Nonlinearity, The *Jeśmanowicz Conjecture* And The *Miyazaki (2013) Conjecture*.

Michael C. Nwogugu
Address: Enugu 400007, Enugu State, Nigeria
Emails: mcn2225@gmail.com; mcn2225@aol.com
Skype: mcn1112
Phone: 234-909-606-8162 or 234-814-906-2100.

Abstract.
In this article, its shown that the *Miyazaki (2013) Conjecture* is wrong and doesn't apply to most Pythagoreans, (and that the *Jesmanowicz Conjecture* remains un-proven) within the context of Sub-Rings (ie. Integers).

Keywords: Nonlinearity; *Jeśmanowicz Conjecture*; Prime Numbers; Dynamical Systems; Mathematical Cryptography; ill-posed problems; *Sub-Rings* And *Ring Theory*; Primitive Pythagorean Triples.

1. Introduction.

On Pythagorean numbers, see: Jeśmanowicz (1955/1956). On other approaches to solving Diophantine Equations, see: Rahmawati, Sugandha, et. al. (2019), Darmon & Merel (1997) and Ibarra & Dang (2006). On the *Jeśmanowicz Conjecture* which has generated substantial debate for decades, see: Guo & Le (1995), Miyazaki (2011; 2013), Miyazaki, Yuan & Wu (2014); Miyazaki & Terai (2015), Takakuwa (1996), and Terai (2014). On various approaches for solving related diophantine equations, see: Bennett & Skinner (2004).

Miyazaki (2013) noted that "………*In 1956 L. Jeśmanowicz conjectured, for any primitive Pythagorean triple (a, b, c) satisfying* $a^2+b^2=c^2$, *that the equation* $a^x+b^y=c^z$ *has the unique solution (x,y,z)=(2,2,2) in positive integers x, y and z. This is a famous unsolved problem on Pythagorean numbers*…..….". Miyazaki (2013) conjectured that: "……..*In this paper we broadly extend many of classical well-known results on the conjecture. As a corollary we can verify that the conjecture is true if a - b = ±1*………".

For the equation $\mathbf{a^x+b^y=c^z}$ in positive integers, the following are combinations of a,b,c, x,y and z; but for each such combination, $\mathbf{(a^x+b^y)/c^z}$ ≈1.0000000000000000000000 (the equation is not exactly equal to 1.000000000000000000000000 like in pythagorean triples):

i) a = 3; b= 5; c = 7; **x= 6**; **y = 7; z= 7;** and $\mathbf{(a^x+b^x)/c^x}$ **= 1.018206700**.
ii) a = 60; b= 80; c = 461; **x= 6**; **y = 7; z= 7;** and $\mathbf{(a^x+b^x)/c^x}$ **= 1.009462982**.
iii) a = 434,500; b= 425,000; c = 75,696,000; **x= 6**; **y = 7; z= 7;** and $\mathbf{(a^x+b^x)/c^x}$ **= 1.007764426**.
iv) a = 37,566; b= 24,844; c = 461; **x= 23**; **y = 40; z= 66;** and $\mathbf{(a^x+b^x)/c^x}$ **= 1.010647596**.
v) a = 567,000; b= 424,410; c = 2,575; **x= 23**; **y = 40; z= 66;** and $\mathbf{(a^x+b^x)/c^x}$ **= 1.000292303**.

Given the foregoing, *Jesmanowicz's Conjecture* can be valid only in the *Domain-Of-Integers*, but not in the *Domain-Of-Real-Numbers*. Lolja (2018) explained the differences between the *Domain-of-Integers* and the *Domain-Of-Lines*.

