$\alpha_{2-9}=\frac{3^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{15}{14})^{29}}}{70}\frac{1}{112-\frac{1}{16}+\frac{1}{11\cdot 43}-\frac{1}{70\cdot 17\cdot(3\cdot 64-1)-\frac{41}{70}}}=1/137.035999111818$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{112}_{48}Cd_{64}$ ${}^{76}_{33}As^{*}_{43}$ ${}^{79}_{35}Br_{44}$ ${}^{93}_{41}Nb_{52}$ ${}^{99}_{43}Td^{*}_{56}$ ${}^{121}_{51}Sb_{70}$ ${}^{157}_{64}Gd_{93}$ ${}^{172}_{70}Yb_{102}$
${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$ ${}^{14\cdot 29}_{160,161=7\cdot 23}Ch^{ie}_{6\cdot 41,5\cdot 49}$ ${}^{24\cdot 17}_{162=2\cdot 81}Ch^{ie}_{6\cdot 41}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$ ${}^{420}_{165=11\cdot 15}Ch^{ie}_{15\cdot 17}$

$$\alpha_{2-10}=\frac{10\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{5\cdot 21}{8\cdot 13})^{11\cdot 19}}}{77}\,\frac{1}{112-\dfrac{1}{3\cdot 14\cdot 19}+\dfrac{1}{14\cdot(4\cdot 27\cdot(2\cdot 15\cdot 19+1)-1)}}=1/137.035999111818$$

${}^{14}_{7}N_{7}$ ${}^{27}_{13}Al_{14}$ ${}^{28}_{14}Si_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{56}_{26}Fe_{30}$ ${}^{97,99}_{42}Mo_{54,56}$ ${}^{131,136}_{54}Xe_{77,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{191}_{77}Ir_{2\cdot 57}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$
${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{418,420}_{168}Ch^{ie}_{250,252}$

$$\alpha_{2-11}=\frac{11\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{33}{32})^{65}}}{85}\,\frac{1}{112-\dfrac{1}{106}+\dfrac{1}{30\cdot(4\cdot 163+1)-\dfrac{35}{52}}}=1/137.035999111818$$

${}^{79}_{35}Br_{44}$ ${}^{79}_{35}Br_{44}$ ${}^{96,99,100,104}_{44}Ru_{52,55,56,60}$ ${}^{125}_{52}Te_{73}$ ${}^{127}_{53}I_{74}$ ${}^{145}_{60}Nd_{85}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$
${}^{2\cdot 163}_{128,129}Ch^{ie}_{198=18\cdot 11,197}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{420}_{165=5\cdot 33}Ch^{ie}_{15\cdot 17}$ ${}^{420}_{170}Ch^{ie}_{250}$

$$\alpha_2=\alpha_{2\text{-}13}=\frac{13\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{9\cdot 31}{2\cdot 139})^{557}}}{10^2}\,\frac{1}{112-\dfrac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

${}^{7}_{3}Li_{4}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}\mathrm{Al}_{14}$ ${}^{29}_{14}\mathrm{Si}_{15}$ ${}^{54,56,57,58}_{26}\mathrm{Fe}_{28,30,31,32}$ ${}^{58}_{28}Ni_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{74}_{32}Ge_{42}$ ${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{96}_{42}Mo_{54}$
${}^{100}_{44}Ru_{56}$ ${}^{112,116}_{48}Cd_{64,68}$ ${}^{116}_{50}Sn_{66}$ ${}^{128}_{52}Te_{76}$ ${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{139}_{57}La_{82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{149}_{62}Sm_{87}$ ${}^{156,157,160}_{64}Gd_{92,93,96}$
${}^{162}_{66}Dy_{96}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175}_{71}Lu_{8\cdot 13}$ ${}^{187}_{75}\mathrm{Re}_{112}$ ${}^{188,189,190,192}_{76}Os_{112,113,114,116}$ ${}^{200}_{80}Hg_{120}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$
${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{247}_{96}Cm^{*}_{151}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$ ${}^{310,312}_{124,125}Ch^{ie}_{186,187}$ ${}^{314}_{126}Ch^{ie}_{188}$ ${}^{330}_{130}Ch^{ie}_{200}$
${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{209}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

$$\alpha_{2-15}=\frac{15\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{29}{28})^{57}}}{2^2\cdot 29}\,\frac{1}{112-\dfrac{1}{4\cdot 13}+\dfrac{1}{12\cdot(36\cdot 43+1)-\dfrac{1}{16}}}=1/137.035999111818$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{31}_{15}P_{16}$ ${}^{56}_{26}Fe_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{84,86}_{36}Kr_{48,50}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{293}_{116=4\cdot 29}Ch^{ie}_{3\cdot 59}$

$$\alpha_{2-17}=\frac{17\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{71}{70})^{3\cdot 47}}}{131}\,\frac{1}{112-\dfrac{1}{6\cdot 101}+\dfrac{1}{23\cdot(30\cdot 35^2-1)+\dfrac{6}{23}}}=1/137.035999111818$$

${}^{35}_{17}Cl_{18}$ ${}^{51}_{23}V_{28}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{81}_{35}Br_{46}$ ${}^{83}_{36}Kr_{47}$ ${}^{106}_{46}Pd_{60}$ ${}^{107}_{47}Ag_{60}$ ${}^{121,122}_{51}Sb_{70,71}$ ${}^{144}_{60}Nd_{84}$ ${}^{171,172}_{70}Yb_{101,102}$
${}^{175}_{71}Lu_{104}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{312,314}_{125,126}Ch^{ie}_{11\cdot 17,4\cdot 47}$ ${}^{330,4\cdot 83}_{130,131}Ch^{ie}_{200,201}$ ${}^{4\cdot 89,358/360}_{141=3\cdot 47,142=2\cdot 71}Ch^{ie}_{5\cdot 43,6\cdot 36/218}$

$$\alpha_{2-18}=\frac{18\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{38}{37})^{75}}}{139}\,\frac{1}{112-\dfrac{1}{2\cdot 47}+\dfrac{1}{2\cdot 31\cdot(2\cdot 136-1)+\dfrac{83}{137}}}=1/137.035999111818$$

${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38}_{18}Ar_{18,20}$ ${}^{69}_{31}Ga_{38}$ ${}^{70,72,74,76}_{32}Ge_{38,40,42,44}$ ${}^{74,76}_{34}Se_{40,42}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{109}_{47}Ag_{62}$
${}^{148,150,152}_{62}Sm_{86,88,90}$ ${}^{130,132,136,137,138}_{56}Ba_{74,76,80,81,82}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{310,312,314}_{124,125,126}Ch^{ie}_{186,187,188}$
${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{348}_{139}Ch^{ie}_{209}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{2-19} = \frac{19 \cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3} \dfrac{e^2}{(\frac{3}{2})^5} \cdots \dfrac{e^2}{(\frac{27}{26})^{53}}}{3 \cdot 49} \frac{1}{112 - \dfrac{1}{44} + \dfrac{1}{16 \cdot (4 \cdot 37 + 1) - \dfrac{23}{6 \cdot 47 + 1}}} = 1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{51}_{23}V_{28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{83}_{36}Kr_{47}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{95}_{42}Mo_{53}$

${}^{98,100}_{44}Ru_{54,56}$ ${}^{107,109}_{47}Ru_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{127}_{53}I_{74}$ ${}^{6\cdot 49}_{118}Og^{ie}_{176=16\cdot 11}$ ${}^{32\cdot 11,4\cdot 89}_{140,141=3\cdot 47}Ch^{ie}_{4\cdot 53,5\cdot 43}$ ${}^{370,22\cdot 17}_{147,148}Ch^{ie}_{223,226}$ ${}^{404/14\cdot 29}_{159/161,160}Ch^{ie}_{5\cdot 49,246}$

$$\alpha_{2-23} = \frac{23 \cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3} \dfrac{e^2}{(\frac{3}{2})^5} \cdots \dfrac{e^2}{(\frac{2 \cdot 81}{7 \cdot 23})^{17 \cdot 19}}}{3 \cdot 59} \frac{1}{112 - \dfrac{1}{2 \cdot (40 \cdot 23 - 1) + \dfrac{9}{32 \cdot 10}}} = 1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39}_{19}K_{20}$ ${}^{40,43}_{20}Ca_{20,23}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{70,72}_{32}Ge_{38,40}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{90,91,94}_{40}Zr_{50,51,54}$ ${}^{102,105,110}_{46}Pd_{56,59,64}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{6\cdot 49}_{117=9\cdot 13}Ts^{ie}_{3\cdot 59}$

$$\alpha_{2-24} = \frac{2^2 \cdot 6 \cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3} \dfrac{e^2}{(\frac{3}{2})^5} \cdots \dfrac{e^2}{(\frac{63}{2 \cdot 31})^{125}}}{5 \cdot 37} \frac{1}{112 - \dfrac{1}{257} + \dfrac{1}{10 \cdot (12 \cdot 13 \cdot 83 + 1) + \dfrac{23}{81}}} = 1/137.035999111818$$

${}^{24,24,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{50,52,54}_{24}Cr_{26,28,30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Rb_{50}$ ${}^{108}_{46}Pd_{62}$

${}^{110,111,112}_{48}Cd_{62,63,64}$ ${}^{124,125,126,130}_{52}Te_{72,73,74,78}$ ${}^{137}_{56}Ba_{81}$ ${}^{184,186}_{74}W_{110,112}$ ${}^{205}_{81}Tl_{124}$ ${}^{208,209}_{83}Bi^{*}_{125,126}$ ${}^{257}_{100}Fm^{*}_{157}$

${}^{310,24\cdot 13}_{124,125}Ch^{ie}_{186,187=11\cdot 17}$ ${}^{32\cdot 13,418}_{166}Ch^{ie}_{250,252}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

$$\alpha_{2-25} = \frac{5^2 \cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3} \dfrac{e^2}{(\frac{3}{2})^5} \cdots \dfrac{e^2}{(\frac{40}{3 \cdot 13})^{79}}}{193} \frac{1}{112 - \dfrac{1}{8 \cdot 43} + \dfrac{1}{18 \cdot 23 \cdot (32 \cdot 27 - 1) - \dfrac{3}{7}}} = 1/137.035999111818$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{40}_{18}Ar_{22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{72,74}_{32}Ge_{40,42}$ ${}^{92,96,98}_{42}Mo_{50,54,56}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{193}_{77}Co_{115}$

${}^{192,194}_{78}Pt_{114,116}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{24\cdot 13,314}_{125,18\cdot 7}Ch^{ie}_{187,188}$ ${}^{318,320}_{127}Ch^{ie}_{191,193}$ ${}^{8\cdot 43,348}_{136,138}Fy^{ie}_{208,210}$ ${}^{3\cdot 128,2\cdot 193}_{153=9\cdot 17,154=7\cdot 22}Ch^{ie}_{7\cdot 33,8\cdot 29}$

$$\alpha_{2-27} = \frac{3 \cdot 3^2 \cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3} \dfrac{e^2}{(\frac{3}{2})^5} \cdots \dfrac{e^2}{(\frac{81}{80})^{7 \cdot 23}}}{4^2 \cdot 13} \frac{1}{112 - \dfrac{1}{10 \cdot 41} + \dfrac{1}{2 \cdot 27 \cdot 43 \cdot (3 \cdot 64 + 1) - \dfrac{19}{26}}} = 1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{56,58}_{26}Fe_{30,32}$ ${}^{59}_{27}Co_{32}$ ${}^{86,87,88}_{38}Sr_{48,49,50}$ ${}^{93}_{41}Nb_{52}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{112}_{48}Cd_{64}$

${}^{126,128,130}_{52}Te_{74,76,78}$ ${}^{16\cdot 13}_{82}Pt_{126}$ ${}^{8\cdot 43,346,348}_{136,137,138=6\cdot 23}Fy^{ie}_{208,209,210}$ ${}^{14\cdot 29}_{161=7\cdot 23,160}Ch^{ie}_{5\cdot 49,6\cdot 41}$ ${}^{24\cdot 17}_{162=2\cdot 81}Ch^{ie}_{6\cdot 41}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{2-29} = \frac{29 \cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3} \dfrac{e^2}{(\frac{3}{2})^5} \cdots \dfrac{e^2}{(\frac{16 \cdot 41}{5 \cdot 131})^{3 \cdot 19 \cdot 23}}}{223} \frac{1}{112 - \dfrac{1}{29 \cdot 59 \cdot (12 \cdot 19 + 1) + \dfrac{19}{29}}} = 1/137.035999111818$$

${}^{39,41}_{19}K_{20,22}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{73}_{32}Ge_{41}$ ${}^{131}_{54}Xe_{77}$ ${}^{139}_{57}La_{82}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{169}_{69}Tm_{100}$ ${}^{9\cdot 23,208}_{82}Pb_{125,126}$ ${}^{223}_{87}Fr^{*}_{136}$

${}^{330,332}_{130,131}Ch^{ie}_{200,201}$ ${}^{16\cdot 23}_{145=5\cdot 29}Ch^{ie}_{223}$ ${}^{14\cdot 29}_{161=7\cdot 23,160}Ch^{ie}_{5\cdot 49,6\cdot 41}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$

$$\alpha_{2-31}=\frac{31\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{59}{58})^{9\cdot 13}}}{7\cdot 34+1}\;\frac{1}{112-\dfrac{1}{7\cdot 43+\dfrac{9}{5\cdot 113}}}=1/137.035999111819$$

${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{76,77,78}_{34}Se_{42,43,44}$ ${}^{89}_{39}Y_{50}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{136,138,140,141}_{58}Ce_{78,80,82,84}$ ${}^{146,147,148,152}_{62}Sm_{84,85,86,90}$

${}^{222}_{86}Rn^{*}_{136}$ ${}^{284}_{113}Nh^{ie}_{9\cdot 19}$ ${}^{310,24\cdot 13}_{124,125}Ch^{ie}_{186,187}$ ${}^{22\cdot 17}_{148}Ch^{ie}_{226}$ ${}^{8\cdot 49,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,7\cdot 34}$

$$\alpha_{2-32}=\frac{2\cdot 4^2\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{42}{41})^{83}}}{13\cdot 19}\;\frac{1}{112-\dfrac{1}{11\cdot 13}+\dfrac{1}{6\cdot 37\cdot(5\cdot 210-1)+\dfrac{10}{11}}}$$

$$=1/137.035999111818$$

${}^{39,41}_{19}K_{20,22}$ ${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{235}_{92}U^{*}_{11\cdot 13}$ ${}^{280}_{111}Rg^{*}_{169}$ ${}^{18\cdot 17,28\cdot 11}_{123=3\cdot 41}Ch^{ie}_{3\cdot 61,5\cdot 37}$

${}^{3\cdot 11^2,28\cdot 13}_{143=11\cdot 13}Ch^{ie}_{220,13\cdot 17}$ ${}^{410}_{163}Ch^{ie}_{247=13\cdot 19}$ ${}^{32\cdot 13}_{164=4\cdot 41}Ch^{ie}_{36\cdot 7}$ $?_{165=11\cdot 15}$ ${}^{420}Ch^{ie}_{15\cdot 17}$ ${}^{418,420}_{166,168}Ch^{ie}_{252}$

$$\alpha_{2-33}=\frac{33\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{139}{138})^{277}}}{2\cdot(2\cdot 8^2-1)}\;\frac{1}{112-\dfrac{1}{32\cdot 48-\dfrac{36}{35\cdot 13}}}=1/137.035999111818$$

${}^{75}_{33}As_{42}$ ${}^{84}_{36}Kr_{48}$ ${}^{112}_{48}Cd_{64}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{326,326/328}_{128,129}Ch^{ie}_{6\cdot 33,197/199}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{209=11\cdot 19}$

$$\alpha_{2-36}=\frac{6^2\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{191}{190})^{3\cdot 127}}}{2\cdot 138+1}\;\frac{1}{112-\dfrac{1}{10\cdot 7\cdot 31+\dfrac{13}{25\cdot 23}}}=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{53}_{24}Cr_{29}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}Rb_{50}$ ${}^{95}_{42}Mo_{53}$ ${}^{108}_{46}Pd_{62}$

${}^{112,115,119,120,122}_{50}Sn_{62,65,69,70,72}$ ${}^{124,126,128,130}_{52}Te_{72,74,76,78}$ ${}^{127}_{53}I_{74}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,190}_{76}Os_{112,114}$ ${}^{191,193}_{77}Ir_{114,116}$

${}^{8\cdot 36,289}_{115=5\cdot 23}Mc^{ie}_{173,174}$ ${}^{310,24\cdot 13}_{124,125}Ch^{ie}_{186,187}$ ${}^{6\cdot 53,320}_{127}Ch^{ie}_{191,193}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{378,380,382}_{150,151,152}Ch^{ie}_{228,229,230}$

$$\alpha_{2-37}=\frac{37\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{2\cdot 43}{5\cdot 17})^{9\cdot 19}}}{3\cdot 5\cdot 19}\;\frac{1}{112-\dfrac{1}{4\cdot 3\cdot 5\cdot 13}+\dfrac{1}{5\cdot 37^2\cdot 149}}=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{39}_{19}K_{20}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{126}_{52}Te_{74}$ ${}^{144,145,146,147,148,150}_{60}Nd_{84,85,86,88,90}$ ${}^{210}_{85}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$

${}^{284}_{113}Nh^{ie}_{171=9\cdot 19}$ ${}^{17^2}_{114=2\cdot 57}Tl^{ie}_{175}$ ${}^{2\cdot 149,300}_{119=7\cdot 17,120}Ch^{ie}_{179,180}$ ${}^{22\cdot 17}_{148=4\cdot 37,149}Ch^{ie}_{226,225}$

$$\alpha_{2-38}=\frac{2\cdot 19\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{3\cdot 19}{56})^{113}}}{6\cdot 7^2-1}\;\frac{1}{112-\dfrac{1}{3\cdot 73}+\dfrac{1}{30(8\cdot 27\cdot 17+1)-\dfrac{12}{13}}}=1/137.035999111816$$

${}^{19}_{9}F_{10}$ ${}^{39}_{19}K_{20}$ ${}^{56}_{26}Fe_{30}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{87,88}_{38}Sr_{49,50}$ ${}^{125}_{52}Te_{73}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{284,15\cdot 19,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,172,173}$ ${}^{17^2}_{114=6\cdot 19}Tl^{ie}_{175}$

$$\alpha_{2-125}=\frac{5\cdot 5^2\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{4294}{4293})^{8587}}}{31^2}\,\frac{1}{112-\dfrac{1}{2159481}}$$

$$=\frac{5\cdot 5^2\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{2\cdot 19\cdot 113}{81\cdot 53})^{31\cdot(12\cdot 23+1)}}}{31^2}\,\frac{1}{112-\dfrac{1}{3\cdot 101\cdot(8\cdot 81\cdot 11-1)}}=1/137.035999111818$$

${}^{39}_{19}K_{20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{53}_{24}Cr_{29}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{84,86,88}_{38}Sr_{46,48,50}$ ${}^{5\cdot 19}_{42}Mo_{53}$ ${}^{100,101}_{44}Ru_{56,57}$ ${}^{108}_{46}Pd_{62}$ ${}^{137}_{56}Ba_{81}$
${}^{150,152}_{62}Sm_{88,90}$ ${}^{9\cdot 19}_{70}Yb_{101}$ ${}^{205}_{81}Tl_{124}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{226}_{88}Ac^{*}_{138}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{9\cdot 19}$ ${}^{310,24\cdot 13}_{124,125}Ch^{ie}_{186,187}$

$$\alpha_{2-253}=\frac{253\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{28187}{28186})^{56373}}}{1945}\,\frac{1}{112-\dfrac{10411}{8\times 10^{11}}}$$

$$=\frac{11\cdot 23\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{[\frac{71(36\cdot 11+1)}{2\cdot 17(36\cdot 23+1)}]^{3\cdot 19\cdot 23\cdot 43}}}{5(4\cdot 97+1)}\,\frac{1}{112-\dfrac{29(360-1)}{8\times 10^{11}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,41}_{19}K_{20,22}$ ${}^{40,42-44,46,48}_{20}Ca_{20,22-24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{77,78,80}_{34}Se_{43,44,46}$ ${}^{80,82,84,86}_{36}Kr_{44,46,48,50}$
${}^{97}_{42}Mo_{55}$ ${}^{97-99}_{43}Tc^{*}_{54-56}$ ${}^{163}_{66}Dy_{97}$ ${}^{175,176}_{71}Lu_{104.105}$ ${}^{194}_{78}Pt_{116}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{9\cdot 19}$? ${}^{32\cdot 11,358/360}_{140,142}Ch^{ie}_{212,6\cdot 36/218}$ ${}^{3\cdot 128,386}_{153,154}Ch^{ie}_{21\cdot 11,8\cdot 29}$

$$\alpha_{2-269}=\frac{(270-1)\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{41655}{41654})^{83309}}}{2068}\,\frac{1}{112-5.317\times 10^{-9}}$$

$$=\frac{(4\cdot 67+1)\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{15\cdot(6\cdot(16\cdot 29-1)-1)}{2\cdot 59\cdot(6\cdot 59-1)})^{227\cdot(6\cdot 61+1)}}}{4\cdot 11\cdot 47}\,\frac{1}{112-\dfrac{13\cdot(24\cdot 17+1)}{10^{12}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{32,33,34}_{16}S_{16,17,18}$ ${}^{46-50}_{22}Ti_{24-28}$ ${}^{50,52,53}_{24}Cr_{26,28,29}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{76-78,80,82}_{34}Se_{42-44,46,48}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$
${}^{107,108}_{47}Ag_{60,62}$ ${}^{134}_{54}Pr_{80}$ ${}^{134-136}_{56}Ba_{78-80}$ ${}^{3\cdot 47}_{59}Pr_{82}$ ${}^{145-147}_{61}Pm^{*}_{84-86}$ ${}^{165}_{67}Ho_{98}$ ${}^{167,168}_{68}Er_{99,100}$ ${}^{188}_{76}Os_{112}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{4\cdot 61}_{94}Pu^{*}_{150}$
${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{293}_{116}Lv^{ie}_{3\cdot 59}$ ${}^{294}_{117}Ts^{ie}_{177}$ ${}^{294}_{118}Og^{ie}_{176}$ ${}^{312,314}_{125,126}Ch^{ie}_{187,188}$ ${}^{340}_{134,135}Ch^{ie}_{206,205}$ ${}^{344}_{136}Fy^{ie}_{208}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$ ${}^{420}_{165}Ch^{ie}_{15\cdot 17}$

In above formulas, there are many amazing coincidences. As 136=8×17 and 138=6×23, 17 and 23 both appear in $\alpha_{1\text{-}1}$, $\alpha_{1\text{-}17}$, $\alpha_{1\text{-}22}$, $\alpha_{1\text{-}23}$, $\alpha_{1\text{-}25}$, $\alpha_{1\text{-}59}$, $\alpha_{1\text{-}103}$, $\alpha_{1\text{-}133}$, $\alpha_{2\text{-}17}$ and $\alpha_{2\text{-}23}$, 17 frequently appears in α_1 and 23 frequently appears in α_2. 157 and 257 in $\alpha_{1\text{-}50}$ should relate to ${}_{100}Fm^{*}_{157}$, 173 in $\alpha_{1\text{-}16}$ should relate to ${}_{112}Cn^{*}_{173}$, and so on. As the factors in formulas of α are reasonably assumed to relate to nuclides, some ideal extended elements such as ${}_{136,137,138}Fy_{208,209,210}$ and ${}_{169}Ch_{257}$ are predicted.

15. Radius of Electron and Proton

The classical electron radius r_e has been calculated very accurately. However, the proton charge radius r_p hasn't yet been determined precisely. Recent two experiments measured r_p and had given the best results up to now which was r_p=0.833(19) fm[9] and r_p=0.831(19) fm[10], and hence CODADA revised its recommended data of r_p to 0.8414(19) fm. Here we give our calculation results of r_e and r_p. And it seems there is α_p similar to α. α_p could be called “the second fine-structure constant”.

Ratio of Bohr radius of hydrogen atom to classical electron radius：

$$\frac{a_0}{r_e}=\frac{1}{\alpha_c^2}=\frac{1}{\alpha_1\alpha_2}=112\times(168-\frac{1}{3}+\frac{1}{2^2\cdot 3\cdot 47}-\frac{1}{2\cdot 3\cdot 29\cdot 53\cdot 59-79/47})=18788.865042381$$

$$r_e=\alpha_c^2 a_0=\alpha_1\alpha_2 a_0=\frac{5.29177210903(80)\times 10^{-11}m}{18788.865042381}=2.81794032658(43)\ fm$$

Comparable to CODATA recomended value $r_e=2.8179403262(13)\ fm$ but more precise.

Ratio of Bohr radius of hydrogen atom to the proton charge radius should have the similiar form, and is assumed to have the following hypothetical formulas:

$$\frac{a_0}{r_p}=\frac{1}{\alpha_{p/c}^2}=\frac{1}{\alpha_{p/1}\alpha_{p/2}}=225\cdot(282+\frac{1}{3}-\frac{1}{12\cdot 47}+\frac{1}{6\cdot 29\cdot 53\cdot 59-79/47})=63524.60147736$$

$$=247\cdot(257+\frac{1}{5}-\frac{1}{5\cdot 13}+\frac{1}{30\cdot(28\cdot(2\cdot 100-1)+1)+\frac{8}{45}})$$

$$=(252+\frac{1}{24}-\frac{1}{2\cdot 17\cdot 37}+\frac{1}{11\cdot 13\cdot 19\cdot(2\cdot 11\cdot 19+1)+\frac{11}{20}})^2=252.040872632515^2$$

$$r_p=\alpha_{p/c}^2 a_0=\alpha_{p/1}\alpha_{p/2}a_0=\frac{5.29177210903(80)\times 10^{-11}m}{63524.60147736}=0.833027202999(13)\ fm$$

$\alpha_{p/c}\approx\alpha_{p/1}\approx\alpha_{p/2}\approx 252.04$, α_p could be called the second fine-structure constant.

2019/12/19-23

${}^{7}_{3}Li_{4}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{63,66}_{26}Cu_{34,36}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{103}_{45}Rh_{58}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{107}_{53}I_{2\cdot 37}$

${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{3\cdot 53}_{65}Tb_{2\cdot 47}$ ${}^{3\cdot 59,178}_{72}Hf_{105,106}$ ${}^{197}_{79}Au_{2\cdot 59}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{293}_{116}Lv^{ie}_{177}$ ${}^{402}_{158}Ch^{ie}_{244}$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{58,60}_{28}Ni_{30,32}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{100}_{44}Ru_{56}$ ${}^{103}_{45}Rh_{58}$

${}^{112,114-120,122,124}_{50}Sn_{62,64-70,72,74}$ ${}^{3\cdot 53}_{65}Tb_{2\cdot 47}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{13\cdot 19=247}_{97}Es^{*}_{150}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{426}_{169}Ch^{ie}_{257}$

${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39-41}_{19}K_{20-22}$ ${}^{42,44,46,48}_{20}Ca_{22,24,26,28}$ ${}^{46-50}_{22}Ti_{24-28}$

${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{74,76-78,80,82}_{34}Se_{40,42-44,46,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{86-88}_{38}Sr_{48-50}$ ${}^{90-92,94,96}_{40}Zr_{50-52,54,56}$ ${}^{126}_{52}Te_{74}$ ${}^{184,186}_{74}W_{110,112}$

${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{235}_{92}U^{*}_{143}$ ${}^{13\cdot 19}_{97}Es^{*}_{150}$ ${}^{252}_{99}Es^{*}_{9\cdot 17}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{314}_{126}Ch^{ie}_{188}$ ${}^{346}_{137}Fy^{ie}_{209}$ ${}^{363,364}_{143}Ch^{ie}_{220,221}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{p/1}=\frac{5\cdot 3^2}{8\cdot(2\pi)_{58}}\frac{1}{225+\frac{1}{4\cdot 112}-\frac{1}{5\cdot 19\cdot 83\cdot 103+\frac{19}{20}}}=1/252.040872632515$$

${}^{19}_{9}F_{10}$ ${}^{39}_{19}K_{20}$ ${}^{55}_{25}Mn_{30}$ ${}^{87}_{38}Sr_{49}$ ${}^{95}_{42}Mo_{53}$ ${}^{112}_{48}Cd_{64}$ ${}^{103}_{45}Rh_{58}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{158,160}_{64}Gd_{94,96}$ ${}^{7\cdot 23}_{66}Dy_{95}$

${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}Rw_{110,112}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{3\cdot 81}_{95}Am^{*}_{148}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{p/2}=\frac{23\cdot(2\pi)_{227}}{2\cdot 9^2}\frac{1}{225-\frac{1}{3\cdot 16\cdot 29-\frac{71}{2\cdot 67}}}=1/252.040872632514$$

2020/1/2

${}^{50,51}_{23}V_{27,28}$ ${}^{50,51,52,53,54}_{24}Cr_{26,27,28,29,30}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44.46}$ ${}^{112}_{48}Cd_{64}$ ${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$ ${}^{138}_{57}La_{81}$

${}^{136,138,140}_{58}Ce_{78,80,82}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{7\cdot 29,205}_{81}Tl_{122,124}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{340}_{134,135}Ch^{ie}_{206,205}$ ${}^{358,360}_{142}Ch^{ie}_{6\cdot 36,218}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$

$$\alpha_{p/2} = \frac{22\cdot(2\pi)_{164}}{5\cdot 31}\frac{1}{225-\frac{1}{7\cdot 137-\frac{7\cdot 13}{197}}} = 1/252.040872632512 \qquad 2020/1/3$$

${}^{47,48,49,50}_{22}Ti_{25.26,27,28}$ ${}^{56,57}_{26}Fe_{30.31}$ ${}^{69,71}_{31}Ga_{38.40}$ ${}^{89}_{39}Y_{50}$ ${}^{99,100}_{44}Ru_{55.56}$ ${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$ ${}^{5\cdot 31}_{64}Gd_{7\cdot 13}$ ${}^{164}_{66}Dy_{98}$ ${}^{196,198}_{78}Pt_{118,120}$

${}^{197}_{79}Au_{118}$ ${}^{206,207,208}_{82}Pb_{124,125,126}$ ${}^{223}_{87}Ra^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{310,312}_{124,125}Ch^{ie}_{186,187}$ ${}^{326,326/328}_{128,129}Ch^{ie}_{198,197/199}$ ${}^{12\cdot 31}_{147}Ch^{ie}_{225}$ ${}^{8\cdot 49,2\cdot 197}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,238}$ ${}^{32\cdot 13}_{164}Ch^{ie}_{252}$

$$\alpha_{p/2} = \frac{21\cdot(2\pi)_{126}}{2^2\cdot 37}\frac{1}{225-\frac{1}{16\cdot 29}+\frac{1}{20\cdot 13^2\cdot 179+\frac{8}{17}}} = 1/252.040872632515 \qquad 2020/1/3$$

${}^{45}_{21}Sc_{24}$ ${}^{63,65}_{29}Cu_{34.36}$ ${}^{85,87}_{37}Rb_{48.50}$ ${}^{126}_{52}Te_{74}$ ${}^{148}_{60}Nd_{88}$ ${}^{169}_{69}Tm_{100}$ ${}^{179}_{72}Hf_{107}$ ${}^{16\cdot 13}_{82}Pb_{126}$ ${}^{298,300}_{119,120}Ch^{ie}_{179,180}$ ${}^{312,314}_{125,126}Ch^{ie}_{187,188}$ ${}^{426}_{169}Ch^{ie}_{257}$

16. Direct Relationships between 2π and Nuclides

In Chen's formulas of the fine-structure constant, there are 2π-e formulas, in which k gets certain numbers and relate to nucleon numbers of some nuclides. So in the end of this paper we feel curious about whether 2π directly relate to nuclides.

$$2\pi = 6.2831853\cdots \approx \frac{4\cdot 157}{100} = 6.28 \approx \frac{3\cdot 7\cdot 44\cdot 68}{100^2} = 6.2832$$

${}^{7}_{3}Li_{4}$ ${}^{100}_{44}Ru_{56}$ ${}^{157}_{64}Gd_{93}$ ${}^{168}_{68}Er_{100}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$2\pi \approx \frac{4\cdot 157}{100} = \frac{157}{25} = 6.28$$

${}^{55}_{25}Mn_{30}$ ${}^{100}_{44}Ru_{56}$ ${}^{157}_{64}Gd_{93}$ ${}^{118,119,120}_{50}Sn_{68,69,70}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{200}_{80}Hg_{120}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{259}_{102}No^{*}_{157}$ ${}^{312,2\cdot 157}_{125,126}Ch^{ie}_{11\cdot 17,188}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$2\pi \approx \frac{16\cdot 3\cdot 7\cdot 11\cdot 17}{100^2} = \frac{48\cdot 7\cdot 11\cdot 17}{100^2} = \frac{3\cdot 7\cdot 44\cdot 68}{100^2} = \frac{3\cdot 112\cdot 11\cdot 17}{100^2} = \frac{2\cdot 168\cdot 11\cdot 17}{100^2} = \frac{2\cdot 3\cdot 7\cdot 11\cdot 136}{100^2} = \cdots = 6.2832$$

${}^{7}_{3}Li_{4}$ ${}^{20-22}_{10}Ne_{10-12}$ ${}^{23}_{11}Na_{12}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,49,50}_{22}Ti_{24,25,27,28}$ ${}^{61}_{28}Ni_{33}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{98,100}_{42}Mo_{56,58}$ ${}^{100}_{44}Ru_{56}$ ${}^{112}_{48}Cd_{64}$

${}^{136}_{56}Ba_{80}$ ${}^{168}_{68}Er_{100}$ ${}^{199,200}_{80}Hg_{119,120}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fa^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344-348}_{136-138}Fy^{ie}_{208-210}$ ${}^{420}_{168}Ch^{ie}_{252}$

$$2\pi \approx \frac{44}{7} = \frac{2\cdot 22}{7} = 6.2857\cdots$$

${}^{50}_{22}Ti_{28}$ ${}^{61}_{28}Ni_{33}$ ${}^{100}_{44}Ru_{56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{294}_{118}Og^{ie}_{176}$ ${}^{8\cdot 44}_{140}Ch^{ie}_{212}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$

$$2\pi \approx \frac{201}{32} = \frac{3\cdot 67}{32} = 6.2812\cdots$$

${}^{32}_{16}S_{16}$ ${}^{59}_{27}Co_{32}$ ${}^{67}_{30}Zn_{37}$ ${}^{112}_{48}Cd_{64}$ ${}^{117}_{50}Sn_{67}$ ${}^{128,134}_{54}Xe_{74,80}$ ${}^{134}_{56}Ba_{78}$ ${}^{165}_{67}Ho_{98}$ ${}^{201}_{80}Hg_{121}$ ${}^{332}_{131}Ch^{ie}_{201}$ ${}^{402}_{158}Ch^{ie}_{244}$

$$2\pi \approx \frac{245}{39} = \frac{5\cdot 7^2}{3\cdot 13} = 6.2820\cdots$$

${}^{7}_{3}Li_{4}$ ${}^{27}_{13}Al_{14}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{89}_{39}Y_{50}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{24\cdot 13,314}_{125,126}Ch^{ie}_{187,188}$

$$2\pi \approx \frac{289}{46} = \frac{17^2}{2\cdot 23} = 6.2826\cdots$$

${}^{3\cdot 17}_{23}V_{28}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{6\cdot 17}_{46}Pd_{56}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{11\cdot 17}_{75}\mathrm{Re}_{112}$ ${}^{222}_{86}Rn^{*}_{136}$

${}^{223}_{87}Fa^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{17\cdot 17}_{114}Fl^{ie}_{175}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$

$$2\pi \approx \frac{333}{53} = \frac{9\cdot 37}{53} = 6.2830\cdots$$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{3\cdot 37=111}_{48}Cd_{7\cdot 9}$ ${}^{127}_{53}I_{74}$ ${}^{180,184,189}_{74}W_{106,110,112}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{280}_{111}Rg^{*}_{169}$ ${}^{22\cdot 17}_{148}Ch^{ie}_{226}$

$$2\pi \approx \frac{377}{60} = \frac{13\cdot 29}{4\cdot 3\cdot 5} = 6.2833\cdots$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{116,120}_{50}Sn_{66,70}$

${}^{140,142}_{58}Ce_{82,84}$ ${}^{144,145,146,148,150}_{60}Nd_{84,85,86,88,90}$ ${}^{200}_{80}Hg_{120}$ ${}^{223}_{87}Fa^{*}_{136}$ ${}^{24\cdot 13,314}_{125,126}Ch^{ie}_{187,188}$

$$2\pi \approx \frac{465}{74} = \frac{30\cdot 31}{4\cdot 37} = 6.2837\cdots$$

${}^{31}_{15}P_{16}$ ${}^{67}_{30}Zn_{37}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{6\cdot 31}_{74}W_{112}$ ${}^{85,67}_{37}Rb_{48,50}$ ${}^{4\cdot 37}_{60}Nd_{88}$ ${}^{157}_{64}Gd_{93}$ ${}^{243}_{95}Am^{*}_{4\cdot 37}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$

$$2\pi \approx \frac{509}{81} = \frac{2\cdot 3\cdot 5\cdot 17-1}{9^2} = 6.2839\cdots$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{64,70}_{30}Zn_{34,40}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{203,205}_{81}Tl_{61,62}$ ${}^{210}_{85}At^{*}_{125}$

${}^{3\cdot 81}_{95}Am^{*}_{148}$ ${}^{344,346,348}_{136,137,138}Fy^{*}_{208,209,210}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$

$$2\pi \approx \frac{622}{99} = \frac{4\cdot(24\cdot 13-1)}{9\cdot 22} = 6.2828\cdots$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,48,49}_{22}Ti_{24,26,27}$ ${}^{50,52,54}_{24}Cr_{26,28,30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{99}_{44}Ru_{55}$ ${}^{167}_{68}Er_{99}$ ${}^{252}_{99}Es^{*}_{153}$

$$2\pi \approx \frac{2\cdot 355}{113} = \frac{4\cdot 5\cdot 71}{2\cdot 113} = 6.2831858\cdots$$

${}^{71}_{31}Ga_{40}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{120,122}_{50}Sn_{70,72}$ ${}^{2\cdot 71}_{60}Nd_{82}$ ${}^{171}_{70}Yb_{101}$ ${}^{175}_{71}Lu$ ${}^{186}_{74}W_{112}$

${}^{187}_{75}\mathrm{Re}_{112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{232}_{90}Th^{*}_{2\cdot 71}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$ ${}^{358/360}_{142=2\cdot 71}Ch^{ie}_{6\cdot 36/218}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$ 2020/1/8-10

The approximate rational numbers of 2π (could be called 2π formulas) relate to nuclides marvelously. This means 2π (along with 2π-e formula) plays important roles in atomic nuclei, and acts as a rational number rather than an irrational number in the world of atomic nuclei.

