Does General Relativity Properly Describe the Perihelion Shift and Black Holes?

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Abstract

The paper proves that General Relativity (GR) is not needed to explain the effects mentioned in the title. It will be shown that Newton's laws are sufficient if the influence of the far distant masses of the universe is included into the calculation, i.e., if Mach's principle is truly obeyed. The quantitative result with the perihelion shift of Mercury is very close to that of the GR, whereas there is possibly a small, but in principle measurable, difference in the size of the event horizon of static black holes.

Key words: perihelion shift, black holes, event horizon, Newton's laws and the universe, action-at-a-distance, Mach's Principle, General Relativity, cosmology, fundamental physics

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1. Introduction

It is commonly accepted within the community of physicists that the correct figures for the event horizon of black holes and for the perihelion shift can only be found within the framework of General Relativity (GR). The calculation based solely on Newton's laws yields the wrong values of the perihelion shift if compared to experimental figures. In the case of Mercury, the calculated shift deviates about 43'' (arc seconds) from the experimental figure. It was one of Einstein's first and amazing successes when he could calculate the correct value using his GR. This was and is one of the reasons that Einstein's GR is considered powerful and as one of the fundamental theories of physics.

But there is an important deficit if planetary or cosmic systems are calculated on the basis of Newton's laws alone: the influence of (in cosmological scales) far distant masses has never been considered up to now. If, for instance, the perihelion shift of the planet mercury is calculated, only the influences of the other planets of the solar system have been and are taken into account.

In an earlier paper[1)], we determined that including the far distant masses in the calculations with Newton's laws leads to decisive and measurable changes compared to the results without this inclusion. For the behavior of a specimen mass positioned somewhere in the universe, one finds exactly the formulas of the Special Theory of Relativity (SR), provided that the mass distribution is homogeneous, i.e. there is no single mass in the close vicinity of the specimen mass. The contrary case that other masses are in the vicinity was mentioned within the paper[1)] but only briefly.

In the present paper we will go into details for two of these cases, namely for the problem of the perihelion shift and for a static black hole. We shall see that the result for the motion of planets leads to particle trajectories that are no longer closed, even if the influence of the other planets within the solar system are not taken into account. For the planet Mercury, the resulting perihelion shift is identical to the value found by the GR. For a static black hole, one finds similar qualitative properties as with GR, but there are small deviations in the size of the event horizon.

2. Short description of the theory applied

We hark back in this paper to a theory developed earlier (see [1),2),3)]). Because this theory is not widely known, we will present the main elements here briefly. The consideration may appear trivial at first view, but its consequences are in no way trivial.

If a meteorite impacts the surface of the earth, its kinetic energy is converted into heat. The meteorite has gained this kinetic energy on its part by the conversion of its potential energy, which it had initially at far distance from earth. We can see that the energy content of the earth after the impact has grown by exactly the same amount as the potential energy of the meteorite has been reduced. If the meteorite does not impinge the surface of the earth but first falls into a hole directed to the center of the earth where it is then stopped, then the total potential energy that the meteorite has lost is given by [4)]:

$$\Delta E_{pot\ tot} = 2\pi\ G\ m_0\ \rho\ R_o^2. \tag{2.1}$$

With the abbreviation

$$b_0^2 = 2\pi\ G\ \rho\ R_o^2 \tag{2.2}$$

this can be written in the form:

$$E_0 = m_0\ b_0^2, \tag{2.3}$$

where G is the gravitational constant, ρ the average mass density of the earth, and R_0 its radius. The relation (2.1) is valid for a homogeneously distributed mass of the earth. In case of a non-homogeneous distribution, e.g. in case of a number of point masses m_i, distributed randomly at the locations r_i, one finds instead of (2.2) and (2.3) [1)]:

$$b_0^2 = G \sum_i \frac{m_i}{r_i} \tag{2.2a}$$

and

$$E_0 = m_0\ b_0^2 = m_0\ G \sum_i \frac{m_i}{r_i}\ . \tag{2.3a}$$

The physical essence of these relations can be described as follows: The (gravitational) energy content of the earth with respect to a mass m_0 at rest at its origin is given by the sum of the losses of the potential energies of the masses m_i having moved from infinity to r_i and being at rest there [1)].

Let us now leave the earth and consider the universe instead, and let us take the following model as a basis: The universe shall be finite in the three dimensional space and shall form a sphere with the radius R_0. On very large scales, the masses shall be distributed homogeneously, i.e. the universe shall show (on this scale) a mean density ρ. In this case, the energy content of the universe, including the mass m_0 at its center, is described by exactly the same formula as described above for a meteorite that was fallen to the center of the earth. Of course, the values ρ and R_0 of the universe have to be set now.

It is important to stress that the energy (2.3) is not merely "assigned" to the mass m_0, but that it is **existentially united** to it: It does not exist, if the mass m_0 is lacking, and it must be imperatively present, if the mass m_0 exists at the origin of the universe and is at rest there. Hence, we can describe this physical state also by saying: A mass at rest being located at the center of the universe "has" (or "possesses") a "rest energy" E_0.

