Circumventing Newton’s third Law through Euler Inertial Forces

Ioannis Xydous\textsuperscript{1}\textsuperscript{*}

\textsuperscript{1}18532 Piraeus, Greece, ORCiD: 0000-0001-5460-2531

Abstract. Newton’s third law of motion would be the cornerstone of physics, were it not for specific experimental results that demonstrate a conflict with conservation of momentum under particular conditions. Such conceptual conflicts appear in systems in which the system’s interacting parts are mediated by a non-equilibrium environment that gives rise to nonreciprocal forces. The present theoretical study of a new mechanism of motion that utilizes internal inertial forces in an isolated system (internally powered) reveals how the limitations of action–reaction symmetry can be circumvented and demonstrated through an experimental apparatus. Furthermore, it explores the potential implications of these findings for Einstein’s special relativity and Lorentz transformations.

PACS numbers: 45.20.df, 45.20.D-, 03.30.+p

I. INTRODUCTION

In classical mechanics, any realization of the action–reaction principle presupposes two bodies exerting equal and opposite forces on each other. The notion of an internally powered isolated system moving in the absence of external forces touches upon the extraordinary idea that object A is attracted to object B while object B is simultaneously repelled by object A. Various physics disciplines explore nonreciprocal forces associated with the breaking of action–reaction symmetry as in statistical mechanics [1–7], gravitation [8] and optics [9–11]. The present paper proposes the Euler inertial forces appear to be the only way to overcome Newton’s third law [12] constraints to realize the above idea. The acceleration of an isolated system that utilizes internal inertial forces leads inevitably to a new phenomenon, namely reducing the effective inertia, which has further and profound implications for Einstein’s special relativity [13] and Lorentz transformations.

II. METHODS

Starting from Newton’s third law, the conservation of momentum in the isolated system in FIG. 1 - Lower, is

\[ \sum F_{\text{ext}} = 0 \text{ and } \sum F_{\text{int}} = \vec{F}_A + \vec{F}_R = 0, \]

\[ dp + dp_{\text{rot}} = 0 \Rightarrow dp = -dp_{\text{rot}}, \]

\[ \frac{dp}{dt} = \sum F_{\text{int}} = -\frac{dp_{\text{rot}}}{dt} = 0, \]

\[ m (u' - u) = -m_T (u'_{cm} - u_{cm}), \]

\[ u' = u \Rightarrow u'_{cm} = u_{cm} \Rightarrow u_{rel} = 0 \Rightarrow a = 0. \]

In addition to forces, the following expression yields the energy conservation for ideal (no dissipation of energy due to frictional forces) isolated systems:

\[ \Delta U_k = W_{F_A} + W_{F_R}, \]

\[ U_s = W_{F_A} = \int_0^d F_A ds \]

\[ \Delta U_k = U_s + \int_0^d F_R ds = 0, \]

\[ U_s : \text{stored energy.} \]

Due to Eq.(6), an isolated system that utilizes collinear internal forces can never accelerate since any action upon any internal part will be counteracted by a reaction force (Newton’s third law) upon the rest of the system. The isolated system, as shown in FIG.1 - Upper is a typical linear actuator device. It consists of a translation screw, a drive nut (mass $m_T$) attached on the translation screw, two linear guides, a front, and a rear housing that, besides holding the translation screw, they have a motor and a power supply enclosed (hidden).

In terms of physics, the linear actuator is classified as a rotating frame inside an inertial one. In general, a rotating frame induces three different types of inertial forces, as seen from an external inertial observer. So,

\[ \sum F_{\text{inertial}} = -F_{\text{Coriolis}} - F_{\text{Centrifugal}} - F_{\text{Euler}}. \]

The construction in FIG.1 - Upper is driven by a couple ($F_A$ and $F_A'$), where the motion of mass $m_T$ is restricted along the axis of rotation of the translation screw. Consequently, the resulting motion of mass $m_T$ can be attributed just to the Euler force. Thus,

\[ F_A = -F_{A'}, \]

\[ F_A \neq \text{const. and } F_A' \neq \text{const.} \Rightarrow \frac{d\omega}{dt} \neq 0, \]

\[ \sum \tau = \tau_A + \tau_A' = (r \times F_A) + (-r \times F_A'), \]

\[ \sum \tau = 2r \times F_A, \]