17. Correlations among α, 2π and nuclides

Some Chen's formulas of the fine-structure constant and 2π formulas correlate with each others with the same factors and all together relate to the same nuclides. For example, $\alpha_{1\text{-}50}$ and $2\pi\approx4\times157/100$ have the same 157 and 100 factors, $\alpha_{1\text{-}50}$ and $2\pi\approx3\times7\times44\times68/100^2$ have the same 100, 7, 11 and 16 factors, and they relate to the same corresponding nuclides. They also have common factors with $\alpha_{1\text{-}7}$ and $\alpha_{2\text{-}13}$ which should relate to $2\pi\approx5\times7^2/3/13$ and $2\pi\approx13\times29/4/3/5$.

$$\alpha_{1-9}=\frac{47}{3^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{10}{9})^{19}}}\frac{1}{112+\dfrac{1}{4\cdot17}-\dfrac{1}{2(8\cdot9\cdot37-1)+\dfrac{3\cdot17}{157}}}$$

$$\alpha_{1-50}=\frac{2\cdot257}{100\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{14\cdot13}{181})^{3\cdot11^2}}}\frac{1}{112+\dfrac{1}{29\cdot61+\dfrac{157}{16\cdot11}}}$$

$$2\pi\approx\frac{4\cdot157}{100}=\frac{157}{25}=6.28 \qquad 2\pi\approx\frac{3\cdot7\cdot44\cdot68}{100^2}=\frac{3\cdot7\cdot11\cdot17}{25^2}=6.2832$$

$$\alpha_{1-7}=\frac{6^2}{7\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{113}{112})^{225}}}\frac{1}{112+\dfrac{1}{75^2}} \qquad \alpha_{2\text{-}13}=\frac{13\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{279}{278})^{557}}}{10^2}\frac{1}{112-\dfrac{1}{3\cdot29\cdot64}}$$

$$2\pi\approx\frac{245}{39}=\frac{5\cdot7^2}{3\cdot13}=6.2820\cdots \qquad 2\pi\approx\frac{377}{60}=\frac{13\cdot29}{4\cdot3\cdot5}=6.2833\cdots$$

$\alpha_{1\text{-}22}$ relates to $2\pi\approx2\times22/7$, $2\pi\approx17^2/7/23$ and $2\pi\approx2\times355/113$ as follows. And $2\pi\approx17^2/7/23$ also relates to $\alpha_{1\text{-}1}$, $\alpha_{1\text{-}17}$, $\alpha_{1\text{-}22}$, $\alpha_{1\text{-}23}$, $\alpha_{1\text{-}25}$, $\alpha_{1\text{-}59}$, $\alpha_{1\text{-}103}$, $\alpha_{1\text{-}133}$, $\alpha_{2\text{-}17}$ and $\alpha_{2\text{-}23}$, in which both 17 and 23 factors appear.

$$\alpha_{1-22}=\frac{113}{22\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{27\cdot29}{2\cdot17\cdot23})^{5\cdot(2\cdot157-1)}}}\frac{1}{112+\dfrac{1}{2\cdot[2\cdot3\cdot17\cdot(10\cdot19+1)+1]+\dfrac{29}{49}}}$$

$$2\pi\approx\frac{2\cdot22}{7}=6.2857\cdots,\ 2\pi\approx\frac{17^2}{2\cdot23}=6.2826\cdots,\ 2\pi\approx\frac{2\cdot355}{113}=\frac{4\cdot5\cdot71}{2\cdot113}=6.2831858\cdots$$

$\alpha_{1\text{-}13}$ and $\alpha_{1\text{-}43}$ relate to $2\pi \approx 3\times 67/32$, $2\pi \approx 5\times 7^2/39$, $2\pi \approx 17^2/46$ and others as follows.

$$\alpha_{1-13}=\frac{67}{13\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{47}{2\cdot 23})^{3\cdot 31}}}\,\frac{1}{112+\dfrac{1}{137}-\dfrac{1}{6(2\cdot 27\cdot 59+1)+\dfrac{9}{50}}}$$

$$\alpha_{1-43}=\frac{13\cdot 17}{43\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{3\cdot 67}{200})^{401}}}\,\frac{1}{112+\dfrac{1}{8\cdot(12\cdot 83+1)+\dfrac{4}{3\cdot 13}}}$$

$$2\pi\approx\frac{3\cdot 67}{32},\ 2\pi\approx\frac{5\cdot 7^2}{3\cdot 13},\ 2\pi\approx\frac{289}{46}=\frac{17^2}{2\cdot 23},\ 2\pi\approx\frac{13\cdot 29}{60},\ 2\pi\approx\frac{30\cdot 31}{4\cdot 37}$$

$\alpha_{1\text{-}11}$, $\alpha_{1\text{-}36}$, $\alpha_{2\text{-}24}$, $\alpha_{2\text{-}23}$, $\alpha_{2\text{-}37}$ and $\alpha_{2\text{-}125}$ relate to $2\pi \approx 9\times 37/53$, $2\pi \approx 15\times 31/2/37$ and $2\pi \approx (30\times 17-1)/81$ as follows.

$$\alpha_{1-11}=\frac{57}{11\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{19}{18})^{37}}}\,\frac{1}{112+\dfrac{1}{35}-\dfrac{1}{88\cdot 41-\dfrac{5\cdot 53}{22\cdot 13}}}$$

$$\alpha_{1-36}=\frac{5\cdot 37}{6^2\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{3\cdot 79}{4\cdot 59})^{11\cdot 43}}}\,\frac{1}{112+\dfrac{1}{5\cdot(31\cdot 42-1)+\dfrac{3\cdot 31}{14\cdot 13}}}$$

$$\alpha_{2-24}=\frac{2^2\cdot 6\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{63}{2\cdot 31})^{125}}}{5\cdot 37}\,\frac{1}{112-\dfrac{1}{257}+\dfrac{1}{10\cdot(12\cdot 13\cdot 83+1)+\dfrac{23}{81}}}$$

$$\alpha_{2-23}=\frac{23\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{2\cdot 81}{7\cdot 23})^{17\cdot 19}}}{3\cdot 59}\,\frac{1}{112-\dfrac{1}{2\cdot(40\cdot 23-1)+\dfrac{9}{32\cdot 10}}}$$

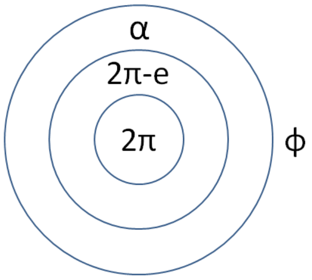
$$\alpha_{2-37}=\frac{37\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{2\cdot 43}{5\cdot 17})^{9\cdot 19}}}{3\cdot 5\cdot 19}\,\frac{1}{112-\dfrac{1}{4\cdot 3\cdot 5\cdot 13}+\dfrac{1}{5\cdot 37^2\cdot 149}}$$

$$\alpha_{2-125}=\frac{5\cdot 5^2\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{2\cdot 19\cdot 113}{81\cdot 53})^{31\cdot(12\cdot 23+1)}}}{31^2}\,\frac{1}{112-\dfrac{1}{101\cdot(20\cdot(12\cdot 89+1)+1)}}$$

$$2\pi\approx\frac{9\cdot 37}{53}=6.2830\cdots\quad 2\pi\approx\frac{15\cdot 31}{2\cdot 37}=6.2837\cdots\quad 2\pi\approx\frac{30\cdot 17-1}{81}=6.2839\cdots$$

18. Chen's Mathematic Shell Model of Nuclides

In overall, there are multi-correlations among α, 2π and nuclides. It seems there should be a mathematical shell model of nuclides, in which the core is 2π formulas and the middle layer is 2π-e formulas and the outer layer is Chen's formulas of α (**Fig. 9, ϕ** is explained in **Section 21**). The nucleon numbers, stability and abundance of nuclides are regulated by these formulas, especially by their integer factors.



Chen's Mathematic Shell Model of Nuclides

Dr. Gang Chen (2020/1/12-13, 3/1)

Fig. 9

19. Ideal Extended Elements

In the deduction of Chen's formulas of the fine-structure constant, it was reasonably assumed the factors in them related to nucleon numbers of nuclides, and it seems this assumption is quite correct. So by somewhat correlation and decoding methodology, all 119th to 170th ideal extended elements were predicted (**Table 7**). In addition, nuclides can even relate to naked 2π's approximate rational numbers (2π formulas). Some typical examples of correlations of ideal extended elements with formulas of α and 2π are listed as follows.

Example 1: Correlations of 100, 121,125,126,157, 257, 169, *et al.*

$$\alpha_{1-9} = \frac{47}{3^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{10}{9})^{19}}}\,\frac{1}{112+\frac{1}{4\cdot 17}-\frac{1}{2(8\cdot 9\cdot 37-1)+\frac{3\cdot 17}{157}}}$$

$$\alpha_{1-50} = \frac{2\cdot 257}{100\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{14\cdot 13}{181})^{3\cdot 11^2}}}\,\frac{1}{112+\frac{1}{29\cdot 61-\frac{157}{16\cdot 11}}}$$

$$\alpha_{2-24} = \frac{2^2\cdot 6\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{63}{62})^{125}}}{5\cdot 37}\,\frac{1}{112-\frac{1}{257}+\frac{1}{10\cdot(12\cdot 13\cdot 83+1)+\frac{23}{81}}}$$

$$2\pi\approx\frac{4\cdot 157}{100}\quad 2\pi\approx\frac{16\cdot 3\cdot 7\cdot 11\cdot 17}{100}\quad 2\pi\approx\frac{4\cdot 11}{7}\quad 2\pi\approx\frac{17^2}{2\cdot 23}\quad 2\pi\approx\frac{30\cdot 31}{4\cdot 37}\quad 2\pi\approx\frac{4\cdot 5\cdot 71}{2\cdot 113}$$

$${}^{100}_{44}Ru_{56}\quad {}^{168,169}_{68,69}Tm_{100}\quad {}^{257}_{100}Fm^{*}_{157}\quad {}^{302}_{121=11^2}Ch^{ie}_{181}\quad {}^{24\cdot 13,2\cdot 157}_{125,126}Ch^{ie}_{11\cdot 17,4\cdot 47}\quad {}^{2\cdot 11\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}\quad {}^{400,402}_{157,158}Ch^{ie}_{3\cdot 81,4\cdot 61}\quad {}^{6\cdot 71}_{169}Ch^{ie}_{257}$$

Example 2: Correlations of 83, 126, 84, 125, 209, 112, 173, 285, 115 and 137

$$\alpha_{1-16}=\frac{83}{4^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{17}{16})^{33}}}\frac{1}{112+\dfrac{1}{28}-\dfrac{1}{6\cdot(18\cdot 41+1)+\dfrac{173}{2\cdot(2\cdot 75-1)}}}$$

$$\alpha_{1-25}=\frac{3\cdot 43}{5^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{35}{34})^{3\cdot 23}}}\frac{1}{112+\dfrac{1}{11\cdot 19}-\dfrac{1}{13^2(2\cdot 136-1)+\dfrac{11}{25}}}$$

$$\alpha_{1-32}=\frac{15\cdot 11}{2\cdot 2^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{41}{40})^{81}}}\frac{1}{112+\dfrac{1}{25\cdot 29-\dfrac{5\cdot 83}{19\cdot 23}}}$$

$$\alpha_{2-10}=\frac{10\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{5\cdot 21}{8\cdot 13})^{11\cdot 19}}}{77}\frac{1}{112-\dfrac{1}{3\cdot 14\cdot 19}+\dfrac{1}{14\cdot(4\cdot 27\cdot(2\cdot 15\cdot 19+1)-1)}}$$

$$\alpha_{2-32}=\frac{2\cdot 4^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{42}{41})^{83}}}{13\cdot 19}\frac{1}{112-\dfrac{1}{11\cdot 13}+\dfrac{1}{6\cdot 37\cdot(5\cdot 210-1)+\dfrac{10}{11}}}$$

$$\alpha_{1-13}=\frac{67}{13\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{47}{2\cdot 23})^{3\cdot 31}}}\frac{1}{112+\dfrac{1}{137}-\dfrac{1}{6(2\cdot 27\cdot 59+1)+\dfrac{9}{50}}}$$

$$\alpha_{1-17}=\frac{2^2\cdot 22}{17\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{21}{20})^{41}}}\frac{1}{112+\dfrac{1}{137}-\dfrac{1}{2\cdot 19\cdot 23\cdot 59-\dfrac{30}{100}}}$$

$$\alpha_{1-27}=\frac{139}{27\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{67}{66})^{7\cdot 19}}}\frac{1}{112+\dfrac{1}{11\cdot 47+\dfrac{18}{23}+\dfrac{1}{6\cdot 23\cdot 137}}}$$

$$\alpha_{2-18}=\frac{18\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{38}{37})^{75}}}{139}\frac{1}{112-\dfrac{1}{2\cdot 47}+\dfrac{1}{2\cdot 31\cdot(16\cdot 17-1)+\dfrac{83}{137}}}$$

$$2\pi\approx\frac{16\cdot 3\cdot 7\cdot 11\cdot 17}{100}\approx\frac{4\cdot 11}{7}\approx\frac{17^2}{2\cdot 23}\quad {}^{8\cdot 17,137,6\cdot 23}_{56}Ba_{80,81,82}\quad {}^{11\cdot 19}_{83}Bi^{*}_{126}\quad {}^{209}_{84}Po^{*}_{125}\quad {}^{210}_{85}At^{*}_{125}\quad {}^{15\cdot 19}_{112}Cn^{*}_{173}\quad {}^{2\cdot 173}_{137}Fy^{ie}_{209}$$

Table 7. Correlations of Ideal Extended Elements (**IEE**) with Formulas of α and 2π.

IEE	Page	α	2π
${}_{113}Nh_{171}$	10 19 21 28 29 31	α_c^2 $\alpha_{1\text{-}5,7}$ $\alpha_{2\text{-}22,23,31,37,38,253}$	2π≈4×355/226
${}_{114}Fl_{175}$	19 23 28 31	$\alpha_{1\text{-}11,133,155}$ $\alpha_{2\text{-}37,38}$	$2\pi\approx 17^2/46$
${}_{115}Mc_{173}$	20 21 25 31	$\alpha_{1\text{-}1,16,23}$ $\alpha_{2\text{-}5}$	$2\pi\approx 17^2/46$
${}_{116}Lv_{177}$ ${}_{117}Ts_{177}$	10 20 22 27 31	$\alpha_{1\text{-}13,59}$ $\alpha_{2\text{-}23}$ $1/\alpha_c^2$	2π≈622/99
${}_{118}Og_{176}$	20 22 23 27	$\alpha_{1\text{-}17,20,50,59,133}$ $\alpha_{2\text{-}19,269}$	2π≈44/7
${}_{119\text{-}122}Ch_{179\text{-}182}$	21-23 28 31 37 39 44	$\alpha_{1\text{-}23,29,50,170}$ $\alpha_{2\text{-}37}$ $\alpha_{p/2}$ c_{au}	2π≈44/7 *et al.*
${}_{123}Ch_{183/185}$	19 20 21 25 28	$\alpha_{1\text{-}4,11,16,17,31}$ $\alpha_{2\text{-}1,4,5,9,32}$	2π≈333/53≈465/74 *et al*

${}_{124,125}Ch_{186,187}$	19 20 21 22 23 25 26 27 28 29 31 37 39	$\alpha_{1-6,13,31,36,59}$ $\alpha_{2-4,13,18,24,31,36,125}$ $\alpha_{p/2}$ c_{au}	2π≈465/74 2π≈622/99 *et al*
${}_{125,126}Ch_{187,188}$	11 19-23 25 27 29 31 37 44	$\alpha_{1-7,9,13,22,25,50,140}$ $\alpha_{2-6,17,18,25,269}$ $\alpha_{p/2}$ c_{au}	2π≈62832/10000 *et al*
${}_{127}Ch_{191-193}$	23 27 28 37 38 39	α_{1-170} $\alpha_{2-25,36}$ $\alpha_{1-9/11}$ $\alpha_{2-20/25}$ c_{au}	$2\pi\approx62831853/100^7$
${}_{128,129}Ch_{198,197/199}$	19 23 26 28 31 39 44	$\alpha_{1-4,96,155}$ $\alpha_{2-11,33}$ $\alpha_{p/2}$ c_{au}	
${}_{130,131}Ch_{200,201}$	20 22 26 27 44	$\alpha_{1-13,43,}$ $\alpha_{2-13,17,27}$ c_{au}	
${}_{132,133}Ch_{202,203}$	21 23 39	$\alpha_{1-27,133}$ c_{au}	
${}_{134,135}Ch_{206,205}$	19 20 21 22 29 30 31	$\alpha_{1-5,13,17,27,43,81}$ α_{2-269} $\alpha_{p/2}$	2π≈3×134/64
${}_{136,137,138}Fy_{208,209,210}$	11 20 21 25-29 42 44	$\alpha_{1-13,16,17,25,27}$ $\alpha_{2-7,10,18,25,27,33,36,269}$ c_{au}	Fibonacci Sequence p_2, n_2
${}_{139}Ch_{209}$	10 21 26 28 30	α_c^2 α_{1-27} $\alpha_{2-13,33}$ α_c^2	
${}_{140,141,142}Ch_{212,215,216/218}$	20 23 25-27 29 30 39	$\alpha_{1-9,140}$ $\alpha_{2-6,17,19,253}$ $\alpha_{p/2}$ c_{au}	2π≈44/7≈710/113
${}_{143}Ch_{220/221}$	19-22 28 30 39	$\alpha_{1-6,11,20,23,25,31,33,50,59}$ α_{2-32} $1/\alpha_{p/c}^2$	c_{au}
${}_{144-149}Ch_{222-227}$	11 19 21 23 27-31 38 39 44	$\alpha_{1-7,22,29,31,96}$ $\alpha_{2-19,29,31,37,269}$ $\alpha_{p/2}$ $\alpha_{2-20/25}$ c_{au}	2π≈930/148 2π≈1420/226 *et al*
${}_{150,151,152}Ch_{228,229,230}$	19 20 25 28 39 44	$\alpha_{1-4,19}$ $\alpha_{2-7,36}$ c_{au}	
${}_{153,154}Ch_{231,232}$	11 19 20-23 27 29 44	$\alpha_{1-6,20,22,23,33,96}$ $\alpha_{2,25,253}$ c_{au}	
${}_{155,156}Ch_{237,238}$	11 19-23 28 39	$\alpha_{1-6,20,31,36,59,155}$ α_{2-31} $\alpha_{p/2}$ c_{au}	
${}_{157}Ch_{243}$	11 20-22 31 38 39 44	$\alpha_{1-9,22,50}$ $\alpha_{1-9/11}$ c_{au}	2π≈628/100≈509/81
${}_{158}Ch_{244}$	10 11 22 25 10 30	$1/\alpha_c^2$ α_{1-50} α_{2-5} $1/\alpha_c^2$	2π≈201/32
${}_{159/161,160}Ch_{245,246}$	20 21 22 23 25 27	$\alpha_{1-11,23,29,33,133,170}$ $\alpha_{2-5,9,19,27,29}$	2π≈245/39≈289/46≈333/53
${}_{162}Ch_{246}$	21 22 25 27	$\alpha_{1-32,59,81}$ $\alpha_{2-1,9,27}$	2π≈509/81
${}_{163}Ch_{247}$	11 23 26 28 30 39 44	α_{1-96} $\alpha_{2-11,111}$ $1/\alpha_{p/c}^2$ c_{au}	
${}_{164}Ch_{252}$	19 20 21 25 27 28 31	$\alpha_{1-4,11,16,17,31}$ $\alpha_{2-1,4,5,9,27,29,32}$ $\alpha_{p/2}$	2π≈62832/10000
${}_{165}Ch_{255}$	19 21 22 23 26 29	$\alpha_{1-2,23,31,32,33,96}$ $\alpha_{2-11,32,269}$	2π≈44/7≈245/39 *et al*
${}_{166/168,167}Ch_{250/252,251}$	11 20 21 22 26 27 28 30 31 38 39 44	$\alpha_{1-16,25,32,33,43}$ $\alpha_{2-10,18,24,32}$ $1/\alpha_{p/c}^2$ $\alpha_{p/1}$ $\alpha_{2-20/25}$ c_{au}	2π≈62832/10000
${}_{169}Ch_{257}$	19 22 26 27 31 39	$\alpha_{1-5,50}$ $\alpha_{2-13,24}$ c_{au}	2π≈710/113 *et al*
${}_{170}Ch_{250}$	23 26 42 44	$\alpha_{1-155,170}$ α_{2-11} c_{au}	Fibonacci Suquenc n_1

20. Chen's Picture of Elements and Ideal Extended Elements

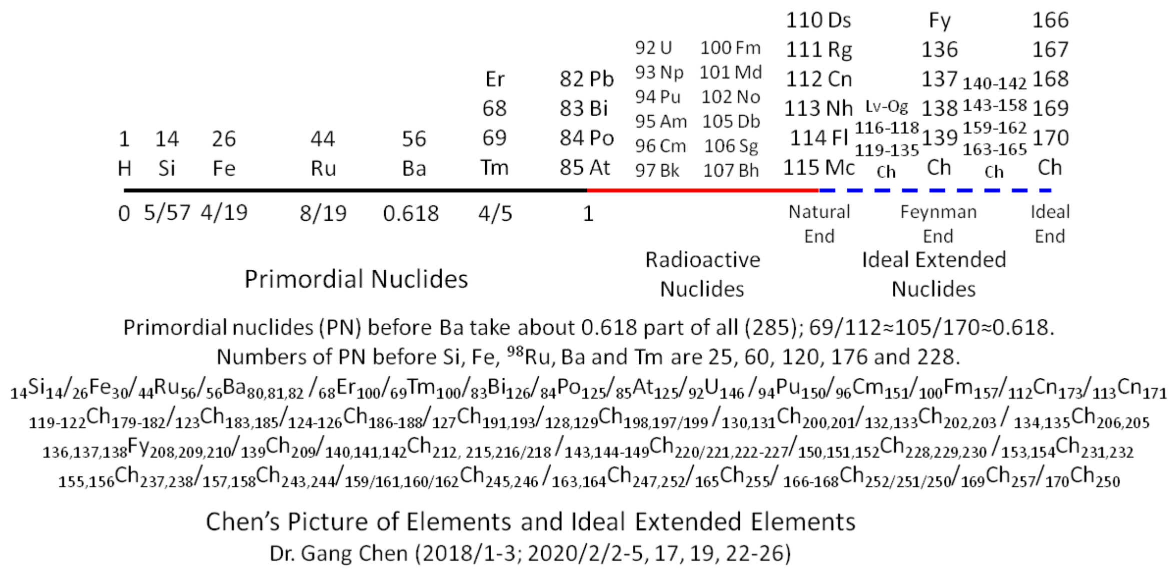


Fig. 10

The relationships between elements and ideal extended elements (the frontier of elements) and an overall picture of them were depicted as above.

21. Some Supplements

Supplement 1:

$$2\pi \approx \frac{16\cdot 3\cdot 7\cdot 11\cdot 17}{100^2} = \frac{3\cdot 7\cdot 44\cdot 68}{100^2} = \frac{3\cdot 112\cdot 11\cdot 17}{100^2} = \frac{2\cdot 168\cdot 11\cdot 17}{100^2} = \frac{2\cdot 3\cdot 7\cdot 11\cdot 136}{100^2} = \cdots = 6.2832$$

Refer to Section 16; Supplements: ${}^{32,33,34}_{16}O_{16,17,18}$ and some of the follows

$$2\pi \approx \frac{9\cdot 7\cdot 127\cdot(4\cdot 13\cdot 151+1)}{10^7} = \frac{63\cdot 127\cdot(52\cdot 151+1)}{10\cdot 100^3} = \frac{63\cdot 127\cdot(2\cdot 3\cdot 7\cdot 11\cdot 17-1)}{10\cdot(8\cdot 125)^2} = \cdots = 6.2831853$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35}_{17}Cl_{18}$ ${}^{46-50}_{22}Ti_{24-28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{76,77,78,80}_{34}Se_{42,43,44,46}$

${}^{98,100}_{42}Mo_{56,58}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{111,112}_{48}Cd_{63,64}$ ${}^{116-120,122}_{50}Sn_{66-70,72}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{125,126,128}_{52}Te_{73,74,76}$ ${}^{151,153}_{63}Eu_{88,90}$

${}^{14\cdot 17}_{64}Gd_{90}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{179}_{72}Hf_{107}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{199,200}_{80}Hg_{119,120}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$

${}^{235,14\cdot 17}_{92}U^{*}_{11\cdot 13,146}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{298}_{119=7\cdot 17}Ch^{ie}_{179}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{187}$ ${}^{6\cdot 53,320}_{127}Ch^{ie}_{191,193}$ ${}^{380}_{151}Ch^{ie}_{229}$ ${}^{2\cdot 193}_{154}Ch^{ie}_{232}$ ${}^{416}_{164}Ch^{ie}_{252}$ ${}^{416-420}_{166,168}Ch^{ie}_{252/250}$

2020/2/11-12

Supplement 2:

$$2\pi \approx \frac{509}{81} = \frac{4\cdot 127+1}{9^2} \qquad 2\pi \approx \frac{201}{32} = \frac{3\cdot 67}{32} \qquad 2\pi \approx \frac{333}{53} = \frac{9\cdot 37}{53}$$

${}^{19}_{9}F_{10}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{203,205}_{81}Tl_{61,62}$ ${}^{6\cdot 53,320}_{127}Ch^{ie}_{191,193}$

$$2\pi \approx \frac{622}{99} = \frac{4\cdot(310+1)}{9\cdot 22} \qquad 2\pi \approx \frac{465}{74} = \frac{30\cdot 31}{4\cdot 37} \qquad 2\pi \approx \frac{44}{7} = \frac{2\cdot 22}{7}$$

$$2\pi \approx \frac{245}{39} = \frac{5\cdot 7^2}{3\cdot 13} \qquad 2\pi \approx \frac{289}{46} = \frac{17^2}{2\cdot 23} \qquad 2\pi \approx \frac{377}{60} = \frac{13\cdot 29}{4\cdot 3\cdot 5} \qquad 2\pi \approx \frac{628}{100} = \frac{4\cdot 157}{100}$$

${}^{23}_{11}Na_{12}$ ${}^{46,48,49,50}_{22}Ti_{24,26,27,28}$ ${}^{99}_{44}Ru_{55}$ ${}^{9\cdot 28}_{99}Es^{*}_{9\cdot 17}$ ${}^{310,24\cdot 13,2\cdot 157}_{124,125,126=18\cdot 7}Ch^{ie}_{6\cdot 31,187=11\cdot 17,188}$

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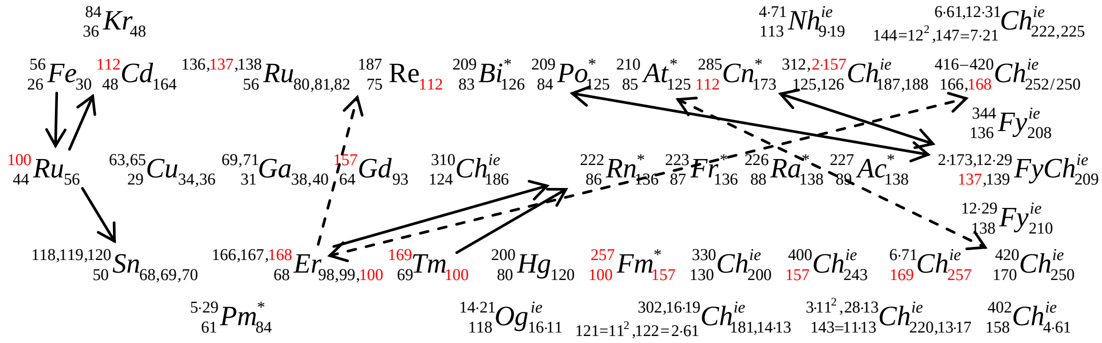
Supplement 3:

Table 8. Relationships of factors in $\alpha_{1\text{-}7}$ and $\alpha_{2\text{-}13}$ with primordial nuclides (2020/2/16-17).

Nuclides	${}_{3}Li_{4}$	${}_{29}Cu_{34}$	${}_{31}Ga_{40}$	${}_{64}Gd_{92}$	${}_{75}Re_{112}$
A	7	65=5×13	71	156=12×13	187=11×17
PN before	5	70	78	209	252
PN all	285	285	285	285	285
Ratios	1/57	14/57	26/95	11/15	84/95

1. 3, 29, 31, 64, 75 and 112 are factors in α_1 and α_2.
2. PN: primordial nuclides; PN all: usually regarded as 286.
3. Nucleon number 285 of ${}_{112}Cn_{173}$ would relate to PN all, or PN all should be 285 rather than 286, and ${}^{235}U$ should not be a primordial nuclide.
4. ${}^{235}U_{143}$ should not be a primordial nuclide, its relative stability (but not much stable) should come from relative stable nucleon numbers 92=96-4 and 143=11×13, so number of PN would become 285 from 286.

Supplement 4: Correlations of factors in α ($\alpha_{1\text{-}7}$, $\alpha_{2\text{-}13}$ and $\alpha_{1\text{-}50}$) and nuclides



Relationships between Formulas of α (α_{1-7}, α_{2-13} and α_{1-50}) and Nuclides

Dr. Gang Chen, 2020/2/18–19

In this scheme there are several important clues based on factors in the formulas of $\alpha_{1\text{-}7}$, $\alpha_{2\text{-}13}$ and $\alpha_{1\text{-}50}$ such as 6 (36, 48, 138, 144, *et al*), 7 (56, 84, 112, 126, 166-168, 210, 252), 10 (30, 50, 70, 100, 120, 130, 170, 200, 210, 220, 250, 310, 330, 400, 420), 11 (44, 88, 121, 134, 176, 187, 209, 220, 330, 363), 13 (26, 143, 169, 221, 364), 29 (87,145, 348), 25 (75, 100, 125, 200, 250, 400), 31 (93 124 186 310, 372), 61 (122, 244), 64 (136 et al), 137 (68, 69, 136,138), 139, 157 (314), 257, *et al*. And these clues correlate each others. These relationships are strong proofs that Chen's formulas of the fine-structure constant are correct, otherwise so many coincidences couldn't be explained.

In addition, numbers 7, 13 and 50 in $\alpha_{1\text{-}7}$, $\alpha_{2\text{-}13}$ and $\alpha_{1\text{-}50}$ may have the following relationships: (13+7)(13-7) =50+70=120 and ${}_{50}Sn_{70}$. And Sn is special, it has the most stable nuclides (up to 10) among which ${}_{50}Sn_{70}$ has the most relative abundance.