Let us now consider the circumstances with a moving mass. And let us assume the general validity of Newton's law of inertia:

$$F = \dot{p} = m_{in}\dot{v} + \dot{m}_{in}v \,. \tag{2.4}$$

Here m_{in} represents the inert mass. F shall be an arbitrary external force, but not a gravitational one (for instance, the mass m_{in} can carry a charge and move under the influence of an electrical field). Other possibly existing masses m_j shall be so small or at far distances that their gravitational forces are very small compared to F and can therefore be neglected in (2.4). If we multiply (2.4) on both sides with the infinitesimal shift ds produced by the force F

$$Fds = \dot{p}\,ds = (m_{in}\dot{v} + \dot{m}_{in}v)ds, \tag{2.5}$$

then we find the energy increase dE when the system "mass within the universe" changes (caused by F) from the status "mass at rest" to the status "mass in motion". According to the consideration above (equations (2.1) to (2.3)), the rest energy of the mass m within the universe is given by $E = m(v{=}0)b_0^2 = m_0\, b_0^2$, where m is the gravitational mass $m{=}m_g$. We know that the inert and the gravitational mass are proportional to each other (or equal if suitable units are chosen). Therefore, the essential relation is valid:

$$m = m_g = m_{in} \,. \tag{2.6}$$

This equivalence principle is experimentally very well confirmed for arbitrary speeds.

Therefore, we can rewrite (2.5) into the form

$$dE = (m\dot{v} + \dot{m}v)\, ds. \tag{2.7}$$

For a mass $m = m_0$ at rest the total energy E_0 is given by (see (2.3))

$$E_0 = m_0\, b_0^2. \tag{2.3a}$$

The following ansatz comes to mind for the system "universe plus moving mass":

$$E = m\, b_0^2. \tag{2.8}$$

Then, accordingly

$$dE = dm\, b_0^2. \tag{2.9}$$

By inserting (2.9) into (2.7) and after some minor conversions we find eventually:

$$dm\, b_0^2 = dm\, v^2 + m\, v\, dv$$

or

$$\frac{dm}{m} = \frac{v\, dv}{{b_0}^2 - v^2} \,. \tag{2.10}$$

Of course, instead of (2.8) we can also try any other ansatz at will. But such another ansatz would then only represent a solution if it would fulfill the differential equation (2.7), and if the rest energy (2.3a) would result for the limit case v = 0. Obviously, the ansatz (2.8) fulfills these requirements.

By integrating (2.10) one ends up with

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{{b_0}^2}}} \,, \tag{2.11}$$

and with (2.8)

$$E = \frac{m_0}{\sqrt{1-\frac{v^2}{{b_0}^2}}}\, b_0^2 \,. \tag{2.12}$$

Surprisingly, the same formulas for a moving mass and its energy result as in the Special Theory of Relativity (SR). In contrast to the SR, the constancy of the light velocity in inertial systems, uniformly moved against each other, does **not** have to be postulated here. Light and the light velocity play no role in the derivation. But we find also a maximum velocity b_0 for moving masses. This is the consequence of the fact that we have taken the distant masses of the universe into consideration. The question as to whether this maximum velocity b_0 and the light velocity c are identical is investigated in the papers [1), 2)] and [3)]. Due to several reasons, this seems to be mandatory.

The results above are remarkable and, of course, have a number of consequences that have already been investigated to some extent in the papers [1), 2)] and [3)]. They are valid under the prerequisite of a homogeneous mass distribution of the universe. In the paper [1)] we mentioned briefly that this mass distribution is generally, of course, not homogeneous and the theory has to be generalized for inhomogeneous distributions. In the following, we perform one step towards such a generalization. We will consider the special case of adding a single mass M, which causes an inhomogeneity in the vicinity of the mass m. As is known, this case of a spherical symmetric mass distribution can be solved exactly in the GR (Schwarzschild metric).

Though our studies [1), 2)] and [3)] suggest almost imperatively that b_0 is identical with the light velocity c, in the following we will continue to use b_0 in order to recall that our theory does not at all need Einstein's postulate of the constancy of the light velocity. Rather, the maximum velocity $v_{max} = b_0$ is already a consequence of Newton's laws if the distant masses of the universe are taken into account appropriately.

3. Extension of the theory to an inhomogeneous mass distribution

As in the chapter before, we start with the consideration of the forces involved, but allow the existence of a mass M located on the x-axis at a distance r_M to the left of the origin (M conceived to be at rest at the beginning). Now, the direct influence of this additional mass M shall not be negligible compared to the other forces. The motion of a mass m under the influence of an additional mass M is therefore to be described.

According to Newtonian theory, the planetary equations result from this. The underlying physical idea is an "absolute, otherwise empty space" with mass M, in which the point mass m moves. In GR, the concept of empty space ("vacuum") is also used, but fields exist within it, which are determined by Einstein's field equations. The motion of a point mass under the influence of M is described by the Schwarzschild solution.