$r$ : translation screw radius.
An alternative way to derive the inertial force that causes the acceleration of mass \( m_T \) is through the conservation of angular momentum. Hence, \( \sum \tau_{\text{ext}} = 0 \) and \( \mathbf{F}_A = -\mathbf{F}_{A'} \), \( \sum \tau_{\text{int}} = (\tau_A + \tau_{A'}) + \tau_{\text{inertial}} = 0 \), \( (2\mathbf{r} \times \mathbf{F}_A) + \tau_{\text{inertial}} = 0 \), \( (2\mathbf{r} \times \mathbf{F}_{A'}) + \tau_{\text{inertial}} = 0 \).

At this point, developing a general expression for the inertial force requires the introduction of the dimensionless factor \( n_r \) (ideal mechanical advantage) along with a definition of the inertial torque. Thus, \( \frac{d\omega}{dt} = n_r \frac{\omega}{u_T} = \frac{2\pi r}{l_T} \), \( n_r \frac{d\omega}{dt} = \frac{2\pi dr}{dt} \), \( n_r (2\mathbf{r} \times \mathbf{F}_A) + n_r \tau_{\text{inertial}} = 0 \), \( n_r (2\mathbf{r} \times \mathbf{F}_{A'}) + \tau_{\text{inertial}} = 0 \).

Dividing Eq. (26) by the position-vector magnitude \( 2|\mathbf{r}| \) yields
\[
\frac{n_r (2\mathbf{r} \times \mathbf{F}_A)}{2|\mathbf{r}|} + \tau_{\text{inertial}} = 0, \quad \text{(27)}
\]
\[
\frac{n_r (2\mathbf{r} \times \mathbf{F}_{A'})}{2|\mathbf{r}|} = n_r \cdot m_T \frac{dm_T}{dt}, \quad \text{(28)}
\]
\[
\mathbf{F}_t = \frac{\tau_{\text{inertial}}}{2|\mathbf{r}|} \Rightarrow \mathbf{F}_t = -n_r \cdot m_T \frac{dm_T}{dt} = -n_r \frac{d\omega}{dt} \times \mathbf{r}. \quad \text{(29)}
\]

Furthermore, from the conservation of energy yields,
\[
n_r^2 \Delta U_{kT} = \Delta U_k, \quad \text{(30)}
\]
\[
\Delta U_k = W_{F_1} = \int_0^d \mathbf{F}_1 ds = \Delta U_k \quad \text{(31)}
\]
\[
\Delta U_s = I_{ns} \int_{\omega_1}^{\omega_2} \omega d\omega = I_{ns} \left( \omega_2^2 - \omega_1^2 \right), \quad \text{(32)}
\]
\[
U_k = \text{stored energy,} \quad I_{ns} : \text{translation screw moment of inertia.} \]

Eq. (29) reveals the inertial force does not possess reaction; therefore, it enables the acceleration of mass \( m_T \) that leads to the redeployment of the center of mass and acceleration of the system as a whole. Thus,
\[
\frac{dp}{dt} = \mathbf{F}_t = -m_T \cdot n_r \frac{dm_T}{dt} = -m_T \frac{d\omega}{dt} \times \mathbf{r}, \quad \text{(33)}
\]
\[
m(T\mathbf{u} - \mathbf{u}) = -m_T (\mathbf{u}_{cm}' - \mathbf{u}_{cm}), \quad \text{(34)}
\]
\[
\frac{dm_T}{dt} = 0 \Rightarrow \mathbf{u}_{rel} = \mathbf{u}_{cm}' - \mathbf{u}_{cm} \neq 0, \quad \text{(35)}
\]
\[
\mathbf{u}' \neq \mathbf{u} \Rightarrow \mathbf{u}_{cm}' \neq \mathbf{u}_{cm} \Rightarrow \mathbf{u} \neq 0, \quad \text{(36)}
\]
\[
\mathbf{u}_{rel} : \text{system’s center of mass relative velocity.} \]