Supplement 5: Other two formulas of the fine-structure constant

$$\alpha_{1-9/11}=\frac{9}{11+\frac{1}{84}-\frac{1}{11\cdot 17\cdot 53+\frac{2\cdot 191}{3\cdot 157}}}\frac{1}{112+\frac{1}{75^2}}=1/137.035999037435$$

$^{19}_{9}F_{10}$ $^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{53}_{24}Cr_{29}$ $^{75}_{33}As_{42}$ $^{84}_{36}Kr_{48}$ $^{95}_{42}Mo_{53}$ $^{127}_{53}I_{74}$ $^{129}_{54}Xe_{75}$ $^{144}_{60}Nd_{84}$

$^{157}_{64}Gd_{93}$ $^{185,11\cdot 17}_{75}Re_{110,112}$ $^{191}_{77}Y_{114}$ $^{11\cdot 19}_{84}Po^{*}_{125}$ $^{257}_{100}Fm^{*}_{157}$ $^{7\cdot 37}_{102=6\cdot 17}No^{*}_{157}$ $^{6\cdot 53}_{127}Ch^{ie}_{191}$ $^{400}_{157}Ch^{ie}_{3\cdot 9^2}$

$$\alpha_{2-20/25}=\frac{20+\frac{1}{2}-\frac{1}{14}+\frac{1}{251}-\frac{1}{8\cdot(12\cdot 43\cdot 227+1)-\frac{8}{37}}}{25}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

$^{40,43,44,48}_{20}Ca_{20,23,24,28}$ $^{55}_{25}Mn_{30}$ $^{85,87}_{37}Rb_{48,50}$ $^{90,96}_{40}Zr_{50,56}$ $^{116,120,124}_{50}Sn_{66,70,74}$ $^{99}_{43}Tc^{*}_{56}$ $^{106,111,112,116}_{48}Cd_{58,63,64,68}$

$^{191,193}_{77}Y_{114,4\cdot 29}$ $^{200}_{80}Hg_{120}$ $^{6\cdot 37}_{86}Rn^{*}_{136}$ $^{223}_{87}Fr^{*}_{136}$ $^{227}_{89}Ac^{*}_{138}$ $^{251}_{98}Cf^{*}_{153}$ $^{11\cdot 29}_{127}Ch^{ie}_{192=3\cdot 64}$ $^{376}_{149}Ch^{ie}_{227}$ $^{22\cdot 19}_{167}Ch^{ie}_{251}$ 2020/2/21

Supplement 6: Other formulas of the speed of light c_{au}

$$
\begin{aligned}
c_{au} &= \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-9/11}\alpha_{2-20/25}}} \\
&= \sqrt{\frac{(11+\frac{1}{84}-\frac{1}{11\cdot 17\cdot 53+\frac{2\cdot 191}{3\cdot 157}})(112+\frac{1}{75^2})\cdot 25\cdot(112-\frac{1}{3\cdot 29\cdot 64})}{9\cdot(20+\frac{1}{2}-\frac{1}{14}+\frac{1}{251}-\frac{1}{8\cdot(12\cdot 43\cdot 227+1)-\frac{8}{37}})}} \\
&= \frac{5}{3}\sqrt{\frac{(11+\frac{1}{84}-\frac{1}{11\cdot 17\cdot 53+\frac{2\cdot 191}{3\cdot 157}})(112^2-\frac{7\cdot 19}{2^2\cdot 3^2\cdot 25^2\cdot 29}-\frac{1}{2^6\cdot 3^3\cdot 25^2\cdot 29})}{20+\frac{1}{2}-\frac{1}{14}+\frac{1}{251}-\frac{1}{8\cdot(12\cdot 43\cdot 227+1)-\frac{8}{37}}}} \\
&= \frac{5}{3}\sqrt{\frac{(11+\frac{1}{84}-\frac{1}{11\cdot 17\cdot 53+\frac{2\cdot 191}{3\cdot 157}})(112^2-\frac{1}{6\cdot 17\cdot 47+\frac{2}{3}})}{20+\frac{1}{2}-\frac{1}{14}+\frac{1}{251}-\frac{1}{8\cdot(12\cdot 43\cdot 227+1)-\frac{8}{37}}}} \\
&= \sqrt{137.035999037435\times 137.035999111818} = 137.035999074627
\end{aligned}
$$

$${}^{23}_{11}Cr_{12}\quad {}^{24-26}_{12}Mg_{12-14}\quad {}^{28-30}_{14}Si_{14-16}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{39-41}_{19}K_{20-22}\quad {}^{47}_{22}Ti_{25}\quad {}^{53}_{24}Cr_{29}\quad {}^{55}_{25}Mn_{30}\quad {}^{63,65}_{29}Cu_{34,36}\quad {}^{84}_{36}Kr_{48}$$

$${}^{84}_{37}Rb^{*}_{47}\quad {}^{85,87}_{37}Rb_{48,50}\quad {}^{90-92,94,96}_{40}Zr_{50-52,54,56}\quad {}^{99}_{43}Tc^{*}_{56}\quad {}^{107,109}_{47}Ag_{60,62}\quad {}^{127}_{53}I_{74}\quad {}^{7\cdot 19}_{55}Cs_{78}\quad {}^{5\cdot 29}_{61}Pm^{*}_{84}\quad {}^{157}_{64}Gd_{93}$$

$${}^{3\cdot 53}_{65}Tb_{94}\quad {}^{5\cdot 37,11\cdot 17}_{75}\mathrm{Re}_{110,112}\quad {}^{191,193}_{77}Y_{114,4\cdot 29}\quad {}^{11\cdot 19}_{84}Po^{*}_{125}\quad {}^{223}_{87}Fr^{*}_{136}\quad {}^{227}_{89}Ac^{*}_{138}\quad {}^{244}_{94}Pu^{*}_{150}\quad {}^{251}_{98}Cf^{*}_{9\cdot 17}\quad {}^{257}_{100}Fm^{*}_{157}$$

$${}^{6\cdot 43}_{101}Md^{*}_{157}\quad {}^{7\cdot 37}_{102}No^{*}_{157}\quad {}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,3\cdot 64,193}\quad {}^{32\cdot 11,4\cdot 89}_{140,141}Ch^{ie}_{4\cdot 53,5\cdot 43}\quad {}^{3\cdot 112}_{133}Ch^{ie}_{7\cdot 29}\quad {}^{8\cdot 47}_{149}Ch^{ie}_{227}\quad {}^{400}_{157}Ch^{ie}_{243}\quad {}^{22\cdot 19}_{167}Ch^{ie}_{251}$$

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$$c_{au} = \frac{1}{\alpha_c} = \frac{25\cdot 112}{3}\sqrt{\frac{37}{11\cdot 12\cdot 13}-\frac{1}{2\cdot 17\cdot 41\cdot 163+\frac{47}{6\cdot 31}}} = 137.035999074628$$

$${}^{23}_{11}Na_{12}\quad {}^{24,25}_{12}Mg_{12,13}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{55}_{25}Mn_{30}\quad {}^{56}_{26}Fe_{30}\quad {}^{69,71}_{31}Ga_{38,40}\quad {}^{74,77,78,82}_{34}Se_{40,43,44,48}\quad {}^{84}_{37}Rb^{*}_{47}\quad {}^{85,87}_{37}Rb_{48,50}\quad {}^{93}_{41}Nb_{52}$$

$${}^{100}_{44}Ru_{56}\quad {}^{107,109}_{47}Ag_{60,62}\quad {}^{112}_{48}Cd_{64}\quad {}^{112,114-120,122,124}_{50}Sn_{62,64-70,72,74}\quad {}^{144,147,148,150,154}_{62}Sm_{82,85,86,88,92}\quad {}^{157,158}_{64}Gd_{93,94}$$

$${}^{163}_{66}Dy_{97}\quad {}^{168}_{68}Er_{100}\quad {}^{169}_{69}Tm_{100}\quad {}^{5\cdot 37,11\cdot 17}_{75}\mathrm{Re}_{110,112}\quad {}^{204,206-16\cdot 13}_{82}Pb_{122,124-126}\quad {}^{237}_{93}Np^{*}_{12^2}\quad {}^{247}_{97}Bk^{*}_{150}\quad {}^{257}_{100}Fm^{*}_{157}\quad {}^{268}_{105}Db^{*}_{163}$$

$${}^{269}_{106}Sg^{*}_{163}\quad {}^{270}_{107}Bh^{*}_{163}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{310}_{124}Ch^{ie}_{186}\quad {}^{334}_{132}Ch^{ie}_{202}\quad {}^{3\cdot 11^2,28\cdot 13}_{143}Ch^{ie}_{220,13\cdot 17}\quad {}^{378}_{150}Ch^{ie}_{228}\quad {}^{394}_{156}Ch^{ie}_{14\cdot 17}\quad {}^{410}_{163}Ch^{ie}_{13\cdot 19}$$

Note: $112\times 5/3\approx 187=11\times 17,\ \ 112\times 25/3\approx 5\times 11\times 17$ 2020 / 2 / 24

$$c_{au} = \frac{1}{\alpha_c} = \frac{25\cdot 112}{3}\sqrt{\frac{1}{47}+\frac{1}{40\cdot 89}-\frac{1}{16\cdot 9\cdot(2\cdot 21\cdot 31\cdot 43+1)+\frac{3}{4}}} = 137.035999074627$$

$${}^{45}_{21}Ti_{24}\quad {}^{47}_{22}Ti_{25}\quad {}^{55}_{25}Mn_{30}\quad {}^{64,66,70}_{30}Zn_{34,36,40}\quad {}^{69,71}_{31}Ga_{38,40}\quad {}^{72}_{32}Ge_{40}\quad {}^{78,80,83,84,86}_{36}Kr_{42,44,47,48,50}\quad {}^{89}_{39}Y_{50}\quad {}^{90,94,96}_{40}Zr_{50,54,56}$$

$${}^{92,94-98,100}_{42}Mo_{50,52-56,58}\quad {}^{98,99}_{43}Tc^{*}_{55,56}\quad {}^{107,109}_{47}Ag_{60,62}\quad {}^{112}_{48}Cd_{64}\quad {}^{144,147,148,150,152}_{62}Sm_{82,85,86,88,90}\quad {}^{151,153}_{63}Eu_{88,90}\quad {}^{178}_{72}Hf_{106}$$

$${}^{185,187}_{75}\mathrm{Re}_{110,112}\quad {}^{222}_{86}Rn^{*}_{136}\quad {}^{227}_{89}Ac^{*}_{138}\quad {}^{237}_{93}Np^{*}_{144}\quad {}^{244}_{94}Pu^{*}_{150}\quad {}^{326,328}_{129}Ch^{ie}_{197/199}\quad {}^{366}_{144}Ch^{ie}_{222}\quad {}^{9\cdot 42}_{150}Ch^{ie}_{228}$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{25\cdot 112}{3}\sqrt{\frac{1}{46}-\frac{1}{55\cdot 100}+\frac{1}{9\cdot 25\cdot 13\cdot(20\cdot 293+1)-\frac{4}{23}}} = 137.035999074627$$

$${}^{50,51}_{23}Mn_{27,28}\quad {}^{55}_{25}Mn_{30}\quad {}^{89}_{39}Y_{50}\quad {}^{99,100}_{44}Ru_{55,56}\quad {}^{106,110}_{46}Pd_{60,64}\quad {}^{117}_{50}Sn_{67}\quad {}^{133}_{55}Cs_{78}\quad {}^{169}_{69}Tm_{100}\quad {}^{185,187}_{75}\mathrm{Re}_{110,112}\quad {}^{195}_{78}Pd_{117}$$

$${}^{5\cdot 47,238}_{92}U^{*}_{11\cdot 13,146}\quad {}^{257}_{100}Fm^{*}_{157}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{293}_{116}Lv^{ie}_{177}\quad {}^{294}_{117}Ts^{ie}_{177}\quad {}^{400}_{157}Ch^{ie}_{243}\quad {}^{426}_{169}Ch^{ie}_{257}$$

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Supplement 7: Comparison of formulas of 1, N, e, 2π, π/2, ϕ, α, α_c, c_{au} and $\alpha_{p/c}$

$$1 = 4\gamma_c + \frac{4\gamma_1}{1(1+1)} + \frac{4\gamma_2}{2(2+1)} + \frac{4\gamma_3}{3(3+1)} + \cdots$$

$$= |B|\frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{|B_{2n}|(\pi/2)^{2n}}{(2n)!} = \sum_{n=1}^{\infty}\frac{|B_{2n}|\pi^{2n}}{(2n)!} = -|B|\frac{3\pi}{2} + \sum_{n=1}^{\infty}\frac{|B_{2n}|(3\pi/2)^{2n}}{(2n)!}$$

$$N \sim -\frac{3}{2}|B| + \sum_{n=1}^{\mathrm{N}}\frac{|B_{2n}|(2\pi)^{2n}}{2(2n)!}$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots, \quad \frac{\pi}{2} = \left(\frac{e}{e^{\gamma_g}}\right)^2 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

$$2\pi \approx \frac{4\cdot 157}{100} \approx \frac{9\cdot 37}{53} \approx \frac{4\cdot 5\cdot 71}{15^2+1} \approx \cdots, \quad \frac{\pi}{2} \approx \frac{157}{25} \approx \frac{9(9+1/4)}{53} \approx \frac{5\cdot 71}{15^2+1} \approx \cdots$$

$$\phi_1 = \frac{\sqrt{5}-1}{2} = 0.618\cdots, \quad \phi_2 = -\frac{\sqrt{5}+1}{2} = -1.618\cdots$$

$$\sqrt{\frac{\sqrt{5}+1}{2} + 2} - \frac{\sqrt{5}+1}{2} = \cfrac{e^{-\frac{2\pi}{5}}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1+\cdots}}}} \qquad 2\pi = \frac{9801}{\sqrt{2}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}}$$

$$\alpha_1 = \frac{6^2}{7\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{113}{112})^{225}}}\;\frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{13\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{9\cdot 31}{278})^{557}}}{10^2}\;\frac{1}{112 - \frac{1}{3\cdot 29\cdot 64}} = 1/137.0359991118181$$

$$\frac{1}{\alpha_c^2} = \frac{1}{\alpha_1\alpha_2} = 112\times\left(168 - \frac{1}{3} + \frac{1}{12\cdot 47} - \frac{1}{6\cdot 29\cdot 53\cdot 59 - 79/47}\right)$$

$$= 137.035999074627^2 = 18778.865042381$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}} = \frac{5}{3}\sqrt{\frac{7}{13}\frac{(2\pi)_{112}}{(2\pi)_{278}}\left(112^2 - \frac{1}{30^2\cdot 5} + \frac{1}{60^2\cdot 15} - \frac{1}{120^2\cdot 15\cdot 29}\right)}$$

$$= \frac{25\cdot 112}{3}\sqrt{\frac{1}{47} + \frac{1}{40\cdot 89} - \frac{1}{16\cdot 9\cdot(2\cdot 21\cdot 31\cdot 43+1) + \frac{3}{4}}}$$

$$= \frac{25\cdot 112}{3}\sqrt{\frac{1}{46} - \frac{1}{55\cdot 100} + \frac{1}{9\cdot 25\cdot 13\cdot(20\cdot 293+1) - \frac{4}{23}}} = \cdots = 137.035999074627$$

$$\frac{1}{\alpha_{p/c}^2} = \frac{1}{\alpha_{p/1}\alpha_{p/2}} = 252.040872632515^2 = 63524.60147736 \quad \text{(Supposed)}$$

The relations of the above formulas are sophisticated. In general, some formulas such as 1, N, e and 2π have similar form (called the natural group form), some formulas such as ϕ, α, α_c and c_{au} can be divided into rational parts and irrational parts for each which may imply they have the same reasonability. In addition, 2π, $\pi/2$, ϕ, α, α_c, c_{au} and $\alpha_{p/c}$ are all proportional constants, so they should have some similar or the same regularities.

Supplement 8: Comparison of pictures of elements and ϕ

With the hints of the above formulas, it is not strange that the gold section ($\phi\approx0.618$) appears in the elements, it should appear in some places with some forms.

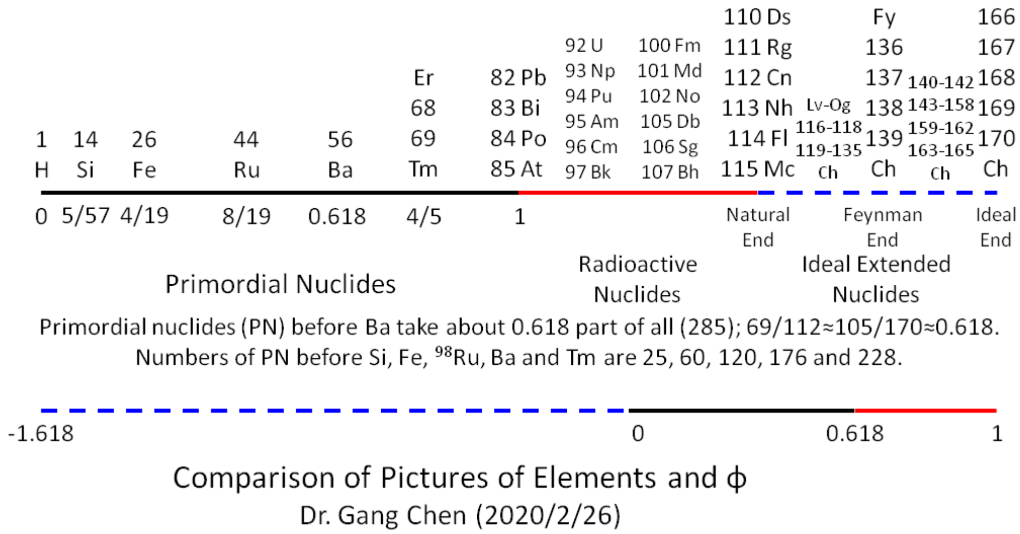


Fig. 11

Imagine a one-dimension creature lives in the line 0-1, he is familiar with 0-0.618 line space and can reach 0.618-1 line space, if he is enough smart, he may feel there should be an ideal extended line space from 0 to -1.618, but he couldn't reach all or can only get the margin of it. The same situation is suitable for us, we live in the space of elements, we mainly utilize the stable elements and can use some radioactive elements before the 112th element Cn, moreover, there should be a space for ideal extended elements from the 119th to the 170th, a few of which we can synthesize, many of which we can't, but this space should exist. This situation is also suitable for our lives in the earth, the solar system and the universe, or even in the mater, dark matter and dark energy, except that the proportion ratios should be different.

Supplement 9: Primordial nuclides and Fibonacci sequences

2π connects to nuclides and 2π also connects to Gold Section ϕ as described by

Ramanujan's formulas, so ϕ should connect to nuclides. And Fibonacci sequences are the integer presentations of ϕ, so Fibonacci sequences should connect to nuclides. These connections are described as follows.

Fibonacci Sequence p_1: 1 8 9 17 26 43 69 112

p ${}^{16,17,18}_{8}\mathrm{O}_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{56}_{26}Fe_{30}$ ${}^{69}_{31}Ga_{38}$ ${}^{100}_{43}Tc^{*}_{56}$ ${}^{112}_{48}Gd_{64}$ ${}^{119}_{50}\mathrm{Re}_{69}$

${}^{169}_{69}Tm_{100}$ ${}^{187}_{75}\mathrm{Re}_{112}$ ${}^{188}_{76}Os_{112}$ ${}^{285}_{112}Cn^{*}_{173}$

Note: ${}^{56}_{26}Fe_{30}$ ${}^{100}_{43}Tc^{*}_{56}$ ${}^{169}_{69}Tm_{100}$, relay of the numbers 56 and 100.

Fibonacci Sequence p_2: 1 10 11 21 32 53 85 138

p ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{45}_{21}Sc_{24}$ ${}^{53}_{24}Cr_{29}$ ${}^{74}_{32}Ge_{42}$ ${}^{85}_{37}Rb_{48}$ ${}^{127}_{53}I_{74}$ ${}^{138}_{56}Ba_{82}$ ${}^{138}_{57}La_{81}$

${}^{138}_{58=2\cdot 29}Ce_{80}$ ${}^{5\cdot 29}_{60}La_{85}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{227}_{87=3\cdot 29}Ac^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{12\cdot 29}_{138}Fy^{ie}_{210}$

Note: ${}^{74}_{32}Ge_{42}$ ${}^{127}_{53}I_{74}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{12\cdot 29}_{138}Fy^{ie}_{210}$, relay of the numbers 29, 74 and 210.

Fibonacci Sequence n_1: 0 8 8 16 24 40 64 104 168

n ${}^{16}_{8}O_{8}$ ${}^{32}_{16}S_{16}$ ${}^{31}_{15}P_{16}$ ${}^{24}_{12}Mg_{12}$ ${}^{40}_{18}Ar_{22}$ ${}^{40,44}_{20}Ca_{20,24}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{58,60}_{28}Ni_{30,32}$ ${}^{63,65}_{29}Cu_{34,36}$

${}^{71}_{31}Ga_{40}$ ${}^{70,72}_{32}Ge_{38,40}$ ${}^{90,92,94,96}_{40}Zr_{50,52,54,56}$ ${}^{104}_{46}Pd_{58}$ ${}^{112}_{48}Cd_{64}$ ${}^{113}_{49}In_{64}$ ${}^{152,154-157,158,160}_{64}Gd_{88,90-94,96}$

${}^{168}_{68}Er_{100}$ ${}^{168,170,173,174}_{70}Yb_{98,100,103,104}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{257}_{100}Ch^{*}_{157}$ ${}^{265}_{104}Rf^{*}_{161}$ ${}^{418}_{168}Ch^{ie}_{250}$ ${}^{426}_{169}Ch^{ie}_{257}$ ${}^{420}_{170}Ch^{ie}_{250}$

Note: ${}^{16}_{8}O_{8}$ ${}^{40,44}_{20}Ca_{20,24}$ ${}^{112}_{48}Cd_{64}$, relay of 29, 71, 90, 92, 94, 96 and 100.

Fibonacci Sequence n_2: 0 4 4 8 12 20 32 52 84 136

n ${}^{4}_{2}He_{2}$ ${}^{9}_{4}Be_{5}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{40,42,44,46}_{20}Ga_{20,22,24,26}$ ${}^{73}_{32}Ge_{41}$ ${}^{84}_{36}Kr_{48}$

${}^{125}_{52}Te_{73}$ ${}^{144}_{60}Nd_{84}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{344,346}_{136,137}Fy^{ie}_{208,209}$

Note: relay of the numbers 24, 41, 73, 125 and 209.

Fibonacci Sequence e: −1 9 8 17 25 42 67 109 176 285

e ${}^{19}_{9}F_{10}$ ${}^{16,17,18}_{8}\mathrm{O}_{8,9,10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{67}_{30}Zn_{37}$ ${}^{97,98}_{42}Mo_{55,56}$ ${}^{109}_{47}Ag_{62}$

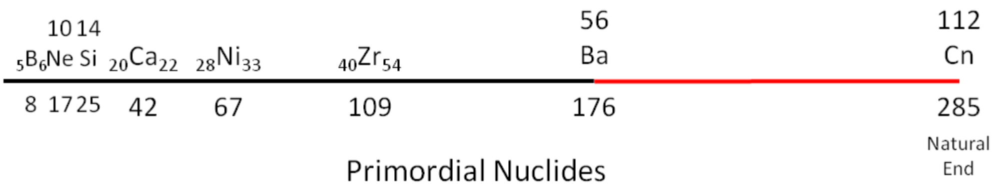
${}^{117}_{50=2\cdot 25}Sn_{67}$ ${}^{165}_{67}Ho_{98}$ ${}^{176}_{71}Lu_{105}$ ${}^{183}_{74}W_{109}$ ${}^{4\cdot 69}_{109}Mt^{*}_{167}$ ${}^{21\cdot 14}_{118}Og_{176}$ ${}^{285}_{112}Cn^{*}_{173}$

Note: ${}^{117}_{50=2\cdot 25}Sn_{67}$, Fibonacci Sequence e is less relevent to specific nuclides.

Numbers of primordial nuclides before ${}_{6}C$ ${}^{11}_{5}B_{6}$ ${}_{10}Ne$ ${}_{14}Si$ ${}^{42}_{20}Ca_{22}$ ${}^{61}_{28}Ni_{33}$ ${}^{94}_{40}Zr_{54}$ ${}_{56}Ba$ and ${}_{112}Cn^{*}$ are 9 8 17 25 42 67 109 176 and 285

2020 / 2 / 27 – 28, 3 / 3 (add Fibonacci Sequence p_2 and n_2)

As stated in **Section 4**, the mathematic expression of chirality is $\pm 2\pi$. There are 10 fingers in a pair of human hands, and there are 14 finger segments in a single hand. It was assumed by us that the numbers 10/20 and 28/56 stand for a pair of "hands" in the world of nuclides. So nuclides ${}_{10}$Ne ${}_{14}$Si ${}_{20}$Ca ${}_{28}$Ni ${}_{40}$Zr ${}_{56}$Ba and ${}_{112}$Cn stand for two pairs of "hands" emerging gradually, and the numbers of primordial nuclides just before them are the numbers of Fibonacci Sequence e. This means chirality or $\pm 2\pi$ is the inner essence and ϕ or Fibonacci sequence is the outer expression of nuclides.

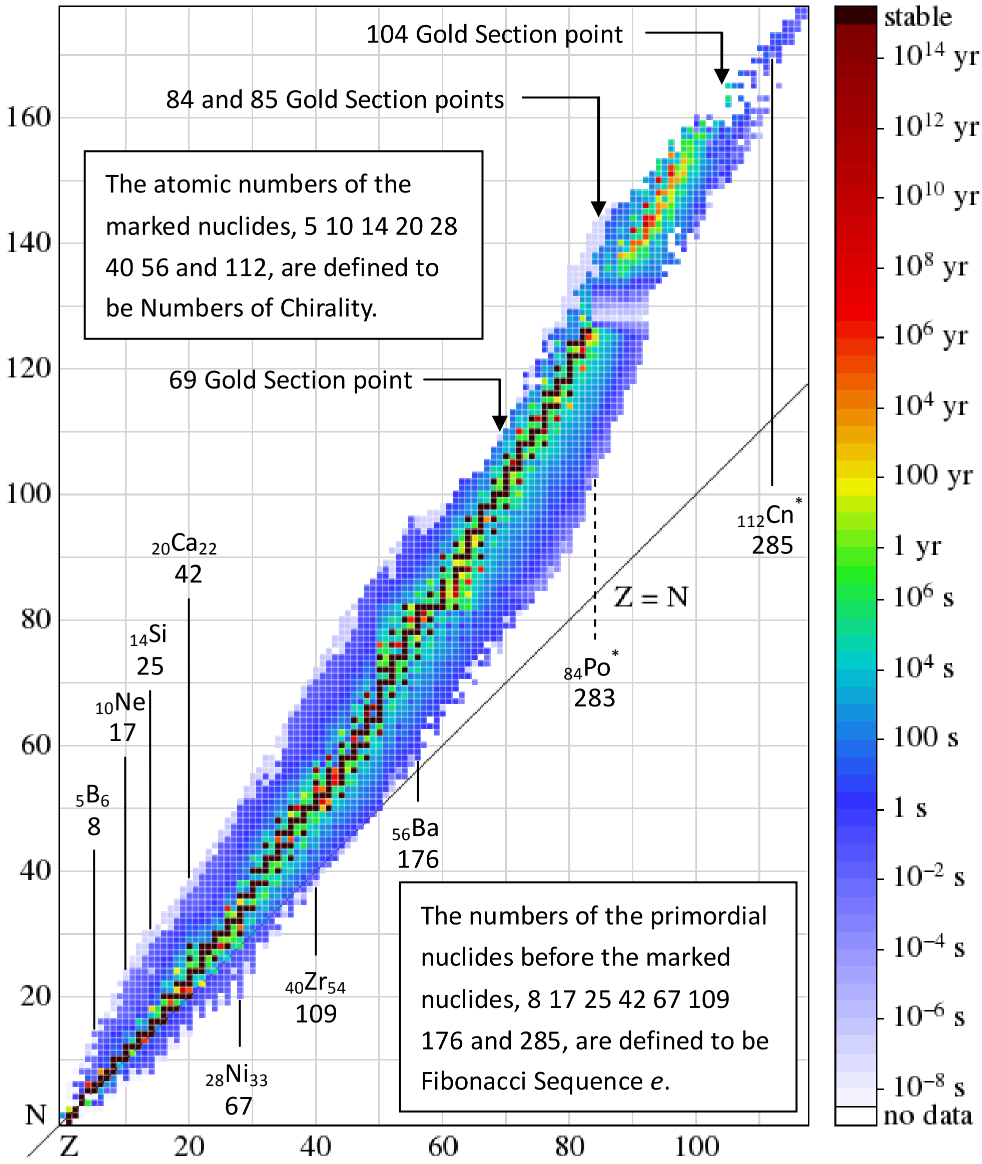


Primordial Nuclides

Primordial nuclides (PN) before Ba take about 0.618 part of all (176/285)
Numbers of PN before $_6C$ $_5B_6$ $_{10}Ne$ $_{14}Si$ $_{20}Ca_{22}$ $_{28}Ni_{33}$ $_{40}Zr_{54}$ $_{56}Ba$ and $_{112}Cn$ are
9 8 17 25 42 67 109 176 and 285 which is Fibonacci Sequence *e*

Primordial Nulclides and Fibonacci Sequence *e*
Dr. Gang Chen (2018/1-3, 2020/2/29-3/1)

Fig. 12



The Integrated Picture of Nuclides and Fibonacci Sequences
The Nuclide Picture was taken from Wikipedia
Dr. Gang Chen (2018/1-3; 2020/3/1-3, 4/24)

Fig. 13

Why are there two pairs of “hands” in the world of nuclides? This should be because a pair of “hands” takes the right “hand” as priority and the other pair takes left “hand” as priority. So, 5 10 14 20 28 40 56 112 could be defined to be Numbers of Chirality, they are connected to Fibonacci Sequence e in nuclides (**Fig 13**).

Supplement 10: Other formulas of the speed of light c_{au}

$$c_{au}=\frac{1}{\alpha_c}=\frac{25\cdot 112}{3\cdot\sqrt{46+\frac{1}{2}-\frac{1}{8}+\frac{1}{77}-\frac{1}{17\cdot(2\cdot 11\cdot 19+1)-\frac{149}{13\cdot 19}}}}=137.035999074627$$

${}^{16,17}_{8}O_{8,9}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39}_{19}K_{20}$ ${}^{55}_{25}Mn_{30}$ ${}^{102,104,105,110}_{46}Pd_{56,58,59,64}$ ${}^{131}_{54}Xe_{77}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{191,192}_{77}Ir_{114,116}$
${}^{209=11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{247=13\cdot 19}_{96}Cm^{*}_{151}$ ${}^{247}_{97}Cm^{*}_{150}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{376}_{149}Ch^{ie}_{227}$ ${}^{410}_{163}Ch^{ie}_{247}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{25\cdot 112}{3\cdot(7-\frac{1}{5}+\frac{1}{4\cdot 23}-\frac{1}{3\cdot(16\cdot 9\cdot 17\cdot 19-1)+\frac{11}{56}\text{ or }\frac{34}{173}})}=137.035999074627$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39}_{19}K_{20}$ ${}^{46-50}_{22}Ti_{24-28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56-58}_{26}Fe_{28,30-32}$ ${}^{78,80,82}_{34}Se_{44,46,48}$
${}^{100}_{44}Ru_{56}$ ${}^{102,105,106,108,110}_{46}Pd_{56,59,60,62,64}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$
${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{235,138}_{92}U^{*}_{143,146}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{284}_{113}Nh^{ie}_{171}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{366}_{144}Ch^{ie}_{222}$ ${}^{384}_{153}Ch^{ie}_{231}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{4\cdot 100}{3}(1+\frac{1}{36}-\frac{1}{63\cdot(8\cdot 15\cdot 17-1)-\frac{34}{173}\text{ or }\frac{157}{17\cdot 47}})=137.035999074627$$

${}^{7}_{3}Li_{4}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,68,70}_{30}Zn_{34,36,38,40}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{82-84,86}_{36}Kr_{46-48,50}$ ${}^{90,91,94}_{40}Zr_{50,51,54}$
${}^{100}_{44}Ru_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{111}_{48}Cd_{63}$ ${}^{118,120,122}_{50}Sn_{68,70,72}$ ${}^{136}_{56}Ba_{80}$ ${}^{144,145,150}_{60}Nd_{84,85,90}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{157}_{64}Gd_{93}$ ${}^{168}_{68}Er_{100}$ ${}^{173}_{70}Yb_{103}$
${}^{199,200,204}_{80}Hg_{119,120,124}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{259}_{102}No^{*}_{157}$ ${}^{300}_{120}Ch^{ie}_{180}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{344,345}_{136}Fy^{ie}_{208,209}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{4\cdot 100}{3\cdot(1-\frac{1}{37}+\frac{1}{29\cdot(10\cdot 13\cdot 36-1)+\frac{31}{81}})}=137.035999074627$$

${}^{7}_{3}Li_{4}$ ${}^{19}_{9}F_{10}$ ${}^{20,22}_{10}Ne_{10,12}$ ${}^{27}_{13}Al_{14}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{40,44,46,48}_{20}Ca_{20,26,24,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{84}_{36}Kr_{48}$
${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{90,91,92,94}_{40}Zr_{50,51,52,54}$ ${}^{137}_{56}Ba_{81}$ ${}^{196,200,204}_{80}Hg_{116,120,124}$ ${}^{7\cdot 29,205}_{81}Tl_{122,4\cdot 31}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{300}_{120}Ch^{ie}_{180}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{4\cdot 100}{3}\sqrt{1+\frac{1}{17}-\frac{1}{2\cdot 199}+\frac{1}{50\cdot(14\cdot 43\cdot 173+1)+\frac{5}{17}}}=137.035999074627$$

${}^{7}_{3}Li_{4}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{20,22}_{10}Ne_{10,12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{40,42,43,44,48}_{20}Ca_{20,22,23,24,28}$ ${}^{50}_{22}Ti_{28}$ ${}^{76,77}_{34}Se_{42,43}$ ${}^{86}_{36}Kr_{50}$ ${}^{90,91}_{40}Zr_{50,51}$
${}^{99}_{43}Tc^{*}_{56}$ ${}^{100}_{44}Ru_{56}$ ${}^{118,120}_{50}Sn_{68,70}$ ${}^{168}_{68}Er_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{199,200}_{80}Hg_{7\cdot 17,120,124}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}Rn^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{300}_{120}Ch^{ie}_{180}$
${}^{312}_{125}Ch^{ie}_{11\cdot 17}$ ${}^{328}_{129}Ch^{ie}_{199}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$ ${}^{420}_{170}Ch^{ie}_{250}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{4\cdot 100}{3\cdot\sqrt{1-\frac{1}{18}+\frac{1}{5\cdot 89}-\frac{1}{75\cdot 10\cdot 19\cdot 79+\frac{11}{20}}}}=137.035999074627$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}K_{20,21,22}$
${}^{40,42,44,48}_{20}Ca_{20,22,24,28}$ ${}^{46-50}_{22}Ti_{24-28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{66,68,70}_{30}Zn_{36,38,40}$ ${}^{75}_{33}As_{42}$ ${}^{79}_{35}Br_{44}$ ${}^{89}_{39}Y_{50}$ ${}^{116,120,122}_{50}Sn_{66,70,72}$
${}^{90,94,96}_{40}Zr_{50,54,56}$ ${}^{99,100,101,102}_{44}Ru_{55,56,57,60}$ ${}^{129,130,131,134,136}_{54}Xe_{75,76,77,80,82}$ ${}^{135}_{56}Ba_{79}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{150}_{62}Sm_{88}$ ${}^{151,153}_{63}Eu_{88,90}$
${}^{161}_{66}Dy_{95}$ ${}^{197}_{79}Au_{118}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{190}_{76}Os_{114}$ ${}^{198,200,201}_{80}Hg_{118,120,121}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{243}_{95}Am^{*}_{148}$
${}^{247}_{97}Bk^{*}_{150}$ ${}^{300}_{120}Ch^{ie}_{180}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{378}_{150}Ch^{ie}_{228}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{418}_{168}Ch^{ie}_{250}$ ${}^{420}_{170}Ch^{ie}_{250}$

Supplement 11: Construct formulas of the fine-structure constant with Wallis formula of π/2 instead of 2π-e formula

Wallis Formula of $\pi/2$:

Traditional format: $\frac{\pi}{2}=\frac{2}{1}\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots=\prod_{n=1}^{\infty}(\frac{2n}{2n-1}\frac{2n}{2n+1})$

Natural group format: $\frac{\pi}{2}=2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots=2\prod_{n=1}^{\infty}(\frac{2n}{2n+1}\frac{2n+2}{2n+1})$

$$(\frac{\pi}{2})_k=2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{2k}{2k+1}\frac{2k+2}{2k+1}=2\prod_{n=1}^{k}(\frac{2n}{2n+1}\frac{2n+2}{2n+1})$$

Comparable and similar to 2π-e formula: $(2\pi)_k=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}$

So, Wallis Formula of $\pi/2$ could be used to construct formulas of α.