Our physical concept differs from both: we assume that all masses in the universe act instantaneously everywhere according to Newton's law of gravitation and are always included in the calculations. A "vacuum" does not exist in principle.

a) Special case: central force

We choose for the description a cartesian co-ordinate system. The external force F on m shall act into the positive x – axis. M shall be placed left of the origin at x = - x_M, and $|-x_M|$ shall be abbreviated as r_M. m shall be placed at x. Then, we find instead of (2.4):

$$-m\dot{v} - \dot{m}v + F - m\frac{MG}{(r_M+x)^2} = 0\,. \tag{3.1}$$

Also, in this case the validity of the equivalence principle (2.6) is implied, again in its "weak form" $m = m_g = m_{in}$ (see (2.6)).

After multiplying (3.1) with dx we find:

$$-dm\,v^2 - m\,v\,dv + Fdx - m\frac{MG}{(r_M+x)^2}\,dx = 0\,. \tag{3.2}$$

We will not attempt to find a general solution to this differential equation in the following, but will try to arrive at solutions through a physically motivated approach, which allows us to answer specific questions, such as about the event horizon of a static black hole or the perihelion shift of Mercury.

To solve (3.2), we can of course take any ansatz we like. However, the resulting solution for $\frac{MG}{(r_M+x)^2} \to 0$ must reduce to the solution for the case without M. Furthermore, in (2.3a) the additional mass M at a distance r_M must be taken into account, i.e., for the rest energy of the mass m_0 at the origin, the following must hold:

$$E_0 = m_0\,G\left(\sum_i \frac{m_i}{r_i} + \frac{M}{r_M}\right) = m_0\left(b_0^2 + \frac{MG}{r_M}\right). \tag{3.3}$$

We will soon see that these requirements are met by the following ansatz:

$$E = m\,(b_0^2 + \frac{2MG}{r_M} - \frac{MG}{(r_M+x)}) = m\,b_0^2\,(1+ \frac{2MG}{b_0{}^2 r_M} - \frac{MG}{b_0{}^2(r_M+x)}). \qquad (3.4)$$

Every change of this total energy is then given by

$$dE = Fdx = dm\,b_0^2\,(1+ \frac{2MG}{b_0{}^2 r_M} - \frac{MG}{b_0{}^2(r_M+x)}) + m\,b_0^2 \frac{MG}{b_0{}^2(r_M+x)^2}\,dx. \qquad (3.5)$$

Let's introduce an abbreviation now

$$y = \frac{2MG}{b_0{}^2 r_M} - \frac{MG}{b_0{}^2(r_M+x)} \qquad (3.6)$$

and then substitute (3.5) with (3.6) into (3.2):

$$dm\,(b_0^2\,(1+y) - v^2\,) + m\frac{MG}{(r_M+x)^2} - m\frac{MG}{(r_M+x)^2} = m\,v\,dv\,. \qquad (3.7)$$

By choosing approach (3.4), the terms with MG stand out, and one finds:

$$\frac{dm}{m} = \frac{v\,dv}{b_0{}^2\,(1+y-\frac{v^2}{b_0{}^2})}\,. \qquad (3.8)$$

This relation applies to arbitrary values of the position x of the mass m. x, and consequently y, should therefore be regarded as independent parameters in this differential equation.

If we integrate (3.8) from $m = m_0$ to m and from v = 0 to v = v we arrive at

$$m = m_0 \frac{\sqrt{(1+y)}}{\sqrt{(1+y) - \frac{v^2}{b_0{}^2}}}\,. \qquad (3.9)$$

and eventually we find for the total energy

$$E = m_0\,b_0^2\,\frac{\sqrt{(1+y)}}{\sqrt{(1+y) - -\frac{v^2}{b_0{}^2}}}\,(1 + y). \qquad (3.10)$$

With respect to later use we re-write (3.9) in in the form

$$m = m_0\,\frac{1}{\sqrt{1 - \frac{v^2}{b_0{}^2(1+y)}}}\,. \qquad (3.11)$$

The result (3.10) describes the energy of the system “universe plus mass m”, whereas m is moving along the connecting line between m and the mass M at rest. This formula for the energy is therefore also applicable for a static black hole, where only the gravitational attraction between m and M is acting and no angular momentum has to be taken into account. In contrast to the classical Newton consideration, now, beside of the influence of the mass M, all the distant masses of the universe are also effective (represented in b_0).

Let us now go ahead with our intention to determine the size of the event horizon of a static black hole following the thought already formulated by John Mitchell in 1783 [5)]. A mass m in a distance r_{eh} (the so-called "event-horizon") from a mass M can only escape, if the velocity v_{eh} is high enough to overcome the gravitational attraction of this mass M. I.e., the mass m can only escape if the amount of $E_{kin}(v=v_{eh})$ exceeds the difference of the potential energy ΔE_{pot} between the "event horizon" and the location of an observer. But if v_{eh} can only reach a limited value (light velocity?) an escape might not be possible. This will be investigated in the following. Unlike Schwarzschild, we will not be examining the path of a light ray in this case. We are rather returning to the above consideration of a mass m under the influence of all (homogeneously distributed and distant) masses of the universe and an additional mass M, but without an external force F. Its motion is described by