However, the way Eq. (33) is formulated, it appears as a momentum exchange that points to a separation between the moving part (\( m_T \)) and the rest of the system, which is not valid. The mass \( m_T \) is an inseparable intrinsic part of the system. To agree with the construction’s integrity and the observations from an external inertial frame of reference, Eq. (33) may have a different and more general form. Therefore,
\[
\frac{dm_T}{dt} = \mathbf{F}_t = -\frac{dm_T}{dt} \cdot n_r \mathbf{u}_{rel}, \quad \text{(37)}
\]
\[
\frac{dp}{dt} = \mathbf{F}_t = -\frac{dm_T}{dt} \cdot n_r \mathbf{u}_{rel}. \quad \text{(38)}
\]

When the relative velocity \( \mathbf{u}_{rel} \) has constant magnitude, the effective inertia of the system becomes
\[
|\mathbf{u}_{rel}| = \text{const.} \Rightarrow \mathbf{u}_{rel} \neq 0 \Rightarrow \mathbf{u}_{cm}' \neq \mathbf{u}_{cm}, \quad \text{(39)}
\]
\[
\frac{dm_T}{dt} = \frac{d\mathbf{u}_{rel}}{dt} \cdot n_r \mathbf{u}_{rel}, \quad \text{(40)}
\]
\[
\int_m^m dm = -\frac{1}{n_r \mathbf{u}_{rel}} \int_0^p dp, \quad \text{(41)}
\]
\[
m_1 = m \left( 1 - \frac{p}{n_r \cdot m\mathbf{u}_{rel}} \right). \quad \text{(42)}
\]
Let us suppose that there is a theoretical quasiparticle (system) that exhibits the property whereby its effective inertia decreases as its velocity $v$ increases. Setting $u_{\text{rel}}$ to be equal to the speed of light, Eq.(42) turns into the nonrelativistic inertia of the system:

$$|u_{\text{rel}}| = c \Rightarrow m_i = m \left(1 - \frac{p}{n_r \cdot m \cdot c}\right). \quad (43)$$

Instead, because of energy conservation, Eq.(43) becomes

$$m_i c^2 - m c^2 = -\left( \frac{p}{n_r} \right)^2 \cdot \frac{1}{2m}, \quad (44)$$

$$m_i c^2 - m c^2 = -\frac{1}{n_r^2} \cdot \frac{m u^2}{2} = -\frac{U_k}{n_r^2}, \quad (45)$$

$$m_i = m \left(1 - \frac{u^2}{n_r^2 \cdot 2c^2}\right). \quad (46)$$

The classical limit of the relativistic inertia for charged particles is obtained using the Taylor-series expansion of the Lorentz factor:

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} = \sum_{n=0}^{\infty} \frac{u^n}{n!} \prod_{k=1}^{2n} \left(\frac{2k - 1}{2k}\right), \quad (47)$$

$$\gamma = 1 + \frac{1}{2} \left(\frac{u}{c}\right)^2 + \frac{3}{8} \left(\frac{u}{c}\right)^4 \cdots \Rightarrow u << c, \quad (48)$$

$$\gamma \approx 1 + \frac{1}{2} \left(\frac{u}{c}\right)^2, \quad (49)$$

$$m_i = m\gamma \approx m(1 + \frac{u^2}{2c^2}). \quad (50)$$

Similarly, Eq.(46) is the classical limit of the relativistic inertia for the theoretical quasiparticle.

