

# Could Newton's gravity law be reconciled with Einstein's general relativity theory?

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## Abstract

It is known that Newton's law of gravitation can be derived as an approximation from Einstein's GR in the weak-field, low-velocity limit. The derivation is done mathematically.

In this paper, I verify this claim by relating to observations of two celestial bodies:

1) The solar system. 2) A neutron star. Newton's law is in good agreement for most of the planets in the solar system, except for the nearest planet to the Sun, Mercury. Mercury's motion is determined by GR.

## General

I rely on the following premise:

- 1) Observations show that all celestial bodies spin. From GR, it is also known that spinning celestial bodies drag the space around them. This phenomenon is called frame dragging, and is described in:

<https://en.wikipedia.org/wiki/Frame-dragging>

Frame dragging is also valid for relatively small celestial bodies such as Earth. This was verified by the Gravity Probe B experiment, albeit the measured frame dragging was minuscule. Frame dragging of massive bodies such as black holes and neutron stars can be calculated only by GR. Newton's law is not applicable.

- 2) I use Rudolf Muradyan's proposal that the angular momentum ( $J$ ) of many astronomical objects, from planets to clusters of galaxies, can be predicted by simple Regge-like laws based on their mass ( $m$ ). This hypothesis suggests a fundamental relationship where the angular momentum scales with the mass of the object. He described the spins of planets and stars using the Regge law for a sphere and the spins of galaxies and clusters of galaxies with the Regge law for a disk. (References 1, 2)

Einstein's Field Equations (EFE) describe the relationship between the geometry of spacetime and the distribution of mass, energy, and momentum within it. The exact solution of EFE depends on the assumptions taken. The first who solve EFE was

Schwarzschild in 1916. He assumed a mass that is spherically symmetric, non-rotating, and that the spacetime outside the mass is static. Newton made the same assumptions.

It can be shown, mathematically, that Newton's law, in the weak gravitational field and slow-motion approximation, can be derived from the Schwarzschild solution of EFE. See:

<https://www.quora.com/How-can-we-derive-Newtons-law-of-gravitation-from-Einsteins-theory-of-relativity?q=can%20Newton%27s%20gravitational%20law%20derived%20from%20general%20relativity>

However, Schwarzschild's solution as well as Newton's gravitational law do not describe exactly the physical world. It is known that all celestial bodies spin on their axis and therefore have a considerable angular momentum. Schwarzschild's solution and Newton's gravitational law do not take into consideration the spinning mass's angular momentum. Therefore, both are approximations.

The real-world solution using EFE was derived in 1963 by Roy Kerr. Kerr's solution takes into consideration the spinning of the mass and is suitable for strong gravitational fields and high-velocity motion. Kerr's solution also describes an additional phenomenon – the frame-dragging of space by the spinning mass. In this sense, Newton's law deviates from Kerr's solution.

I think that the majority of the readers will find Kerr's solution derivation quite difficult. Therefore, in this paper, I suggest a simpler way to compare Newton's gravity and Kerr's solution. To this end, I use the final frame dragging equation derived from Kerr's solution of EFE, without relating to the way how it was derived.

Newton's gravitational law describes gravity as the force exerted between two bodies. The force is dependent on the masses of the two bodies and the distance between the centers of the two bodies. The formula is:

$$F = \frac{G \cdot M_1 \cdot M_2}{R^2}$$

Although it was his idea, Newton was baffled by the gravitational force and could not explain its origin. However, Newton's law has been proven to be valid and sufficiently accurate in our solar system. It is used in calculating the motion of the planets and trajectories of spacecraft being sent, e.g., from Earth to Mars.

On the other hand, some observations cannot be explained by Newton's law, for example, the precession of Mercury, the bending of light by massive celestial bodies, or the dynamics near neutron stars or black holes. Einstein's GR explains these observations quite accurately. Einstein described gravity as a distortion of the fabric of spacetime by a celestial body. The trajectory of a small celestial body will be along the geodesics in space created by a bigger body.