$$-\mathrm{dm}\,v^2 - m\,v\,dv = m\frac{MG}{(r_M+x)^2}\,dx\,. \tag{3.12}$$

For the mass m itself, we have determined from the considerations and calculations (3.2) to (3.9):

$$m = m_0\frac{1}{\sqrt{1-\frac{v^2}{{b_0}^2(1+y)}}}\,. \tag{3.11}$$

We divide by m, set $r = r_M + x$, and integrate:

$$-\int\frac{dm}{m}v^2 - \int v\,dv = \int\frac{MG}{r^2}\,dr\,. \tag{3.13}$$

With this transformation, no separation of variables was achieved, because under the first integral there is a function v^2 that depends on m and r (cf. (3.11)). However, according to the mean value theorem of integral calculus, we can bring the mean value $<v^2>$ outside the integral, and we know that its value must lie between 0 and b_0^2. Let's write

$$<v^2> = a^2 b_0^2\,, \qquad \text{then is} \qquad 0 < a^2 < 1. \tag{3.14}$$

Now we can approximately integrate (3.13) from $r = r_{eh}$ to $r \to \infty$, v from $v(r=r_{eh})$ to $v((r \to \infty)$ and m from $m(r=reh, v=v(r=r_{eh}))$ to $m((r=r \to \infty)\,, v= v((r \to \infty))$. Instead of (3.13) we write:

$$<v^2> \int_{m_{eh}}^{m_\infty}\frac{dm}{m} + \int_{v_{eh}}^{v_\infty} v\,dv = -\int_{r_{eh}}^{\infty}\frac{MG}{r^2}dr \tag{3.15}$$

We can assume that v becomes smaller and smaller as the distance of mass m from mass M increases, and $v(r \to \infty)$ approaches zero in a good approximation. According to (3.9), $m \to m_0$ also works. Thus, it follows from (3.15):

$$a^2 b_0^2 \ln\frac{m_0}{m_{eh}} - \frac{1}{2}{v_{eh}}^2 = -\frac{MG}{r_{eh}}\,. \tag{3.16}$$

Let's introduce an abbreviation:

$$\beta^2 = \frac{{v_{eh}}^2}{{b_0}^2}\,, \tag{3.16a}$$

Then we can write:

$$r_{eh} = \frac{2MG}{b_0^{\,2}(a^2\ln(\frac{m_{eh}}{m_0})^2 + \beta^2)} \, . \qquad (3.17)$$

If the mass m starts at a velocity v_{eh} much smaller than the speed of light from the event horizon r_{eh}, then $m_{eh} \approx m_0$ and the ln-term becomes zero. Accordingly, in this case

$$r_{eh} = \frac{2MG}{v_{eh}^{\,2}} \, . \qquad (3.18)$$

This is the classic limiting case underlying John Mitchell's 1783 consideration. However, (as he did not know) the formula is **precisely not** applicable to speeds on the order of the speed of light. In this case, (3.17) must be used.

We now want to do this using (3.9). The mass m is defined to have the velocity $v = v_{eh} = b_0$ at the event horizon. Then, according to (3.9)

$$m = m_{eh} = m_0 \frac{\sqrt{(1+y_{eh})}}{\sqrt{(1+y_{eh}) - \frac{v_{eh}^{\,2}}{b_0^{\,2}}}} = m_0 \frac{\sqrt{(1+y_{eh})}}{\sqrt{(y_{eh})}}. \qquad (3.19a)$$

At an (infinitely) large distance from M, the velocity of the mass m approaches zero, and from (3.9) we obtain in this case:

$$m = m_\infty = m_0 \, . \qquad (3.19b)$$

With $r = r_m + x$ (see (3.6)) we find

$$y_{eh} = \frac{2MG}{b_0^{\,2} r_M} - \frac{MG}{b_0^{\,2} r_{eh}} \, . \qquad (3.20)$$

We can arbitrarily define the location where the mass M is situated. We make the definition

$$r_M = \alpha \, r_{eh} \, , \qquad (3.21)$$

where the following applies: $0 < \alpha < 1.$ (3.21a)

Then is $y_{eh} = \frac{MG}{b_0^{\,2}\, r_{eh}} (\frac{2-\alpha}{\alpha}) \, .$ (3.20a)

This now results for the square of the mass ratios m_{eh} and m_∞:

$$(\frac{m_{eh}}{m_\infty})^2 = \frac{b_0^{\,2} r_{eh} \alpha}{(2-\alpha)} + 1. \qquad (3.21)$$

In the choice of α, we are again completely free, i.e. we can choose the spatial coordinate of M as we wish, e.g. very close to the origin of the coordinate system we have chosen. The requirements for our approach (3.4) (cf. (3.2) and (3.3)) are not violated. With this choice, α becomes close to zero, and it results in

$$(\frac{m_{eh}}{m_\infty})^2 = 1. \qquad (3.22)$$

In this case, the logarithmic term in (3.17) also tends to zero, and we eventually find

$$r_{eh} = \frac{2MG}{{b_0}^2(a^2\ln(\frac{m_{eh}}{m_0})^2 + \beta^2)} = \frac{2MG}{{v_{eh}}^2} = \frac{2MG}{{b_0}^2} = r_s \,. \tag{3.23}$$

This is exactly the same result that Karl Schwarzschild found through the analytical solution of Einstein's field equation for a spherically symmetric mass distribution.