Note: There should be $(2\pi)_k\sim4(\frac{\pi}{2})_{3k/2}$, or $(2\pi)_k$ and $(\frac{\pi}{2})_{3k/2}$ keep the same accuracy.

$$\alpha_{1-7-Wallis}=\frac{9}{7\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{338}{339}\frac{2\cdot10\cdot17}{2\cdot169+1})}\frac{1}{112+\frac{1}{11\cdot137+\frac{13\cdot41}{2\cdot19\cdot49-1}}}$$

$=1/137.035999037435$

${}^{14}_{7}N_{7}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,36}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,42,44,46,48}_{20}Ca_{20,22,24,26,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{76,77,78,80,82}_{34}Se_{42,43,44,46,48}$ ${}^{87,88}_{38}Sr_{49,50}$ ${}^{93}_{41}Nb_{52}$ ${}^{98}_{42}Mo_{56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{151,9\cdot17}_{63}Eu_{88,90}$ ${}^{166,167,168,170}_{68}Er_{98,99,100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{188,3\cdot7\cdot9}_{76=4\cdot19}Os_{112,113}$ ${}^{9\cdot23,16\cdot13}_{82}Pb_{125,126}$ ${}^{2\cdot113}_{88}Ra^{*}_{138}$ ${}^{251}_{98}Cf^{*}_{9\cdot17}$ ${}^{277}_{108}Hs^{*}_{169}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{284}_{113}Nh^{ie}_{9\cdot19}$ ${}^{344,2\cdot173}_{136,137}Fy^{ie}_{16\cdot13,11\cdot19}$ ${}^{340}_{134,135}Ch^{ie}_{206,5\cdot41}$ ${}^{348}_{139}Ch^{ie}_{11\cdot19}$ ${}^{426}_{169}Ch^{ie}_{257}$ ${}^{420}_{170}Ch^{ie}_{250}$

$$\alpha_{1-22-Wallis}=\frac{113}{4\cdot22\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{2346}{2347}\frac{4\cdot(12\cdot49-1)}{2\cdot3\cdot17\cdot23+1})}\frac{1}{112+\frac{1}{2\cdot103\cdot(24\cdot13+1)+\frac{1}{14}}}$$

$=1/137.035999037435$

${}^{23}_{11}Na_{12}$ ${}^{46,48-50}_{22}Ti_{24,26-28}$ ${}^{99,100,102,104}_{44}Ru_{55,56,58,60}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{173}_{70}Yb_{103}$ ${}^{189}_{76}Os_{113}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{284}_{113}Nh^{ie}_{171}$

$$\alpha_{1-29-Wallis}=\frac{149}{4\cdot29\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{964}{965}\frac{6\cdot7\cdot23}{2\cdot2\cdot(2\cdot11^2-1)+1})}\frac{1}{112+\frac{1}{27\cdot7\cdot73-\frac{5}{9}}}$$

$=1/137.035999037435$

${}^{7}_{3}Li_{4}$ ${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Al_{34,36}$ ${}^{92,96,98,100}_{42}Mo_{50,54,56,58}$ ${}^{102}_{46}Pd_{58}$ ${}^{125}_{52}Te_{73}$ ${}^{138}_{58}Ce_{80}$ ${}^{149}_{62}Sm_{87}$ ${}^{181}_{73}Ta_{108}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{12\cdot29}_{138}Fy^{ie}_{210}$

$$\alpha_{1-36-Wallis}=\frac{5\cdot37}{16\cdot9\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{708}{709}\frac{10\cdot71}{12\cdot59+1})}\frac{1}{112+\frac{1}{15\cdot(2\cdot19\cdot41+1)+\frac{4}{3\cdot15}}}$$

$=1/137.035999037435$

${}^{7}_{3}Li_{4}$ ${}^{19}_{9}F_{10}$ ${}^{31}_{15}P_{16}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{80,82,84,86}_{36}Kr_{44,46,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{86,88}_{38}Sr_{48,50}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{175,176}_{71}Lu_{104,105}$

$$\alpha_{1-43-Wallis}=\frac{13\cdot17}{4\cdot43\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{602}{603}\frac{4\cdot(8\cdot19-1)}{2\cdot7\cdot43+1})}\frac{1}{112+\frac{1}{18\cdot(2\cdot9\cdot11+1)}-\frac{41\cdot(2\cdot11\cdot17-1)}{2\cdot10^{11}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{73}_{42}Ge_{41}$ ${}^{93}_{41}Nb_{52}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{222}_{86}Rn^{*}_{136}$

$$\alpha_{1-50-Wallis}=\frac{257}{4\cdot50\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{542}{543}\frac{32\cdot17}{2\cdot(270+1)+1})}\frac{1}{112+\frac{1}{3\cdot29\cdot61+\frac{7\cdot11}{107}}}$$

$$=1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{87}_{37}Rb_{50}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{106,108}_{46}Pd_{60,62}$ ${}^{114,116-120,122}_{50}Sn_{64,66-70,72}$ ${}^{136}_{56}Ba_{80}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{145.146,147}_{61}Pm^{*}_{84,85,86}$ ${}^{166-168,170}_{68}Er_{98-100,102}$ ${}^{200}_{80}Hg_{120}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{302,304}_{121,122}Ch^{ie}_{181,182}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{344}_{136}Ch^{ie}_{208}$

$$\alpha_{1-59-Wallis}=\frac{3\cdot101}{4\cdot59\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{7926}{7927}\frac{8\cdot(9\cdot10\cdot11+1)}{2\cdot3\cdot(10\cdot11\cdot12+1)+1})}\frac{1}{112+\frac{1}{40\cdot67\cdot233+\frac{16}{25}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{12}_{6}C_{6}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{55}_{25}Mn_{30}$ ${}^{90,96}_{40}Zr_{50,56}$ ${}^{116,117,118,120,122}_{50}Sn_{66,67,68,70,72}$ ${}^{165}_{67}Ho_{98}$ ${}^{171}_{70}Yb_{101}$

$$\alpha_{1-81-Wallis}=\frac{4\cdot26}{81\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{4814}{4815}\frac{43\cdot112}{2\cdot29\cdot83+1})}\frac{1}{112+\frac{1}{8\cdot27\cdot37\cdot(4\cdot53-1)}}$$

$$=1/137.035999037435$$

${}^{9}_{4}Be_{5}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{126}_{52}Te_{74}$ ${}^{127}_{53}I_{74}$ ${}^{128}_{54}Xe_{74}$ ${}^{137}_{56}Ba_{81}$ ${}^{138,140,142}_{58}Ce_{80,82,84}$ ${}^{186}_{74}W_{112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{265}_{104}Rf^{*}_{161}$ ${}^{285}_{112}Cn^{*}_{173}$

$$\alpha_{1-96-Wallis}=\frac{17\cdot29}{4\cdot96\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{17418}{17419}\frac{10\cdot26\cdot67}{6\cdot(4\cdot6\cdot11^{2}-1)})}\frac{1}{112+\frac{17^{2}\cdot23}{25\cdot10^{10}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{20-22}_{10}Ne_{10-12}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{50,52-54}_{24}Cr_{26,28-30}$ ${}^{55}_{25}Mn_{30}$ ${}^{56,58}_{26}Fe_{30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{165}_{67}Ho_{98}$

$$\alpha_{1-103-Wallis}=\frac{23^{2}}{4\cdot103\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{3930}{3931}\frac{4\cdot(24\cdot41-1)}{2\cdot15\cdot(12\cdot11-1)+1})}\frac{1}{112+\frac{23\cdot(4\cdot(8\cdot7\cdot19+1)+1)}{4\cdot10^{11}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{50,52,54}_{24}Cr_{26,28,30}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{103}_{45}Rh_{58}$ ${}^{138}_{56}Ba_{82}$ ${}^{173}_{70}Yb_{103}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$

$$\alpha_{1-133-Wallis}=\frac{36\cdot19-1}{4\cdot7\cdot19\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{37170}{37171}\frac{4\cdot(4\cdot23\cdot101+1)}{2\cdot9\cdot5\cdot7\cdot59+1})}\frac{1}{112+\frac{12\cdot37\cdot(420+1)}{25\cdot10^{11}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{39,40}_{19}K_{20,21}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{58,60}_{28}Ni_{30,32}$ ${}^{84}_{36}Kr_{48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{88}_{38}Sr_{50}$ ${}^{105}_{46}Pd_{59}$ ${}^{138}_{56}Ba_{82}$ ${}^{188}_{76}Os_{112}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{280}_{111}Rg^{*}_{169}$

$$\alpha_{1-140-Wallis}=\frac{6^{2}\cdot20-1}{4\cdot140\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{5768}{5769}\frac{10\cdot(2\cdot17^{2}-1)}{56\cdot103+1})}\frac{1}{112+\frac{11\cdot23\cdot67}{5\cdot10^{11}}}=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{40,42,23,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{116-120,122}_{50}Sn_{66-70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{165}_{67}Ho_{98}$ ${}^{173}_{70}Yb_{103}$

$$\alpha_{1-155-Wallis}=\frac{199}{5\cdot 31\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{11964}{11965}\frac{2\cdot 31\cdot 193}{12\cdot(12\cdot 83+1)+1})}\frac{1}{112+\frac{1}{8\cdot 71\cdot(10\cdot 13\cdot 29-1)}}$$

$=1/137.035999037435$

${}^{69,71}_{31}Ga_{38,40}$ ${}^{155}_{64}Gd_{91}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{193}_{77}Ir_{116}$ ${}^{199}_{80}Hg_{119}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{320}_{127}Ch^{ie}_{193}$ ${}^{328}_{129}Ch^{ie}_{199}$ ${}^{392}_{155}Ch^{ie}_{237}$

$$\alpha_{1-170-Wallis}=\frac{9\cdot 97}{4\cdot 10\cdot 17\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{103324}{103325}\frac{6\cdot 17\cdot(4\cdot 11\cdot 23+1)}{2\cdot 26\cdot(4\cdot 7\cdot 71-1)+1})}\frac{1}{112+\frac{7\cdot 19\cdot 83}{25\times 10^{11}}}$$

$=1/137.035999037435$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,42-44,46,48}_{20}Ca_{20,22-24,26,28}$

${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56-58}_{26}Fe_{28,30-32}$ ${}^{58,60-62,64}_{28}Ni_{30,32-34,36}$ ${}^{74,76-78,80}_{34}Se_{40,42-44,46}$ ${}^{90-92,94,96}_{40}Zr_{50-52,54,56}$

${}^{97}_{42}Mo_{55}$ ${}^{96,98-102,104}_{44}Ru_{52,54-58,60}$ ${}^{112,114-120,122}_{50}Sn_{62,64-70,72}$ ${}^{120,122,124-126,128,130}_{52}Te_{68,70,72-74,76,78}$ ${}^{133}_{55}Cs_{78}$

${}^{130,132,134-138}_{56}Ba_{74,76,78-82}$ ${}^{142,143}_{60}Nd_{82,83}$ ${}^{163}_{66}Dy_{97}$ ${}^{166-168,170}_{68}Er_{98-100,102}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$

${}^{210}_{85}At^{*}_{125}$ ${}^{232}_{90}Th^{*}_{142}$ ${}^{235,238}_{92}U^{*}_{143,146}$ ${}^{13\cdot 19}_{97}Bk^{*}_{150}$ ${}^{284}_{113}Nh^{ie}_{171}$ ${}^{336}_{133}Ch^{ie}_{7\cdot 29}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{358,360}_{142}Ch^{ie}_{216,218}$

$$\alpha_{2-10-Wallis}=\frac{4\cdot 10\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{312}{313}\frac{2\cdot 157}{2\cdot 12\cdot 13+1})}{7\cdot 11}\frac{1}{112-\frac{1}{2\cdot 27^{2}}+\frac{32\cdot 19\cdot(4\cdot(4\cdot 7\cdot 11-1)+1)}{25\cdot 10^{11}}}$$

$=1/137.035999111818$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{70,72,74,76}_{32}Ge_{38,40,42,44}$

${}^{131}_{54}Xe_{77}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{157}_{64}Gd_{93}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$\alpha_{2-13-Wallis}=\frac{13\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{834}{835}\frac{4\cdot 11\cdot 19}{6\cdot 139+1})}{25}\frac{1}{112-\frac{1}{5\cdot 41\cdot 49-\frac{53}{11\cdot 19}}}=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{79}_{35}Br_{44}$ ${}^{93}_{41}Nb_{52}$ ${}^{5\cdot 19}_{42}Mo_{53}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{113,115}_{49}In_{64,66}$

${}^{114,116,117,120}_{50}Sn_{64,66,67,70}$ ${}^{125}_{52}Te_{73}$ ${}^{139}_{57}La_{82}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$

${}^{24\cdot 13}_{125}Ch^{ie}_{11\cdot 17}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{11\cdot 19}$ ${}^{348}_{139}Ch^{ie}_{11\cdot 19}$ ${}^{2\cdot 11\cdot 19}_{167}Ch^{ie}_{251}$

$$\alpha_{2-23-Wallis}=\frac{4\cdot 23\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{482}{483}\frac{4\cdot 11^{2}}{2\cdot(12\cdot 20+1)+1})}{3\cdot 59}\frac{1}{112-\frac{1}{16\cdot 3\cdot(4\cdot 7\cdot 11-1)+\frac{3\cdot 89}{2\cdot 11\cdot 13}}}$$

$=1/137.035999111818$

${}^{7}_{3}Li_{4}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{48,50}_{22}Ti_{26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{89}_{39}Y_{50}$ ${}^{105}_{46}Pd_{59}$ ${}^{112}_{48}Cd_{64}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{235}_{92}U^{*}_{143}$

$$\alpha_{2-29-Wallis}=\frac{4\cdot 29\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{1966}{1967}\frac{3\cdot 16\cdot 41}{2\cdot(24\cdot 41-1)+1})}{223}\frac{1}{112-\frac{1}{4\cdot 41\cdot 239+\frac{7}{16}}}$$

$=1/137.035999111818$

${}^{63,65}_{29}Cu_{34,36}$ ${}^{73}_{32}Ge_{41}$ ${}^{93}_{41}Nb_{52}$ ${}^{112}_{48}Cd_{64}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{208}_{82}Pt_{126}$ ${}^{223}_{87}Ra^{*}_{136}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{293}_{116}Lv^{ie}_{177}$ ${}^{368}_{145}Ch^{ie}_{223}$ ${}^{416}_{164}Ch^{ie}_{252}$

$$\alpha_{2-33-Wallis}=\frac{4\cdot 33\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{414}{415}\frac{32\cdot 13}{2\cdot 9\cdot 23+1})}{2\cdot 127}\frac{1}{112-\frac{1}{24\cdot 127-\frac{139}{16\cdot 13}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Li_{14}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{46,48,50}_{22}Ti_{24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{50,52}_{24}Cr_{26,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{320}_{127}Ch^{ie}_{193}$

$$\alpha_{2-36-Wallis}=\frac{4\cdot 36\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{570}{571}\frac{4\cdot 11\cdot 13}{2\cdot 15\cdot 19+1})}{2\cdot 138+1}\frac{1}{112-\frac{1}{10\cdot 15\cdot 23-\frac{3\cdot 11\cdot 17}{5\cdot 131}}}=1/137.035999111818$$

${}^{20-22}_{10}Ne_{10-12}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{84}_{36}Kr_{48}$ ${}^{131}_{54}Xe_{77}$ ${}^{143,144}_{60}Nd_{83,84}$ ${}^{187}_{75}Re_{112}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{235}_{92}U^{*}_{143}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{187}$

$$\alpha_{2-125-Wallis}=\frac{5\cdot 10^{2}\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{12880}{12881}\frac{6\cdot 19\cdot 113}{20\cdot 23\cdot 28+1})}{31^{2}}\frac{1}{112-\frac{1}{32\cdot 239\cdot 281}}=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{39,40}_{19}K_{20,21}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{68,71}_{31}Ga_{38,40}$ ${}^{70,72,74,76}_{32}Ge_{38,40,42,44}$ ${}^{112,114,115,120,124}_{50}Sn_{62,64,65,70,74}$
${}^{238}_{92}U^{*}_{146}$ ${}^{281}_{110}Ds^{*}_{9\cdot 19}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{384}_{113}Nh^{ie}_{9\cdot 19}$

$$\alpha_{2-253-Wallis}=\frac{4\cdot 11\cdot 23\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{84550}{84551}\frac{24\cdot 13\cdot 271}{2\cdot 19\cdot 25\cdot 89+1})}{5\cdot(4\cdot 97+1)}\frac{1}{112-\frac{27\cdot 19\cdot 61}{20\times 10^{11}}}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{89}_{39}Y_{50}$ ${}^{97}_{42}Mo_{55}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{163}_{66}Dy_{97}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{13\cdot 19}_{97}Bk^{*}_{150}$

$$\alpha_{2-269-Wallis}=\frac{269\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{124926}{124927}\frac{2^{11}\cdot 61}{2\cdot 3\cdot 47\cdot(4\cdot 3\cdot 37-1)+1})}{11\cdot 47}\frac{1}{112-\frac{9\cdot 19\cdot 37}{10^{12}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{280}_{111}Rg^{*}_{169}$

Supplement 12: Comparison of two kinds of formulas of the speed of light

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-7}\alpha_{2-13}}}=\frac{5}{3}\sqrt{\frac{7}{13}\frac{(2\pi)_{112}}{(2\pi)_{278}}(112^{2}-\frac{1}{30^{2}\cdot 5}+\frac{1}{60^{2}\cdot 15}-\frac{1}{120^{2}\cdot 15\cdot 29})}$$

$=137.035999074627$ (Refer to Page 12)

$$c_{au}=\frac{1}{\alpha_c}==\frac{1}{\sqrt{\alpha_{1-7}\alpha_{2-13}}}=\frac{5}{3}\sqrt{\frac{7}{13}\frac{(2\pi)_{112}}{(2\pi)_{278}}(112^{2}-\frac{1}{9\cdot 5\cdot 109+\frac{1}{4}})}=137.035999074627$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66}_{30}Zn_{34,36}$
${}^{79,80}_{35}Br_{44,46}$ ${}^{103}_{45}Rh_{58}$ ${}^{107,109}_{47}Ag_{60,2\cdot 31}$ ${}^{142-145,150}_{60}Nd_{82-85,90}$ ${}^{182,183}_{74}W_{108,109}$ ${}^{2\cdot 138}_{109}Mt^{*}_{167}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{30}_{120}Ch^{ie}_{180}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{209}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-7-Wallis}\alpha_{2-13-Wallis}}}=\frac{5}{3}\sqrt{\frac{7}{13}\frac{(\frac{\pi}{2})_{169}}{(\frac{\pi}{2})_{3\cdot 139}}(112^{2}+\frac{1}{15}-\frac{1}{4\cdot 71}+\frac{1}{6\cdot(48\cdot(12\cdot 29-1)+1)})}$$

$$=137.035999074627$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$
${}^{63,65}_{29}Cu_{34,36}$ ${}^{84}_{36}Kr_{48}$ ${}^{106,108,112,113,116}_{48}Cd_{58,60,64,65,68}$ ${}^{139}_{57}La_{82}$ ${}^{2\cdot 71}_{58}Ce_{84}$ ${}^{142}_{60}Nd_{82}$ ${}^{155}_{64}Gd_{91}$ ${}^{169}_{69}Tm_{100}$
${}^{175,176}_{71}Lu_{104,105}$ ${}^{8\cdot 29}_{90}Th^{*}_{2\cdot 71}$ ${}^{231}_{91}Pa^{*}_{140}$ ${}^{277}_{108}Hs^{*}_{169}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{9\cdot 19}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{11\cdot 19}$ ${}^{360}_{142}Ch^{ie}_{218}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-29}\alpha_{2-29}}}=\sqrt{\frac{223}{149}\frac{(2\pi)_{3\cdot107}}{(2\pi)_{5\cdot131}}(112^2+\frac{1}{6\cdot23}-\frac{1}{5\cdot67\cdot100+\frac{17}{25}})}=137.035999074627$$

${}^{50,51}_{23}V_{27,28}$ ${}^{136,138}_{56}Ba_{80,82}$ ${}^{165}_{67}Ho_{98}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{344,346,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{332}_{131}Ch^{ie}_{3\cdot67}$ ${}^{16\cdot23}_{145}Ch^{ie}_{223}$ ${}^{376}_{149}Ch^{ie}_{227}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-29-Wallis}\alpha_{2-29-Wallis}}}=\sqrt{\frac{223}{149}\frac{(\frac{\pi}{2})_{2\cdot(2\cdot11^2-1)}}{(\frac{\pi}{2})_{24\cdot41-1}}(112^2+\frac{1}{6\cdot23}-\frac{1}{14\cdot17\cdot137+\frac{29}{5\cdot17}})}$$

$$=137.035999074627$$

${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{28,29}_{14}Si_{14,15}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{80}_{34}Se_{46}$ ${}^{102,104-106,108,110}_{46}Pd_{56,58-60,62,64}$
${}^{136,137,138}_{46}Ba_{80,81,82}$ ${}^{149}_{62}Sm_{87}$ ${}^{169}_{69}Tm_{100}$ ${}^{210}_{85}Po^{*}_{125}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{344,346,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{16\cdot23}_{145=5\cdot29}Ch^{ie}_{223}$ ${}^{376}_{149}Ch^{ie}_{227}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-36}\alpha_{2-36}}}=\sqrt{\frac{277}{5\cdot37}\frac{(2\pi)_{4\cdot59}}{(2\pi)_{10\cdot19}}(112^2-\frac{1}{29}+\frac{1}{6\cdot41\cdot47-\frac{3\cdot29}{139}})}=137.035999074627$$

${}^{19}_{9}F_{10}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{82}_{36}Kr_{46}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{138}_{56}Ba_{82}$ ${}^{139}_{57}La_{82}$ ${}^{3\cdot47}_{59}Pr_{2\cdot41}$ ${}^{184,186}_{74}W_{110,112}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{12\cdot29}_{139}Ch^{ie}_{209}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-36-Wallis}\alpha_{2-36-Wallis}}}=\sqrt{\frac{277}{5\cdot37}\frac{(\frac{\pi}{2})_{6\cdot59}}{(\frac{\pi}{2})_{15\cdot19}}(112^2-\frac{1}{36}+\frac{1}{4\cdot43\cdot61-\frac{9}{43}})}=137.035999074627$$

${}^{9}_{4}Be_{5}$ ${}^{19}_{9}F_{10}$ ${}^{39}_{19}K_{20}$ ${}^{84,86}_{36}Kr_{48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{2\cdot49,99}_{43}Tc_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{145,3\cdot49}_{61}Pm^{*}_{84,86}$ ${}^{277}_{108}Hs^{*}_{13^2}$ ${}^{6\cdot37}_{86}Rn^{*}_{136}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$
${}^{293}_{116}Lv^{ie}_{3\cdot59}$ ${}^{6\cdot49}_{117=9\cdot13}Ts^{ie}_{3\cdot59}$ ${}^{16\cdot19}_{122}Ch^{ie}_{14\cdot13}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-81}\alpha_{2-125}}}=\frac{9\cdot31}{40}\sqrt{\frac{2}{5\cdot13}\frac{(2\pi)_{15\cdot107}}{(2\pi)_{53\cdot81}}(112^2+\frac{1}{2\cdot17\cdot53-\frac{29}{49}})}=137.035999074627$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{53}_{24}Cr_{29}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{68,71}_{31}Ga_{38,40}$ ${}^{74,76,78,80}_{34}Se_{40,42,44,46}$ ${}^{87}_{38}Sr_{49}$ ${}^{93}_{41}Nb_{52}$
${}^{90-92,94,96}_{40}Zr_{50-52,54,56}$ ${}^{95}_{42}Mo_{53}$ ${}^{103,105}_{49}In_{64,66}$ ${}^{130}_{52}Te_{78}$ ${}^{127}_{53}I_{74}$ ${}^{3\cdot53}_{65}Tb_{94}$ ${}^{200}_{80}Hg_{120}$ ${}^{205}_{81}Tl_{124}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-81-Wallis}\alpha_{2-125-Wallis}}}=\frac{9\cdot31}{40}\sqrt{\frac{2}{5\cdot13}\frac{(\frac{\pi}{2})_{29\cdot83}}{(\frac{\pi}{2})_{10\cdot23\cdot28}}(112^2+\frac{1}{10\cdot16\cdot19\cdot23})}=137.035999074627$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{39,40}_{19}K_{20,21}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57,32}_{26}Fe_{28,30,31,32}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$
${}^{68,71}_{31}Ga_{38,40}$ ${}^{90,92,96}_{40}Zr_{50,52,56}$ ${}^{93}_{41}Nb_{52}$ ${}^{136-138}_{56}Ba_{80-82}$ ${}^{143}_{60}Nd_{83}$ ${}^{200}_{80}Hg_{120}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{14\cdot29}_{160}Ch^{ie}_{246}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-170}\alpha_{2-253}}}=\frac{5}{3}\sqrt{(\frac{8\cdot17}{11\cdot23}+\frac{2\cdot17}{11\cdot23\cdot97})\frac{(2\pi)_{2\cdot25\cdot13\cdot53}}{(2\pi)_{2\cdot17\cdot(36\cdot23+1)}}(112^2-\frac{18\cdot97+1}{2\cdot10^9})}$$

$$=137.035999074627$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{97}_{42}Mo_{55}$ ${}^{163}_{66}Dy_{97}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{13\cdot19}_{97}Bk^{*}_{150}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-170-Wallis}\alpha_{2-253-Wallis}}}=\frac{5}{3}\sqrt{(\frac{8\cdot17}{11\cdot23}+\frac{2\cdot17}{11\cdot23\cdot97})\frac{(\frac{\pi}{2})_{26\cdot(4\cdot7\cdot71-1)}}{(\frac{\pi}{2})_{19\cdot25\cdot89}}(112^2-\frac{19\cdot(4\cdot83-1)}{5\cdot10^9})}$$

$$=137.035999074627$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{56,57}_{26}Fe_{30,31}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{97}_{42}Mo_{55}$ ${}^{163}_{66}Dy_{97}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$
${}^{142}_{60}Nd_{82}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{232}_{90}Th^{*}_{142}$ ${}^{13\cdot19}_{97}Bk^{*}_{150}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{4\cdot71}_{113}Nh^{ie}_{9\cdot19}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-133}\alpha_{2-253}}}=\sqrt{\frac{\frac{5\cdot 17}{11}-\frac{10}{11\cdot 23}}{\frac{36}{7}-\frac{1}{7\cdot 19}}\frac{(2\pi)_{13\cdot(8\cdot 7\cdot 17+1)}}{(2\pi)_{2\cdot 17\cdot(36\cdot 23+1)}}(112^2-\frac{23\cdot(12\cdot 41-1)}{10^{10}})}=137.035999074627$$

${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{80,82,84}_{36}Kr_{44,46,48}$
${}^{93}_{41}Nb_{52}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{145}_{60}Nd_{85}$ ${}^{204,207,208}_{82}Pt_{122,125,126}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{223}_{87}Ra^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{336}_{133}Ch^{ie}_{203}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-133}\alpha_{2-253}}}=\sqrt{\frac{\frac{5\cdot 17}{11}-\frac{10}{11\cdot 23}}{\frac{36}{7}-\frac{1}{7\cdot 19}}\frac{(\frac{\pi}{2})_{9\cdot 5\cdot 7\cdot 59}}{(\frac{\pi}{2})_{19\cdot 25\cdot 89}}(112^2+\frac{19\cdot(8\cdot 9\cdot 11^2+1)}{25\cdot 10^9})}=137.035999074627$$

${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{55}_{25}Mn_{30}$ ${}^{89}_{39}Y_{50}$
${}^{80,82,84}_{36}Kr_{44,46,48}$ ${}^{105,110}_{46}Pd_{59,64}$ ${}^{136-138}_{56}Ba_{80-82}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{336}_{133}Ch^{ie}_{203}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208-210}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-170}\alpha_{2-269}}}=\frac{2}{3}\sqrt{\frac{11\cdot 47\cdot 170}{97\cdot(270-1)}\frac{(2\pi)_{2\cdot 25\cdot 13\cdot 53}}{(2\pi)_{2\cdot 59\cdot(6\cdot 59-1)}}(112^2+\frac{23}{2\cdot 10^9})}=137.035999074627$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{59}_{27}Co_{32}$
${}^{77,78,80,82}_{34}Se_{43,44,46,48}$ ${}^{97}_{42}Mo_{55}$ ${}^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ ${}^{102,104,105,106,108,110}_{46}Pd_{56,58,59,60,62,64}$ ${}^{107,109}_{47}Ag_{60,62}$
${}^{116,118,119,120}_{50}Sn_{66,68,69,70}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{163}_{66}Dy_{97}$ ${}^{162,167,168,170}_{68}Er_{94,99,100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{247}_{97}Bk^{*}_{150}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-170-Wallis}\alpha_{2-269-Wallis}}}=\frac{2}{3}\sqrt{\frac{11\cdot 47\cdot 170}{97\cdot(270-1)}\frac{(\frac{\pi}{2})_{26\cdot(4\cdot 7\cdot 71-1)}}{(\frac{\pi}{2})_{3\cdot 47\cdot(4\cdot 3\cdot 37-1)}}(112^2-\frac{107}{5\cdot 10^8})}=137.035999074627$$

${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47,48-50}_{22}Ti_{25,26-28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$
${}^{77,78,80,82}_{34}Se_{43,44,46,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{97}_{42}Mo_{55}$ ${}^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{116,118,119,120,114}_{50}Sn_{66,68,69,70,74}$
${}^{121,123}_{51}Sb_{70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{163}_{66}Dy_{97}$ ${}^{162,167,168,170}_{68}Er_{94,99,100,102}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$ ${}^{420}_{170}Ch^{ie}_{250}$

Supplement 13: Construct formulas of the fine-structure constant with Gregory-Leibliz formula of π/4 instead of 2π-e formula

Gregory-Leibniz Formula of $\pi/4$:

Traditional format: $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$

Natural group format: $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=1+\sum_{n=1}^{\infty}\frac{(-1)^{n}}{2n+1}$

$$(\frac{\pi}{4})_k=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\frac{1}{k}=1+\sum_{n=1}^{k}\frac{(-1)^{n}}{2n+1}$$

Comparable and similar to 2π - e formula: $(2\pi)_k=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}$

So, Leibniz Formula of $\pi/4$ could be used to construct formulas of α.

Note: There should be $(2\pi)_k\sim 8(\frac{\pi}{4})_{\frac{3}{2}\sqrt{\frac{\sqrt{5}+1}{2}}k}$.

That means $(2\pi)_k$ and $(\frac{\pi}{4})_{\frac{3}{2}\sqrt{\frac{\sqrt{5}+1}{2}}k}$ become covergent at the same speed.

The larger the k is, the better is the accuracy and the less is the error.

$$\alpha_{1-7-GL}=\frac{36}{8\cdot7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot6\cdot36+1})}\frac{1}{112+\frac{1}{64\cdot13}-\frac{1}{128\cdot3\cdot(4\cdot3\cdot17\cdot23-1)}}$$

$=1/137.035999037435$

${}^{12,13}_{6}C_{6,7}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{62,64}_{28}Ni_{34,36}$ ${}^{84}_{36}Kr_{48}$ ${}^{112}_{48}Cd_{64}$ ${}^{136-138}_{56}Ba_{80-82}$ ${}^{155,156,160}_{64}Gd_{91,92,96}$

$$\alpha_{1-22-GL}=\frac{113}{8\cdot22\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot5\cdot13\cdot23+1})}\frac{1}{112+\frac{1}{5\cdot17\cdot(2\cdot3\cdot7\cdot13-1)+\frac{3}{44}}}$$

$=1/137.035999037435$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{48}_{22}Ti_{26}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{96,98-100,102,104}_{44}Ru_{52,54-56,58,60}$ ${}^{110-114}_{48}Cd_{62-66}$ ${}^{189}_{76}Os_{113}$ ${}^{210}_{85}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\alpha_{1-29-GL}=\frac{149}{8\cdot29\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot(4\cdot7\cdot11-1)+1})}\frac{1}{112+\frac{1}{6\cdot25\cdot79+\frac{28}{5\cdot23}}}$$

$=1/137.035999037435$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{47,50}_{22}Ti_{25,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79}_{35}Br_{44}$ ${}^{100}_{44}Ru_{56}$ ${}^{115}_{50}Sn_{65}$ ${}^{135}_{56}Ba_{79}$ ${}^{149}_{62}Sm_{87}$ ${}^{191,193}_{77}Ir_{114,116}$

$$\alpha_{1-36-GL}=\frac{5\cdot37}{8\cdot36\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot113+1})}\frac{1}{112+\frac{1}{3\cdot29\cdot47-\frac{2\cdot47}{4\cdot83-1}}}$$

$=1/137.035999037435$

${}^{24}_{12}Mg_{12}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{83}_{36}Kr_{47}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{184,186}_{74}W_{110,112}$ ${}^{223}_{87}Fr^{*}_{136}$

${}^{284}_{113}Nh^{ie}_{9\cdot19}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{223}_{144}Ch^{ie}_{6\cdot37}$

$$\alpha_{1-43-GL}=\frac{13\cdot17}{8\cdot43\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot128\cdot3+1})}\frac{1}{112+\frac{1}{4\cdot(64\cdot9+1)-\frac{136}{137}}}$$

$$\text{or }=\frac{13\cdot17}{8\cdot43\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot128\cdot3+1})}\frac{1}{112+\frac{1}{4\cdot(64\cdot9+1)}-\frac{12\cdot11\cdot13}{25\cdot10^{11}}}$$

$=1/137.035999037437$ or $1/137.035999037435$

${}^{19}_{9}F_{10}$ ${}^{77}_{34}Se_{43}$ ${}^{84}_{36}Kr_{48}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{8\cdot43,2\cdot173}_{136,137}Fy^{ie}_{16\cdot13,209}$

$$\alpha_{1-50-GL}=\frac{257}{8\cdot50\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot173+1})}\frac{1}{112+\frac{1}{16\cdot11\cdot13-\frac{28}{6\cdot31}}}$$

$=1/137.035999037436$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{47,48,50}_{22}Ti_{25,26,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60}_{28}Ni_{30,32}$ ${}^{69,71}_{31}Ga_{38,40}$

${}^{116,119,120}_{50}Sn_{66,69,70}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{150}_{62}Sm_{88}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{400}_{80}Hg_{120}$ ${}^{209}_{84}Po^{*}_{125}$

${}^{237}_{93}Np^{*}_{144}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{209}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$

$$\alpha_{1-59-GL}=\frac{3\cdot 101}{8\cdot 59\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 6\cdot 29^{2}+1})}\frac{1}{112+\frac{1}{27\cdot(8\cdot 9\cdot(8\cdot 3\cdot 13-1)-1)+\frac{10}{23}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39-41}_{19}K_{20-22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{82}_{36}Kr_{46}$ ${}^{101}_{44}Ru_{57}$ ${}^{105}_{46}Pd_{59}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{171}_{70}Yb_{101}$

$$\alpha_{1-81-GL}=\frac{4\cdot 13}{81\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 6\cdot 7\cdot 73+1})}\frac{1}{112+\frac{1}{5\cdot 23\cdot(36\cdot 47+1)+\frac{19}{25}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{47}_{22}Ti_{25}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83}_{36}Kr_{47}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{125,128,130}_{52}Te_{73,76,78}$ ${}^{181}_{73}Ta_{108}$

$$\alpha_{1-96-GL}=\frac{7\cdot 29}{8\cdot 96\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 10\cdot(30\cdot 37-1)+1})}\frac{1}{112+\frac{1}{9\cdot 103\cdot(4\cdot 17\cdot 41+1)}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,67}_{30}Zn_{34,37}$ ${}^{74,82}_{34}Se_{40,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{103}_{45}Rh_{58}$ ${}^{168}_{68}Er_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{206}_{82}Pb_{124}$

$$\alpha_{1-103-GL}=\frac{23^{2}}{8\cdot 103\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 18\cdot 139+1})}\frac{1}{112+\frac{2\cdot(10\cdot 23\cdot 41+1)}{25\cdot 10^{10}}}$$

$$=\frac{23^{2}}{8\cdot 103\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 18\cdot 139+1})}\frac{1}{112+\frac{103\cdot 293}{4\cdot 10^{11}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{93}_{41}Nb_{52}$ ${}^{93}_{41}Nb_{52}$ ${}^{103}_{45}Rh_{58}$ ${}^{102,104}_{46}Pd_{56,58}$ ${}^{138,139}_{57}La_{81,82}$
${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{293}_{116}Lv^{ie}_{177}$ ${}^{348}_{139}Ch^{ie}_{209}$

$$\alpha_{1-133-GL}=\frac{36\cdot 19-1}{8\cdot 7\cdot 19\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 16\cdot 3\cdot 17\cdot 29+1})}\frac{1}{112+\frac{3\cdot 107\cdot 151}{4\cdot 10^{11}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{84}_{36}Kr_{48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$ ${}^{136}_{56}Ba_{80}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{179,180}_{72}Hf_{107,108}$

$$\alpha_{1-140-GL}=\frac{36\cdot 20-1}{8\cdot 7\cdot 20\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 22\cdot 167+1})}\frac{1}{112+\frac{1}{128\cdot 19\cdot 83+\frac{2}{17}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{47}_{22}Ti_{25}$ ${}^{78}_{34}Se_{44}$ ${}^{83}_{36}Kr_{47}$ ${}^{100,102,104}_{44}Ru_{56,58,60}$ ${}^{143}_{60}Nd_{83}$ ${}^{167}_{68}Er_{99}$ ${}^{209}_{83}Bi^{*}_{126}$

$$\alpha_{1-155-GL}=\frac{199}{10\cdot 31\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 64\cdot 7\cdot 17+1})}\frac{1}{112+\frac{9\cdot 7\cdot 29\cdot 107}{25\cdot 10^{11}}\text{ or }\frac{113\cdot 173}{25\cdot 10^{10}}}$$

${}^{68,71}_{31}Ga_{38,40}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{157}_{64}Gd_{93}$ ${}^{173}_{70}Yb_{103}$ ${}^{186,187}_{75}\mathrm{Re}_{110,112}$ ${}^{3\cdot 63}_{76}Os_{113}$ ${}^{199}_{80}Hg_{7\cdot 17}$
${}^{185}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{9\cdot 19}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\alpha_{1-170-GL}=\frac{9\cdot 97}{8\cdot 170\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 6\cdot 19\cdot(2\cdot 17^2-1)+1})}\frac{1}{112+\frac{7\cdot 23\cdot 41}{25\cdot 10^{11}}}$$

$=1/137.035999037435$

${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{84,88}_{38}Sr_{46,50}$
${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{164,166}_{68}Er_{96,98}$ ${}^{247}_{97}Bk^{*}_{150}$

$$\alpha_{2-10-GL}=\frac{8\cdot 10\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 18\cdot 11+1})}{7\cdot 11}\frac{1}{112-\frac{1}{3\cdot 23\cdot(2\cdot 13\cdot 23+1)}+\frac{30}{47}\,or\,\frac{37}{58}}$$

