

Conventional Calculation of Potential Energy Fundamentally Incorrect

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Abstract-This article proves that the conventional calculation of potential energy leads to absurd consequences. The presented alternative calculation corrects all these consequences.

1 Conventional versus alternative calculation of potential energy

Potential energy is defined as $E_p = \int F(r) dr$, with $F(r)$ a force along the path r , of which the boundaries have yet to be defined.

1.1 Conventional calculation

Reference [1] calculates that, given a distance d between masses M and m , their mutual *conventional* potential energy $E_{PC} = -GMm/d$. The negative sign is explained as follows: "The negative sign follows the convention that work is gained from a loss of potential energy." It can be deduced that the result has been obtained by moving the masses further away from each other, starting the variable r in $F(r)$ at the mutual distance d and ending at $r \rightarrow \infty$.

1.2 Alternative calculation

The smallest mutual distance r between these two masses, with radii r_M resp. r_m , is $r_i = r_M + r_m$. The *alternative* calculation of their mutual potential energy is to move them away from each other, starting at $r = r_i$ until $r = d$, and assuming a *positive* force $F(r)$. The result is:

$$E_{PA} = \int_{r_i}^d GMm/r^2 dr = GMm (1/r_i - 1/d)$$

This expression has the following remarkable property compared to the conventional one:

E_{PA} is only equal to E_{PC} , regarding its absolute value as well as its sign, if $1/r_i \ll 1/d$, so if $r_i \gg d$. An impossible configuration!

The expression for the alternative energy also shows that for $r_M \gg r_m$ and $d \gg r_M$, $E_{PA} = GMm/r_M$. At first sight a surprising result, because the distance d doesn't play any role in it, while the conventional calculation leads to a zero potential energy for $d \gg r_M$. The explanation is as follows: An object at distance $d \gg r_M$ has the potential to be transformed into kinetic energy, just by "releasing" it there. Coming back at distance r_M this kinetic energy is $\frac{1}{2}mv_{r_M}^2$. So $v_{r_M} = \sqrt{2GM/r_M}$, the so-called escape velocity of mass M with radius r_M . The formal meaning of this variable is that if an arbitrary mass m is shot into space with the escape velocity, it will become out of the gravitational influence of mass M at distance $d \gg r_M$.

Just for information: the escape velocity of earth is $\sqrt{2 \cdot 6.7 \cdot 10^{-11} \cdot 6.0 \cdot 10^{24} / 6.4 \cdot 10^6} = 11200$ m/s.

Another way of calculating this escape velocity is to start at earth with a kinetic energy of $\frac{1}{2}mv^2$. The pushing force that is applied to m is GMm/s^2 , with s starting at r_M and ending at ∞ ($v=0$). The total work of this force is $GMm(1/r_M - 1/d) = GMm/r_M$, being equal to $\frac{1}{2}mv^2$. N.B. $r_M \gg r_m$.

Quote from [2]: ".....escape velocity is the minimum speed needed for a free, non-propelled object to achieve an infinite distance from it."

The well-known expression for the potential energy of an object w.r.t. earth is mgh , with h the distance w.r.t. earth's *surface*, so in terms of d and r_M : $d = r_M + h$. The restriction is: $h \ll r_M$. Applying this in the expression of the alternative potential energy, with again $r_M \gg r_m$, results in:

$$E_{PA} = GMm\{1/r_M - 1/(r_M+h)\} \sim GMmh/r_M^2 = mgh$$

The parameter 'g' is the so-called free-fall acceleration GM/r_M^2 .

2 Conventional and alternative potential energy in circular orbital systems

The force between the two objects in a circular orbital system, from now on shortly orbital system, is in general terms $F(r) = C/r^2$ and known under the name centripetal force.

Taking the orbital radius r_o as the distance in the calculation of the two kinds of potential energies, then: $E_{PC} = -C/r_o$ and $E_{PA} = C/r_M$ for $r_M \gg r_m$ and $r_o \gg r_M$, as shown above.

The kinetic energy E_K of mass m is $\frac{1}{2}mv_o^2$, with v_o the orbital velocity. The centrifugal force is mv_o^2/r_o and the centripetal force C/r_o^2 . These forces are by definition equal in an orbital system. So it follows from $mv_o^2/r_o = C/r_o^2$ that $\frac{1}{2}mv_o^2 = \frac{1}{2}C/r_o = -\frac{1}{2}E_{PC}$, resulting in $E_{PC} = -2E_K$. An absurd outcome at least regarding the sign.

Applying the alternative calculation of the potential energy in such a situation ($E_{PA} = C/r_M$) shows that this *realistic* potential energy doesn't have a meaningful relation at all with the kinetic energy $E_K = C/2r_o$. The reason is the following.

In an orbiting system the centripetal and centrifugal force are continuously in balance. Otherwise the orbiting object would escape out of its orbit in whatever direction. Due to this balance of forces the potential energy does not play any role anymore in an orbiting system.

3 Intrinsic atomic energy

The total energy $E_K + E_P$ of an orbital system, applying the conventional calculation, would be $-E_K$, given the relation $E_P = -2E_K$. Again an absurd consequence leading to the description below copied from [3]. The wrong words have been scratched out and the correct words, obeying the alternative calculation, written behind them in italics.

"In atoms with a single electron, the energy of an orbital is determined exclusively by n . The $n=1$ orbital has the ~~lowest~~ *highest* possible energy in the atom. Each successively higher value of n has a ~~higher~~ *lower* level of energy, but the difference decreases as n increases *. For high n , the level of energy becomes so ~~high~~ *low* that the electron can easily escape from the atom."

* The examples below show the correctness of this statement by means of the Rydberg expression: $E = hf = hc * R_\infty (1/n_1^2 - 1/n_2^2)$.

n_1	n_2	$1/n_1^2 - 1/n_2^2$
1	2	0,75
5	6	0,012
10	11	0,0017

Conclusions

1 The conventional calculation of potential energy leads to the wrong result $E_{PC} = -GMm/d$ in case of mass M and m at mutual distance d of their centres.

In the alternative calculation $E_{PA} = GMm (1/r_s - 1/d)$, with r_s the sum of the radii of the masses.

2 Applying the conventional calculation the absurd consequence is that the sum of potential and kinetic energy of an atom with a single electron is negative, resulting in an equivalent absurd consequence that the smallest orbit contains the lowest energy.

3 The alternative calculation leads to the conclusion that the energy of a *gravitational* orbital system is only determined by its kinetic energy.

In a Coulomb force orbital system the energy of magnetic fields has to be added. See [4].

References

[1] https://en.wikipedia.org/wiki/Potential_energy

[2] https://en.wikipedia.org/wiki/Escape_velocity

[3] http://en.wikipedia.org/wiki/Atomic_orbital

[4] <https://vixra.org/abs/1505.0225> Why a Photon is not a Particle