The choice of "α nearly zero" is allowed, but other values of α can also be chosen. Then the result (that is, the value for the event horizon r_{eh}) depends on α and thus on the choice of the spatial coordinate of M in the coordinate system we have chosen. This cannot be ruled out because we are considering a finite (not homogeneous in the x-direction) system. If we allow arbitrary values of α, we must continue calculations with (3.17) and (3.20a). Then we obtain

$$\frac{r_{eh}}{r_s} = \frac{1}{a^2\ln(2\frac{r_{eh}}{r_s}\frac{2\alpha}{(2-\alpha)}+1)+1} \,. \tag{3.24}$$

The largest possible denominator and thus the smallest possible value for $\frac{r_{eh}}{r_s}$ is obtained for the two (fundamentally) allowed values

$$\alpha = 1 \text{ and } a = 1. \tag{3.25}$$

By numerical solution of (3.24), we find for the smallest possible value of r_{eh}:

$$r_{eh} = 0{,}663\ r_s \,. \tag{3.26}$$

However, the fact is that although we do not know the exact r-dependence of $\langle v^2\rangle = a^2\, {b_0}^2$, we can safely assume that a^2 becomes very small for $\frac{r}{r_s} \gg 1$, or (for $r \to R_u$):

$$\frac{R_u {b_0}^2}{2MG} \gg 1. \tag{3.27}$$

This condition could only be violated for extremely large masses M, i.e., for extremely heavy black holes (on the order of the total mass of the universe), whose existence is not currently assumed.

Let's assume anyway that a = 0.1, then for r_{eh} we would get:

$$r_{eh} \approx 0{,}99\ r_s \,. \tag{3.28}$$

Our approximation approach (3.15) therefore does not give us a completely precise value for the event horizon, but it does narrowly limit the possible values for it and shows that they are very close to the Schwarzschild radius for the conditions in our universe.

b) Special case: Force perpendicular to the connecting line

Now, we will investigate the case that the force $F = F\varphi$, which acts onto the mass m, is oriented perpendicular to the connecting line between m and M. In this case, the force $F\varphi$ effects a change of the location of m into the azimuthal direction $ds = r\, d\varphi$. We allow an additional force F_r to act on m, which shall always be oriented into the direction of M. Newton's laws of motion have then to be written in the form:

for the motion in φ – direction

$$- \dot{m}v_{\varphi} - m\,\dot{v}_{\varphi} + F_{\varphi} = 0 \qquad (3.29)$$

and in the r – direction

$$- \dot{m}v_r - m\,\dot{v}_r + F_r - Z(v_{\varphi}) + m\frac{MG}{r^2} = 0. \qquad (3.30)$$

Here $Z(v_{\varphi})$ is the centrifugal force at the velocity v_{φ}. Let us now choose the amount of F_r in such a way, that it compensates for the difference between the forces $Z(v_{\varphi})$ and m $\frac{MG}{r^2}$ for each φ. This could be effectuated e.g. by charging the mass m electrically and letting it move within a magnetic field of suitable strength, which is also oriented perpendicular to the r – φ – plane. In this case, a motion into the radial direction is not possible (dr, v_r and $\dot{v}_r$ = 0). Therefore, an energy alteration through forces into that direction is not possible. The radial forces are also always directed perpendicular to the direction of ds and cannot cause an energy change into this direction, too. F_r becomes zero when the system has reached the stationary state. Then, the centrifugal force completely compensates the attraction of M onto m.

If we again multiply (3.29) with ds, we find:

$$-dm\,{v_{\varphi}}^2 - m v_{\varphi}\,dv_{\varphi} + F_{\varphi}\,ds = 0\,. \qquad (3.31)$$

The only one of the forces appearing in (3.30) and (3.31) which is able to add energy to the system is F_{φ}. It causes the energy alteration F_{φ} ds = d$E\varphi$. And we can, analogous to (3.4), try again the ansatz

$$F_{\varphi}\,ds = dE\varphi = dm\,(b_0^2 + \frac{MG}{r}), \qquad (3.32)$$

since we have to set again the following, of course, for the rest energy:

$$E_0 = m_0\,G(\textstyle\sum_i \frac{m_i}{r_i} + \frac{M}{r}) = m_0\,(b_0^2 + \frac{MG}{r}). \qquad (3.3)$$

(3.32) in (3.31) yields:

$$dm\,(b_0^2 - {v_{\varphi}}^2 + \frac{MG}{r}) = m\,v_{\varphi}\,dv_{\varphi} \qquad (3.33)$$

or, if we replace for simplicity v_{φ} by v:

$$\frac{dm}{m} = \frac{v\,dv}{(b_0^2 - v^2 + \frac{MG}{r})}\,. \qquad (3.34)$$

This is identical to (3.8), and for m and E there are again the relations (3.9) and (3.10). The only difference consists of the direction of v, which is now perpendicular to the connecting line between m and M. We denote the respective energy by $E\varphi$.