$=1/137.035999111818$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{47}_{22}Ti_{25}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{66,67}_{30}Zn_{36,37}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{169}_{69}Tm_{100}$ ${}^{191,193}_{77}Ir_{114,116}$

$$\alpha_{2-13-GL}=\frac{8\cdot 13\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 28\cdot 19+1})}{100}\frac{1}{112-\frac{1}{10\cdot(10\cdot 23-1)+\frac{7\cdot(36\cdot 11+1)}{100^2}}}$$

$=1/137.035999111818$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{58,60,61,64}_{28}Ni_{30,32,33,36}$ ${}^{100}_{44}Ru_{56}$ ${}^{169}_{69}Tm_{100}$

$$\alpha_{2-23-GL}=\frac{8\cdot 23\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 4\cdot 7\cdot 11+1})}{3\cdot 59}\frac{1}{112-\frac{1}{36\cdot 61-\frac{89}{10\cdot 17}}}$$

$=1/137.035999111818$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{80,82,84}_{36}Kr_{44,46,48}$
${}^{89}_{39}Y_{50}$ ${}^{99,100,102,104}_{44}Ru_{55,56,58,60}$ ${}^{105}_{46}Pd_{59}$ ${}^{145,146}_{61}Pm^{*}_{84,85}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$

$$\alpha_{2-29-GL}=\frac{8\cdot 29\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 4\cdot(24\cdot 13+1)+1})}{223}\frac{1}{112-\frac{1}{25\cdot 49\cdot 89+\frac{4}{11}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{87}_{38}Sr_{49}$ ${}^{89}_{39}Y_{50}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$

$$\alpha_{2-33-GL}=\frac{4\cdot 33\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 24\cdot 11+1})}{127}\frac{1}{112-\frac{1}{81\cdot 25}+\frac{1}{2\cdot 43\cdot 61\cdot(2\cdot 11\cdot 49-1)}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{55}_{25}Mn_{30}$ ${}^{75}_{33}As_{42}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{145,146,3\cdot 49}_{61}Pm^{*}_{84,85,86}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

$$\alpha_{2-36-GL}=\frac{8\cdot 36\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 181+1})}{2\cdot 138+1}\frac{1}{112-\frac{1}{16\cdot(2\cdot 7\cdot 11\cdot 17-1)+\frac{6}{7}}}$$

$$=1/137.035999111818$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{46,48,50}_{22}Ti_{24,26,28}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{181}_{73}Ta_{103}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{302}_{121}Ch^{ie}_{181}$

$$\alpha_{2-125-GL}=\frac{8\cdot 125\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 8\cdot 25\cdot 41+1})}{31^{2}}\frac{1}{112-\frac{1}{2\cdot 13^{2}\cdot 37\cdot 137}}$$

$$=1/137.035999111818$$

${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{3\cdot 31}_{41}Nb_{52}$ ${}^{136,137,137}_{56}Ba_{80,81,82}$ ${}^{157}_{64}Gd_{3\cdot 31}$ ${}^{169}_{69}Tm_{100}$ ${}^{16\cdot 13}_{82}Pt_{126}$ ${}^{210}_{85}At^{*}_{125}$
${}^{223}_{87}Hs^{*}_{136}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{277}_{108}Hs^{*}_{169}$ ${}^{280}_{111}Rg^{*}_{169}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

$$\alpha_{2-253-GL}=\frac{8\cdot 11\cdot 23\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 6\cdot(30\cdot 13\cdot 23+1)+1})}{5\cdot(4\cdot 49+1)}\frac{1}{112-\frac{6\cdot 49}{25\cdot 10^{9}}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$
${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{66}_{30}Zn_{36}$ ${}^{89}_{39}Y_{50}$ ${}^{98,99,100,104}_{44}Ru_{54,55,56,60}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{169}_{69}Tm_{100}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\alpha_{2-269-GL}=\frac{2\cdot 269\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{4\cdot 3\cdot 5\cdot 11\cdot(2\cdot 11^{2}-1)+1})}{11\cdot 47}\frac{1}{112-\frac{2\cdot 47\cdot 97}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{47}_{22}Ti_{25}$ ${}^{40,42,44}_{20}Ca_{20,22,24}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{66}_{30}Zn_{36}$
${}^{75}_{33}As_{42}$ ${}^{97}_{42}Mo_{55}$ ${}^{96,98,99,100,104}_{44}Ru_{52,54,55,56,60}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{116}_{50}Sn_{66}$ ${}^{163}_{66}Dy_{97}$ ${}^{244}_{94}Pu^{*}_{150}$ ${}^{247}_{97}Bk^{*}_{150}$

Supplement 14: Construct formulas of the speed of light with $\alpha_{1\text{-}GL}$ and $\alpha_{2\text{-}GL}$

$$c_{au}=\frac{1}{\alpha_{c}}=\frac{1}{\sqrt{\alpha_{1-7-GL}\alpha_{2-13-GL}}}=\frac{5}{3}\sqrt{\frac{7}{13}\frac{(\frac{\pi}{4})_{6\cdot 36}}{(\frac{\pi}{4})_{4\cdot 7\cdot 19}}(112^{2}+\frac{1}{9}-\frac{1}{23\cdot 43+\frac{37}{3\cdot 43}})}=137.035999074627$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Fe_{38,30,31,32}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{208}_{56}Ba_{82}$
${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{8\cdot 43,346}_{136,137}Fy^{ie}_{16\cdot 13,209}$

$$c_{au}=\frac{1}{\alpha_{c}}=\frac{1}{\sqrt{\alpha_{1-29-GL}\alpha_{2-29-GL}}}=\sqrt{\frac{223}{149}\frac{(\frac{\pi}{4})_{2\cdot(28\cdot 11-1)}}{(\frac{\pi}{4})_{4\cdot(24\cdot 13+1)}}(112^{2}+\frac{1}{2\cdot 59}-\frac{1}{17\cdot(14\cdot 83+1)+\frac{4}{11}})}$$

$$=137.035999074627$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{19,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{141,143}_{58}Ce_{82,84}$ ${}^{141}_{59}Pr_{82}$ ${}^{149}_{62}Sm_{87}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-36-GL}\alpha_{2-36-GL}}}=\sqrt{\frac{277}{5\cdot 37}\frac{(\frac{\pi}{4})_{4\cdot 113}}{(\frac{\pi}{4})_{2\cdot 181}}(112^2+\frac{1}{40}-\frac{1}{2\cdot 7\cdot 11\cdot 23+\frac{9}{37}})}$$

$$=137.035999074627$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{85,86}_{37}Rb_{48,50}$ ${}^{90,91,92}_{40}Zr_{50,51,52}$ ${}^{113}_{48}Cd_{65}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{189}_{76}Os_{113}$ ${}^{277}_{108}Hs^{*}_{169}$ ${}^{284}_{113}Nh^{ie}_{171}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-81-GL}\alpha_{2-125-GL}}}=\frac{9\cdot 31}{40}\sqrt{\frac{2}{5\cdot 13}\frac{(\frac{\pi}{4})_{6\cdot 7\cdot 73}}{(\frac{\pi}{4})_{8\cdot 25\cdot 41}}(112^2+\frac{1}{37\cdot 53+\frac{19}{2\cdot 43}})}$$

$$=137.035999074627$$

${}^{12,13}_{6}C_{6,7}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,86}_{37}Rb_{48,50}$ ${}^{93}_{41}Nb_{52}$ ${}^{127}_{53}I_{74}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-170-GL}\alpha_{2-253-GL}}}=\frac{5}{3}\sqrt{(\frac{8\cdot 17}{11\cdot 23}+\frac{2\cdot 17}{11\cdot 23\cdot 97})\frac{(\frac{\pi}{4})_{6\cdot 19\cdot(2\cdot 17^2-1)}}{(\frac{\pi}{4})_{6\cdot(30\cdot 13\cdot 23+1)}}(112^2-\frac{19\cdot 43}{8\cdot 10^8})}$$

$$=137.035999074627$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{56,57}_{26}Fe_{30,31}$
${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{76,77,78,80}_{34}Se_{42,43,44,46}$ ${}^{97}_{42}Mo_{55}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{163}_{66}Dy_{97}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{222}_{86}Rn^{*}_{136}$
${}^{13\cdot 19}_{97}Bk^{*}_{150}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{8\cdot 43,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-170-GL}\alpha_{2-269-GL}}}=\frac{2}{3}\sqrt{\frac{11\cdot 47\cdot 170}{97\cdot(270-1)}\frac{(\frac{\pi}{4})_{6\cdot 19\cdot(2\cdot 17^2-1)}}{(\frac{\pi}{4})_{2\cdot 30\cdot 11\cdot(2\cdot 11^2-1)}}(112^2-\frac{23\cdot 49}{10^{10}})}$$

$$=137.035999074627$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$
${}^{55}_{25}Mn_{30}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{76,77,78,80}_{34}Se_{42,43,44,46,48}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{86,88}_{37}Rb_{48,50}$ ${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$
${}^{97}_{42}Mo_{55}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{163}_{66}Dy_{97}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$
${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{13\cdot 19}_{97}Bk^{*}_{150}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

Supplement 15: DNA-Protein model of formulas of α and nuclides

$$\text{DNA: } \alpha_1=\alpha_{1-7}=\frac{6^2}{7\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{113}{112})^{225}}}\,\frac{1}{112+\frac{1}{75^2}}=1/137.035999037435$$

Gene Factors: 3 4 5 6 7 9 12 18 25 36 44($7\cdot 2\pi\approx 44$) 75 112 113 225

Direct Derived Factors: 30 35 42 88 150 226;

7 14 28 56 224; 42 82-84 126 166-168; 68 69 136 137 138.

Emerging Factors: 11 13 19 48 71 173 187 209 *et al*.

Protein: ${}^{12,13}_{6}C_{6,7}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{24-26}_{12}Mg_{12-14}$ ${}^{28-30}_{14}Si_{14-16}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$
${}^{58,60,61,64}_{28}Ni_{30,32,33,36}$ ${}^{75}_{33}As_{42}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{88}_{38}Sr_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{125}_{52}Te_{73}$ ${}^{129}_{54}Xe_{75}$
${}^{136-138}_{56}\mathrm{Ba}_{80-82}$ ${}^{142-146,148,150}_{60}Nd_{82-86,88,90}$ ${}^{150}_{62}Sm_{88}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{207,208}_{82}Pt_{125,126}$ ${}^{209}_{83}Bi^{*}_{126}$
${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$ ${}^{312,314}_{125,126}Ch^{ie}_{187,188}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{366,372}_{144,147}Ch^{ie}_{222,225}$

$$\text{DNA: } \alpha_2=\alpha_{2\text{-}13}=\frac{13\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{9\cdot 31}{2\cdot 139})^{557}}}{10^2}\,\frac{1}{112-\dfrac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

Gene Factors: 2 3 5 9 10 13 29 31 64 82($13\cdot 2\pi\approx 82$) 100 112 139

Direct Derived Factors: 26 39 52 104 169, 50 200, 58 87 116, 32 96 128 192, 93;

7 14 28 56 224; 42 82-84 126 166-168; 68 69 136 137 138.

Emerging Factors: 19 38 57 71 76 113 114 157 173 187 208 209 210 *et al*.

Protein: ${}^{7}_{3}Li_{4}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}\mathrm{Al}_{14}$ ${}^{29}_{14}\mathrm{Si}_{15}$ ${}^{54,56,57,58}_{26}\mathrm{Fe}_{28,30,31,32}$ ${}^{58}_{28}Ni_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{74}_{32}Ge_{42}$ ${}^{89}_{39}Y_{50}$

${}^{93}_{41}Nb_{52}$ ${}^{96}_{42}Mo_{54}$ ${}^{100}_{44}Ru_{56}$ ${}^{112,116}_{48}Cd_{64,68}$ ${}^{116}_{50}Sn_{66}$ ${}^{128}_{52}Te_{76}$ ${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{139}_{57}La_{82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$

${}^{149}_{62}Sm_{87}$ ${}^{156,157,160}_{64}Gd_{92,93,96}$ ${}^{162}_{66}Dy_{96}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175}_{71}Lu_{104}$ ${}^{187}_{75}\mathrm{Re}_{112}$ ${}^{188\text{–}190,192}_{76}Os_{112\text{-}114,116}$

${}^{200}_{80}Hg_{120}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{210}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{247}_{96}Cm^{*}_{151}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$

${}^{310,312}_{124,125}Ch^{ie}_{186,187,192}$ ${}^{318}_{126}Ch^{ie}_{192}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{344,346,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{209}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

Supplement 16: Some formulas of α_2

$$\alpha_{2\text{-}45}=\frac{5\cdot 9\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{30\cdot 36}{13\cdot 83})^{17\cdot 127}}}{2\cdot 173}\,\frac{1}{112-\dfrac{1}{2\cdot(32\cdot 13\cdot 97-1)-\dfrac{15}{16}}}$$

$=1/137.035999111818$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{103}_{45}Rh_{58}$

${}^{126,128}_{52}Te_{74,76}$ ${}^{163}_{66}Dy_{97}$ ${}^{173}_{70}Yb_{103}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173}_{136,137}Fy^{ie}_{208,209}$ ${}^{318\text{–}320}_{127}Ch^{ie}_{191\text{–}193}$

$$\alpha_{2\text{-}45\text{-}Wallis}=\frac{2\cdot 5\cdot 9\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{3234}{3235}\frac{4\cdot(8\cdot 101+1)}{2\cdot 3\cdot 49\cdot 11+1})}{173}\,\frac{1}{112-\dfrac{1}{12\cdot 37\cdot(2\cdot 126-1)-\dfrac{25}{31}}}$$

$=1/137.035999111818$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{101}_{44}Ru_{57}$ ${}^{103}_{45}Rh_{58}$ ${}^{113,115}_{49}In_{64,66}$

${}^{171,173}_{70}Yb_{101,103}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{2\cdot 137}_{137}Fy^{ie}_{209}$

$$\alpha_{2\text{-}45\text{-}GL}=\frac{4\cdot 5\cdot 9\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot(24\cdot 43-1)+1})}{173}\,\frac{1}{112-\dfrac{1}{25\cdot(22\cdot 109+1)+\dfrac{31}{59}}}$$

$=1/137.035999111818$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{47}_{22}Ti_{25}$ ${}^{68,71}_{31}Ga_{38,40}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{103}_{45}Rh_{58}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118,120}_{50}Sn_{68,70}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$

${}^{148,150}_{62}Sm_{86,88}$ ${}^{173}_{70}Yb_{103}$ ${}^{183}_{74}W_{109}$ ${}^{222}_{86}Lr^{*}_{136}$ ${}^{276}_{109}Mt^{*}_{167}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{294}_{118}Og^{ie}_{176}$ ${}^{8\cdot 43,2\cdot 173}_{136,137}Fy^{ie}_{208,11\cdot 19}$

$$\alpha_{2\text{-}173}=\frac{173\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{8\cdot17\cdot59}{71\cdot113})^{9\cdot(2\cdot9^2\cdot11+1)}}}{10\cdot7\cdot19}\frac{1}{112-\frac{53\cdot(42\cdot41-1)}{4\cdot10^{11}}}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,42,44,48}_{20}Ca_{20,22,24,28}$ ${}^{95}_{42}Mo_{53}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{113,115}_{49}In_{64,66}$ ${}^{127}_{53}I_{74}$ ${}^{133}_{55}Cs_{78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{161}_{66}Dy_{95}$ ${}^{173}_{70}Yb_{103}$ ${}^{180,184,186}_{74}W_{106,110,112}$ ${}^{175,187}_{75}\mathrm{Re}_{110,112}$

${}^{188,189}_{76}Os_{112,113}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{84}Bi^{*}_{125}$ ${}^{243}_{95}Am^{*}_{148}$ ${}^{15\cdot19}_{112}Cn^{ie}_{173}$ ${}^{4\cdot71}_{113}Nh^{ie}_{9\cdot19}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{336}_{133}Ch^{ie}_{203}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2\text{-}173\text{-}Wallis}=\frac{2\cdot173\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{24068}{24069}\frac{10\cdot29\cdot83}{2\cdot22\cdot(42\cdot13+1)+1})}{5\cdot7\cdot19}\frac{1}{112-\frac{3\cdot53\cdot(20\cdot27+1)}{25\cdot10^{11}}}$$

$$=1/137.035999111818$$

${}^{95}_{42}Mo_{53}$ ${}^{133}_{55}Cs_{78}$ ${}^{103}_{45}Rh_{58}$ ${}^{143}_{60}Nd_{83}$ ${}^{173}_{70}Yb_{103}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{262}_{103}Lr^{*}_{3\cdot53}$ ${}^{336}_{133}Ch^{ie}_{7\cdot29}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2\text{-}173\text{-}GL}=\frac{4\cdot173\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot12\cdot(4\cdot11\cdot29+1)+1})}{5\cdot7\cdot19}\frac{1}{112-\frac{1}{4\cdot7\cdot23\cdot59\cdot89}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{173}_{70}Yb_{103}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{336}_{133}Ch^{ie}_{7\cdot29}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2\text{-}49}=\frac{49\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{16\cdot13}{9\cdot23})^{5\cdot83}}}{13\cdot29}\frac{1}{112-\frac{1}{7\cdot(6\cdot11^2+1)+\frac{11\cdot19}{3\cdot97}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{97}_{42}Mo_{55}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{140,142}_{58}Ce_{82,84}$

${}^{163}_{66}Dy_{97}$ ${}^{169}_{69}Tm_{100}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{13\cdot19}_{97}Bk^{*}_{150}$

$$\alpha_{2\text{-}49\text{-}Wallis}=\frac{4\cdot49\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{622}{623}\frac{48\cdot13}{2\cdot(10\cdot31+1)+1})}{13\cdot29}\frac{1}{112-\frac{1}{10\cdot(30\cdot17-1)+\frac{2\cdot7\cdot11}{300}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15.16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{48}_{22}Ti_{26}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{121,123}_{51}Sb_{70,72}$

$$\alpha_{2\text{-}49\text{-}GL}=\frac{8\cdot49\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot36\cdot11+1})}{13\cdot29}\frac{1}{112-\frac{1}{11^2\cdot47-\frac{36}{125}}}$$

$$=1/137.035999111818$$

${}^{16,17,8}_{8}O_{8,9,10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$

${}^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{125}_{52}Te_{73}$ ${}^{132,134,135,136,137,138}_{56}Ba_{74,76,78,79,80,81,82}$

${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{24\cdot13}_{125}Ch^{ie}_{11\cdot17}$

Supplement 17: Some formulas of the speed of light

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-81}\alpha_{2-49}}}=\frac{9}{4\cdot 7}\sqrt{\frac{29}{2}\frac{(2\pi)_{15\cdot 107}}{(2\pi)_{9\cdot 23}}(112^2-\frac{1}{2\cdot 23}+\frac{1}{7\cdot(2\cdot 11\cdot 19+1)-\frac{13}{7\cdot 11}})}$$

$=137.035999074627$

${}^{17}_{3}Li_{4}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{104}_{46}Pd_{58}$ ${}^{179}_{72}Hf_{107}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{270}_{107}Bh^{*}_{163}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-81-Wallis}\alpha_{2-49-Wallis}}}=\frac{9}{4\cdot 7}\sqrt{\frac{29}{2}\frac{(\frac{\pi}{2})_{29\cdot 83}}{(\frac{\pi}{2})_{10\cdot 31+1}}(112^2-\frac{1}{9\cdot 5}+\frac{1}{5\cdot 17\cdot 41+\frac{11}{60}})}$$

$=137.035999074627$

${}^{17}_{3}Li_{4}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{31}_{15}P_{16}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{93}_{41}Nb_{52}$ ${}^{103}_{45}Rh_{58}$ ${}^{143,146}_{60}Nd_{83,85}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{210}_{85}Bi^{*}_{125}$

$$c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_{1-81-GL}\alpha_{2-49-GL}}}=\frac{9}{4\cdot 7}\sqrt{\frac{29}{2}\frac{(\frac{\pi}{4})_{6\cdot 7\cdot 73}}{(\frac{\pi}{4})_{36\cdot 11}}(112^2-\frac{1}{4\cdot 13}+\frac{1}{9\cdot 7\cdot 11\cdot 13+\frac{17}{18}})}$$

$=137.035999074627$

${}^{17}_{3}Li_{4}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{94,98,100}_{42}Mo_{52,56,58}$ ${}^{120,124,125,128}_{52}Te_{68,72,73,76}$ ${}^{151,153}_{63}Eu_{88,90}$

Supplement 18: Other formulas of α_2

$$\alpha_{2\text{-}42}=\frac{2\cdot 3\cdot 7\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{42\cdot 11}{20\cdot 23+1})^{13\cdot 71}}}{17\cdot 19}\frac{1}{112-\frac{1}{12\cdot(10\cdot 11\cdot 67-1)+\frac{5}{12}}}$$

$=1/137.035999111818$

${}^{7}_{3}Li_{4}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{67}_{30}Zn_{37}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{92,95,97,98,100}_{42}Mo_{50,53,55,56,58}$ ${}^{117}_{50}Sn_{67}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{165}_{67}Ho_{98}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{209}_{84}Po^{*}_{125}$

$$\alpha_{2-42-Wallis}=\frac{8\cdot 3\cdot 7\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{1384}{1385}\frac{2\cdot 9\cdot 7\cdot 11}{2\cdot 4\cdot 173+1})}{17\cdot 19}\frac{1}{112-\frac{1}{2\cdot 25\cdot 29\cdot 61-\frac{37}{3\cdot 43}}}$$

$=1/137.035999111818$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{55}_{25}Mn_{30}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{137}_{56}Ba_{81}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{173}_{70}Yb_{103}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\alpha_{2-42-Wallis}=\frac{16\cdot 3\cdot 7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 3^2\cdot 7^2+1})}{17\cdot 19}\frac{1}{112-\frac{1}{4\cdot(2\cdot 23\cdot 113-1)+\frac{10}{59}}}$$

$=1/137.035999111818$

${}^{50,51}_{23}V_{27,28}$ ${}^{84,86}_{36}Kr_{48,50}$ ${}^{105}_{46}Pd_{59}$ ${}^{113}_{49}In_{64}$ ${}^{118}_{50}Sn_{68}$ ${}^{141}_{59}Pr_{82}$ ${}^{4\cdot 49}_{78}Pt_{118}$ ${}^{188,3\cdot 63}_{76}Os_{112,113}$ ${}^{284}_{113}Nh^{ie}_{9\cdot 19}$ ${}^{6\cdot 49}_{118}Og^{ie}_{176}$

$$\alpha_{2\text{-}61}=\frac{61\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{9\cdot173}{4\cdot(4\cdot97+1)})^{11\cdot(6\cdot47+1)}}}{7\cdot67}\frac{1}{112-\frac{1}{2\cdot6\cdot13\cdot(8\cdot7\cdot11\cdot13+1)}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{47}_{22}Ti_{25}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{61}_{28}Ni_{33}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{97}_{42}Mo_{55}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{117}_{50}Sn_{67}$ ${}^{134,136-138}_{56}Ba_{78,80-82}$

${}^{145}_{61}Pm^{*}_{84}$ ${}^{163}_{66}Dy_{97}$ ${}^{165}_{67}Ho_{98}$ ${}^{173}_{70}Yb_{103}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{209}$

$$\alpha_{2-61-Wallis}=\frac{4\cdot61\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{5670}{4671}\frac{8\cdot(12\cdot59+1)}{2\cdot10\cdot(36\cdot13-1)+1})}{7\cdot67}\frac{1}{112-\frac{1}{2\cdot97\cdot(10\cdot9\cdot17+1)-\frac{19}{28}}}$$

$=1/137.035999111818$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{82,84}_{36}Kr_{46,48}$ ${}^{118}_{50}Sn_{68}$ ${}^{139}_{57}La_{82}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{196}_{78}\mathrm{Pt}_{118}$ ${}^{209}_{84}Po^{*}_{125}$

$$\alpha_{2-61-GL}=\frac{8\cdot61\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot(16\cdot3\cdot31-1)+1})}{7\cdot67}\frac{1}{112-\frac{1}{4\cdot3\cdot5\cdot(2\cdot7\cdot173+1)-\frac{37}{62}}}$$

$=1/137.035999111818$

${}^{31}_{15}P_{16}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{157}_{64}Gd_{93}$

${}^{165}_{67}Ho_{98}$ ${}^{173}_{70}Yb_{103}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{209}$

$$\alpha_{2\text{-}77}=\frac{7\cdot11\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{11\cdot191}{3\cdot7\cdot100})^{4201}}}{16\cdot37}\frac{1}{112-\frac{1}{5\cdot(4\cdot9\cdot7\cdot13\cdot17-1)+\frac{5}{16}}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{32-34,36}_{16}S_{16-18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{45}_{21}Sc_{24}$ ${}^{64,66-68,70}_{30}Zn_{34,36-38,40}$ ${}^{74,76-78,80,82}_{34}Se_{40,42-44,46,48}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{100}_{44}Ru_{56}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{168}_{68}Tm_{100}$ ${}^{184}_{74}W_{110}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{200}_{80}Hg_{120}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$

$$\alpha_{2-77-Wallis}=\frac{7\cdot11\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{6300}{6301}\frac{2\cdot16\cdot(2\cdot9\cdot11-1)}{2\cdot2\cdot9\cdot25\cdot7+1})}{4\cdot37}\frac{1}{112-\frac{1}{2\cdot15\cdot17\cdot29\cdot31-\frac{5}{6}}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47,50}_{22}Ti_{25,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$

${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{75}_{33}As_{42}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{182,184,186}_{74}W_{108,110,112}$ ${}^{191,193}_{77}Ir_{114,116}$

$$\alpha_{2-77-GL}=\frac{7\cdot11\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot10\cdot(6\cdot67-1)+1})}{2\cdot37}\frac{1}{112-\frac{1}{28\cdot(10\cdot28\cdot(2\cdot173+1)-1)}}$$

$=1/137.035999111818$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{58,60,61}_{28}Ni_{30,32,33}$ ${}^{66,67,70}_{30}Zn_{36,37,40}$ ${}^{117}_{50}Sn_{67}$ ${}^{173}_{70}Yb_{103}$ ${}^{165}_{67}Ho_{98}$ ${}^{201}_{80}Hg_{121}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{209}$

$$\alpha_{2-93}=\frac{93\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{\left(\dfrac{12\cdot 227}{7\cdot(4\cdot 97+1)}\right)^{13\cdot(10\cdot 42-1)}}}{5\cdot 11\cdot 13}\,\frac{1}{112-\dfrac{1}{10\cdot 12\cdot(8\cdot 9\cdot 7\cdot 13+1)+\dfrac{7}{12}}}=1/137.035999111818$$

${}^{19}_{9}F_{10}\ {}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{54,56-58}_{26}Fe_{28,30-32}\ {}^{69,71}_{31}Ga_{38,40}\ {}^{93}_{41}Nb_{52}\ {}^{97}_{42}Mo_{55}\ {}^{157}_{64}Gd_{93}\ {}^{163}_{66}Dy_{97}\ {}^{209}_{84}Po^{*}_{125}\ {}^{227}_{89}Ac^{*}_{138}\ {}^{237}_{93}Np^{*}_{144}\ {}^{247}_{97}Bk^{*}_{150}$

$$\alpha_{2-93-Wallis}=\frac{4\cdot 93\cdot\left(2\,\frac{2}{3}\,\frac{4}{3}\,\frac{4}{5}\,\frac{6}{5}\,\frac{6}{7}\,\frac{8}{7}\cdots\frac{8170}{8171}\,\frac{36\cdot 227}{2\cdot 5\cdot 19\cdot 43+1}\right)}{5\cdot 11\cdot 13}\,\frac{1}{112-\dfrac{1}{45\cdot(30\cdot 11\cdot 53+1)+\dfrac{4}{13}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}\ {}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{54,56,57}_{26}Fe_{28,30,31}\ {}^{66,68}_{30}Zn_{36,38}\ {}^{83,84}_{36}Kr_{47,48}\ {}^{95}_{42}Mo_{53}\ {}^{98,99}_{43}Tc^{*}_{55,56}\ {}^{127}_{53}I_{74}\ {}^{209}_{83}Bi^{*}_{126}\ {}^{209}_{84}Po^{*}_{125}\ {}^{227}_{89}Ac^{*}_{138}$

$$\alpha_{2-93-GL}=\frac{8\cdot 93\cdot\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 9\cdot 17^2+1}\right)}{5\cdot 11\cdot 13}\,\frac{1}{112-\dfrac{1}{60\cdot(30\cdot 11\cdot 23+1)+\dfrac{14}{33}}}=1/137.035999111818$$

${}^{19}_{9}F_{10}\ {}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{28,29,30}_{14}Si_{14,15,16}\ {}^{35,37}_{17}Cl_{18,20}\ {}^{50,51}_{23}V_{27,28}\ {}^{54,56-58}_{26}Fe_{28,30-32}\ {}^{64,66,68}_{30}Zn_{34,36,38}\ {}^{78,80}_{34}Se_{44,46}\ {}^{167,168}_{68}Er_{99,100}$

$$\alpha_{2-109}=\frac{109\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{\left(\dfrac{9\cdot(3\cdot 128-1)}{2\cdot(42\cdot 41+1)}\right)^{61\cdot 113}}}{2\cdot(2\cdot 11\cdot 19+1)}\,\frac{1}{112-\dfrac{1}{3\cdot 47\cdot(4\cdot 7\cdot 13^2+1)+\dfrac{3}{4}}}=1/137.035999111818$$

${}^{19}_{9}F_{10}\ {}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{39,40,41}_{19}K_{20,21,22}\ {}^{46-50}_{22}Ti_{24-28}\ {}^{83,84}_{36}Kr_{47,48}\ {}^{93}_{41}Nb_{52}\ {}^{107,109}_{47}Ag_{60,62}\ {}^{112,113}_{48}Cd_{64,65}\ {}^{145}_{61}Pm^{*}_{84}\ {}^{3\cdot 61}_{74}W_{109}$
${}^{188,189}_{76}Os_{112,113}\ {}^{209}_{83}Bi^{*}_{126}\ {}^{209}_{84}Po^{*}_{125}\ {}^{276}_{109}Mt^{*}_{167}\ {}^{285}_{112}Cn^{*}_{173}\ {}^{284}_{113}Nh^{ie}_{171}\ {}^{346}_{137}Fy^{ie}_{209}$

$$\alpha_{2-109-Wallis}=\frac{2\cdot 109\cdot\left(2\,\frac{2}{3}\,\frac{4}{3}\,\frac{4}{5}\,\frac{6}{5}\,\frac{6}{7}\,\frac{8}{7}\cdots\frac{10338}{10339}\,\frac{2\cdot 10\cdot 11\cdot 47}{2\cdot 3\cdot(42\cdot 41+1)+1}\right)}{2\cdot 11\cdot 19+1}\,\frac{1}{112-\dfrac{1}{49\cdot 67\cdot 313}}=1/137.035999111818$$

${}^{19}_{9}F_{10}\ {}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{39-41}_{19}K_{20-22}\ {}^{46-50}_{22}Ti_{24-28}\ {}^{93}_{41}Nb_{52}\ {}^{107,109}_{47}Ag_{60,62}\ {}^{113,115}_{49}In_{64,66}\ {}^{165}_{67}Ho_{98}\ {}^{209}_{83}Bi^{*}_{126}\ {}^{209}_{84}Po^{*}_{125}\ {}^{346}_{137}Fy^{ie}_{209}$

$$\alpha_{2-109-GL}=\frac{4\cdot 109\cdot\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 6\cdot(8\cdot 137+1)+1}\right)}{2\cdot 11\cdot 19+1}\,\frac{1}{112-\dfrac{1}{49\cdot 97\cdot 151-\dfrac{5}{13}}}=1/137.035999111818$$

Note: $8\cdot 137+1=18\cdot 61-1$

${}^{47}_{22}Ti_{25}\ {}^{107,109}_{47}Ag_{60,62}\ {}^{113,115}_{49}In_{64,65}\ {}^{145}_{61}Pm^{*}_{84}\ {}^{150,152}_{62}Sm_{88,90}\ {}^{163}_{66}Dy_{97}\ {}^{3\cdot 61}_{74}W_{109}\ {}^{209}_{83}Bi^{*}_{126}\ {}^{209}_{84}Po^{*}_{125}\ {}^{13\cdot 19}_{97}Bk^{*}_{150}\ {}^{276}_{109}Mt^{*}_{167}\ {}^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\alpha_{2-141}=\frac{3\cdot 47\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{\left(\dfrac{22\cdot(2\cdot 11^2-1)}{9\cdot 19\cdot 31}\right)^{23\cdot(42\cdot 11-1)}}}{4\cdot(16\cdot 17-1)}\,\frac{1}{112-\dfrac{1}{18\cdot 17\cdot(6\cdot 11\cdot 137-1)}}=1/137.035999111818$$

${}^{19}_{9}F_{10}\ {}^{23}_{11}Na_{12}\ {}^{46-50}_{22}Ti_{24-28}\ {}^{35,37}_{17}Cl_{18,20}\ {}^{36,38,40}_{18}Ar_{18,20,22}\ {}^{39-41}_{19}K_{20-22}\ {}^{50,51}_{23}V_{27,28}\ {}^{69,71}_{31}Ga_{38,40}\ {}^{107,109}_{47}Ag_{60,62}\ {}^{163}_{66}Dy_{93}\ {}^{346}_{137}Fy^{ie}_{209}$

$$\alpha_{2-141-Wallis}=\frac{3\cdot 47\cdot\left(2\,\frac{2}{3}\,\frac{4}{3}\,\frac{4}{5}\,\frac{6}{5}\,\frac{6}{7}\,\frac{8}{7}\cdots\frac{15904}{15905}\,\frac{6\cdot 11\cdot(2\cdot 11^2-1)}{2\cdot 16\cdot 7\cdot 71+1}\right)}{16\cdot 17-1}\,\frac{1}{112-\delta_2}=1/137.035999111818$$

$$\delta_2=\frac{71\cdot(10\cdot 11\cdot 47+1)}{10^{12}}\approx\frac{1}{7\cdot 43\cdot(13\cdot 24\cdot 29+1)}\approx\frac{1}{125\cdot(20\cdot 109-1)}\approx\frac{1}{8\cdot 23\cdot 113\cdot(12\cdot 11-1)}$$

${}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{63,65}_{29}Cu_{34,36}\ {}^{71}_{31}Ga_{40}\ {}^{98,99}_{43}Tc^{*}_{55,56}\ {}^{107,109}_{47}Ag_{60,62}\ {}^{136}_{56}Ba_{80}\ {}^{142}_{60}Nd_{82}\ {}^{175,176}_{71}Lu_{104,105}\ {}^{209}_{84}Po^{*}_{125}\ {}^{223}_{87}Fr^{*}_{136}\ {}^{4\cdot 71}_{113}Nh^{ie}_{171}$

$$\alpha_{2-141-GL}=\frac{2\cdot 3\cdot 47\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 4\cdot(10\cdot 11\cdot 23+1)+1})}{16\cdot 17-1}\frac{1}{112-\frac{4\cdot 109\cdot(2\cdot 3\cdot 47-1)}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{46-50}_{22}Ti_{24-28}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{183}_{74}W_{109}$ ${}^{2\cdot 138}_{109}Mt^{*}_{167}$

$$\alpha_{2-189}=\frac{27\cdot 7\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{12\cdot(36\cdot 23-1)\,\text{or}\,(14\cdot 59+1)}{2\cdot 11^2\cdot 41+1})^{89\cdot 223}}}{12\cdot 11^2+1}\frac{1}{112-\frac{2\cdot 3\cdot 7\cdot 113}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{45}_{21}Sc_{24}$ ${}^{46-50}_{22}Ti_{24-28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{78,80,82,83,84,86}_{36}Kr_{42,44,46,47,48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{105}_{46}Pd_{59}$