On Homomorphisms, see: Wang & Chin (2012). Chu (2008) and Lu & Wu (2016) studied dynamical systems pertaining to Diophantine equations (and *equations such as* $a^2+b^2=c^2$ can approximate Dynamical Systems). Luca, Moree & Weger (2011) discussed *Group Theory*. Elia (2005), Jones, Sato, et. al. (1976) and Matijasevič (1981) noted that primes can be represented as Diophantine equations or as polynomials (ie. and the equation $a^2+b^2=c^2$ can represent a prime). On uses of Diophantine Equations in Cryptography, see: Ding, Kudo, et. al. (2018), Okumura (2015), and Ogura (2012) (the equation $a^x+b^y=c^z$ can be used in cryptoanalysis and in creation of public-keys). Zadeh (2019) notes that Diophantine equations have been used in analytic functions.

The *Miyazaki (2013) Conjecture* and the *Jesmanowicz Conjecture* are not valid for all or many primitive pythagorean triples in positive integers. The problem is an ill-posed problem because the equation $\mathbf{a^x+b^y=c^z}$ varies dramatically over the interval (0,+∞). A primitive Pythagorean triple is where *a*, *b* and *c* are co-prime (ie. there is no common divisor larger than 1). In the following simplest cases of Pythagorean-Triples where $\mathbf{a^2+b^2=c^2}$ is valid in positive integers and *a*, *b* and *c* are relatively-prime/co-prime, the condition (a-b)=±1, doesn't hold:

$5^2 + 12^2 = 13^2$, but $5-12 \neq \pm1$;
$7^2+24^2=25^2$, but $24-7 \neq \pm1$;
$20^2+21^2=29^2$, but $20-21 \neq \pm1$;
$12^2+35^2=37^2$, but $12-35 \neq \pm1$;
$9^2+40^2=41^2$, but $9-40 \neq \pm1$;
$28^2+45^2=53^2$, but $28-45 \neq \pm1$;
$11^2+60^2=61^2$, but $11-60 \neq \pm1$;
$33^2+56^2=65^2$, but $33-65 \neq \pm1$;
$16^2+63^2=65^2$, but $16-63 \neq \pm1$;
$48^2+55^2=73^2$, but $48-55 \neq \pm1$;
$36^2+77^2=85^2$, but $36-77 \neq \pm1$;
$13^2 + 84^2 = 85^2$, but $13-84 \neq \pm1$;
$65^2+72^2=97^2$, but $65-72 \neq \pm1$;

In the case of $3^2+4^2=5^2$, but 3-4= -1, but not +1.

Furthermore:

If a,b,c=1,2,3 and x,y,z= 3,3,2, then $\mathbf{a^x+b^y=c^z}$**;**
If a,b,c=3,3,6 and x,y,z= 2,3,2, then $\mathbf{a^x+b^y=c^z}$**;**
If a,b,c=2,3,5 and x,y,z= 4,2,2, then $\mathbf{a^x+b^y=c^z}$**;**
If a,b,c=5,7,24 and x,y,z= 4,2,2, then $\mathbf{a^x+b^y=c^z}$**; and** also (a,b,c,x,y,z)=(7,24,25,2,2,2);
If a,b,c=3,40,41 and x,y,z= 4,2,2, then $\mathbf{a^x+b^y=c^z}$**; and** also (a,b,c,x,y,z)=(9,40,41,2,2,2);
If a,b,c=2,63,65 and x,y,z= 8,2,2, then $\mathbf{a^x+b^y=c^z}$**; and** also (a,b,c,x,y,z)=(8,63,65,2,2,2);
If a,b,c=2,15,17 and x,y,z= 6,2,2, then $\mathbf{a^x+b^y=c^z}$**; and** also (a,b,c,x,y,z)=(8,15,17,2,2,2);

and also in all these foregoing mentioned equations, the condition (a-b)=±1, doesn't hold. Thus, the *Miyazaki (2013) Conjecture* and *Jesmanowicz Conjecture* are wrong or don't apply to all pythagoreans.