c) General Case: Force and motion in arbitrary direction

Until now we have treated the two cases of a pure radial and a pure azimuthal motion completely separately. The total energy of a mixed radial and azimuthal motion is not the plain addition of E_r and $E\varphi$, because the rest energy is contained in both energy forms and may not be counted twice. Hence, we have to subtract the rest energy once, when adding E_r and $E\varphi$, to compose the total energy:

$$E_g = m_0\, b_0^2\,(1+x)\ \left(\frac{\sqrt{(1+x)}}{\sqrt{(1-\frac{v_r^2}{b_0^2}+x)}} + \frac{\sqrt{(1+x)}}{\sqrt{(1-\frac{v_\varphi^2}{b_0^2}+x)}}\right) - m_0\, b_0^2(1+x). \qquad (3.35)$$

There we have abbreviated: $x = \frac{MG}{b_0^2 r}$. (3.35a)

For the total velocity v the following relation is valid:

$$v^2 = v_r^2 + v_\varphi^2. \qquad (3.36)$$

The energy E_g described by (3.35) fulfills the requirements

$$E_g\,(v_\varphi=0) = E_r \quad \text{und} \quad E_g\,(v_r=0) = E\varphi. \qquad (3.37)$$

For a differential alteration of the total energy, the following requirement has to also be satisfied:

$$dE_g = \frac{\partial E_g}{\partial v_r}\, dv_r + \frac{\partial E_g}{\partial v_\varphi}\, dv_\varphi, \qquad (3.38)$$

or respectively $\int dEg = \int \frac{\partial E}{\partial v_r}\, dv_r + \int \frac{\partial E}{\partial v_\varphi}\, dv_\varphi$, (3.39)

which is also met by E_g defined by (3.35).

Since F_r is a central force, it has no influence on the amount of the angular momentum J, defined by

$$m\, r^2\, \dot{\varphi} = J. \qquad (3.40)$$

J can only be changed by $F\varphi$ and is determined by v_φ or $\dot{\varphi}$ respectively (see (3.22).

As usual we define J = mh, then (3.29) assumes the shape

$$r^2\, \dot{\varphi} = h. \qquad (3.41)$$

This can also be written in the form $dt = \frac{r^2}{h}\, d\varphi$. (3.42)

We would like now to gain the path curvature of the mass m around the central mass M. For this purpose, we transform (3.35):

$$\left(\frac{Eg}{m_0 b_0^2} + 1 + x\right)(1+x)^{-3/2} - \frac{1}{\sqrt{(1-\frac{v_\varphi^2}{b_0^2}+x)}} = \frac{1}{\sqrt{(1-\frac{v_r^2}{b_0^2}+x)}}\,. \qquad (3.41)$$

To carry out the calculation clearly arranged, we introduce the abbreviations Γ_1 and Γ_2:

$$\Gamma_1 = \left(\frac{Eg}{m_0 b_0^2} + 1 + x\right) (1+x)^{-3/2} - \frac{1}{\sqrt{\left(1 - \frac{v_\varphi^2}{b_0^2} + x\right)}} , \qquad (3.42)$$

$$\Gamma_2 = \frac{1}{\sqrt{\left(1 - \frac{v_r^2}{b_0^2} + x\right)}} . \qquad (3.43)$$

Then (3.41) turns into the simple form

$$\Gamma_1 = \Gamma_2. \qquad (3.44)$$

With the definition $\frac{v_r^2}{b_0^2} = y^2$ (3.45)

we find $\frac{1}{\Gamma_2^{\ 2}} = 1 - y^2 + x,$ (3.46)

or with (3.44): $y^2 = \frac{(1+x)\,\Gamma_1^{\ 2} - 1}{\Gamma_1^{\ 2}}$, (3.47)

or in other form $\frac{1}{y} = \frac{\Gamma_1}{\sqrt{(1+x)\,\Gamma_1^{\ 2} - 1}}$. (3.48)

Using (3.42) and (3.45) we arrive at

$$\frac{d\varphi}{dr} = \frac{h}{r^2 b_0} \frac{\Gamma_1}{\sqrt{(1+x)\,\Gamma_1^{\ 2} - 1}} . \qquad (3.49)$$

This relation is still fully exact. In order to identify the quantitative amount of the perihelion shift of the planet Mercury, we start now to carry out a series expansion. During this procedure we have to evaluate which terms may be neglected. Therefore, we first compile the order of magnitude for the relevant terms related to the conditions of the planet Mercury (to be found in [6)]).