${}^{112,113}_{48}Cd_{64,65}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{208}_{82}Pt_{126}$ ${}^{189}_{76}Cs_{113}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{284}_{113}Nh^{ie}_{171}$

$$\alpha_{2-189-Wallis}=\frac{4\cdot 27\cdot 7\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{29770}{19771}\frac{36\cdot(36\cdot 23-1)\,\text{or}(14\cdot 59+1)}{2\cdot 5\cdot 13\cdot(12\cdot 19+1)+1})}{12\cdot 11^2+1}\frac{1}{112-\delta_2}$$

$$=1/137.035999111818 \qquad \delta_2=\frac{13^2\cdot 29\cdot 31}{25\cdot 10^{11}}\approx\frac{9\cdot 25\cdot 37\cdot 73}{4\cdot 10^{11}}$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39-41}_{19}Al_{20-22}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{169}_{69}Tm_{100}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{277}_{108}Hs^{*}_{169}$ ${}^{280}_{111}Rg^{*}_{169}$

$$\alpha_{2-189-GL}=\frac{8\cdot 27\cdot 7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 8\cdot 23\cdot 103+1})}{12\cdot 11^2+1}\frac{1}{112-\frac{16\cdot(6\cdot 7\cdot 11^2-1)}{25\cdot 10^{11}}\,\text{or}\,\frac{3\cdot 7\cdot 19\cdot 163}{2\cdot 10^{12}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{96}_{42}Mo_{54}$ ${}^{102,104}_{44}Ru_{58,60}$ ${}^{163}_{66}Dy_{97}$ ${}^{173}_{70}Yb_{103}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$

$$\alpha_{2-205}=\frac{5\cdot 41\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{79\cdot 157}{2\cdot 9\cdot 13\cdot 53})^{5\cdot 11^2\cdot 41}}}{8\cdot 197}\frac{1}{112-\frac{2\cdot(4\cdot 11\cdot(16\cdot 17-1)-1)}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{53}_{24}Cr_{29}$ ${}^{55}_{25}Mn_{30}$ ${}^{56,58}_{26}Fe_{30,32}$ ${}^{93}_{41}Nb_{52}$ ${}^{79}_{35}Br_{44}$ ${}^{95}_{42}Mo_{53}$ ${}^{100}_{44}Ru_{56}$ ${}^{157}_{64}Nb_{93}$ ${}^{197}_{79}Au_{118}$ ${}^{226}_{88}Ra^{*}_{136}$ ${}^{257}_{100}Fm^{*}_{157}$

$$\alpha_{2-205-Wallis}=\frac{5\cdot 41\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{37208}{37209}\frac{10\cdot 61^2}{2\cdot 4\cdot(6\cdot 25\cdot 31+1)+1})}{2\cdot 197}\frac{1}{112-\frac{4\cdot 3\cdot 5\cdot 37^2+1}{2\cdot 10^{12}}}$$

$$=1/137.035999111818$$

${}^{31}_{15}P_{16}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{93}_{41}Nb_{52}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{145,146,147}_{61}Pm^{*}_{84,85,86}$

$$\alpha_{2-205-GL}=\frac{5\cdot 41\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 8\cdot 9\cdot 7\cdot 47+1})}{197}\frac{1}{112-\frac{3\cdot 41\cdot(16\cdot 7\cdot 11+1)}{2\cdot 10^{12}}\,\text{or}\,\frac{23\cdot 83\cdot 197}{5\cdot 10^{12}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{78,80,82,83,84,86}_{36}Kr_{42,44,46,47,48,50}$ ${}^{112}_{48}Cd_{64}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{197}_{79}Au_{118}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$

$$\alpha_{2-221}=\frac{13\cdot17\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{6\cdot11\cdot(2\cdot7\cdot17+1)-1}{4\cdot(8\cdot17\cdot29-1)})^{9\cdot5\cdot(7\cdot100+1)}}}{17\cdot100-1}\frac{1}{112-\frac{3\cdot7\cdot139}{4\cdot10^{11}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{138,140}_{58}Cs_{80,82}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{12\cdot29}_{139}Ch^{ie}_{209}$

$$\alpha_{2-221-Wallis}=\frac{4\cdot13\cdot17\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{47318}{47319}\frac{10\cdot4\cdot7\cdot13^2}{2\cdot59\cdot401+1})}{17\cdot100-1}\frac{1}{112-\frac{19^2\cdot103}{25\cdot10^{11}}}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{59}_{27}Co_{32}$ ${}^{103}_{45}Rh_{58}$ ${}^{105}_{46}Pd_{59}$ ${}^{141}_{59}Mn_{82}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{285}_{112}Cn^{*}_{173}$

$$\alpha_{2-221-GL}=\frac{8\cdot13\cdot17\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot17\cdot(2\cdot13\cdot17+1)+1})}{17\cdot100-1}\frac{1}{112-\frac{7\cdot23\cdot(6\cdot7\cdot10+1)}{25\cdot10^{11}}}$$

$=1/137.035999111818$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{56}_{26}Fe_{30}$ ${}^{76,80}_{34}Se_{42,46}$ ${}^{92,94,95,96,97,98,100}_{42}Mo_{50,52,53,54,55,56,58}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$

$$\alpha_{2\text{-}234}=\frac{2\cdot9\cdot13\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot41\cdot47-1}{36\cdot107})^{5\cdot23\cdot67}}}{7\cdot257}\frac{1}{112-\frac{1}{7\cdot29\cdot(2^9-1)+\frac{11}{14}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29}_{14}Si_{14,15}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{93}_{41}Nb_{52}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{165}_{67}Ho_{98}$ ${}^{257}_{100}Fm^{*}_{157}$

$$\alpha_{2-234-Wallis}=\frac{8\cdot9\cdot13\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{11556}{11557}\frac{2\cdot(20\cdot17^2-1)}{2\cdot2\cdot27\cdot107+1})}{7\cdot257}\frac{1}{112-\frac{1}{5\cdot(12\cdot47\cdot(4\cdot163+1)+1)}}$$

$=1/137.035999111818$ (Note: $4\cdot163+1=6\cdot109-1$)

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{168}_{68}Er_{100}$ ${}^{179}_{72}Hf_{107}$ ${}^{183}_{74}W_{109}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{276}_{109}Mt^{*}_{167}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$

$$\alpha_{2-234-GL}=\frac{16\cdot9\cdot13\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot13\cdot(4\cdot71-1)+1})}{7\cdot257}\frac{1}{112-\frac{1}{5\cdot11^2\cdot(13\cdot100-1)+\frac{14}{19}\text{ or }\frac{17}{23}}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$
${}^{175,176}_{71}Lu_{104,105}$ ${}^{188,7\cdot27}_{76}Os_{112,113}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{4\cdot71}_{113}Nh^{ie}_{9\cdot19}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$

$$\alpha_{2\text{-}237}=\frac{3\cdot79\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{20\cdot(24\cdot43-1)}{2\cdot13^2\cdot61+1})^{11\cdot23\cdot163}}}{2\cdot(10\cdot7\cdot13+1)}\frac{1}{112-\frac{163\cdot(4\cdot79+1)}{2\cdot10^{12}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{97}_{43}Tc^{*}_{54}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{135}_{56}Ba_{79}$ ${}^{163}_{66}Dy_{97}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{410}_{163}Ch^{ie}_{13\cdot19}$

$$\alpha_{2-237-Wallis}=\frac{6\cdot79\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{61858}{61859}\frac{60\cdot(24\cdot43-1)}{2\cdot157\cdot197+1})}{10\cdot7\cdot13+1}\frac{1}{112-\frac{18\cdot(8\cdot(12\cdot41-1)+1)}{25\cdot10^{11}}}$$

$=1/137.035999111818$

${}^{27}_{13}Al_{14}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{73}_{32}Ge_{41}$ ${}^{93}_{41}Nb_{42}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{157}_{64}Gd_{93}$ ${}^{208}_{82}Pt_{126}$ ${}^{197}_{79}Au_{118}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{426}_{169}Ch^{ie}_{257}$

$$\alpha_{2-237-GL}=\frac{12\cdot 79\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 20\cdot 11\cdot 179+1})}{10\cdot 7\cdot 13+1 \text{ or } 48\cdot 19-1}\frac{1}{112-\frac{3\cdot 61\cdot 137}{10^{12}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{39-41}_{19}K_{20-22}$ ${}^{46-50}_{22}Ti_{24-28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{86,88}_{38}Sr_{48,50}$ ${}^{89}_{39}Y_{50}$
${}^{99,100}_{44}Ru_{55,56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{134,135-137,138}_{56}Ba_{78,79-81,82}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{179}_{72}Hf_{107}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{346}_{137}Fy^{ie}_{209}$

Supplement 19: Relationships between Ramanujan-Sato series for 1/π and nuclides

$$\text{Ramanujan Series (1914):}\quad \frac{1}{\pi}=\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

$$\text{Ramanujan-Sato-Chen Series:}\quad 2\pi=\frac{(9\cdot 11)^2\sin\frac{\pi}{4}}{\sum_{k=0}^{\infty}\frac{(4k)!(1+2\cdot 19\cdot 29+2\cdot 5\cdot 7\cdot 13\cdot 29\cdot k)}{(k!)^4(4\cdot 9\cdot 11)^{4k}}}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$
${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{40}_{18}Ar_{22}$ ${}^{39-41}_{19}K_{20-22}$ ${}^{45}_{21}Sc_{24}$ ${}^{48,49,50}_{22}Ti_{26,27,28}$ ${}^{54,56-58}_{26}Fe_{28,30-32}$
${}^{59}_{27}Co_{32}$ ${}^{58,60,61,64}_{28}Ni_{30,32,33,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,68,70}_{30}Zn_{34,36,38,40}$ ${}^{75}_{33}As_{42}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{78,80,82-84}_{36}Kr_{42,44,46-48}$
${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{103}_{45}Rh_{58}$ ${}^{102,104,105,106,108,110}_{46}Pd_{56,58,59,60,62,64}$
${}^{122,124,126,128,130}_{52}Te_{70,72,74,76,78}$ ${}^{130,132,134,135,136,137,138}_{56}Ba_{74,76,78,79,80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$
${}^{159}_{65}Tb_{94}$ ${}^{161,162,163}_{66}Dy_{95,96,97}$ ${}^{165}_{67}Ho_{98}$ ${}^{167}_{68}Er_{99}$ ${}^{171,174,176}_{70}Yb_{101,104,106}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{174,176,177,180}_{72}Hf_{102,104,105,108}$
${}^{196,198,200,201}_{80}Hg_{116,118,120,121}$ ${}^{202}_{81}Tl^{*}_{121}$ ${}^{7\cdot 29,205}_{81}Tl_{122,124}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{3\cdot 7\cdot 11}_{91}Pa^{*}_{140}$
${}^{36\cdot 7}_{99}Es^{*}_{153}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{302}_{121}Ch^{ie}_{181}$ ${}^{2\cdot 163}_{128}Ch^{ie}_{198}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{2\cdot 173}_{137}Ch^{ie}_{209}$ ${}^{3\cdot 121}_{143}Ch^{ie}_{220}$ ${}^{368}_{145}Ch^{ie}_{223}$

Supplement 20: Rewriting of Einstain's E=mc^2

$$\text{Einstein Formula: } E=mc^2 \qquad \text{Maxwell Formula: } c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\text{In atomic units, } c_{au}=\frac{1}{\sqrt{\mu_{0/au}\varepsilon_{0/\mathrm{au}}}}=\frac{1}{\sqrt{\frac{\mu_{0/au}}{4\pi}}}=\frac{1}{\sqrt{\alpha_1\alpha_2}} \text{ or } {c_{au}}^2=\frac{1}{\alpha_1\alpha_2}$$

$$\text{So: } E_{au}=m_{au}{c_{au}}^2=\frac{m_{au}}{\alpha_1\alpha_2},\ \text{or } E_{au}=\frac{m_{au}}{\alpha_1\alpha_2},\ \text{or } m_{au}=E_{au}\alpha_1\alpha_2$$

Supplement 21: Other formulas of the fine-structure constant

$$\alpha_{1-13-Wallis}=\frac{67}{4\cdot 13\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{138}{139}\frac{140}{2\cdot 3\cdot 23+1})}\frac{1}{112+\frac{1}{9\cdot 25}-\frac{1}{103\cdot(24\cdot 37-1)+\frac{4}{5}}}$$

$$=1/137.035999037435$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{55}_{25}Mn_{30}$ ${}^{67}_{30}Zn_{37}$ ${}^{103}_{45}Rh_{58}$ ${}^{117,119,120}_{50}Sn_{67,69,70}$
${}^{126,128}_{52}Te_{74,76}$ ${}^{138}_{56}Ba_{82}$ ${}^{165}_{67}Ho_{98}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

$$\alpha_{1-13-GL} = \frac{67}{8\cdot 13\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 88+1})}\frac{1}{112+\frac{1}{7\cdot 31}-\frac{1}{20\cdot 19\cdot 127-\frac{14}{99}}}$$

$$=1/137.035999037435$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,42,44.46,48}_{20}Ca_{20,22,24,26,28}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$
${}^{69,71}_{31}Ga_{38,40}$ ${}^{84,88}_{38}Sr_{46,50}$ ${}^{99,100,104}_{44}Ru_{55,56,60}$ ${}^{126,128}_{52}Te_{74,76}$ ${}^{165}_{67}Ho_{98}$ ${}^{174}_{70}Yb_{104}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\alpha_{2-7} = \frac{7\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{49}{48})^{97}}}{6\cdot 9}\,\frac{1}{112-\frac{1}{16\cdot 11+\frac{89}{8\cdot 5\cdot 7\cdot 71}}} = 1/137.035999111819$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{48,49}_{22}Ti_{26,27}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{84,86}_{36}Kr_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{96,97,98}_{42}Mo_{54,55,56}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$
${}^{110,111,112,113,114}_{48}Cd_{62,63,64,65,66}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{163}_{66}Dy_{97}$ ${}^{175,176}_{71}Lu_{104,105}$
${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$

$$\alpha_{2-7-Wallis} = \frac{14\cdot(2\,\frac{2}{3}\,\frac{4}{3}\,\frac{4}{5}\,\frac{6}{5}\,\frac{6}{7}\,\frac{8}{7}\cdots\frac{144}{145}\,\frac{2\cdot 73}{2\cdot 2\cdot 36+1})}{27}\,\frac{1}{112-\frac{1}{25\cdot 13}+\frac{1}{29\cdot(36\cdot 11\cdot 13-1)-\frac{3}{23}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{84}_{36}Kr_{48}$
${}^{106,108,110,111,112,113,114,116}_{48}Cd_{58,60,62,63,64,65,66,68}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{115,116,117,119,120,122}_{50}Sn_{65,66,67,69,70,72}$ ${}^{125}_{52}Te_{73}$
${}^{169}_{69}Tm_{100}$ ${}^{180,181}_{73}Ta_{107,108}$

$$\alpha_{2-7-GL} = \frac{28\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 4\cdot 23+1})}{27}\,\frac{1}{112-\frac{1}{36\cdot 7}+\frac{1}{40\cdot 36\cdot 47+\frac{9}{31}}}$$

$$=1/137.035999111818$$

${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{40,42,43,44.46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{52,54}_{24}Cr_{28,30}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{69,71}_{31}Ga_{38,40}$
${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{90,92,94}_{40}Zr_{50,52,54}$ ${}^{92,94,96,98}_{42}Mo_{50,52,54,56}$ ${}^{102,106,108}_{46}Pd_{56,60,62}$ ${}^{107,108}_{47}Ag_{60,62}$ ${}^{144}_{60}Nd_{84}$

Supplement 22: The fine-structure constant 13 billion years ago

In a recent paper[12], Wilczynska and Webb *et al* reported measurements of the fine-structure constant in the location of the universe 13 billion light years away, the results indicated that there would be deviation from the terrestrial value, i.e., $\Delta\alpha/\alpha=(\alpha_z-\alpha_0)/\alpha_0=(2.18\pm 7.27)\times 10^{-5}$ in this direction or location at the age of the univers of 0.8 billion years old.

According to our theories about the fine-structure constant, the formulas of it are related to nuclides, and the universe should have different nuclide contents and distribution in different ages or even in different directions of the universe which should result in a little different values of the fine-structure constant, so this deviation could be explained to be reasonable. We here try to give some different hypothetical formulas and values of the fine-structure constant in reference to Webb's results as follows.

$$\Delta\alpha = (\alpha_Z - \alpha_0)/\alpha_0 = (-2.18 \pm 7.27)\times 10^{-5}$$

$$\alpha_Z \approx (1-2.18\times 10^{-5})\times \frac{1}{137.035999} = 1/137.03899$$

$$\alpha_{Z-2-7} = \frac{7\cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{49}{48})^{97}}}{6\cdot 9}\frac{1}{112} = 1/137.04295$$

$$\alpha_{Z-2-7\text{-}Wallis} = \frac{14\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{144}{145}\frac{2\cdot 73}{2\cdot 2\cdot 36+1})}{27}\frac{1}{112} = 1/137.03976$$

$$\alpha_{Z-2-7\text{-}GL} = \frac{28\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 4\cdot 23+1})}{27}\frac{1}{112} = 1/137.04084$$

Supplement 23: The most possible elements to be synthesized after the 118th element

Up to now human beings have already discovered and synthesized 118 elements which fully fill 7 periods in Periodic Table of Elements. In this paper, we predicted the 119-170th elements, and called the 113-170th elements were ideal extended elements which implied some of them could be synthesized and many of them shouldn't. Some questions at present are whether people could synthesize new elements to open the 8th period in Periodic Table of Elements, what would be the next element to be synthesized after the 118th element and so on. As 126, 128 and 137 are special numbers according to our theories, we here predict the following ideal extended elements could be relatively easier to be synthesized in future.

$$^{310}_{124}Ch^{ie}_{186} \quad ^{312}_{125}Ch^{ie}_{187} \quad ^{314}_{126}Ch^{ie}_{188} \quad ^{318-320}_{127}Ch^{ie}_{191-193} \quad ^{326}_{128}Ch^{ie}_{198}$$

$$^{344}_{136}Fy^{ie}_{208} \quad ^{346}_{137}Fy^{ie}_{209} \quad ^{348}_{138}Fy^{ie}_{210}$$

Among them $_{126}Ch_{188}$ should be the easiest to be synthesized. So, we predict one of these ideal extended elements (most likely $_{126}Ch_{188}$) would open the 8th period in Periodic Table of Elements.

Supplement 24: Relationships between Chudnovsky algorithm for 1/π and nuclides

Chudnovsky algorithm (1987): $$\frac{1}{\pi}=12\sum_{k=0}^{\infty}\frac{(-1)^k(6k)!(13591409+545140134k)}{(3k)!(k!)^3 640320^{3k+3/2}}$$

Chudnovsky-Chen algorithm:

$$2\pi=\frac{1}{6\sum_{k=0}^{\infty}\frac{(-1)^k(6k)!(-1+2\cdot3\cdot5\cdot7\cdot61\cdot(4\cdot5\cdot53+1)+2\cdot9\cdot7\cdot11\cdot19\cdot127\cdot163\cdot k)}{(3k)!(k!)^3(64\cdot3\cdot5\cdot23\cdot29)^{3k+3/2}}}$$

$$=\frac{1}{6\sum_{k=0}^{\infty}\frac{(-1)^k(6k)!(-1+2\cdot3\cdot5\cdot7\cdot61\cdot(2\cdot9\cdot59-1)+2\cdot9\cdot7\cdot11\cdot19\cdot127\cdot163\cdot k)}{(3k)!(k!)^3(64\cdot3\cdot5\cdot23\cdot29)^{3k+3/2}}}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,39,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}K_{20,21,22}$

${}^{40,42-44,46,48}_{20}Ca_{20,22-24,26,28}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{55}_{25}Mn_{30}$

${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{69,71}_{31}Ga_{38,40}$

${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{75}_{33}As_{42}$ ${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{78,80,82,83,84,86}_{36}Kr_{42,44,46,47,48,50}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{90,91,92,94,96}_{40}Zr_{50,51,52,54,56}$ ${}^{92,94,95,96,97,98,100}_{42}Mo_{50,52,53,54,55,56,58}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$

${}^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{103}_{45}Rh_{58}$ ${}^{102,104,105,106,108,110}_{46}Pd_{56,58,59,60,62,64}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$

${}^{106,108,110-114,116}_{48}Cd_{58,60,62-66,68}$ ${}^{112,114-120,122,124}_{50}Sn_{62,64-70,72,74}$ ${}^{121,123}_{51}In_{70,72}$ ${}^{126,128}_{52}Te_{74,76}$ ${}^{127}_{53}I_{74}$ ${}^{133}_{55}Cs_{78}$

${}^{129-132,134,136}_{54}Xe_{75-78,80,82}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{138,140}_{58}Ce_{80,82}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{5\cdot29}_{61}Pm^{*}_{84}$ ${}^{157,160}_{64}Gd_{93,96}$

${}^{161,162,163,164}_{66}Dy_{95,96,97,98}$ ${}^{169}_{69}Tm_{100}$ ${}^{171,173,174,176}_{70}Yb_{101,103,104,106}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{177,178}_{72}Hf_{105,106}$ ${}^{183,184,186}_{74}W_{109,110,112}$

${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,190,192}_{76}Os_{112,114,116}$ ${}^{191,192}_{77}Ir_{114,116}$ ${}^{197}_{79}Au_{118}$ ${}^{196,198,200-202}_{80}Hg_{116,118,120-122}$ ${}^{204,208}_{82}Pt_{122,126}$

${}^{209,210}_{83}Bi^{*}_{126,127}$ ${}^{208,209,210}_{84}Po^{*}_{124,125,126}$ ${}^{207-209,210,211}_{85}At^{*}_{122-124,125,126}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{8\cdot29}_{90}Th^{*}_{142}$ ${}^{7\cdot33}_{91}Pa^{*}_{140}$

${}^{235,238}_{92}U^{*}_{143,146}$ ${}^{4\cdot61}_{94}Pu^{*}_{150}$ ${}^{243}_{95}Am^{*}_{148}$ ${}^{13\cdot19}_{96}Cm^{*}_{151}$ ${}^{13\cdot19}_{97}Bk^{*}_{150}$ ${}^{252}_{99}Es^{*}_{153}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{268}_{105}Dh^{*}_{163}$ ${}^{269,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$

${}^{281}_{110}Ds^{*}_{171}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{284,286}_{113}Nh^{ie}_{171,173}$ ${}^{289}_{114}Fl^{ie}_{175}$ ${}^{289}_{115}Mc^{ie}_{174}$ ${}^{293}_{116}Lv^{ie}_{177}$ ${}^{294}_{117}Ts^{ie}_{177}$ ${}^{294}_{118}Og^{ie}_{176}$ ${}^{16\cdot19}_{122}Ch^{ie}_{182}$ ${}^{314}_{126}Ch^{ie}_{188}$

${}^{6\cdot53,11\cdot29,320}_{127}Ch^{ie}_{191,3\cdot64,193}$ ${}^{2\cdot163}_{128}Ch^{ie}_{198}$ ${}^{336}_{133}Ch^{ie}_{7\cdot29}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{12\cdot29}_{139}Ch^{ie}_{209}$ ${}^{386}_{154}Ch^{ie}_{8\cdot29}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{410}_{163}Ch^{ie}_{13\cdot19}$

According to our theories, the formulas of the fine-structure constant are related to nuclides. As some formulas and values of 2π are incorporated in formulas of the fine-structure constant, they would be related to nuclides directly. In **Supplement 19**, **24** and **28**, we gave two examples of Ramanujan-Sato series for 1/π which are related to nuclides. In 2012, G. Almkvist and J. Guillera[13] stated that they used the theory of Calabi-Yau differential equations to obtain all the parameters of Ramanujan-Sato-like series for $1/\pi^2$ and along with a conjecture to find new examples of series of non-hypergeometric type. So, we here guess that Calabi-Yau differential equations would be somewhat related to the formulas of the fine-structure constant and nuclides.

Supplement 25: Other formulas of α2

$$\alpha_{2\text{-}157}=\frac{157\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{6521}{8\cdot 5\cdot 163})^{81\cdot 7\cdot 23}}}{17\cdot 71}\;\frac{1}{112-\dfrac{1}{8\cdot 27\cdot 7\cdot 13\cdot 127}}$$

$$=1/137.035999111818$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$
${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,19}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}Kr_{20,21,22}$
${}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$
${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$
${}^{69,71}_{31}Ga_{38,40}$ ${}^{69,72,74}_{32}Ge_{38,40,42}$ ${}^{75}_{33}As_{42}$ ${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{78,82,84}_{36}Kr_{42,46,48}$
${}^{89}_{39}Y_{50}$ ${}^{90,91,92,94,96}_{40}Zr_{50,51,52,54,56}$ ${}^{92,94,96,97,98,100}_{42}Mo_{50,52,54,55,56,58}$ ${}^{100}_{44}Ru_{56}$ ${}^{104}_{46}Pd_{58}$ ${}^{113,115}_{49}In_{64,66}$
${}^{121,123}_{51}In_{70,72}$ ${}^{112,114-120,122}_{50}Sn_{62,64-70,72}$ ${}^{120,123,126,128,130}_{52}Te_{68,71,74,76,78}$ ${}^{127}_{53}I_{74}$ ${}^{132,134,136}_{54}Xe_{78,80,82}$
${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{157}_{64}Gd_{93}$ ${}^{161,163}_{66}Dy_{95,97}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$
${}^{176,177,180}_{72}Hf_{104,105,108}$ ${}^{195}_{78}Pt_{117}$ ${}^{199,200}_{80}Hg_{119,120}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{208}_{82}Pt_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{208,209,210}_{84}Po^{*}_{124,125,126}$
${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{257}_{102}No^{*}_{157}$ ${}^{263,265}_{104}Rf^{*}_{159,161}$ ${}^{268}_{105}Dh^{*}_{163}$ ${}^{269,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$
${}^{314}_{126}Ch^{ie}_{188}$ ${}^{318-320}_{127}Ch^{ie}_{191-193}$ ${}^{326}_{129}Ch^{ie}_{197}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$ ${}^{406}_{161}Ch^{ie}_{245}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

$$\alpha_{2-157-Wallis}=\frac{4\cdot 157\cdot(2\,\frac{2}{3}\,\frac{4}{3}\,\frac{4}{5}\,\frac{6}{5}\,\frac{6}{7}\,\frac{8}{7}\cdots\frac{19560}{19561}\,\frac{19562}{2\cdot 3\cdot 4\cdot 5\cdot 163+1})}{17\cdot 71}\;\frac{1}{112-\dfrac{1}{8\cdot 9\cdot 11\cdot 37\cdot(4\cdot 3\cdot 11-1)}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,19}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{100}_{44}Ru_{56}$ ${}^{142-146,148,150}_{60}Nd_{82-86,88,90}$ ${}^{163}_{66}Dy_{97}$

$$\alpha_{2-157-GL}=\frac{8\cdot 157\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 4\cdot 11\cdot(4\cdot 71-1)+1})}{17\cdot 71}\;\frac{1}{112-\dfrac{7\cdot 17\cdot(8\cdot 71+1)}{4\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{100}_{44}Ru_{56}$ ${}^{118,119,120}_{50}Sn_{68,69,70}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{199}_{80}Hg_{119}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{2\cdot 113}_{88}Ra^{*}_{138}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$ ${}^{298}_{119}Ch^{ie}_{179}$ ${}^{8\cdot 43}_{136}Fy^{ie}_{208}$

Supplement 26: Other formulas of α1

$$\alpha_{1-73}=\frac{3\cdot 125}{73\cdot e^{2}\,\dfrac{e^{2}}{(\frac{2}{1})^{3}}\,\dfrac{e^{2}}{(\frac{3}{2})^{5}}\cdots\dfrac{e^{2}}{(\frac{500-1}{6\cdot 83})^{997}}}\;\frac{1}{112+\dfrac{1}{2\cdot 7\cdot(2100-1)+\frac{30}{67}}}=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{45}_{21}Sc_{24}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{67}_{30}Zn_{37}$ ${}^{75}_{33}As_{42}$ ${}^{73,74}_{32}Ge_{41,42}$
${}^{92,97,98,100}_{42}Mo_{50,55,56,58}$ ${}^{100}_{44}Ru_{56}$ ${}^{9\cdot 13,120,124}_{50}Sn_{67,70,74}$ ${}^{122,125,126,130}_{52}Te_{70,73,74,78}$ ${}^{165}_{67}Ho_{98}$ ${}^{166,168}_{68}Er_{98,100}$ ${}^{180,181}_{73}Ta_{107,108}$
${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{208,209,210}_{83}Bi^{*}_{125,126,127}$ ${}^{208,209,210}_{84}Po^{*}_{124,125,126}$ ${}^{210,211}_{85}At^{*}_{125,126}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{24\cdot 13,314}_{125,126}Ch^{ie}_{187,188}$

$$\alpha_{1-73-Wallis}=\frac{3\cdot125}{4\cdot73\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{1494}{1495}\frac{8\cdot11\cdot17}{2\cdot9\cdot83+1})}\frac{1}{112+\frac{1}{4\cdot9\cdot(8\cdot5\cdot7\cdot11-1)-\frac{1}{2}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}Al_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{75}_{33}As_{42}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{83,84}_{36}Kr_{47,48}$ ${}^{166,168}_{68}Er_{98,100}$ ${}^{100}_{44}Ru_{56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{166,168}_{68}Er_{98,100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{24\cdot13,314}_{125,126}Ch^{ie}_{187,188}$

$$\alpha_{1-73-Wallis}=\frac{3\cdot125}{8\cdot73\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot8\cdot7\cdot17+1})}\frac{1}{112+\frac{1}{5\cdot61\cdot83-\frac{29}{35}}}$$

$$=1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{115,118,119,120,122}_{50}Sn_{66,68,69,70,72}$ ${}^{136}_{56}Ba_{80}$ ${}^{145,146}_{61}Pm^{*}_{84,85}$ ${}^{199}_{80}Hg^{ie}_{119}$ ${}^{223}_{87}Ra^{*}_{136}$

$$\alpha_{1-199}=\frac{2\cdot7\cdot73}{199\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{7\cdot13\cdot23}{4\cdot(18\cdot29+1)})^{27\cdot5\cdot31}}}\frac{1}{112+\frac{1}{32\cdot13\cdot(2\cdot13\cdot23+1)-\frac{4}{5}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{73}_{32}Ge_{41}$

${}^{91,92,94,96}_{40}Zr_{51,52,54,56}$ ${}^{93}_{41}Nb_{52}$ ${}^{125}_{52}Te_{73}$ ${}^{155,156,157,160}_{64}Gd_{91,92,93,96}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{199}_{80}Hg^{ie}_{119}$ ${}^{223}_{87}Ra^{*}_{136}$

$$\alpha_{1-199-Wallis}=\frac{2\cdot7\cdot73}{4\cdot199\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{6276}{6277}\frac{2\cdot43\cdot73}{2\cdot6\cdot523+1})}\frac{1}{112+\frac{1}{11\cdot17\cdot(16\cdot3\cdot43-1)-\frac{4}{5}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{84}_{38}Kr_{48}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{8\cdot43,12\cdot29}_{136,138}Fy^{ie}_{208,210}$

$$\alpha_{1-199-GL}=\frac{2\cdot7\cdot73}{8\cdot199\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot27\cdot37+1})}\frac{1}{112+\frac{1}{2\cdot11\cdot19\cdot(60\cdot11+1)+\frac{1}{20}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,49,50}_{22}Ti_{24,27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{346}_{137}Fy^{ie}_{209}$

Supplement 27: The analogies between graph of the approximate formulas of the fine-structure constant and graph of the stability of nuclides

The stability of nuclides (**Fig. 14**) determines the abundance of elements in the universe (**Fig. 15**). For example, ^{56}Fe is the most stable nuclide, so the abundance of Fe in the universe is relatively high (called Fe peak) because massive stars create it; ^{4}He is a much stable nuclide, so He is the second rich element in the universe because the Big Bang and stars create it.

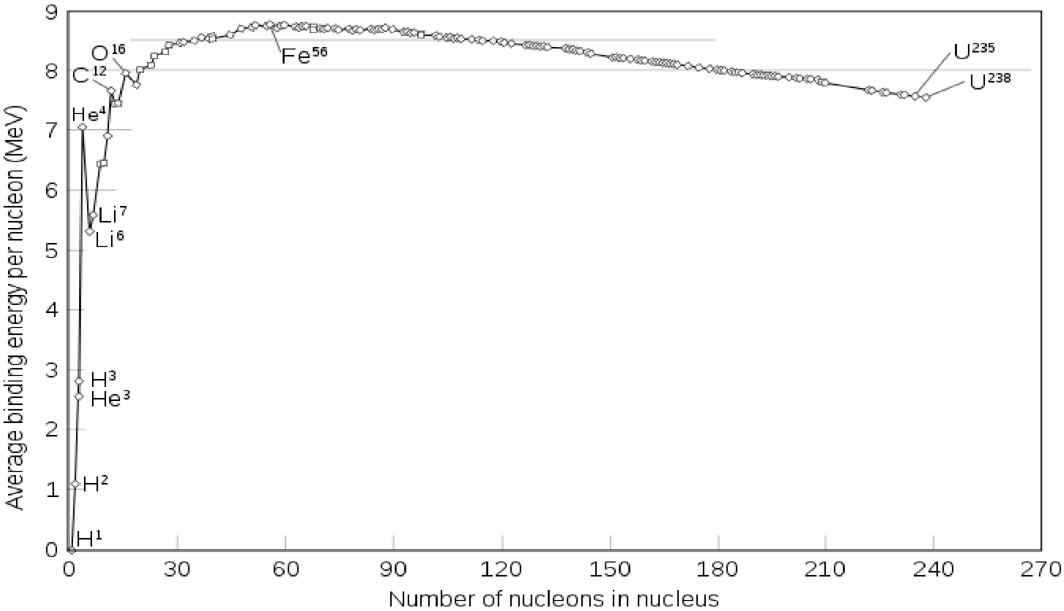


Fig. 14. Graph of stability of nuclides

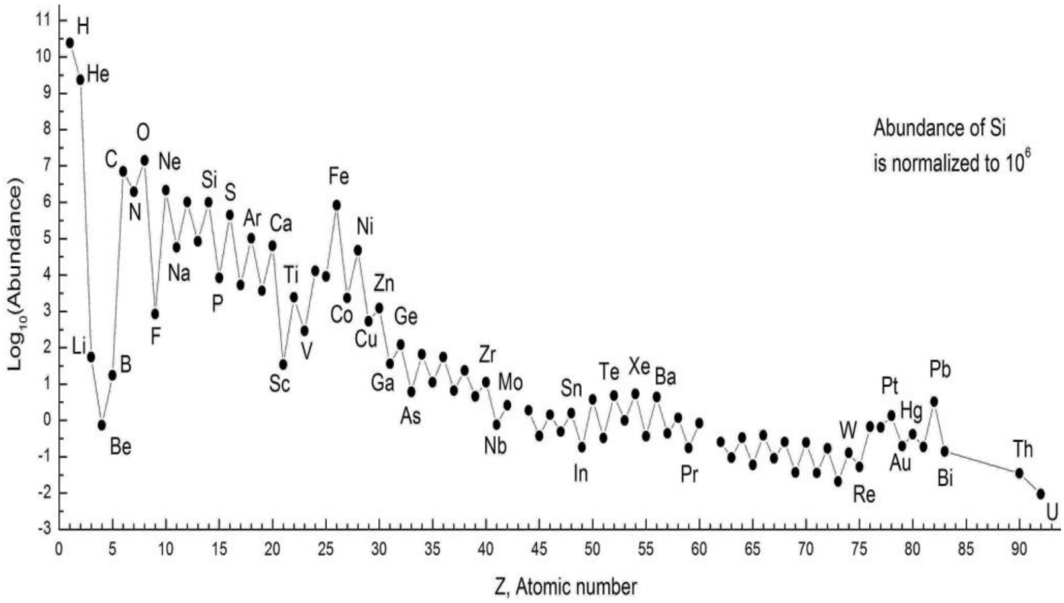


Fig. 15. Graph of abundance of elements in the universe

We noticed that there are morphological analogies between graph of approximate formulas of the fine-structure constant (**Fig. 7** and **Fig. 8**) and graph of the stability of nuclides (**Fig. 14**). So we suppose that there would be corresponding relationships between the formulas of the fine-structure constant and elements (**Table 9**).

The corresponding relationships described in **table 9** are approximate rather than accurate. Among them, $\alpha_{1\text{-}7}$ to $_2$He and $\alpha_{2\text{-}13}$ to $_2$He should be the most important. $\alpha_{1\text{-}22}$ to $_6$C, $\alpha_{2\text{-}29}$ to $_6$C, $\alpha_{1\text{-}170}$ to $_{26}$Fe and $\alpha_{1\text{-}253}$ to $_{26}$Fe are also special. Essentially, these formulas of α and the corresponding elements ($_2$He and $_{26}$Fe) are special respectively.