The Miyazaki (2013) conjecture is based on the condition/equation (a–b)= ±1 which is henceforth collectively referred to as the *(a-b) Conditions*, which are:
i) (a–b)= +1, the "*First (a-b) Condition*"; and
ii) (a–b)= -1, the "Second *(a-b) Condition*".

2. The Theorems.

Theorem-1: **Jeśmanowicz Conjectured That For Any Primitive Pythagorean Triple (*a*, *b*, *c*), The Equation $\mathbf{a^x + b^y = c^z}$ Has The Unique Solution (x, y, z)=(2, 2, 2) In Positive Integers; But For All *a*, *b*, *c*, *x*, *y* And *z* In Positive Integers, The *First (a-b) Condition* Is Wrong And The *Miyazaki Conjecture* Is Wrong.**
Proof: To test the first *(a-b) Condition*, assume that (x,y,z) = (2,2,2); then substitute (a–b)= 1, or a=(1+b) into $a^2+b^2=c^2$, and the result is: $(1+b)^2+b^2=c^2$, which is equivalent to: $(1+2b+ b^2+b^2)= c^2$; which is equivalent to: $(1+2b+ 2b^2)=c^2$, which is equivalent to: $[1+2b+2(c^2-a^2)]=c^2$; and $a^2=(1+2b+b^2)$. The following are "sub-theorems" each of which can be presented as a separate/independent Theorem.

Sub-Theorem-1:
In equation $a^2+b^2=c^2$, $c>b>a$, and (c-b)≤(b-a) and for small values of *a*, *b* and *c* (eg. integers that are single-digits), 2b can be equal to, or greater than b^2 (eg. $2*2=2^2$; and $2*1>1^2$); and thus in such instances, $[1+2b+2(c^2-a^2)] \neq c^2$ (that is, $[1+2b+2b^2] \neq c^2$), and the *First (a-b) Condition* [ie. (a–b)=1], is wrong.

Sub-Theorem-2:
In equation $a^2+b^2= c^2$, *c>b>a*, and (c-b)≤(b-a) and for large values of *a*, *b* and *c* (eg. integers that are greater than single-digits), $2b<b^2$; and as (a,b,c) →+∞, $2(c^2-a^2) \geq c^2$, and like above, $1+2b+2(c^2-a^2) \neq c^2$; and the *First (a-b) Condition* [ie. (a–b)=1], is wrong.

Sub-Theorem-3:
For most pythagoreans, *c>b>a*, and (c-b)≤(b-a). The *First (a-b) Condition* requires that $[1+2b+ b^2+b^2]=c^2$ exist, but then $(1+2b+b^2) \neq a^2$, for most pythagoreans. Thus the *First (a-b) Condition* (ie. (a–b)=1), is wrong (in order for the equation $a^2+b^2=c^2$ to be valid, the condition $(1+2b+b^2)=a^2$ must exist).

Sub-Theorem-4:
For all or most pythagoreans, in equation $a^2+b^2=c^2$, c>b>a, and (c-b)≤(b-a) and hence, $(c^2-b^2) \leq (b^2-a^2)$; and from above, $a^2=[1+2b+b^2]$. If $[1+2b+2b^2]= c^2$, then the condition $(b^2-[1+2b+b^2]) \geq (c^2-b^2)$, should exist but it doesn't because that condition/inequality is equivalent to: $[b^2-1-2b-b^2] \geq (c^2-b^2)$, which is equivalent to: $[-1-2b] \geq (c^2-b^2)$, which is impossible because for most pythagoreans, the RHS of the inequality $[-1-2b] \geq (c^2-b^2)$, will always produce a positive integer, while the LHS of that inequality will always produce a negative integer. Therefore, the *First (a-b) Condition* [ie. (a–b)=1], is wrong.