The question now is how two theories that are based on different assumptions can be reconciled.

**Hypothesis: I claim that Newton's law is a good approximation to GR, except that the distance  $R$  between the centers of the two bodies is not a straight line connecting both centers, but should be replaced by the geodesic length derived from general relativity  $L_{geodesic}$ .** The geodesic length for two celestial bodies is calculated below.

Therefore, Newton's modified equation should be:

$$F = \frac{G \cdot M_1 \cdot M_2}{L_{geodesic}^2}$$

To explain the geodesic length, I would like to relate it to the phenomenon of frame-dragging of spacetime around any rotating celestial body. The rotational frame-dragging effect was first derived in 1918 by Lense-Thirring based on GR. Later, in 1963, frame dragging was also derived from the solution of a rotating Kerr black hole. This effect was validated in 2011 by the Gravity Probe B experiment. The effect was measured near Earth, and the results were minuscule but validated the frame-dragging effect.

The equation of frame-dragging around a spinning celestial body is given by:

<https://en.wikipedia.org/wiki/Frame-dragging>

$$\Omega = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{r_s \alpha r c}{\rho^2 (r^2 + \alpha^2) + r_s \alpha^2 r \sin^2 \theta}$$

**Equation (1)**

Where  $\Omega$  - angular velocity depends on the radius  $r$  and the colatitude  $\theta$ .

$$V_{\text{sun}}(r, \theta) := \frac{R_{H\_sun} \cdot \alpha_{\text{sun}} \cdot r^2 \cdot c}{\left(r^2 + \alpha_{\text{sun}}^2 \cdot \cos(\theta)^2\right) \cdot \left(r^2 + \alpha_{\text{sun}}^2\right) + R_{H\_sun} \cdot \alpha_{\text{sun}}^2 \cdot r \cdot \sin(\theta)^2}$$

For simplicity, I use the equation of the angular velocity in the equatorial plane of a celestial body:

$$\Omega(r) = \frac{R_H \cdot \alpha \cdot C}{r^3 + \alpha^2 \cdot r + R_H \cdot \alpha^2} \quad \dots \text{The angular velocity}$$

Where:

$$c = 2.9979 \cdot 10^8 \cdot \frac{m}{s} \quad \dots \text{Light velocity}$$

$$R_H = \frac{2 \cdot G \cdot M}{C^2} \quad \dots \text{Schwarzschild's radius}$$

$$G = 6.67 \cdot 10^{-11} \cdot \frac{m^3}{kg \cdot s^2} \quad \dots \text{Gravitational constant}$$

$$M \cdot kg \quad \dots \text{Mass of celestial body}$$

$$m_{\text{neutron}} = 1.674927 \cdot 10^{-27} \cdot kg \quad \dots \text{The Neutron's mass}$$

$$\hbar = 1.054571 \cdot 10^{-34} \cdot J \cdot s \quad \dots \text{Reduced Planck constant}$$

$$J = \hbar \cdot \left( \frac{M}{m_{\text{neutron}}} \right)^{\frac{4}{3}} \quad \dots \text{Angular momentum of a celestial body}$$

- according to Muradian (See Reference 1, 2)

$$\alpha = \frac{J}{M \cdot c} \quad \dots \text{Definition of spin parameter}$$

## Verification of the hypothesis ( two examples are given):

### 1. Frame – dragging around the Sun

Given:

$$M_{sun} = 1.99 \cdot 10^{30} \cdot kg.$$

....The Sun mass

$$R_{sun} = 696342 \cdot km.$$

.... The Radius of the Sun

$$T_{sun} = 27 \cdot day.$$

.... Spin of Sun around its axis

$$R_{earth\_sun} = 149.5 \cdot 10^6 \cdot km.$$

....Average Sun-Earth distance

$$T_{earth\_sun} = 1 \cdot yr.$$

.... Time of Earth rotation around Sun

$$J_{sun} = \hbar \cdot \left( \frac{M_{sun}}{m_{neutron}} \right)^{4/3} = 1.327 \cdot 10^{42} J \cdot s \dots \text{The angular momentum of Sun}$$

