Proving of 4 kinds of earthquake clouds caused by an electric dipole and Guess of the epicenter

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1. Abstract

When different solid substances are touched and pressed, it’s good an electron moves to one substance by
the friction, and that plus-minus static electricity does electrification (i.e. contact triboelectric charging).
Thus, solid substances make plus-minus static electricity occur before they come to destruction, and the
other take place electric currents, sound waves (AE), radio waves and ions in addition to an electric dipole.

Plus-minus static electricity with compression of solid substances causes the energy of the electric field,
so that movement of the electrified cloud droplet which floats in the air undergoes the influence, and the belt
of cloud will gather in the stable specific orbit and track like the energy stable. Derivation of earthquake
clouds and method of epicenter guess are investigated from an orbit and Newton equation of motion using
the simplest model in an epicenter here.

2. Fundamental equation of motion

As a calculation model, the horizontal plane x-y face, the vertical
direction z axis and electric dipole moment 2ℓ_oQ sets the angle α
inclines toward the origin of x-y plane. An electric charge of an
electrified cloud droplet is +q.

The wind velocity of the atmosphere: Vx, Vy, Vz and the
air density:  ρ and the cloud droplet radius: r and the gravity: g and the coefficient of
viscosity of the air: c and energy correction: Co .

Newton equation of motion of the cloud droplet in
which mechanical and electrical elements exist, is as
follows about x, y, z direction.

\[
\begin{align*}
m \frac{d^2x}{dt^2} + c \rho r^2 \left( \frac{dx}{dt} - V_x \right) & + \frac{Qq}{4\pi \varepsilon_0 \varepsilon_r} \cdot \frac{\partial}{\partial x} \left( \frac{-1}{\sqrt{(x - \ell_o \cos \alpha)^2 + y^2 + (z - \ell_o \sin \alpha)^2}} + \frac{1}{\sqrt{(x + \ell_o \cos \alpha)^2 + y^2 + (z + \ell_o \sin \alpha)^2}} \right) = 0 \\
m \frac{d^2y}{dt^2} + c \rho r^2 \left( \frac{dy}{dt} - V_y \right) & + \frac{Qq}{4\pi \varepsilon_0 \varepsilon_r} \cdot \frac{\partial}{\partial y} \left( \frac{-1}{\sqrt{(x - \ell_o \cos \alpha)^2 + y^2 + (z - \ell_o \sin \alpha)^2}} + \frac{1}{\sqrt{(x + \ell_o \cos \alpha)^2 + y^2 + (z + \ell_o \sin \alpha)^2}} \right) = 0 \\
m \frac{d^2z}{dt^2} + c \rho r^2 \left( \frac{dz}{dt} - V_z \right) & + \left( m - \frac{4\pi \rho^3 r}{3} \right) g + \frac{\partial}{\partial z} C_o(z) \\
& + \frac{Qq}{4\pi \varepsilon_0 \varepsilon_r} \cdot \frac{\partial}{\partial z} \left( \frac{-1}{\sqrt{(x - \ell_o \cos \alpha)^2 + y^2 + (z - \ell_o \sin \alpha)^2}} + \frac{1}{\sqrt{(x + \ell_o \cos \alpha)^2 + y^2 + (z + \ell_o \sin \alpha)^2}} \right) = 0
\end{align*}
\]
Mentioning of a symbol is simplified as follows from here.

\[ Q = \frac{Qq}{4\pi \varepsilon_0 \varepsilon_r}, \quad G = \left( \frac{m - 4\pi r^3 \rho}{3} \right) g, \quad C = c \rho \pi r^2 \]

\[
\sqrt{\frac{-1}{\sqrt{(x - \ell_o \cos a)^2 + y^2 + (z - \ell_o \sin a)^2} + \sqrt{(x + \ell_o \cos a)^2 + y^2 + (z + \ell_o \sin a)^2}}} + \frac{1}{2} \]

When a quadratic equation about \( \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \) is solved, it is as follows.