Thus, the *Miyazaki (2013) conjecture* is wrong. ■

Theorem-2: **Jeśmanowicz Conjectured That For Any Primitive Pythagorean Triple (*a*, *b*, *c*), The Equation $a^x + b^y = c^z$ Has The Unique Solution (x, y, z)=(2, 2, 2) In Positive Integers; But For All *a, b, c, x, y* And *z* In Positive Integers, The *Second (a-b) Condition* Is Wrong And The *Miyazaki Conjecture* Is Wrong.**
Proof: To test the *Second (a-b) Condition* (which is: (a–b)=-1), assume that (x,y,z) = (2,2,2); then substitute (a–b)=-1, or a=(b-1) into $a^2+b^2= c^2$, and the result is: $(b-1)^2+b^2= c^2$. Thus, $b^2-2b+1+b^2= c^2$; and $a^2= (1-2b+b^2)$; and $2b^2-2b+1= c^2$; and by substituting $b^2= c^2-a^2$ into the equation, that is equivalent to: $1-2b+2(c^2-a^2) = c^2$. The following are "sub-theorems" each of which can be presented as a separate/independent Theorem.

Sub-Theorem-1:
For most pythagoreans, c>b>a, and (c-b)≤(b-a). In equation $a^2+b^2= c^2$, for small values of a, b and c (eg. single-digit integers), 2b can be equal to, or greater than b^2 (eg. $1^2=1$, while 2*1=2>1; and 2*2=4, while $2^2=4$). In such instances, $[1-2b+ 2(c^2-a^2)] \neq c^2$ (that is, $[1-2b+2b^2] \neq c^2$) and the *Second (a-b) Condition* (ie. [a–b]= -1), is wrong.

Sub-Theorem-2:
For most pythagoreans, c>b>a, and (c-b)≤(b-a). In the equation $a^2+b^2=c^2$, and for large values of *a*,*b* and *c* (eg. integers that are greater than single-digits), $2b<b^2$; and as (a,b,c)→+∞, $2(c^2-a^2) \geq c^2$, for some large values of *a*, *b* and *c*; and thus like above, $[1-2b+2(c^2-a^2)] \neq c^2$ (that is, $[1-2b+2b^2] \neq c^2$). The equation $[1-2b+2(c^2-a^2)] = c^2$ erroneously implies that $[1-2b+c^2-2a^2]=0$, or that $[1-2b+b^2-a^2]=0$. Thus, the *Second (a-b) Condition* (ie. [a–b]=-1), is wrong.

Sub-Theorem-3:
For all or most pythagoreans, in equation $a^2+b^2=c^2$, c>b>a, and (c-b)≤(b-a). The *Second (a-b) Condition* requires that the condition $[1-2b+b^2+b^2]=c^2$ exist, but $(1-2b+b^2) \neq a^2$, and $(1-2b+2b^2) \neq c^2$, and like above, $[1-2b+2(c^2-a^2)] \neq c^2$. Therefore, the *Second (a-b) Condition* (ie. [a–b]= -1), is wrong.

Sub-Theorem-4:
For all pythagoreans, in equation $a^2+b^2=c^2$, c>b>a, and (c-b)≤(b-a) and hence, $(c^2-b^2)<(b^2-a^2)$; and from above, $a^2=[1-2b+b^2]$. If $[1-2b+2b^2]= c^2$ (as required by the *Second [a-b] Condition*), then the condition $(b^2-[1-2b+b^2]) \geq$

(c^2-b^2) should exist but it doesn't because the condition $(b^2-[1-2b+b^2]) \geq (c^2-b^2)$, is equivalent to $[b^2-1+2b-b^2] \geq (c^2-b^2)$, which is equivalent to $[-1+2b] \geq (c^2-b^2)$, which is impossible because for most pythagoreans:

i) $(b^2-a^2) \geq [-1+2b]$ and as stated above, $(b^2-a^2) \geq (c^2-b^2)$;

ii) $(c^2-b^2) \geq [-1+2b]$;

and therefore, the *Second (a-b) Condition* (ie. [a–b]= -1), is wrong.