$$\alpha_{sun} = \frac{J_{sun}}{M_{sun} \cdot c} = 2.2439 \cdot km \quad \dots \text{Spin parameter}$$

$$R_{H\_sun} = \frac{2 \cdot G \cdot M_{sun}}{c^2} = 2.954 \cdot km. \quad \dots \text{Schwarzschild radius of Sun}$$

$$\Omega(r) = \frac{R_{H\_sun} \cdot \alpha_{sun} \cdot c}{r^3 + \alpha_{sun}^2 \cdot r + R_{H\_sun} \cdot \alpha_{sun}^2} \quad \dots \text{The angular velocity around the Sun}$$

$$\theta(r) = \Omega(r) \cdot T_{earth} \cdot \frac{r}{R_{earth\_sun}} \quad \dots \text{The rotation angle of Earth around the Sun}$$

Finding the length of a geodesic  $L_{geodesic}$  is done by using the formula of curve length:

$$L_{geodesic} = \int_{R_{sun}}^{R_{earth\_sun}} \sqrt{\left[ 1 + \left( \frac{d}{dr} (r \cdot \Omega(r) \cdot T_{earth}) \right)^2 \right]} \cdot dr = 148.8 \cdot 10^6 \text{ km} \quad \dots \text{Length of-}$$

-geodesic Earth to Sun

And the result is  $\frac{L_{geodesic}}{R_{earth\_sun}} \cong 1$  , therefore Newton's approximation is good.

Fig. 1 shows the straight line connecting the Sun to Earth as formulated by Newton's law.