Table 9. The corresponding relationships between the formulas of α and elements

α_1	n	k	Elements	α_2	n	k	Elements
$\alpha_{1\text{-}7}$	36	112	$_2$He	$\alpha_{2\text{-}13}$	100	278	$_2$He
$\alpha_{1\text{-}22}$	113	782	$_6$C	$\alpha_{2\text{-}29}$	223	655	$_6$C
$\alpha_{1\text{-}29}$	149	321	$_7$N	$\alpha_{2\text{-}36}$	277	190	$_7$N
$\alpha_{1\text{-}36}$	185	236	$_7$N	$\alpha_{2\text{-}45}$	346	1079	$_7$N
$\alpha_{1\text{-}43}$	221	200	$_8$O	$\alpha_{2\text{-}61}$	469	1556	$_8$O
$\alpha_{1\text{-}50}$	257	181	$_{10}$Ne	$\alpha_{2\text{-}77}$	592	2100	$_{10}$Ne
$\alpha_{1\text{-}59}$	303	2645	$_{12}$Mg	$\alpha_{2\text{-}93}$	715	2723	$_{12}$Mg
$\alpha_{1\text{-}73}$	375	498	$_{14}$Si	$\alpha_{2\text{-}109}$	838	3446	
$\alpha_{1\text{-}81}$	416	1605	$_{14}$Si	$\alpha_{2\text{-}125}$	961	4293	$_{14}$Si
$\alpha_{1\text{-}96}$	493	5806	$_{16}$S	$\alpha_{2\text{-}141}$	1084	5301	
$\alpha_{1\text{-}103}$	529	1310	$_{18}$Ar	$\alpha_{2\text{-}157}$	1207	6520	$_{16}$S
$\alpha_{1\text{-}133}$	683	12389	$_{20}$Ca	$\alpha_{2\text{-}173}$	1330	8023	$_{18}$Ar
$\alpha_{1\text{-}140}$	719	1923	$_{22}$Ti	$\alpha_{2\text{-}189}$	1453	9923	
$\alpha_{1\text{-}155}$	796	3988	$_{24}$Cr	$\alpha_{2\text{-}205}$	1576	12402	$_{20}$Ca
$\alpha_{1\text{-}170}$	873	34450	$_{26}$Fe	$\alpha_{2\text{-}221}$	1699	15772	$_{22}$Ti
$\alpha_{1\text{-}199}$	1022	2092	$_{28}$Ni	$\alpha_{2\text{-}237}$	1822	20619	$_{24}$Cr
$\alpha_{1\text{-}207}$	1063			$\alpha_{2\text{-}253}$	1945	28186	$_{26}$Fe
				$\alpha_{2\text{-}269}$	2068	41654	$_{28}$Ni

Supplement 28: Relationships between Ramanujan series for 1/π (1914) and nuclides

$$\frac{1}{\pi}=\frac{1}{4}\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(21460n+1123)}{(n!)^4 441^{2n+1}2^{10n+1}}=\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(-1+2\cdot3\cdot11\cdot17+4\cdot5\cdot29\cdot37\cdot n)}{(n!)^4(3^2\cdot7^2)^{2n+1}2^{10n+1}}$$

$$2\pi=\frac{8}{\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(-1+2\cdot3\cdot11\cdot17+4\cdot5\cdot29\cdot37n)}{(n!)^4 2^{10n+1}(3^2\cdot7^2)^{2n+1}}}$$

$$\frac{1}{\pi}=\frac{1}{3528}\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(21460n+1123)}{(n!)^4 4^{4n}882^{2n}}=\frac{1}{42\cdot84}\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(-1+6\cdot11\cdot17+20\cdot29\cdot37n)}{(n!)^4 4^{4n}(2\cdot3^2\cdot7^2)^{2n}}$$

$$2\pi=\frac{(84)^2}{\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(-1+2\cdot3\cdot11\cdot17+4\cdot5\cdot29\cdot37n)}{(n!)^4 4^{4n}(2\cdot3^2\cdot7^2)^{2n}}}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16-18}_{8}O_{8-10}$ ${}^{19}_{9}F_{10}$ ${}^{20-22}_{10}Ne_{10-12}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{45}_{21}Sc_{24}$

${}^{40,42,44,48}_{20}Ca_{20,22,24,28}$ ${}^{46,48-50}_{22}Ti_{24,26-28}$ ${}^{58,60-62,64}_{28}Ni_{30,32-34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{74,76-78,80,82}_{34}Se_{40,42-44,46,48}$

${}^{78,80,84,86}_{36}Kr_{42,44,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{90,91,94,96}_{40}Zr_{50,51,54,56}$ ${}^{92,96,97,98,100}_{42}Mo_{50,54,55,56,58}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$

${}^{96,98,99,100,102,104}_{44}Ru_{52.54,55,56,58,60}$ ${}^{103}_{45}Rh_{58}$ ${}^{106,108,110,111,112,114,116}_{48}Cd_{58,60,62,63,64,66,68}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$

${}^{124,126,128,131,132.134,136}_{54}Xe_{70,72,74,77,78,80,82}$ ${}^{133}_{55}Cs_{78}$ ${}^{130,132,135,136,137,138}_{56}Ba_{74,76,79,80,81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$

${}^{144,145,148,150}_{60}Nd_{84,85,88,90}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{156,160,162,164}_{66}Dy_{90,94,96,98}$ ${}^{162,164,166,167,168,170}_{68}Er_{94,96,98,99,100,102}$

${}^{168,170,171,172,174,176}_{70}Yb_{98,100,101,102,104,106}$ ${}^{174,176,177,180}_{72}Hf_{102,104,105,108}$ ${}^{180,182,184,186}_{74}W_{106,108,110,112}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{187,188,189,192}_{76}Os_{111,112,113,116}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{196,198,199,200,201,204}_{80}Hg_{116,118,119,120,121,124}$ ${}^{7\cdot29,205}_{81}Tl_{122,124}$ ${}^{207,208}_{82}Pt_{125,126}$

${}^{209}_{83}Bi^{*}_{126}$ ${}^{209,210}_{84}Po^{*}_{125,126}$ ${}^{210,211}_{85}Po^{*}_{125,126}$ ${}^{6\cdot37}_{88}Ra^{*}_{138}$ ${}^{243}_{95}Am^{*}_{148}$ ${}^{308}_{123}Ch^{ie}_{185}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{357,368}_{145}Ch^{ie}_{6\cdot37,223}$ ${}^{22\cdot17}_{148}Ch^{ie}_{226}$

$$\frac{1}{\pi}=\frac{1}{72}\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(260n+23)}{(n!)^4 4^{4n}\cdot 18^{2n}}=\frac{1}{8\cdot 9}\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(-1+8\cdot 3+4\cdot 5\cdot 13\cdot n)}{(n!)^4 4^{4n}\cdot(2\cdot 9)^{2n}}$$

$$2\pi=\frac{(4\cdot 3)^2}{\sum_{n=0}^{\infty}\frac{(-1)^n(4n)!(-1+8\cdot 3+4\cdot 5\cdot 13\cdot n)}{(n!)^4 4^{4n}\cdot(2\cdot 9)^{2n}}}$$

${}^{4}_{2}He_{2}$ ${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$
${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{40,44,46,48}_{20}Ca_{20,24,26,28}$ ${}^{50,52,44,54}_{24}Cr_{26,28,30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$
${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,70}_{30}Zn_{34,36,40}$ ${}^{90,92,94,96}_{40}Zr_{50,52,54,56}$ ${}^{108,110,112,113}_{48}Cd_{60,62,64,65}$ ${}^{113,115}_{49}In_{64,66}$
${}^{120,124,126,128,130}_{52}Te_{68,72,74,76,78}$ ${}^{144,150}_{60}Nd_{84,90}$ ${}^{154,156,160}_{64}Gd_{90,92,96}$ ${}^{200}_{80}Hg_{120}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{330}_{130}Ch^{ie}_{200}$ ${}^{366}_{144}Ch^{ie}_{222}$

Supplement 29: Other formulas of α_1 with Nilakantha formula for π

Nilakantha (1444-1544) formula for π: $\pi=3+\frac{4}{2\cdot 3\cdot 4}-\frac{4}{4\cdot 5\cdot 6}+\frac{4}{6\cdot 7\cdot 8}-\frac{4}{8\cdot 9\cdot 10}+\cdots$

Nilakantha-Chen formula for 2π:

$$2\pi=6+\frac{1}{1(1+1/2)(1+1)}-\frac{1}{2(2+1/2)(2+1)}+\frac{1}{3(3+1/2)(3+1)}-\frac{1}{4(4+1/2)(4+1)}+\cdots$$

$$=6+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}$$

$$(2\pi)_{NC-k}=6+\sum_{n=1}^{k}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}$$

$$\alpha_{1-7-NC}=\frac{36}{7\cdot(2\pi)_{NC-3}}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{1}{2\cdot 5\cdot(3\cdot 7)^2-\frac{113}{7\cdot 17}}}=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{45}_{21}Sc_{24}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{84}_{36}Kr_{48}$ ${}^{98}_{42}Mo_{56}$ ${}^{112,113}_{48}Cd_{64,65}$
${}^{113,115}_{49}In_{64,66}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284,286}_{113}Nh^{ie}_{171,173}$

$$\alpha_{1-22-NC}=\frac{113}{2\cdot 11\cdot(2\pi)_{NC-7}}\frac{1}{112+\frac{1}{3\cdot 49}-\frac{1}{2\cdot 19\cdot(12\cdot 11^2-1)-\frac{2\cdot 7}{43}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{49}_{22}Ti_{27}$ ${}^{86,87,88}_{38}Sr_{48,49,50}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{147}_{61}Pm^{*}_{86}$ ${}^{186-190}_{76}Os_{110-114}$

$$\alpha_{1-29-NC}=\frac{149}{29\cdot(2\pi)_{NC-5}}\frac{1}{112+\frac{1}{5\cdot 11}-\frac{1}{3\cdot 17\cdot(4\cdot 3\cdot 23+1)-\frac{31}{61}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{99,100,102}_{44}Ru_{55,56,58}$ ${}^{116,118,119}_{50}Sn_{66,68,69}$ ${}^{121,123}_{51}Sb_{70,72}$
${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{149}_{62}Sm_{87}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{344,346,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-59-NC}=\frac{3\cdot 101}{59\cdot(2\pi)_{NC-11}}\frac{1}{112+\frac{1}{2^9}-\frac{1}{4\cdot 27\cdot 11\cdot(32\cdot 3\cdot 5-1)}}=1/137.035999037435$$

${}^{16-18}_{8}O_{8-10}$ ${}^{32-34,36}_{16}S_{16-18,20}$ ${}^{49}_{22}Ti_{27}$ ${}^{59}_{27}Co_{32}$ ${}^{96,98-102,104}_{44}Ru_{52,54-58,60}$ ${}^{102,104-106,108,110}_{46}Pd_{56,58-60,62,64}$ ${}^{177}_{72}Hf_{105}$

$$\alpha_{1-36-NC}=\frac{5\cdot 37}{4\cdot 9\cdot(2\pi)_{NC-5}}\frac{1}{112+\dfrac{1}{25}-\dfrac{1}{2\cdot 7\cdot 71}+\dfrac{1}{C}}=1/137.035999037435$$

$$C=4\cdot 13\cdot(16\cdot 5\cdot 17\cdot 37+1)\approx 7\cdot 11\cdot 17\cdot(2\cdot 27\cdot 37+1)\approx 2\cdot 193\cdot(4\cdot 3\cdot 5\cdot 113-1)$$

${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{78,80,83,84,86}_{36}Kr_{42,44,47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{112,114-120,122,124}_{50}Sn_{62,64-70,72,74}$

${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{120,122-126,128,130}_{52}Te_{68,70-74,76,78}$ ${}^{170,172,174,176}_{70}Yb_{100,102,104,106}$

${}^{175,176}_{71}Lu_{104,105}$ ${}^{174,176,180}_{72}Hf_{102,104,108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{187,189}_{76}Sb_{111-113}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{210}_{85}At^{*}_{125}$

${}^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$ ${}^{297,298}_{119}Ch^{ie}_{178,179}$ ${}^{306,308}_{123}Ch^{ie}_{183/185}$ ${}^{318-320}_{127}Ch^{ie}_{191-193}$

$$\alpha_{1-81-NC}=\frac{4^2\cdot 26}{9^2\cdot(2\pi)_{NC-9}}\frac{1}{112+\dfrac{1}{6\cdot 59}-\dfrac{1}{4\cdot 17\cdot(42\cdot(16\cdot 47-1)+1)}}=1/137.035999037435$$

${}^{9}_{4}Be_{5}$ ${}^{27}_{13}Al_{14}$ ${}^{32-34,36}_{16}S_{16-18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{76,78}_{34}Se_{42,44}$ ${}^{78,83,84}_{36}Kr_{42,47,48}$

${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118}_{50}Sn_{68}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{168}_{68}Er_{100}$ ${}^{174,176,177,180}_{72}Hf_{102,104,105,108}$ ${}^{196}_{78}Pt_{118}$ ${}^{294}_{117}Ts^{ie}_{177}$

$$\alpha_{1-96-NC}=\frac{17\cdot 29}{32\cdot 3\cdot(2\pi)_{NC-15}}\frac{1}{112+\dfrac{1}{8\cdot 7\cdot 17+1}-\dfrac{37\cdot(6\cdot 173+1)}{25\cdot 10^{11}}}=1/137.035999037435$$

${}^{16,17}_{8}O_{8,9}$ ${}^{32,33,34}_{16}S_{16,17,18}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{136-138}_{56}Ba_{80-82}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{346}_{137}Fy^{ie}_{209}$

$$\alpha_{1-103-NC}=\frac{23^2}{103\cdot(2\pi)_{NC-15}}\frac{1}{112+\dfrac{1}{3\cdot 61}-\dfrac{1}{49\cdot 11\cdot 71-\dfrac{49}{52}}}=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{49}_{22}Ti_{27}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{103}_{45}Rh_{58}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{122,123,130}_{52}Te_{70,71,78}$

${}^{147}_{61}Pm^{*}_{86}$ ${}^{168,173,174,176}_{70}Yb_{98,103,104,106}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{262}_{103}Lr^{*}_{159}$

$$\alpha_{1-133-NC}=\frac{4\cdot 9\cdot 19-1}{7\cdot 19\cdot(2\pi)_{NC-19}}\frac{1}{112+\dfrac{1}{25\cdot 101}-\dfrac{(84)^2\cdot 11}{10^{12}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{39.40,41}_{19}K_{20,21,22}$ ${}^{55}_{25}Mn_{30}$ ${}^{84,86}_{36}Kr_{48,50}$ ${}^{84,88}_{38}Sr_{46,50}$ ${}^{99,100,101}_{44}Ru_{55,56,57}$ ${}^{171}_{70}Yb_{101}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$

$$\alpha_{1-140-NC}=\frac{4\cdot 9\cdot 20-1}{7\cdot 20\cdot(2\pi)_{NC-9}}\frac{1}{112+\dfrac{1}{30\cdot 37-1}-\dfrac{1}{120\cdot(6\cdot 11\cdot 283+1)}}=1/137.035999037435$$

${}^{20-22}_{10}Ne_{10-12}$ ${}^{40,42,44,48}_{20}Ca_{20,22,24,28}$ ${}^{64,66,67,70}_{30}Ga_{34,36,37,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{116,120,124}_{50}Sn_{66,70,74}$ ${}^{200}_{80}Hg_{120}$ ${}^{300}_{120}Ch^{ie}_{180}$

$$\alpha_{1-155-NC}=\frac{4\cdot 199}{5\cdot 31\cdot(2\pi)_{NC-13}}\frac{1}{112+\dfrac{1}{16\cdot 43}-\dfrac{1}{32\cdot 47\cdot(8\cdot 7\cdot 17+1)}}=1/137.035999037435$$

${}^{31}_{15}P_{16}$ ${}^{32-34}_{16}S_{16-18}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{74,76}_{32}Ge_{42,44}$ ${}^{77,80,82}_{34}Se_{43,46,48}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{199}_{80}Hg_{119}$ ${}^{222}_{86}Rn^{*}_{136}$

$$\alpha_{1-170-NC}=\frac{9\cdot 97}{170\cdot(2\pi)_{NC-25}}\frac{1}{112+\dfrac{1}{2\cdot 27\cdot 11\cdot 47-\dfrac{6}{35}}}=1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83}_{36}Kr_{47}$ ${}^{97}_{42}Mo_{55}$ ${}^{97-99}_{43}Tc^{*}_{54-56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{163}_{66}Dy_{97}$ ${}^{170}_{70}Yb_{100}$ ${}^{210}_{85}At^{*}_{125}$

$$\alpha_{1-199-NC} = \frac{2\cdot 7\cdot 73}{199\cdot(2\pi)_{NC-9}} \frac{1}{112+\cfrac{1}{29\cdot(4\cdot 73-1)-\cfrac{27\cdot 11}{32\cdot 13}}} = 1/137.035999037435$$

$${}^{27}_{13}Al_{14}\quad {}^{28-30}_{14}Si_{14-16}\quad {}^{54,56,58}_{26}Fe_{28,30,32}\quad {}^{59}_{27}Co_{32}\quad {}^{63,65}_{29}Cu_{34,36}\quad {}^{73}_{32}Ge_{41}\quad {}^{125}_{52}Te_{73}\quad {}^{180,181}_{73}Ta_{107,108}\quad {}^{199}_{80}Hg_{119}$$

$$\alpha_{1-207-NC} = \frac{2\cdot 9\cdot 59+1}{9\cdot 23\cdot(2\pi)_{NC-46}} \frac{1}{112+\cfrac{1}{8\cdot 3\cdot(8\cdot 3\cdot 7\cdot 31+1)+\cfrac{3}{46}}} = 1/137.035999037435$$

$${}^{23}_{11}Na_{12}\quad {}^{46,50}_{22}Ti_{24,28}\quad {}^{50,51}_{23}V_{27,28}\quad {}^{52}_{24}Cr_{28}\quad {}^{59}_{27}Co_{32}\quad {}^{69}_{31}Ga_{38}\quad {}^{79,81}_{35}Br_{44,46}\quad {}^{102,105,108,110}_{46}Pd_{56,59,62,64}$$

Supplement 30: Other formulas of α_2 with Nilakantha series for π

$$\alpha_{2-13-NC} = \frac{13\cdot(2\pi)_{NC-5}}{100} \frac{1}{112-\cfrac{1}{4\cdot 9}+\cfrac{1}{2\cdot 9\cdot 89-1}-\cfrac{1}{3\cdot 173\cdot(16\cdot 17\cdot 23+1)}}$$

$$= 1/137.035999111818$$

$${}^{9}_{4}Be_{5}\quad {}^{19}_{9}F_{10}\quad {}^{27}_{13}Al_{14}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{36,38,40}_{18}Ar_{18,20,22}\quad {}^{50,51}_{23}V_{27,28}\quad {}^{82,84,86}_{36}Kr_{46,48,50}\quad {}^{89}_{39}Y_{50}\quad {}^{115,118,119}_{50}Sn_{65,68,69}$$

$${}^{168}_{68}Er_{100}\quad {}^{169}_{69}Tm_{100}\quad {}^{173}_{70}Yb_{103}\quad {}^{178}_{72}Hf_{106}\quad {}^{227}_{89}Ac^{*}_{138}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{286}_{113}Nh^{ie}_{173}\quad {}^{288}_{115}Mc^{ie}_{173}\quad {}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$$

$$\alpha_{2-29-NC} = \frac{29\cdot(2\pi)_{NC-5}}{223} \frac{1}{112-\cfrac{1}{3\cdot 29}+\cfrac{1}{8\cdot 3\cdot 5\cdot 19^2+\cfrac{8}{15}}} = 1/137.035999111818$$

$${}^{14,15}_{7}N_{7,8}\quad {}^{28,29,30}_{14}Si_{14,15,16}\quad {}^{31}_{15}P_{16}\quad {}^{39,40,41}_{19}K_{20,21,22}\quad {}^{63,65}_{29}Cu_{34,36}\quad {}^{84,86,87,88}_{38}Sr_{46,48,49,50}\quad {}^{223}_{87}Fr^{*}_{136}\quad {}^{368}_{145}Ch^{ie}_{223}$$

$$\alpha_{2-36-NC} = \frac{36\cdot(2\pi)_{NC-5}}{12\cdot 23+1} \frac{1}{112-\cfrac{1}{17}+\cfrac{1}{2\cdot 11\cdot 79}-\cfrac{1}{25\cdot 17\cdot(2\cdot 25\cdot 11\cdot 17-1)}}$$

$$= 1/137.035999111818$$

$${}^{23}_{11}Na_{12}\quad {}^{24,25,26}_{12}Mg_{12,13,14}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{47}_{22}Ti_{25}\quad {}^{55}_{25}Mn_{30}\quad {}^{79,81}_{35}Br_{44,46}\quad {}^{135,136}_{56}Ba_{79,80}\quad {}^{197}_{79}Au_{118}\quad {}^{210}_{85}At^{*}_{125}$$

$$\alpha_{2-45-NC} = \frac{45\cdot(2\pi)_{NC-5}}{2\cdot 173} \frac{1}{112-\cfrac{1}{4\cdot 11}+\cfrac{1}{47\cdot(10\cdot 23-1)-\cfrac{10}{199}}} = 1/137.035999111818$$

$${}^{10,11}_{5}B_{5,6}\quad {}^{19}_{9}F_{10}\quad {}^{23}_{11}Na_{12}\quad {}^{47}_{22}Ti_{25}\quad {}^{50,51}_{23}V_{27,28}\quad {}^{79,81}_{35}Br_{44,46}\quad {}^{83}_{36}Kr_{47}\quad {}^{99,100}_{44}Ru_{55,56}\quad {}^{103}_{45}Rh_{58}\quad {}^{106,109}_{47}Ag_{60,62}$$

$${}^{113,115}_{49}In_{64,66}\quad {}^{169}_{69}Tm_{100}\quad {}^{173}_{70}Ta_{103}\quad {}^{199,200}_{80}Hg_{119,120}\quad {}^{209}_{83}Bi^{*}_{126}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{286}_{113}Nh^{ie}_{173}\quad {}^{288}_{115}Mc^{ie}_{173}\quad {}^{2\cdot 173}_{137}Fy^{ie}_{209}$$

$$\alpha_{2-61-NC} = \frac{61\cdot(2\pi)_{NC-9}}{7\cdot 67} \frac{1}{112-\cfrac{1}{2\cdot 157-1}+\cfrac{1}{B}} = 1/137.035999111818$$

$$B = 16\cdot 7\cdot 13\cdot 73+\frac{23}{27} = 157\cdot(4\cdot 13^2+1)-\frac{4}{27} = 157\cdot(6\cdot 113-1)-\frac{4}{27}$$

$${}^{27}_{13}Al_{14}\quad {}^{50,51}_{23}V_{27,28}\quad {}^{59}_{27}Co_{32}\quad {}^{61}_{28}Ni_{33}\quad {}^{73}_{32}Ge_{41}\quad {}^{106,108}_{46}Pd_{60,62}\quad {}^{107,108}_{47}Ag_{60,62}\quad {}^{112,113}_{48}Cd_{64,65}\quad {}^{117,119}_{50}Sn_{67,69}$$

$${}^{125}_{52}Te_{73}\quad {}^{137}_{56}Ba_{81}\quad {}^{146}_{61}Pm^{*}_{85}\quad {}^{157}_{64}Gd_{93}\quad {}^{165}_{67}Ho_{98}\quad {}^{169}_{69}Tm_{100}\quad {}^{180,181}_{73}Ta_{107,108}\quad {}^{203}_{81}Tl_{122}\quad {}^{257}_{100}Fm^{*}_{157}\quad {}^{400}_{157}Ch^{ie}_{243}\quad {}^{426}_{169}Ch^{ie}_{257}$$

$$\alpha_{2-77-NC}=\frac{77\cdot(2\pi)_{NC-9}}{16\cdot 37}\frac{1}{112-\dfrac{1}{9\cdot(8\cdot 3\cdot 5\cdot 11+1)+\dfrac{3\cdot 37}{5\cdot 29}}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{63,65}_{29}Cu_{34,36}$ $^{67}_{30}Zn_{37}$ $^{85,87}_{37}Rb_{48,50}$ $^{111}_{48}Cd_{63}$ $^{145}_{60}Nd_{85}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{191,193}_{77}Ir_{114,116}$

$$\alpha_{2-93-NC}=\frac{3\cdot 31\cdot(2\pi)_{NC-11}}{5\cdot 11\cdot 13}\frac{1}{112-\dfrac{1}{2\cdot 7\cdot 41}+\dfrac{1}{4\cdot 5\cdot 107\cdot 227}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{93}_{41}Nb_{52}$ $^{107,109}_{47}Ag_{60,62}$ $^{133}_{55}Cs_{78}$ $^{159}_{65}Tb_{94}$ $^{180,181}_{73}Ta_{107,108}$ $^{227}_{89}Ac^{*}_{138}$ $^{8\cdot 47}_{149}Ch^{ie}_{227}$

$$\alpha_{2-109-NC}=\frac{109\cdot(2\pi)_{NC-11}}{2\cdot(2\cdot 11\cdot 19+1)}\frac{1}{112-\dfrac{1}{3\cdot 11\cdot 100}+\dfrac{41\cdot(1200+1)}{25\cdot 10^{11}}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{75}_{33}As_{42}$ $^{100}_{44}Ru_{56}$ $^{168}_{68}Er_{100}$ $^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{330}_{130}Ch^{ie}_{200}$ $^{346}_{137}Fy^{ie}_{209}$

$$\alpha_{2-125-NC}=\frac{125\cdot(2\pi)_{NC-13}}{31^2}\frac{1}{112-\dfrac{1}{4\cdot 223}+\dfrac{1}{18\cdot 19\cdot(24\cdot 113+1)}}=1/137.035999111818$$

$^{19}_{9}F_{10}$ $^{23}_{11}Na_{12}$ $^{38}_{18}Ar_{20}$ $^{39}_{19}K_{20}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{68,71}_{31}Ga_{38,40}$ $^{189}_{76}Os_{113}$ $^{223}_{87}Fr^{*}_{136}$ $^{284}_{113}Nh^{ie}_{171}$ $^{368}_{145}Ch^{ie}_{223}$

$$\alpha_{2-141-NC}=\frac{3\cdot 47\cdot(2\pi)_{NC-13}}{4\cdot(16\cdot 17-1)}\frac{1}{112-\dfrac{1}{16\cdot 3\cdot 71-\dfrac{2\cdot 31}{25\cdot 19}\text{ or }\dfrac{5\cdot 13}{6\cdot 83}}}=1/137.035999111818$$

$^{27}_{13}Al_{14}$ $^{35,37}_{17}Cl_{18,20}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{47,48}_{22}Ti_{25,26}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{68,71}_{31}Ga_{38,40}$ $^{83,84}_{36}Kr_{47,48}$
$^{107,109}_{47}Ag_{60,62}$ $^{113}_{48}Cd_{65}$ $^{121,123}_{51}Sb_{70,72}$ $^{159}_{65}Tb_{94}$ $^{175,176}_{71}Lu_{104,105}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{4\cdot 71}_{113}Nh^{ie}_{171}$

$$\alpha_{2-157-NC}=\frac{157\cdot(2\pi)_{NC-15}}{17\cdot 71}\frac{1}{112-\dfrac{1}{6\cdot(2\cdot 7\cdot 17+1)-1}+\dfrac{1}{2\cdot 59\cdot(64\cdot 7\cdot 61+1)}}$$
$$=1/137.035999111818$$

$^{59}_{27}Co_{32}$ $^{105}_{46}Pd_{59}$ $^{107,109}_{47}Ag_{60,62}$ $^{121,123}_{51}Sb_{70,72}$ $^{141}_{59}\mathrm{Pr}_{82}$ $^{157}_{64}Gd_{93}$ $^{175,176}_{71}Lu_{104,105}$ $^{257}_{100}Fm^{*}_{157}$ $^{400}_{157}Ch^{ie}_{243}$

$$\alpha_{2-173-NC}=\frac{173\cdot(2\pi)_{NC-15}}{2\cdot 5\cdot 7\cdot 19}\frac{1}{112-\dfrac{1}{5\cdot 17\cdot 73+(\dfrac{7}{20})^2}}=1/137.035999111818$$

$^{35,37}_{17}Cl_{18,20}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{79,81}_{35}Br_{44,46}$ $^{125}_{52}Te_{73}$ $^{173}_{70}Yb_{103}$ $^{180,181}_{73}Ta_{107,108}$ $^{285}_{112}Cn^{*}_{173}$ $^{210}_{85}At^{*}_{125}$ $^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\alpha_{2-189-NC}=\frac{27\cdot 7\cdot(2\pi)_{NC-17}}{4\cdot 3\cdot 11^2+1}\frac{1}{112-\dfrac{1}{4\cdot 17\cdot 41}+\dfrac{23\cdot 59\cdot 79}{25\cdot 10^{11}}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{50,51}_{23}V_{27,28}$ $^{59}_{27}Co_{32}$ $^{79,81}_{35}Br_{44,46}$ $^{82}_{36}Kr_{46}$ $^{105}_{46}Pd_{59}$ $^{135-138}_{56}Ba_{79-82}$ $^{141}_{59}\mathrm{Pr}_{82}$ $^{118}_{50}Sn_{68}$ $^{197}_{79}Au_{118}$

$$\alpha_{2-205-NC}=\frac{5\cdot 41\cdot(2\pi)_{NC-19}}{8\cdot 197}\frac{1}{112-\dfrac{1}{3\cdot 5\cdot 13^2}+\dfrac{23\cdot(84\cdot 23+1)}{5\cdot 10^{11}}}=1/137.035999111818$$

$^{27}_{13}Al_{14}$ $^{31}_{15}P_{16}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{50,51}_{23}V_{27,28}$ $^{89}_{39}Y_{50}$ $^{73}_{41}Nb_{52}$ $^{169}_{69}Tm_{100}$ $^{197}_{79}Au_{118}$ $^{209}_{84}Po^{*}_{125}$ $^{326,328}_{129}Ch^{ie}_{197,199}$

$$\alpha_{2-221-NC}=\frac{13\cdot 17\cdot(2\pi)_{NC-19}}{1700-1}\frac{1}{112-\cfrac{1}{3\cdot 23\cdot(2\cdot 9\cdot 11+1)+\cfrac{4\cdot 47}{5\cdot 41}}}=1/137.035999111818$$

$^{19}_{9}F_{10}$ $^{27}_{13}Al_{14}$ $^{35,37}_{17}Cl_{18,20}$ $^{47}_{22}Ti_{25}$ $^{50,51}_{23}V_{27,28}$ $^{93}_{41}Nb_{52}$ $^{99,100}_{44}Ru_{55,56}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{168}_{68}Er_{100}$

$^{169}_{69}Tm_{100}$ $^{188}_{76}Os_{112}$ $^{203,205}_{81}Tl_{122,124}$ $^{204,206,207,208}_{82}Pt_{122,124,125,126}$ $^{257}_{100}Fm^{*}_{157}$ $^{314}_{126}Ch^{ie}_{188}$ $^{340}_{135}Ch^{ie}_{205}$

$$\alpha_{2-237-NC}=\frac{3\cdot 79\cdot(2\pi)_{NC-21}}{2\cdot(2\cdot 5\cdot 7\cdot 13-1)}\frac{1}{112-\cfrac{1}{8\cdot 25\cdot 71+\cfrac{49}{11\cdot 19}\text{ or }\cfrac{53}{2\cdot 113}}}=1/137.035999111818$$

$^{27}_{13}Al_{14}$ $^{47,48,49}_{22}Ti_{25,26,27}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{79,81}_{35}Br_{44,46}$ $^{87,88}_{38}\mathrm{Sr}_{49,50}$ $^{95}_{42}Mo_{53}$ $^{113,115}_{49}In_{64,66}$ $^{127}_{53}I_{74}$

$^{135-138}_{56}Ba_{79-82}$ $^{175,176}_{71}Lu_{104,105}$ $^{189}_{76}Os_{113}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{226}_{88}Ra^{*}_{138}$ $^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$ $^{346}_{137}Fy^{ie}_{209}$ $^{374}_{148}Ch^{ie}_{226}$

$$\alpha_{2-253-NC}=\frac{11\cdot 23\cdot(2\pi)_{NC-23}}{5\cdot(4\cdot 97+1)}\frac{1}{112-\cfrac{1}{3\cdot 5\cdot 11^{2}\cdot 29+\cfrac{3}{29}}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{50,51}_{23}V_{27,28}$ $^{63,65}_{29}Cu_{34,36}$ $^{92,95,97,98,100}_{42}Mo_{50,53,55,56,58}$ $^{163}_{66}Dy_{97}$ $^{247}_{97}Bk^{*}_{150}$

$$\alpha_{2-269-NC}=\frac{(4\cdot 67+1)\cdot(2\pi)_{NC-27}}{4\cdot 11\cdot 47}\frac{1}{112-\cfrac{1}{11\cdot 13\cdot 163-\cfrac{83}{13\cdot 17}}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{47,49}_{22}Ti_{25,27}$ $^{83}_{36}Kr_{47}$ $^{99,100,104}_{44}Ru_{55,56,60}$ $^{143}_{60}Nd_{83}$ $^{117}_{50}Sn_{67}$ $^{163}_{66}Dy_{97}$ $^{209}_{83}Bi^{*}_{126}$

$^{235}_{92}U^{*}_{143}$ $^{4\cdot 67}_{105}Db^{*}_{163}$ $^{269}_{106}Sg^{*}_{163}$ $^{270}_{107}Bh^{*}_{163}$ $^{364}_{143}Ch^{ie}_{221}$ $^{410}_{163}Ch^{ie}_{247}$

Supplement 31: Other formulas of α_1 with Beeler formula for $\pi/2$

$$\text{Beeler formula for }\pi/2:\ \frac{\pi}{2}=1+\sum_{n=1}^{\infty}\frac{n!}{(2n+1)!!}=1+\frac{1}{3}+\frac{1\cdot 2}{3\cdot 5}+\frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}+\cdots$$

$$\left(\frac{\pi}{2}\right)_{Beeler-k}=1+\sum_{n=0}^{k}\frac{n!}{(2n+1)!!}=1+\frac{1}{3}+\frac{1\cdot 2}{3\cdot 5}+\frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}+\cdots+\frac{k!}{(2k+1)!!}$$

$$\alpha_{1-207-Beeler}=\frac{2\cdot 9\cdot 59+1}{4\cdot 9\cdot 23\cdot\left(\frac{\pi}{2}\right)_{Beeler-17}}\frac{1}{112+\cfrac{1}{2\cdot 97\cdot 167+\cfrac{4}{37}\text{ or }\cfrac{7}{5\cdot 13}}}$$

$$=\frac{2\cdot 9\cdot 59+1}{4\cdot 9\cdot 23\cdot\left(\frac{\pi}{2}\right)_{Beeler-17}}\frac{1}{112+\cfrac{1}{179\cdot 181-\cfrac{2\cdot 29}{5\cdot 13}}}=137.035999037435$$

$^{56,58}_{26}Fe_{30,32}$ $^{59}_{27}Cr_{32}$ $^{63,65}_{29}Cu_{34,36}$ $^{105}_{46}\mathrm{Pd}_{59}$ $^{136,137,138}_{59}Ba_{80,81,82}$ $^{141}_{59}\mathrm{Pr}_{82}$ $^{159}_{65}Ta_{94}$ $^{179}_{72}Hf_{107}$

$^{180,181}_{73}Ta_{107,108}$ $^{222}_{86}Rn^{*}_{136}$ $^{223}_{87}Fr^{*}_{136}$ $^{226}_{88}Ra^{*}_{138}$ $^{227}_{89}Ac^{*}_{138}$ $^{298}_{119}Ch^{ie}_{179}$ $^{302}_{121}Ch^{ie}_{181}$ $^{334-338}_{136-138}Fy^{ie}_{208-210}$