$\overline{v_r} \approx 6{,}2 \ \frac{km}{s}$, $\dot{\varphi} \approx 8{,}3\ 10^{-7} \frac{1}{s}$, $\frac{v_r^2}{b_0^2} = y^2 \approx 4{,}3\ 10^{-10}$, $\frac{v_\varphi^2}{b_0^2} = z^2 \approx 2{,}5\ 10^{-8}$,

$\bar{h} = \bar{r}\ \overline{v_\varphi} \approx 2{,}8\ 10^9 \frac{km^2}{s}$, $\frac{\bar{h}^2}{\bar{r}^2 b_0^2} \approx 2{,}5\ 10^{-8}$, $r_s \approx 2{,}9\ 10^3$ m , $\frac{r_s}{\bar{r}} \approx 5{,}1\ 10^{-8}$,

$x \approx 2{,}5\ 10^{-8}$, $v^2 \approx 2{,}3\ 10^3 \frac{km^2}{s^2}$. (3.50)

Using, additionally, the abbreviation $E' = \frac{Eg}{m_0 b_0^2}$, we find the series expansion for Γ_1:

$$\Gamma_1 \approx E' - \frac{3}{2} E'x - \frac{1}{2} z^2 + \frac{3}{4} z^2 x . \qquad (3.51)$$

In this relation, we have neglected terms of higher order than z^2 and x, because, due to (3.40), we find as a very good approximation: $z^4 \ll z^2$ und $x^2 \ll x$.

If we neglect higher order terms in the same way, we get:

$$\Gamma_1{}^2 \approx E' (E' - 3 E'x - z^2 + 3 z^2 x) \tag{3.52}$$

and
$$\Gamma_1{}^2 (1+x) \approx E' (E' - 2 E'x - z^2 + 2 z^2 x). \tag{3.53}$$

Inserting this into (3.49) yields eventually:

$$\frac{d\varphi}{dr} = \pm \frac{h}{r^2 b_0} \frac{E' - \frac{3}{2}E'x - \frac{1}{2}z^2 + \frac{3}{4}z^2 x}{\sqrt{E'}\sqrt{(E' - 2E'x - z^2 + 2z^2 x) - \frac{1}{E'}}} . \tag{3.54}$$

We would like to further simplify this relation. For this purpose we divide E_g into the constituent parts "rest energy" and "kinetic energy":

$$E_g = m_0 b_0^2 (1+x) + E_{kin} . \tag{3.55}$$

Then we can write
$$E' = 1+x + \frac{E_{kin}}{m_0 b_0{}^2} = 1+x+E''. \tag{3.56}$$

With this definition, we continue the series expansion in (3.54):

$$\frac{d\varphi}{dr} = \pm \frac{h}{r^2} \frac{\left(1 - \frac{1}{2}x - \frac{1}{2}E''\right)(1+x+E'')\left(1 - \frac{3}{2}x - \frac{1}{2}\frac{z^2}{E'} + \frac{3}{4}\frac{z^2}{E'}x\right)}{\sqrt{2E''b^2 - 2E''b^2 x - b_0{}^2 z^2 + 2b_0{}^2 z^2 x}} . \tag{3.57}$$

With (3.35), (3.55), (3.56) and the quantities (3.50) for the circumstances of Mercury, we can see that E' is of the order of 1, whereas x and E'' are smaller by a factor of 10^{-8}. If we substitute now r by 1/u and x by $\frac{1}{2}\frac{r_s}{r}$ (see (3.35a)), we can write in a very good approximation:

$$d\varphi = \pm \frac{h\,du}{\sqrt{2E''b_0{}^2 - E''b_0{}^2 r_s u - h^2 u^2 + h^2 r_s u^3}}. \tag{3.58}$$

We would like to compare this result with the path curvature of a planet, which is calculated solely on the basis of Newton's laws **without consideration of the other planets and the distant masses**. It is given by

$$d\varphi = \pm \frac{h\,du}{\sqrt{\frac{2E_N}{m_0} + b_0{}^2 r_s u - h^2 u^2}}. \tag{3.59}$$

Here E_N is Newton's total energy

$$E_N = \frac{1}{2} m_0 (v_r{}^2 + v_\varphi{}^2) - \frac{m_0 MG}{r} = \frac{1}{2} m_0 v^2 - \frac{m_0 MG}{r} ,$$

or written in another form

$$\frac{1}{2} m_0 v^2 = E_N + \frac{m_0 MG}{r} . \tag{3.60}$$

To compare (3.59) with (3.58) we remember, how we defined E'' in (3.56):

$$E'' = \frac{E_{kin}}{m_0 {b_0}^2} . \qquad (3.56a)$$

Here E_{kin} has to be determined according to (3.55) referring to the total energy E_g (3.35). If we again neglect terms of higher order then $\frac{{v_r}^2}{{b_0}^2}$, $\frac{{v_\varphi}^2}{{b_0}^2}$ and x (which due to (3.50) is again a good approximation), we find:

$$E_g - m_0\, {b_0}^2\,(1+x) = \quad E_{kin} = \frac{1}{2}\, m_0 v^2 + \frac{3}{4}\, m_0 v^2 x \approx \frac{1}{2}\, m_0 v^2. \qquad (3.61)$$

Hence, the kinetic energy calculated on the basis of the theory used here is, in case of Mercury, in very good accord with the classical theory of Newton. Because of (3.60) and (3.61) we, therefore, can write:

$$E'' = \frac{E_N}{m_0 {b_0}^2} + \frac{MG}{r {b_0}^2} = \frac{E_N}{m_0 {b_0}^2} + x. \qquad (3.62)$$