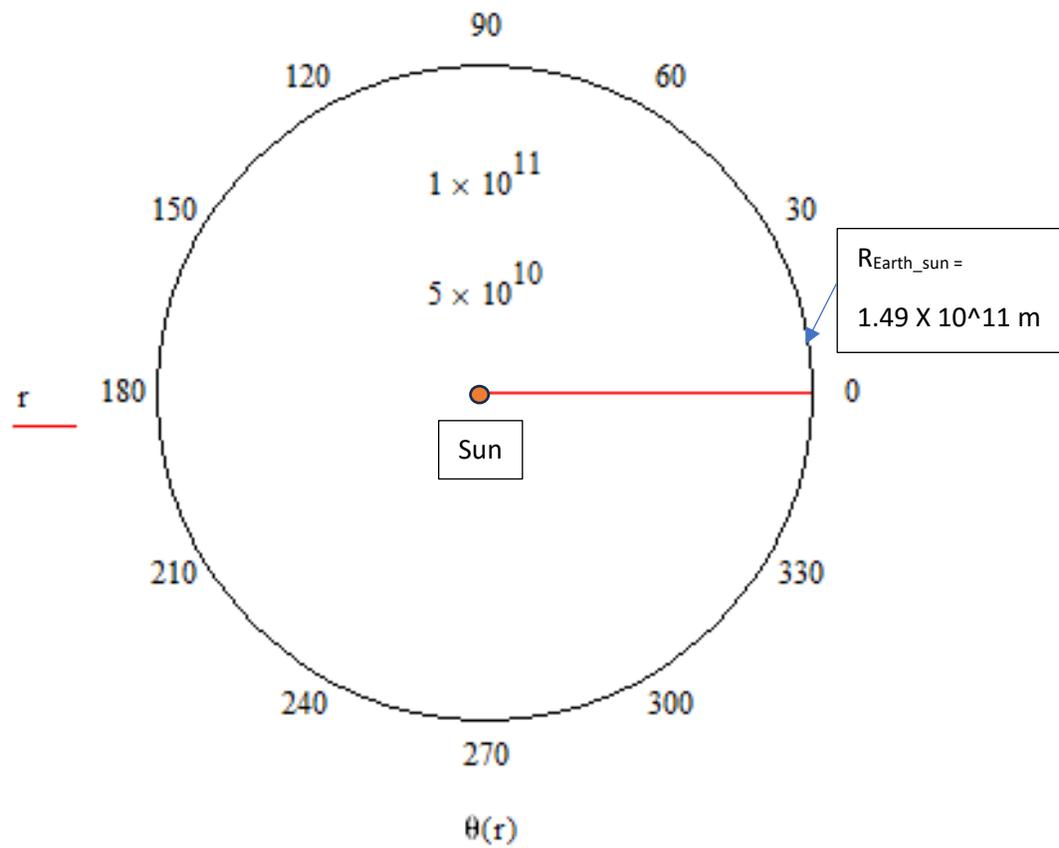


Fig. 1- The line connecting Earth- Sun – According to Newton

However, as GR predicts, the frame-dragging is minuscule. To see the geodesic, the angle  $\theta(r)$  is scaled up by a factor of  $10^8$ . See Fig. 2. The geodesic becomes evident near the Sun.

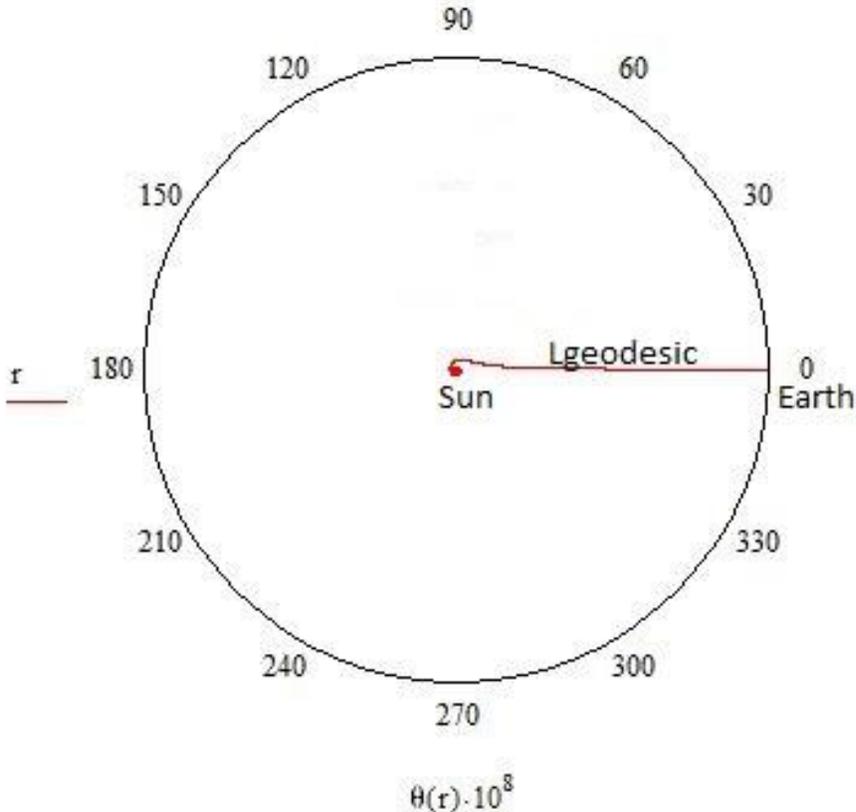


Fig. 2- The geodesic around the Sun – According to GR

## 2) Frame – dragging around Neutron star PSR J1748 – 2446ad

Given

$$M_{ns} = 2 \cdot 1.99 \cdot 10^{30} \cdot \text{kg}. \quad \dots \text{The Neutron star mass: 2x Sun mass}$$

$$R_{ns} = R_{neutron} \cdot \left( \frac{M_{ns}}{m_{neutron}} \right)^{1/3} = 10.67 \cdot \text{km} \dots \text{Calculated radius of the Neutron star}$$

$$f_{ns} = 716 \frac{1}{s}. \quad \dots \text{Measured spin of neutron star}$$

$$T_{ns} = \frac{1}{f_{ns}} = 1.4 \cdot 10^{-3} \cdot s$$

$$J_{ns} = \hbar \cdot \left( \frac{M_{ns}}{m_{neutron}} \right)^{\frac{4}{3}} = 3.34 \cdot 10^{42} J \cdot s. \quad \dots \text{The angular momentum of Neutron star}$$