\[
\frac{dx}{dt} = -\frac{-Cx + \sqrt{(Cx)^2 + 2m \left( CxV_x - Q \int \frac{\partial \sqrt{}}{\partial x} \ dx + K_x \right)}}{m} \quad \frac{dy}{dt} = -\frac{-Cy + \sqrt{(Cy)^2 + 2m \left( CyV_y - Q \int \frac{\partial \sqrt{}}{\partial y} \ dy + K_y \right)}}{m} \quad \frac{dz}{dt} = -\frac{-Cz + \sqrt{(Cz)^2 + 2m \left( CzV_z - Q \int \frac{\partial \sqrt{}}{\partial z} \ dz + Gz - C_o + K_z \right)}}{m} 
\]

When there are no electric charges in particular, it's \( Q = 0 \), a cloud droplet moves with a wind, so that \( 2mK_x = (mV_x)^2 \), \( 2mK_y = (mV_y)^2 \), a cloud trails along levelly and it's \( CzV_x - Gz - C_o + K_z = 0 \) in the purpose which is \( dz = 0 \). Therefore, fundamental equation is as follows.

\[
\frac{dy}{dx} = -\frac{-Cy + \sqrt{(Cy + mV_y)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial y} \ dy}}{-Cx + \sqrt{(Cx + mV_x)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial x} \ dx}} 
\]

\[
\frac{dz}{dx} = -\frac{-Cz + \sqrt{(Cz)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial z} \ dz}}{-Cx + \sqrt{(Cz + mV_z)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial x} \ dx}} - \frac{-Cx + \sqrt{(Cx + mV_x)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial x} \ dx}}{m} \]

I’ll make \( \frac{dy}{dx}, \frac{dz}{dx} \) above-mentioned coalesced and reduce \( y(x) \) and \( z(x) \), but the analytic solution is difficult to reduce in many cases.

3. The belt of cloud far from the epicenter

When an occurring place of a cloud droplet is far from the epicenter \( (x^2 + y^2 \gg z^2, \ \ell_o \sim) \) as a special analytical calculation, \( \frac{\partial \sqrt{}}{\partial x}, \frac{\partial \sqrt{}}{\partial y}, \text{ and } \frac{\partial \sqrt{}}{\partial z} \) can resemble as follows by series expansion.

\[
\frac{\partial \sqrt{}}{\partial x} = \frac{x - \ell_o \cos a}{\sqrt{\left( (x - \ell_o \cos a)^2 + y^2 + (z - \ell_o \sin a)^2 \right)^{3/2}}} = \frac{x + \ell_o \cos a}{\sqrt{\left( (x + \ell_o \cos a)^2 + y^2 + (z + \ell_o \sin a)^2 \right)^{3/2}}} = \frac{2(2x^2 - y^2)\ell_o \cos a}{(x^2 + y^2)^{5/2}}
\]

\[
\frac{\partial \sqrt{}}{\partial y} = \frac{y}{\sqrt{\left( (x - \ell_o \cos a)^2 + y^2 + (z - \ell_o \sin a)^2 \right)^{3/2}}} = \frac{y}{\sqrt{\left( (x + \ell_o \cos a)^2 + y^2 + (z + \ell_o \sin a)^2 \right)^{3/2}}} = \frac{6xy\ell_o \cos a}{(x^2 + y^2)^{5/2}}
\]

\[
\frac{\partial \sqrt{}}{\partial z} = \frac{z - \ell_o \sin a}{\sqrt{\left( (x - \ell_o \cos a)^2 + y^2 + (z - \ell_o \sin a)^2 \right)^{3/2}}} = \frac{z + \ell_o \sin a}{\sqrt{\left( (x + \ell_o \cos a)^2 + y^2 + (z + \ell_o \sin a)^2 \right)^{3/2}}} = \frac{2\ell_o \sin a}{(x^2 + y^2)^{3/2}}
\]