Thus, the *Miyazaki (2013) Conjecture* is wrong. ■

3. Conclusion.

The *Miyazaki (2013) Conjecture* is wrong for all or most primitive pythagorean triples (and by extension, the *Jesmanowicz Conjecture* remains un-proven).

4. Bibliography.

Bennett, M. & Skinner, C. (2004). Ternary Diophantine equations via Galois representations and modular forms. *Canadian Journal Of Mathematics*, 56, 23–54.

Darmon, H. & Merel, L. (1997). Winding quotients and some variants of Fermat's last theorem. *J. Reine Angew. Math.*, 490, 81–100.

Ding, J., Kudo, M., et. al. (2018). Cryptanalysis of a public key cryptosystem based on Diophantine equations via weighted LLL reduction. *Japan Journal of Industrial and Applied Mathematics*, 35, 1123–1152.

Guo, Y. & Le, M. (1995). A note on Jeśmanowicz' conjecture concerning Pythagorean numbers. *Comment. Mat., Univ. St. Pauli*, 44, 225–228.

Ibarra, O. & Dang, Z. (2006). On the solvability of a class of diophantine equations and applications. *Theoretical Computer Science*, 352(1–3), 342-346.

Jeśmanowicz, L. (1955/1956). Some remarks on Pythagorean numbers, *Wiadom. Mat.*, 1 (1955/1956), 196–202 (in Polish).

Jones, J. P., Sato, D., et. al. (1976). Diophantine Representation of the Set of Prime Numbers. *American Mathematical Monthly*, 83, 449-464.

Lolja, S. (2018). The Proof of the Fermat's Conjecture in the Correct Domain. *Ratio Mathematica*, 35, 53-74

Matijasevič, Y. (1981). Primes are nonnegative values of a polynomial in 10 variables. *Journal of Soviet Mathematics*, _________.

Miyazaki, T. (2011). Jeśmanowicz' conjecture on exponential Diophantine equations. *Functional Approximation, Comment. Math.*, 45, 207–229.

Miyazaki T. (2013). Generalizations of classical results on Jeśmanowicz' conjecture concerning primitive Pythagorean triples. *Journal Of Number Theory*, 133, 583–595.

Miyazaki, T., Yuan, P. & Wu, D. (2014). Generalizations of classical results on Jeśmanowicz's conjecture concerning Pythagorean triples II. *Journal of Number Theory*, 141, 184–201.

Miyazaki, T. & Terai, N. (2015). On Jeśmanowicz' conjecture concerning primitive Pythagorean triples, II. *Acta Mathematica Hungarica*, 147(2), 286–293.

Ogura, N. (2012). *On Multivariate Public-key Cryptosystems*. PhD thesis, Tokyo Metropolitan University, Japan.

Okumura, S. A (2015). Public key cryptosystem based on diophantine equations of degree increasing type. *Pacific Journal of Industrial Mathematics*, 7(4), 33–45.

Rahmawati, R., Sugandha, S., et. al. (2019). The Solution for the Nonlinear Diophantine Equation $(7k-1)^x + (7k)^y = z^2$ with k as the positive even whole number. *Journal of Physics: Conference Series*, Volume 1179. The 1st International Conference on Computer, Science, Engineering and Technology 27–28 November 2018, Tasikmalaya, Indonesia.

Takakuwa, K. (1996). A remark on Jeśmanowicz' conjecture. *Proc. Japan Acad. Ser. A Math. Sci.*, 72, 109–110.

Terai, N. (2014). On Jeśmanowicz' conjecture concerning primitive Pythagorean triples. *Journal Of Number Theory*, 141, 316–323.

Wang, L. & Chin, C. (2012). Some property-preserving homomorphisms. *Journal of Discrete Mathematical Sciences and Cryptography*, 15(2-3).

Zadeh, S. (2019). Diophantine equations for analytic functions. *Online Journal of Analytic Combinatorics*, 14, 1-7.