$$\alpha_{ns} = \frac{J_{ns}}{M_{ns} \cdot c} = 2.8 \cdot \text{km} \quad \dots \text{Spin parameter}$$

$$R_{H\_ns} = \frac{2 \cdot G \cdot M_{ns}}{c^2} = 5.9 \cdot \text{km}. \quad \dots \text{The Schwarzschild radius of the Neutron star}$$

$$\Omega(r) = \frac{R_{H\_ns} \cdot \alpha_{ns} \cdot c}{r^3 + \alpha_{ns}^2 \cdot r + R_{H\_ns} \cdot \alpha_{ns}^2} \quad \dots \text{The angular velocity around the Neutron star}$$

$$R_{out} = 20 \cdot \text{km}. \quad \dots \text{Assumption: distance of a particle from Neutron star}$$

$$L_{geodesic\_ns} = \int_{R_{ns}}^{R_{out}} \sqrt{\left[ 1 + \left( \frac{d}{dr} (r \cdot \Omega(r) \cdot T_{ns}) \right)^2 \right]} \cdot dr = 39.4 \cdot \text{km} \quad \dots \text{Length of geodesic-}$$

-particle to Neutron star

$$\text{The ratio is: } \frac{L_{geodesic\_ns}}{R} = 1.97$$

$$\theta(r) = \Omega(r) \cdot T_{ns} \cdot \frac{r}{R_{ns}} \quad \dots \text{Polar plot around the neutron star}$$

Fig. 3 shows frame-dragging around a Neutron star, on the equatorial plane. It shows  $L_{geodesic\_ns}$  Vs. Newton's distance  $R$ . The gravitational force between the Neutron star and a particle orbiting it differs significantly from Newton's law.

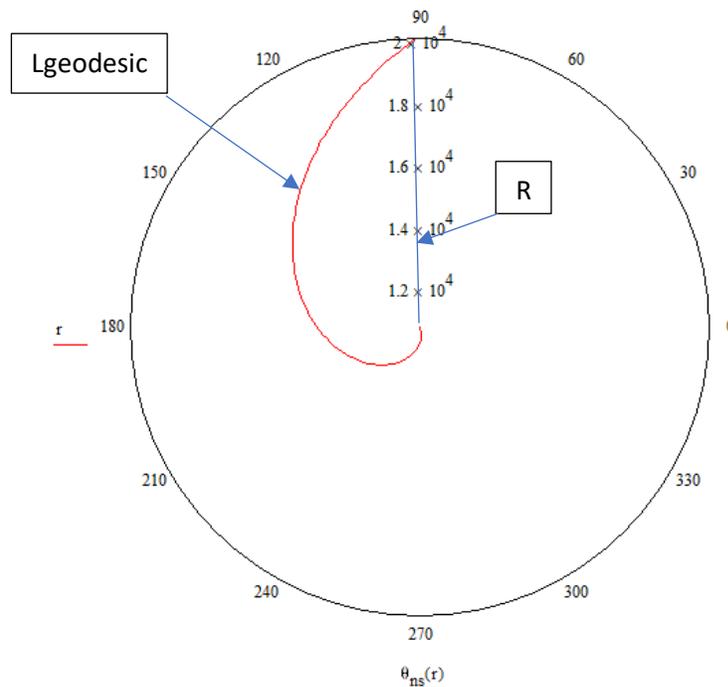


Fig. 3 - The geodesic around the Neutron star per GR.

## References

1. R. M. Muradian (1980). "The primeval hadron: origin of stars, Galaxies, and astronomical Universe" <https://lib-extopc.kek.jp/preprints/PDF/1979/7911/7911323.pdf>
2. R. M. Muradian "SCALING LAWS IN PARTICLE PHYSICS AND ASTROPHYSICS" <https://arxiv.org/ftp/arxiv/papers/1106/1106.1270.pdf>