Supplement 32: Other formulas of α_2

$$\alpha_{2-522}=\frac{(18\cdot 223-1)\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{\left(\dfrac{149\cdot 227}{18\cdot(40\cdot 47-1)}\right)^{5\cdot 83\cdot 163}}}{18\cdot 29}\,\frac{1}{112-\delta_2}$$

$$=1/137.035999111818$$

$$\delta_2=\frac{27\cdot 13\cdot 47}{25\cdot 10^{11}}\ \text{or}\ \frac{3\cdot 53\cdot 83}{2\cdot 10^{12}}\ \text{or}\ \frac{2\cdot 73\cdot 113}{25\cdot 10^{11}}$$

${}^{27}_{13}Al_{14}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{47}_{22}Ti_{25}$ ${}^{53}_{24}Cr_{29}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{83}_{36}Kr_{47}$ ${}^{89}_{39}Y_{50}$ ${}^{94}_{40}Zr_{54}$ ${}^{94}_{42}Mo_{52}$

${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{117}_{50}Sn_{67}$ ${}^{125}_{52}Te_{73}$ ${}^{127}_{53}I_{74}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{146}_{60}Nd_{86}$ ${}^{149}_{62}Sm_{87}$

${}^{159}_{65}Dy_{94}$ ${}^{106,181}_{73}Ta_{107,108}$ ${}^{163}_{66}Dy_{97}$ ${}^{174}_{70}Yb_{104}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{195}_{78}Pt_{117}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$

${}^{226}_{88}Ac^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{5\cdot 47,238}_{92}U^{*}_{143,146}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{289}_{115}Mc^{ie}_{174}$

${}^{314}_{126}Ch^{ie}_{188}$ ${}^{4\cdot 89}_{141}Ch^{ie}_{215}$ ${}^{368}_{145}Ch^{ie}_{223}$ ${}^{370}_{146}Ch^{ie}_{224}$ ${}^{374}_{148}Ch^{ie}_{226}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$

$$\alpha_{2-522-Wallis}=\frac{(18\cdot 223-1)\cdot\left(2\,\frac{2}{3}\,\frac{4}{3}\,\frac{4}{5}\,\frac{6}{5}\cdots\frac{101446}{101447}\,\dfrac{8\cdot 9\cdot(30\cdot 47-1)}{2\cdot(2\cdot 7\cdot(24\cdot 151-1)+1)+1}\right)}{18\cdot 29}$$

$$\frac{1}{112-\dfrac{4\cdot(12\cdot(8\cdot 9\cdot 7-1)+1)}{25\cdot 10^{11}}}=1/137.035999111818$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{66}_{30}Zn_{36}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{151,153}_{63}Eu_{88,89}$ ${}^{247}_{96}Cm^{*}_{151}$ ${}^{380}_{151}Ch^{ie}_{229}$

$$\alpha_{2-522-GL}=$$

$$\frac{(18\cdot 223-1)\cdot\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\dfrac{1}{2\cdot 2\cdot 49\cdot(14\cdot 47+1)+1}\right)}{18\cdot 29}\,\frac{1}{112-\dfrac{37\cdot(2\cdot 3\cdot 47-1)}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{83}_{36}Kr_{47}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{87}_{38}Sr_{49}$ ${}^{113,115}_{49}In_{64,66}$

$$\alpha_{2-522-NC}=\frac{(18\cdot 223-1)\cdot(2\pi)_{NC-25}}{18\cdot 29}\,\frac{1}{112-\dfrac{1}{9\cdot(2\cdot 7\cdot 173+1)-\dfrac{5\cdot 7}{3\cdot 31}}}$$

$$=\frac{(18\cdot 223-1)\cdot(2\pi)_{NC-25}}{18\cdot 29}\,\frac{1}{112-\dfrac{1}{9\cdot(8\cdot 3\cdot 101-1)-\dfrac{2\cdot 19}{101}}}$$

$$=1/137.035999111818$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{31}_{15}P_{16}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{69,71}_{31}Ga_{38,40}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{86,87,88}_{38}Sr_{48,49,50}$ ${}^{101}_{44}Ru_{57}$ ${}^{171,173,174}_{70}Yb_{101,103,104}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{289}_{115}Mc^{ie}_{174}$ ${}^{2\cdot 173}_{137}Nh^{ie}_{209}$

$$\alpha_{2-791}=\frac{3\cdot(4\cdot3\cdot13^2-1)\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot7\cdot29\cdot89}{23\cdot(10\cdot157+1)})^{3\cdot13\cdot17\cdot109}}}{7\cdot113}\frac{1}{112-\frac{7\cdot(4\cdot100+1)}{4\cdot10^{11}}}$$

$=1/137.035999111818$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$

${}^{136-138}_{56}Ba_{80-82}$ ${}^{138}_{58}Ce_{82}$ ${}^{157}_{64}Gd_{93}$ ${}^{169}_{69}Tm_{100}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{344-348}_{136-138}Fy^{ie}_{208-210}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{426}_{169}Ch^{ie}_{257}$

$$\alpha_{2-791-Wallis}=\frac{3\cdot(4\cdot3\cdot13^2-1)\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{108372}{108373}\frac{2\cdot7\cdot(2\cdot49\cdot79-1)}{2\cdot6\cdot11\cdot(6\cdot137-1)+1})}{7\cdot113}\frac{1}{112-\frac{81\cdot97}{2\cdot10^{12}}}$$

$=1/137.035999111818$

${}^{79,81}_{35}Br_{44,46}$ ${}^{97}_{42}Mo_{55}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{135,137}_{56}Ba_{79,81}$ ${}^{163}_{66}Dy_{97}$ ${}^{197}_{79}Au_{118}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{346}_{137}Fy^{ie}_{209}$

$$\alpha_{2-791-GL}=\frac{3\cdot(4\cdot3\cdot13^2-1)\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot128\cdot49\cdot11+1})}{7\cdot113}\frac{1}{112-\frac{17\cdot23}{8\cdot10^{10}}}$$

$=1/137.035999111818$

${}^{49}_{22}Ti_{27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{100}_{44}Ru_{56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{169}_{69}Tm_{100}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\alpha_{2-791-NC}=\frac{3\cdot(4\cdot3\cdot13^2-1)\cdot(2\pi)_{NC-25}}{7\cdot113}\frac{1}{112-\frac{1}{2\cdot5\cdot49\cdot193+\frac{3\cdot7}{2\cdot11}}}=1/137.035999111818$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{55}_{25}Mn_{30}$ ${}^{75}_{33}As_{42}$ ${}^{98}_{42}Mo_{56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{165}_{67}Ho_{98}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{320}_{127}Ch^{ie}_{193}$

$$\alpha_{2-1092}=\frac{5\cdot23\cdot73\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{4\cdot23\cdot(2\cdot3\cdot5\cdot17-1)}{3^2\cdot11^2\cdot43})^{5\cdot(42\cdot223-1)}}}{4\cdot3\cdot7\cdot13}\frac{1}{112-\frac{2\cdot29\cdot103}{5\cdot10^{11}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{103}_{45}Rh_{58}$ ${}^{125}_{52}Te_{73}$ ${}^{145}_{60}Nd_{85}$

${}^{136,138,140,142}_{58}Cs_{78,80,82,84}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{173}_{70}Yb_{103}$ ${}^{180,181}_{70}Ta_{107,108}$ ${}^{209}_{84}Bi^{*}_{125}$ ${}^{210}_{85}Po^{*}_{125}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{9\cdot29}_{103}Lr^{*}_{159}$ ${}^{289}_{115}Mc^{ie}_{174}$ ${}^{368}_{145}Ch^{ie}_{223}$

$$\alpha_{2-1092-Wallis}=\frac{5\cdot23\cdot73\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{140466}{140467}\frac{140668}{2\cdot41\cdot(16\cdot107+1)+1})}{4\cdot3\cdot7\cdot13}\frac{1}{112-\frac{2\cdot7\cdot(2\cdot125+1)}{25\cdot10^{11}}}$$

$=1/137.035999111818$

${}^{73}_{32}Ge_{41}$ ${}^{82,84}_{36}Kr_{46,48}$ ${}^{93}_{41}Nb_{52}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{125}_{52}Te_{73}$ ${}^{138}_{56}Ba_{82}$ ${}^{207,208}_{82}Pt_{125,126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{24\cdot13}_{125}Ch^{ie}_{187}$

$$\alpha_{2-1092-GL}=\frac{5\cdot23\cdot73\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2^4\cdot3^5\cdot23+1})}{4\cdot3\cdot7\cdot13}\frac{1}{112-\frac{8\cdot107+1}{25\cdot10^{11}}\text{ or }\frac{61\cdot(6\cdot47-1)}{5\cdot10^{12}}}$$

$=1/137.035999111818$

${}^{106,108}_{46}Pd_{60,62}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{125}_{52}Te_{73}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{169}_{69}Tm_{100}$ ${}^{180,181}_{73}Ta_{107,108}$

$$\alpha_{2-1092-NC}=\frac{5\cdot 23\cdot 73\cdot (2\pi)_{NC-29}}{4\cdot 3\cdot 7\cdot 13}\frac{1}{112-\cfrac{1}{3\cdot 11\cdot (6\cdot 73+1)+\frac{17}{6\cdot 31}\text{ or }\frac{18}{197}}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{24,25,16}_{12}Mg_{12,13,14}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{36,38,40}_{18}Ar_{18,20,22}$ $^{50,51}_{23}V_{27,28}$ $^{54,56,57,58}_{26}Fe_{28,30,31.32}$

$^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ $^{63,65}_{29}Cu_{34,36}$ $^{68,71}_{31}Ga_{38,40}$ $^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ $^{75}_{33}As_{42}$ $^{113,115}_{49}In_{64,66}$ $^{125}_{52}Te_{73}$

$^{144,145,146}_{60}Nd_{84,85,86}$ $^{163}_{66}Dy_{97}$ $^{180,181}_{73}Ta_{107,108}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{197}_{79}Pt_{118}$ $^{209}_{84}Po^{*}_{125}$ $^{289}_{115}Mc^{ie}_{174}$ $^{310}_{124}Ch^{ie}_{186}$ $^{326}_{129}Ch^{ie}_{197}$

22. Discussion and Conclusion

Regarding the fine-structure constant, Richard Feynman said: “is it related to π or perhaps to the base of natural logarithms?”[4] Our answer is that it relate to 2π-e, 2π, π/2 and π/4 formulas. He also deduced that the maximum element should be the 137th element Fynmanium (Fy) based on the analyses of the electron line velocity of his ideal hydrogen-like atoms. Our answer is that the natural end of the elements is the 112th element Copernicium (Cn*) , but the elements could have some ideal extensions, and above all, the fine-structure constant does relate to elements.

So, based on the analyses of ideal and real natural maximum element, Chen’s Chirality and Poetry Model of Atomic Nuclei[7] and 2π-e formula[6,7,8], we deduced two series of Chen’s formulas of the fine-structure constant which gave two values α_1=1/137.035999037435 and α_2=1/137.035999111818. The factors in the formulas are much coincident to nucleon numbers of some nuclides, this means the formulas should be correct (too many coincidences mean too few possibilities to be wrong, or too many coincidences imply science). And we indicate the reason of $\alpha\approx 1/137.036$ is that it’s almost the equal ratio factor between 112 and 168 (more precisely 168-1/3) which are the key stable numbers (magic numbers) in Chen’s Chirality and Poetry Model of Atomic Nuclei[7].

With Chen’s formulas of the fine-structure constant, we predicted the nucleon numbers of all 119th to 170th ideal extended elements; we theoretically or mathematically calculated the speed of light in atomic units, i.e., $c_{au}=1/\alpha_c= 1/(\alpha_1\alpha_2)^{1/2}$ =137.035999074627; we deduced a concise Schrödinger-Chen equation of hydrogen atom which included α_1/α_2 factor; in analogy to α and its formulas, α_p (the second fine-structure constant) and its formulas were hypothesized, and the proton charge radius r_p was supposed to be 0.833027203 fm; in the end we discovered that the approximate rational numbers of 2π marvelously and directly related to nuclides.

Based on these, a mathematic shell model of elements was established and a picture of elements and ideal extended elements was depicted.

In their relations to nuclides, 2π formulas can only be certain approximate rational numbers and 2π-e formulas in Chen's formulas of the fine-structure constant can only take certain k values. So we also believe the two values of the fine-structure constant should be rational numbers with definite digits rather than irrational numbers with infinite digits, and actually the fine-structure constant is transformed to nucleon numbers of 136,137 and 138 in the world of nuclides.

In a recent paper[11], physicist Nicolas Gisin commented that in 1920s there once was a debate between mathematicians David Hilbert and Luitzen Egbertus Jan Brouwer. Hilbert was promoting formalized mathematics, in which every real number with its infinite series of digits is a completed individual object. On the other side the Luitzen Egbertus Jan Brouwer was defending the view that each point on the line should be represented as a never-ending process that develops in time, a view known as intuitionistic mathematics. Hilbert and his supporters clearly won the debate. In consequence, formalized mathematics has been adopted as the language of physics. In the end of his paper, Nicolas Gisin said: "Physics can be as successful if built on intuitionistic mathematics, even if this breaks its marriage to determinism. Contrary to usual expectations, I bet that the next physical theory will not be even more abstract than quantum field theory, but might well be closer to human experience."

In this paper we adopted mathematical language like intuitionistic mathematics, but we go ahead even more. The formulas of 2π, 2π-e and the fine-structure constant consist of integer factors and relate to nucleon numbers of nuclides, and hence correlate with each others. So in this paper we may use super-intuitionistic mathematics or decoding methodology with features of multi-correlations of integer factors or rational numbers which relate to nucleon numbers of nuclides, and it seems it is the real language in the world of nuclides. As we know an atomic nucleus is a N-body system and chaos should be its real state, so it seems N-body chaos returns to integers. In overall, Leopold Kronecker's famous saying "God made the integers, all else is the work of man" should be correct in the world of nuclides or even in other fields of the real world. It seems an irrational number can only be a rational number to play roles in the real world.

“God is a pure mathematician!” declared British astronomer Sir James Jeans(1877-1946). The physical Universe does seem to be 80rganized around elegant mathematical relationships[3]. The fine-structure constant may be the most important number in physics. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. And we have successfully given reasonable and precise formulas of it. In some sense, we explain the bridge between mathematics and physics, or we may realize the unification of mathematics and physics. It seems we prove the saying “God is a pure mathematician”. At least, it seems that good mathematics means good physics, and some pure mathematical numbers do have scientific meanings.

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Acknowledgements

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Appendix I: Research History

Items	Page	Discover/Create	Revise/Supplement
2π-e formula	3	2013/4-12	
Formulas related to 2π-e formula	4	2013/4-12	
Preliminary applications of 2π-e formula and its related formulas	6	2013/4-12	
Chen's Periodic Table of Elements and Natural Group Theory	6	2014-2017/12	
Chirality and Poetry Model of Atomic Nuclei	6	2017/12-2018/3	
Chen's theory of the fine-structure constant	6	2018/4-6	2018/7-2020/1
Original Inspiration for Formulas of the Fine-structure Constant	6	2018/4/12	
Logical deduction of Chen's formulas of the fine-structure constant	6 7	2018/4/12-24	
α_1 ($\alpha_{1\text{-}7}$)	6 7	2018/4/12	2018/4/20 (+1/75^2)
α_2 ($\alpha_{2\text{-}13}$)	7	2018/4/24	2018/9/18-20 (280→278 *et al.*)
Calculation tables and diagrams of α	8	2018/4/12-24	2018/9/18-20
$\alpha_{1\text{-}(3/2)}$	9	2019/4/25	
$\alpha_{2\text{-}(3/2)}$	9	2019/4/25	
α_c^2	10	2018/6/8-9, 9/18-19; 2019/4/17-19	
$1/\alpha_c^2$	10	2019/12/14	
112/137≈137/168 *et al.*	11	2018/4-6	
$_{136,137,138}\mathrm{Fy}_{208,209,210}$	11	2019/12-2020/1	
$_{125,126}\mathrm{Ch}$, $_{144\text{-}149}\mathrm{Ch}$, $_{153,154}\mathrm{Ch}$, $_{155,156}\mathrm{Ch}$, $_{157}\mathrm{Ch}$, $_{163}\mathrm{Ch}$, $_{164\text{-}168\text{-}169}\mathrm{Ch}$	11	2019/12-2020/1	
c_{au} formulas	12	2019/12/16	2020/1/5-8, 2/24, 3/29
The special 29 and 75 factors	12-15	2019/4/22-24	
α_1/α_2 in Schrödinger Equation of Hydrogen Atom	16	2018/4-6	2019/12/13
α_1/α_2	16	2019/8/28-29	
The two kinds of general formulas of the fine-structure constant	17	2019/6/27	2019/7/2-3
Parameters and results of $\alpha_{1\text{-}m'}$	18	2019/7/2	
$\alpha_{1\text{-}1}$	19	2019/7/2	
$\alpha_{1\text{-}2}$	19	2019/6/26	
$\alpha_{1\text{-}3}$	19	2019/5/26	2019/6/26
$\alpha_{1\text{-}4}$	19	2019/6/27	
$\alpha_{1\text{-}5}$	19	2019/6/27	

$\alpha_{1\text{-}6}$	19	2019/6/27	
$\alpha_{1\text{-}7}$ (α_1)	19	2018/4/12	2018/4/20 (+1/75^2)
$\alpha_{1\text{-}9}$	20	2019/6/28	
$\alpha_{1\text{-}11}$	20	2019/6/29	
$\alpha_{1\text{-}13}$	20	2019/6/29	
$\alpha_{1\text{-}16}$	20	2019/6/29-30	
$\alpha_{1\text{-}17}$	20	2019/6/30	2019/10/29
$\alpha_{1\text{-}19}$	20	2019/7/1	
$\alpha_{1\text{-}20}$	20	2018/4-6	2019/6/26
$\alpha_{1\text{-}22}$	21	2019/5/25	2019/12/12
$\alpha_{1\text{-}23}$	21	2019/7/4	
$\alpha_{1\text{-}25}$	21	2019/7/4	
$\alpha_{1\text{-}27}$	21	2019/5/25-26	
$\alpha_{1\text{-}29}$	21	2019/5/25	
$\alpha_{1\text{-}31}$	21	2019/7/4	
$\alpha_{1\text{-}32}$	21	2019/7/5	
$\alpha_{1\text{-}33}$	22	2019/7/5	
$\alpha_{1\text{-}34}$	22	2019/5/24	
$\alpha_{1\text{-}36}$	22	2019/5/24	
$\alpha_{1\text{-}43}$	22	2019/12/15	
$\alpha_{1\text{-}50}$	22	2018/4-6	2019/6/26
$\alpha_{1\text{-}59}$	22	2019/5/25	
$\alpha_{1\text{-}81}$	22	2019/5/25-26	
$\alpha_{1\text{-}96}$	23	2019/5/25-26	2019/12/29
$\alpha_{1\text{-}103}$	23	2019/12/15	
$\alpha_{1\text{-}133}$	23	2019/5/26	2019/12/29
$\alpha_{1\text{-}140}$	23	2019/5/26	
$\alpha_{1\text{-}155}$	23	2019/12/15	
$\alpha_{1\text{-}170}$	23	2019/7/2	2019/7/9
Parameters and results of $\alpha_{2\text{-}m'}$	24	2019/7/3	
$\alpha_{2\text{-}1}$	25	2018/4-6	2019/5/26-27
$\alpha_{2\text{-}4}$	25	2019/5/28	
$\alpha_{2\text{-}5}$	25	2019/6/22	
$\alpha_{2\text{-}6}$	25	2019/6/23	
$\alpha_{2\text{-}7}$	25	2019/5/24	2020/3/5
$\alpha_{2\text{-}9}$	25	2019/6/21	
$\alpha_{2\text{-}10}$	26	2019/5/28	
$\alpha_{2\text{-}11}$	26	2019/6/22	
$\alpha_{2\text{-}13}$ (α_2)	26	2018/4/24	2018/9/18-20 (280→278 *et al.*)
$\alpha_{2\text{-}15}$	26	2019/6/19	
$\alpha_{2\text{-}17}$	26	2019/6/24	

$\alpha_{2\text{-}18}$	26	2019/6/24	
$\alpha_{2\text{-}19}$	27	2019/6/24	
$\alpha_{2\text{-}23}$	27	2019/6/23	
$\alpha_{2\text{-}24}$	27	2019/6/23	
$\alpha_{2\text{-}25}$	27	2019/6/24	
$\alpha_{2\text{-}27}$	27	2019/6/21	
$\alpha_{2\text{-}29}$	27	2019/6/20	
$\alpha_{2\text{-}31}$	28	2019/7/6	
$\alpha_{2\text{-}32}$	28	2019/7/6	
$\alpha_{2\text{-}33}$	28	2019/7/6	
$\alpha_{2\text{-}36}$	28	2019/6/25	
$\alpha_{2\text{-}37}$	28	2019/7/7	
$\alpha_{2\text{-}38}$	28	2019/7/7	
$\alpha_{2\text{-}125}$	29	2019/5/25	2019/12/20
$\alpha_{2\text{-}253}$	29	2019/7/3	2019/7/9
$\alpha_{2\text{-}269}$	29	2019/7/7	2019/12/19 2020/3/18-19
a_0/r_e, r_e; a_0/r_p, r_p	30	2019/12/19-23	
$\alpha_{p/1}$	30	2020/1/2	
$\alpha_{p/2}$	30 31	2020/1/2-3	
${}_{164}Ch_{252}$	31	2020/1/3	
Direct relationships between 2π and nuclides	31	2020/1/8-10	
Some correlations of formulas of 2π and α	32 33	2020/1/11-13	
Chen's Mathematic Shell Model of Nuclides	33	2020/1/12-13	
${}_{130,131}Ch_{200,201}$	20 22 26 27	2020/1/26-28	
${}_{119,120,121}Ch_{179,180,181}$	21 28 31	2020/1/28-29	2020/2/5 (add 121)
${}_{128,129}Ch_{198,197/199}$	23 26 31	2020/1/28-29	2020/1/31
${}_{139}Ch_{209}$	21 26 28	2020/1/29	
${}_{132,133}Ch_{202,203}$	21 23	2020/1/31	
${}_{169}Ch_{257}$	11 22 27	2020/1/29-30	
${}_{157}Ch_{243}$	11 20 21 22 31	2019/1/8	
${}_{158}Ch_{243}$	11 20 22 25 30 31	2019/1/31	
${}_{169}Ch_{257}$	11 22 26 27	2020/1/29-30	
${}_{134,135}Ch_{206,205}$	20	2020/1/31	
${}_{127}Ch_{191,193}$	27	2020/1/31	2020/2/1 (add 191)
${}_{150,151,152}Fy_{228,229,230}$	19 28	2020/2/2	
${}_{170}Ch_{250}$	23 26	2020/2/3	
Chen's Picture of Elements and Ideal Extended Elements	41	2018/1-3 2020/2/2-5	2020/2/12, 16, 17, 19, 22-24

Supplement 1	37	2020/2/11-12	
$2\pi=62831853/10^{7}$	37	2020/2/11-12	
Supplement 2	37	2020/2/12	
${}_{124}Ch_{186}$	37	2020/2/12	
Supplement 3	37	2020/2/16-17	
${}_{143}Ch_{220,221}$	36	2020/2/17	
Supplement 4	38	2020/2/18-21	
${}_{122}Ch_{182}$	22	2020/2/19	
Supplement 5	38	2020/2/21	
${}_{127}Ch_{192}$ ${}_{167}Ch_{251}$	38	2020/2/21	
Supplement 6	39	2020/2/21-22, 24-25	
${}_{140,\ 141}Ch_{212,215}$	39	2020/2/22	
${}_{123}Ch_{183,185}$	36	2020/2/23	
${}_{159/161,160}Ch_{245,246}$	36	2020/2/23	
${}_{165}Ch_{255}$	36	2020/2/23	
${}_{162}Ch_{246}$	36	2020/2023	
Supplement 7	40	2020/2/25-26	
Supplement 8	41	2020/2/26	
Supplement 9	41-44	2020/2/27-3/3	2020/4/24
Supplement 10	44	2020/3/7-8	2020/3/12-14
Supplement 11	45-48	2020/3/21-27	
$\alpha_{1\text{-}7\text{-Wallis}}$	45	2020/3/21	
$\alpha_{1\text{-}22\text{-Wallis}}$	45	2020/3/25	
$\alpha_{1\text{-}29\text{-Wallis}}$	45	2020/3/25-26	
$\alpha_{1\text{-}36\text{-Wallis}}$	45	2020/3/26	
$\alpha_{1\text{-}43\text{-Wallis}}$	46	2020/3/26	
$\alpha_{1\text{-}50\text{-Wallis}}$	46	2020/3/22	
$\alpha_{1\text{-}59\text{-Wallis}}$	46	2020/3/26	
$\alpha_{1\text{-}81\text{-Wallis}}$	46	2020/3/23	
$\alpha_{1\text{-}96\text{-Wallis}}$	46	2020/3/26	
$\alpha_{1\text{-}103\text{-Wallis}}$	46	2020/3/26	
$\alpha_{1\text{-}133\text{-Wallis}}$	46	2020/3/25	
$\alpha_{1\text{-}140\text{-Wallis}}$	46	2020/3/26	
$\alpha_{1\text{-}155\text{-Wallis}}$	47	2020/3/23	
$\alpha_{1\text{-}170\text{-Wallis}}$	47	2020/3/24	
$\alpha_{2\text{-}10\text{-Wallis}}$	47	2020/3/26-27	
$\alpha_{2\text{-}13\text{-Wallis}}$	47	2020/3/22	
$\alpha_{2\text{-}23\text{-Wallis}}$	47	2020/3/26	
$\alpha_{2\text{-}29\text{-Wallis}}$	47	2020/3/23	
$\alpha_{2\text{-}33\text{-Wallis}}$	48	2020/3/27	
$\alpha_{2\text{-}36\text{-Wallis}}$	48	2020/3/25	

$\alpha_{2\text{-}125\text{-Wallis}}$	48	2020/3/23
$\alpha_{2\text{-}253\text{-Wallis}}$	48	2020/3/25
$\alpha_{2\text{-}269\text{-Wallis}}$	48	2020/3/25
Supplement 12	48-50	2020/3/22-30
$C_{au}=(\alpha_{1\text{-}7}\alpha_{2\text{-}13})^{-1/2}$	48	2020/3/27
$C_{au}=(\alpha_{1\text{-}7\text{-Wallis}}\alpha_{2\text{-}13\text{-Wallis}})^{-1/2}$	48	2020/3/22
$C_{au}=(\alpha_{1\text{-}29}\alpha_{2\text{-}29})^{-1/2}$	49	2020/3/27
$C_{au}=(\alpha_{1\text{-}29\text{-Wallis}}\alpha_{2\text{-}29\text{-Wallis}})^{-1/2}$	49	2020/3/27
$C_{au}=(\alpha_{1\text{-}36}\alpha_{2\text{-}36})^{-1/2}$	49	2020/3/27-28
$C_{au}=(\alpha_{1\text{-}36\text{-Wallis}}\alpha_{2\text{-}36\text{-Wallis}})^{-1/2}$	49	2020/3/27
$C_{au}=(\alpha_{1\text{-}81}\alpha_{2\text{-}125})^{-1/2}$	49	2020/3/23
$C_{au}=(\alpha_{1\text{-}81\text{-Wallis}}\alpha_{2\text{-}125\text{-Wallis}})^{-1/2}$	49	2020/3/23
$C_{au}=(\alpha_{1\text{-}70}\alpha_{2\text{-}253})^{-1/2}$	49	2020/3/28
$C_{au}=(\alpha_{1\text{-}170\text{-Wallis}}\alpha_{2\text{-}253\text{-Wallis}})^{-1/2}$	49	2020/3/25
$C_{au}=(\alpha_{1\text{-}170}\alpha_{2\text{-}269})^{-1/2}$	50	2020/3/29-30
$C_{au}=(\alpha_{1\text{-}170\text{-Wallis}}\alpha_{2\text{-}269\text{-Wallis}})^{-1/2}$	50	2020/3/29-30
$C_{au}=(\alpha_{1\text{-}133}\alpha_{2\text{-}253})^{-1/2}$	50	2020/3/30
$C_{au}=(\alpha_{1\text{-}133\text{-Wallis}}\alpha_{2\text{-}253\text{-Wallis}})^{-1/2}$	50	2020/3/30
Supplement 13	50-54	2020/3/31-4/4
$\alpha_{1\text{-}7\text{- GL}}$	51	2020/4/1
$\alpha_{1\text{-}22\text{- GL}}$	51	2020/4/1
$\alpha_{1\text{-}29\text{- GL}}$	51	2020/4/1
$\alpha_{1\text{-}36\text{-GL}}$	51	2020/4/1
$\alpha_{1\text{-}43\text{- GL}}$	51	2020/4/2
$\alpha_{1\text{-}50\text{- GL}}$	51	2020/4/2
$\alpha_{1\text{-}59\text{- GL}}$	52	2020/4/2
$\alpha_{1\text{-}81\text{- GL}}$	52	2020/4/2
$\alpha_{1\text{-}96\text{- GL}}$	52	2020/4/2
$\alpha_{1\text{-}103\text{-GL}}$	52	2020/4/2
$\alpha_{1\text{-}133\text{- GL}}$	52	2020/4/2
$\alpha_{1\text{-}140\text{- GL}}$	52	2020/4/2
$\alpha_{1\text{-}155\text{- GL}}$	52	2020/4/2
$\alpha_{1\text{-}170\text{- GL}}$	53	2020/4/1
$\alpha_{2\text{-}10\text{- GL}}$	53	2020/4/3
$\alpha_{2\text{-}13\text{- GL}}$	53	2020/4/1
$\alpha_{2\text{-}23\text{- GL}}$	53	2020/4/3
$\alpha_{2\text{-}29\text{- GL}}$	53	2020/4/1
$\alpha_{2\text{-}33\text{- GL}}$	53	2020/4/3-4
$\alpha_{2\text{-}36\text{- GL}}$	54	2020/4/3-4
$\alpha_{2\text{-}125\text{- GL}}$	54	2020/4/2
$\alpha_{2\text{-}253\text{- GL}}$	54	2020/4/3

$\alpha_{2\text{-}269\text{- GL}}$	54	2020/4/1
Supplement 14	54-55	2020/4/3-
$C_{au}=(\alpha_{1\text{-}7\text{-GL}}\alpha_{2\text{-}13\text{-GL}})^{-1/2}$	54	2020/4/3
$C_{au}=(\alpha_{1\text{-}29\text{-GL}}\alpha_{2\text{-}29\text{-GL}})^{-1/2}$	54	2020/4/4
$C_{au}=(\alpha_{1\text{-}36\text{-GL}}\alpha_{2\text{-}36\text{-GL}})^{-1/2}$	55	2020/4/4
$C_{au}=(\alpha_{1\text{-}81\text{-GL}}\alpha_{2\text{-}125\text{-GL}})^{-1/2}$	55	2020/4/3
$C_{au}=(\alpha_{1\text{-}170\text{-GL}}\alpha_{2\text{-}253\text{-GL}})^{-1/2}$	55	2020/4/3
$C_{au}=(\alpha_{1\text{-}170\text{-GL}}\alpha_{2\text{-}269\text{-GL}})^{-1/2}$	55	2020/4/3
Supplement 15	55-56	2020/4/3-4
Supplement 16	55	2020/4/2-7
$\alpha_{2\text{-}45}$	56	2020/4/5
$\alpha_{2\text{-}45\text{- Wallis}}$	56	2020/4/8
$\alpha_{2\text{-}45\text{- GL}}$	56	2020/4/7-8
$\alpha_{2\text{-}173}$	57	2020/4/6
$\alpha_{2\text{-}173\text{- Wallis}}$	57	2020/4/7
$\alpha_{2\text{-}173\text{- GL}}$	57	2020/4/7
$\alpha_{2\text{-}49}$	57	2020/4/8
$\alpha_{2\text{-}49\text{-Wallis}}$	57	2020/4/8
$\alpha_{2\text{-}49\text{-GL}}$	57	2020/4/8
Supplement 17	58	2020/4/8-9
$C_{au}=(\alpha_{1\text{-}81}\alpha_{2\text{-}49})^{-1/2}$	58	2020/4/8
$C_{au}=(\alpha_{1\text{-}81\text{-Wllis}}\alpha_{2\text{-}49\text{-Wallis}})^{-1/2}$	58	2020/4/9
$C_{au}=(\alpha_{1\text{-}81\text{-GL}}\alpha_{2\text{-}49\text{-GL}})^{-1/2}$	58	2020/4/9
$\alpha_{2\text{-}42}$	58	2020/4/9
$\alpha_{2\text{-}42\text{-Wallis}}$	58	2020/4/11-12
$\alpha_{2\text{-}42\text{-GL}}$	58	2020/4/12
$\alpha_{2\text{-}61}$	58	2020/4/9
$\alpha_{2\text{-}61\text{-Wallic}}$	59	2020/4/11
$\alpha_{2\text{-}61\text{-GL}}$	59	2020/4/11
$\alpha_{2\text{-}77}$	59	2020/4/9
$\alpha_{2\text{-}77\text{-Wallic}}$	59	2020/4/12
$\alpha_{2\text{-}77\text{-GL}}$	59	2020/4/12
$\alpha_{2\text{-}93}$	60	2020/4/10
$\alpha_{2\text{-}93\text{-Wallic}}$	60	2020/4/12
$\alpha_{2\text{-}93\text{-GL}}$	60	2020/4/12
$\alpha_{2\text{-}109}$	60	2020/4/10
$\alpha_{2\text{-}109\text{-Wallis}}$	60	2020/4/13
$\alpha_{2\text{-}109\text{-GL}}$	60	2020/4/12-13
$\alpha_{2\text{-}141}$	60	2020/4/10
$\alpha_{2\text{-}141\text{-Wallis}}$	60	2020/4/13
$\alpha_{2\text{-}141\text{-GL}}$	61	2020/4/13

$\alpha_{2\text{-}189}$	61	2020/4/10
$\alpha_{2\text{-}189\text{-Wallis}}$	61	2020/4/13
$\alpha_{2\text{-}189\text{-GL}}$	61	2020/4/13
$\alpha_{2\text{-}205}$	61	2020/4/10
$\alpha_{2\text{-}205\text{-Walis}}$	61	2020/4/13-14
$\alpha_{2\text{-}205\text{-GL}}$	61	2020/4/13
$\alpha_{2\text{-}221}$	62	2020/4/10
$\alpha_{2\text{-}221\text{-Wallis}}$	62	2020/4/14
$\alpha_{2\text{-}221\text{-GL}}$	62	2020/4/14
$\alpha_{2\text{-}234}$	62	2020/4/10
$\alpha_{2\text{-}234\text{-Wallic}}$	62	2020/4/12
$\alpha_{2\text{-}234\text{-GL}}$	62	2020/4/12
$\alpha_{2\text{-}237}$	62	2020/4/10-11
$\alpha_{2\text{-}237\text{-Wallic}}$	62	2020/4/12
$\alpha_{2\text{-}237\text{-GL}}$	63	2020/4/12
Supplement 19	63	2020/4/29-30
Supplement 20	63	2020/5/1
Supplement 21	63-64	2020/5/4
$\alpha_{1\text{-}13\text{-Wallis}}$	63	2020/5/4
$\alpha_{1\text{-}13\text{-GL}}$	64	2020/5/4
$\alpha_{2\text{-}7}$ (revised)	64	2020/5/4
$\alpha_{2\text{-}7\text{-Wallis}}$	64	2020/5/4
$\alpha_{2\text{-}7\text{-GL}}$	64	2020/5/4
Supplement 22	65	2020/5/4-5
$\alpha_{Z\text{-}2\text{-}7}$	65	2020/5/4-5
$\alpha_{Z\text{-}2\text{-}7\text{-Wallis}}$	65	2020/5/4-5
$\alpha_{Z\text{-}2\text{-}7\text{-GL}}$	65	2020/5/4-5
Supplement 23	65	2020/5/22
Supplement 24	66	2020/5/29-5/31
Supplement 25	67	2020/6/3-4
Supplement 26	67-68	2020/6/4-5
Supplement 27	68-70	2020/6/2-5
Supplement 28	70-71	2020/6/7-8
Supplement 29	71-73	2020/6/10-11
Supplement 30	73-75	2020/6/10-12
Supplement 31	75	2020/6/12
Supplement 32	76-78	2020/6/12-14
Preparing this paper	1-88	2019/12/1-2020/6/18

Note: Dates were recorded according to Beijing Time; *ie* means ideal extended elements; GL means Gregory-Leibniz formula.