Hidden relations in the FRW cosmological model.

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Abstract

This article presents the relations occurring in the FRW cosmological model:
the radius of the visible universe is \( \pi \) times the age of the universe,
Hubble's law is expressed by the relation \( \ln (1 + Z) \),
The deceleration parameter \( q = 1 - \frac{\pi}{2} \).

Keywords: Cosmology, Hubble law.

1. Introduction.

The photon is predicted to be massless, and to have zero electric charge and integer spin. The particular form of the electromagnetic interaction specifies that the photon must have spin \( \pm 1 \). These two spin components correspond to the classical concepts of right-handed and left-handed circularly polarized light.

Electromagnetic wave has two polarities, which means that the photons move in three-dimensional manifold.

The Poincaré conjecture (proved by Grigori Perelman) states: every simply connected, closed three-dimensional manifold is homeomorphic to the three-dimensional sphere.

Assuming that the visible universe is simply connected and closed, the photons propagate on the surface of the three-dimensional sphere.

The rest of the article presents the derivation of the photon trajectory in the visible universe.

2. An alternative to the Alan Guth Inflationary universe.

The cosmological principle implies that the spatial part of the FRW metric is a 3D hypersphere (informal approach).

three-dimensional sphere is a set of points \( \{x y z w\} \) satisfying:

\[ x^2 + y^2 + z^2 + w^2 = R \]  (1)
for $R > 0$

$$x^2 + y^2 + z^2 + w^2 = R^2 \quad (2)$$

Assuming that the visible universe is simply connected and closed, the photons propagate on
the surface of the three-dimensional sphere from formula (2).

With the above assumption Hubble law indicates that the sphere is expanding at the rate of
light speed (see chapter 2.1):

$$R = cT \quad (3)$$

where $R$ is the radius of the sphere, and $T$ is the age of the visible universe.

We can neglect the short (compared to the current age of the universe) the initial period when
the universe was opaque and accept that CMB has been issued before 13.8 billion years.

It follows that presented in 4D Euclidean space trajectory of CMB radiation would be
logarithmic spiral from formula (6).

Figure 2.1 shows an embedding diagram of a 3D sphere in 4D Euclidean space. The
directions in the diagram represent the normal two spatial x y dimensions, with dimensions z
and w suppressed.

An observer is sitting at the north pole of the diagram, at $\{0 \ 1 \ 0 \ 0\}$, A is a path traveled by
CMB, B is a path traveled by CMB in comoving coordinates. $R$ is equal to the radius of the
sphere from the formula (3).
As can be seen the path B is approximately equal to the radius of the visible universe in comoving coordinates and can be expressed by the formula:

$$\text{dist} \approx \pi c T$$  \hspace{1cm} (4)

after substituting $T = 13.8$ bln y [1] [2] we get

$$\text{dist} = 43.4 \text{ bln ly}$$

which represents 95% of the value estimated at 45.7 [3]

Presented intuitive model with constant rate of expansion of the universe has a surprisingly correlation with the current estimates of the age and size of the visible universe. (Assumed constant rate of expansion of the universe since the beginning.)
2.1 Hubble law

Figure 2.1.1 Example of distance determination for ULAS J1120+0641 and C3 9.

From the assumption (7) the distance traveled by the photon is equal $\sqrt{2}t$:

$$Z = \sqrt{\frac{2}{2t}} = \frac{T}{t} - 1$$

$$Z = \frac{1}{Ht} - 1$$  \hspace{1cm} (5)

where: $T$ is the age of the visible universe, $t$ is the age of the object emitting photon, $H$ is the Hubble constant.

Photon trajectory is a logarithmic spiral:

$$t = ae^{-be}$$  \hspace{1cm} (6)

assuming the sphere is expanding at the constant rate of light speed

$$b = ctg\left(\frac{\pi}{4}\right) = 1,$$  \hspace{1cm} (7)

$$T = \frac{1}{H}e^0, \quad a = \frac{1}{H}$$

$$\phi = \ln(\frac{1}{Ht})$$, \hspace{1cm} from (5): \hspace{1cm} $$Z + 1 = \frac{1}{Ht}$$

$$\phi = \ln(1 + Z)$$
The distance in comoving coordinates (8) is determined by the intersection point of the circle of radius $t$ with the photon trajectory (6) from figure 2.1.1:

$$\text{dist} = \varphi T = \frac{\varphi}{H}$$

$$\text{dist} = \frac{\ln(1 + Z)}{H}$$  \hspace{1cm} (8)

3. The accelerating expansion of the universe.

The first evidence of accelerating the expansion of the visible universe came from observations of type Ia supernovae.

Observations prefer: $q \approx -0.55$

For the cosmological model described in Chapter 2, the average distance from the observer to the supernova is expressed by the relationship:

$$d(t) = \frac{\pi}{2}ct$$

This means that in the described model the average supernova moves away from the observer $\frac{\pi}{2}$ times faster than in the FRW model with constant expansion of the universe.

So we can assume that the universe with a constant expansion rate described in Chapter 2 seemingly "accelerates" compared to the universe from the FRW model (FRW model with constant expansion of the universe).

That compared to the FRW universe we get “accelerating” expansion of the universe:

$$q = 1 - \frac{\pi}{2} \approx -0.57$$

which corresponds to 95% of the value calculated from supernovae observations.

4. Conclusions

This article presents the relations occurring in the FRW cosmological model: the radius of the visible universe is $\pi$ times the age of the universe, Hubble's law is expressed by the relation $\ln (1 + Z)$, The deceleration parameter $q = 1 - \frac{\pi}{2}$. 


5. References