The binomial series expansion of the inverse Lorentz factor yields

$$u_n = u/n_r, \quad (51)$$

$$\frac{1}{\gamma_r} = \left(1 - \frac{u_n^2}{c^2}\right)^{1/2} = 1 - \frac{1}{2} \left(\frac{u_n}{c}\right)^2 - \frac{1}{8} \left(\frac{u_n}{c}\right)^4, \quad (52)$$

$$u_n << c \Rightarrow \frac{1}{\gamma_r} \approx 1 - \frac{1}{2} \left(\frac{u_n}{c}\right)^2, \quad (53)$$

$$m_i = m\gamma \approx m(1 - \frac{u^2}{n_r^2 \cdot 2c^2}). \quad (54)$$

Consequently, the relativistic inertia of the theoretical quasiparticle is

$$m_i = m \left(1 - \frac{u^2}{n_r^2 \cdot c^2}\right)^{1/2} = \xi_c \cdot m \left(1 - \frac{u^2}{n_r^2 \cdot c^2}\right)^{-1/2}. \quad (55)$$

A general expression that incorporates Einstein’s special relativity (see FIG. 2) is derived as follows:

$$\xi_c = \frac{u_c}{c} = \left(1 - \frac{u_{sw}^2}{n_r^2 \cdot c^2}\right) \Rightarrow m_i = \xi_c \cdot m\gamma, \quad (56)$$

where $u_{sw}$ is the travelling speed of the translation mechanism in the quasiparticle structure. For a particle in Einstein’s theory of special relativity, we have

$$\textbf{F}_1 = 0 \text{ and } \sum F_{\text{ext}} \geq 0, \quad (56)$$

$$u_{sw} = 0 \Rightarrow \xi = 1 \Rightarrow n_r = 1, \quad (57)$$

$$u_c = c \Rightarrow 0 \leq u < c, m_i = m \left(\frac{1}{1 - \frac{u^2}{c^2}}\right)^{-1/2}, \quad (58)$$

$$m_i c^2 - m c^2 = U_k, \quad (59)$$
The proper time is defined as the time measured in the rest frame of the theoretical quasiparticle, thus

$$dx' = 0 \Rightarrow dx = (u/n_t)dt,$$  \hspace{1cm} (81)

$$dt_x = dt' = \gamma_t \left(1 - \frac{u^2}{n_t^2c^2}\right) dt = \frac{dt}{\xi \gamma_t} = \gamma_t dt.$$  \hspace{1cm} (82)

Similarly, the proper length is

$$dt = 0 \Rightarrow dx_r = dx' = \xi \gamma dt = \frac{dx}{\gamma_t}.$$  \hspace{1cm} (83)

The general Lorentz transformations [Eqs. (78) and (80)] can be verified by deriving the momentum and energy using the proper time, hence

$$p = \frac{dx}{dt} \cdot \frac{dt}{dt} = \frac{mu}{n_t} : \xi \gamma_t,$$  \hspace{1cm} (84)

$$p = \frac{1}{n_t} \gamma \mu = \frac{mu}{n_t} \sqrt{1 - \frac{u^2}{n_t^2c^2}},$$  \hspace{1cm} (85)

$$m \gamma c^2 = mc^2 \cdot \xi \gamma_t = \frac{mc^2}{\gamma_t} = 1 - \frac{u^2}{n_t^2c^2}.$$  \hspace{1cm} (86)

Note that the original expressions of Einstein and Lorentz are recovered when there are no inertial internal forces (no translation mechanism) in the system:

$$F_1 = 0 \text{ and } \sum F_{ext} \geq 0,$$  \hspace{1cm} (87)

$$u_{sw} = 0 \Rightarrow \xi \gamma_t = 1 \Rightarrow n_t = 1 \Rightarrow \gamma_t = \gamma.$$  \hspace{1cm} (88)

III. CONCLUSIONS

Newton’s laws of motion and Einstein’s theory of special relativity overlook the nature of the inertial (fictitious) force and apparently fail to anticipate the motion of an isolated system due to inertial internal forces. Besides the present results imply a paradigm shift in the way that motion is conducted, the new phenomenon whereby the effective inertia decreases while the speed of a system increases led to the discovery of a wider framework— which predicts the existence of quasiparticles with group velocities that may reach and even surpass the speed of light.

Acknowledgments

I thank my wife, who encouraged me to initiate and complete this work.