If we insert this in (3.48) we find:

$$d\varphi = \pm \frac{h\,du}{\sqrt{2(\frac{E_N}{m_0 {b_0}^2} + x){b_0}^2 - (\frac{E_N}{m_0 {b_0}^2} + x\,){b_0}^2 r_s u - h^2u^2 + h^2 r_s u^3}},$$

or

$$d\varphi = \pm \frac{h\,du}{\sqrt{2\frac{E_N}{m_0} + {b_0}^2 r_s u - (x{b_0}^2 + \frac{E_N}{m_0})\, r_s u - h^2u^2 + h^2 r_s u^3}} . \qquad (3.63)$$

We assess again the order of magnitude for the circumstances of Mercury, here concerning the term $(x{b_0}^2 + \frac{E_N}{m_0})$:

$$(x{b_0}^2 + \frac{E_N}{m_0}) = \frac{1}{2}\frac{r_s}{r}{b_0}^2 + \frac{1}{2}v^2 - \frac{1}{2}\frac{2MG}{r{b_0}^2}{b_0}^2 = \frac{1}{2}{b_0}^2 (\frac{v^2}{{b_0}^2}) . \qquad (3.64)$$

It is to be seen that the third term under the root (3.63) is to a factor of 10^{-8} smaller than the second one and we can neglect it. It would additionally take account of a small influence, which can be attributed to the influence of the SR alone [8], which is as well incorporated in the theory presented here (see section 2 above and [1]).

With this final approximation, we find eventually:

$$d\varphi = \pm \frac{h\,du}{\sqrt{2\frac{E_N}{m_0} + {b_0}^2 r_s u - h^2u^2 + h^2 r_s u^3}} . \qquad (3.65)$$

It can be seen that this relation migrates for small u (i.e. large r) into the classical Newtonian case, and it is also in very good accordance with the respective relation of the GR [7), 8)]. It contains the important fourth term under the root, which brings in a dependency of u^3 and, therefore, a closed path is no longer possible. Such a closed path is allowed if Newton's theory is applied without considering the influence of the other planets and also neglecting the influence of the distant masses. For Mercury, the term with u^3 is responsible for the famous additional perihelion shift of 43.03'' per century (3.8" for the earth and 8.6" for Venus)

[7), 8)]. These figures resulting from the GR are exactly the same figures that result from the theory presented here.

4. Summary and conclusion

It is shown that General Relativity is not needed for the explanation and quantitative description of the perihelion shift of the planet Mercury as well as of the event horizon of black holes. This explanation succeeds on the basis of Newton's laws alone if the distant masses of the universe are included into the consideration, i.e. if Mach's principle is duly respected. Quantitative calculations partly show deviations to the respective results of the GR. In the case of black holes, there is a small, but probably measurable, difference with the size of the event horizon. The result for the perihelion shift of Mercury obtained here is nearly identical with that of the GR.

The connections revealed here, together with the results found in [1), 2)] and [3)], lead to the conclusion that the theory applied here digs deeper than the Theory of Relativity, and that means both the Special as well as the General Theory. Compared to the SR and the GR, the theory applied here exhibits the advantage that the constancy of the light velocity does **not** have to be postulated. Rather, this constancy is one of the results of the theory, and it turns out to be not generally valid but only valid within spatial volumes that are small compared to the extent of the universe.

A further and fundamental difference to the SR and GR appears in the fact that the long-range-order (or action-at-a-distance principle) of Newton's force of gravity constitutes an essential part of the theory here, though it leads to a maximum velocity for moving masses. Hence, the theory is, with respect to the fundamental law of gravitation and the distant masses, non-local, but it describes some essential properties of the nature as local, becoming manifest with the maximum velocity of moving masses and light. Thus, it seems not to be in contrast to the non-local character of nature, which has been demonstrated in a huge number of experiments with entangled quantum states (recently even on cosmic dimensions [9)]). Yet, the principle of causality is preserved in the theory, due to the maximum velocity for masses and light (see [8)], p. 29).

The theory applied in the present paper introduces, together with the preceding papers [1), 2)] and [3)], the basic principles of a fundamental theory which, apparently, forges ahead to a deeper cognizance than the Theory of Relativity. Still, the geometrical description of the nature by the GR is, of course, mathematically elegant as well as aesthetic. However, it seems that the GR is not exactly valid, but it will prove as an approximation of an even better-grounded theory. The instantaneous influence of the distant masses of the universe, i.e. the observance of Mach's principle, seems to be fundamental in this respect.

It has to be investigated whether and how the theory demonstrated and applied here for some (but important) examples could be better formalized and generalized (e.g. as a field theory as in [2)]). And of course, it has to be further discussed whether it represents a deeper insight compared to the Theory of Relativity.

Remark: A further prominent result of the GR is the bending of light (e.g. around the sun). This can probably not be investigated by the theory presented here, because this theory is based upon a specimen mass, which has a finite rest energy at velocity zero, and shows a singularity for v = c. This does not apply to the propagation of light. Therefore, the bending of light shall be investigated in a separate paper.

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