I transform fundamental differential equation and \( \frac{\partial \sqrt{}}{\partial x}, \frac{\partial \sqrt{}}{\partial y}, \frac{\partial \sqrt{}}{\partial z} \) in \( \sqrt{\cdots} \) omits because it’s minute. It is as follows about \( (x,y) \).

\[
\frac{Cx + \sqrt{(Cx + mV_x)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial x} \ dx}}{mV_x(2Cx + mV_x) - 2mQ \int \frac{\partial \sqrt{}}{\partial x} \ dx} \quad \frac{Cy + \sqrt{(Cy + mV_y)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial y} \ dy}}{mV_y(2Cy + mV_y) - 2mQ \int \frac{\partial \sqrt{}}{\partial y} \ dy}
\]

\[
\frac{Cz + \sqrt{(Cz + mV_z)^2 - 2mQ \int \frac{\partial \sqrt{}}{\partial z} \ dz}}{mV_z(2Cz + mV_z) - 2mQ \int \frac{\partial \sqrt{}}{\partial z} \ dz}
\]

\[2\]
\[ \frac{dy}{dx} = \frac{V_y}{V_x} \quad y = \frac{V_y}{V_x} x + K_y \quad (K_y = 0) \]

The belt of cloud will be generated along the direction of the wind as like this. Also about \((x,z)\),
\[ \frac{dz}{dx} = \frac{-Q}{V_x} \int \frac{\sqrt{-z}}{dz} \quad \text{when it substitutes} \quad \int \frac{\sqrt{-z}}{dz}, \quad \text{It’ll be} \quad \frac{dz}{dx} = \frac{2Q \ell_s \sin \alpha}{CV_x(x^2 + y^2)^{3/2}} \]

When more \(y = \frac{V_y}{V_x} x\) is substituted, it follows \(dz/dx = 2Q \ell_s \sin \alpha / x^3 \left(1 + \frac{\nu_z}{\nu_x}\right)^{3/2} \)

\(C, Q, \ell_s, \alpha, V\) in this equation makes unknown fixed number \(A\).

When the denominator of the system of \(x\) is 0 at \(z_{cr}\), the critical height of the belt of cloud will be as follows \(z_{cr} = 1/Ax^2 + z_i\).

The belt of cloud has the aptitude most develops straight horizontally along the direction of the wind at a distant place.

3.1. Guess of the epicenter by the belt of cloud

The point of 3 cloud droplets on a belt of cloud is made \((u_1, w_1), (u_2, w_2)\) and \((u_3, w_3)\). A difference on the map of the cloud droplet seems to be \(\Delta u_2 = u_2 - u_1, \Delta u_3 = u_3 - u_1, \Delta w_2 = w_2 - w_1, \Delta w_3 = w_3 - w_1\).

\(A = (x^2 - x_1^2) / x^2 \cdot 1 / x_1^2(z - z_1)\) derived by upper equation is common to cloud droplet 1,2,3 and is changed from electric dipole standard \(a\) axis to the belt of cloud standard \(u\) axis.

\[ A = \frac{(x^2 - x_1^2)}{x^2} \cdot \frac{1}{x_1^2(z - z_1)} \quad \Delta u_2(2u_1 + \Delta u_2) = \frac{\Delta u_3(2u_1 + \Delta u_3)}{(u_1 + \Delta u_1)^2\Delta w_3} \]

About the epicenter where this is unknown quantity \(u_i\), it’s a cubic equation and solved like algebra using the formula of Cardano.

\[ 2u_1^3 \left[\Delta u_2\Delta w_3 - \Delta u_3\Delta w_2\right] + u_1^2 \left[\Delta u_2\Delta w_3(4\Delta u_3 + \Delta u_2) - \Delta u_3\Delta w_2(4\Delta u_2 + \Delta u_3)\right] + 2u_1 \left[\Delta u_2\Delta u_3 \left(\Delta u_2 + \Delta u_3\right)\left(\Delta w_3 - \Delta w_2\right)\right] + \Delta u_2^2 \Delta u_3^2 \left(\Delta w_3 - \Delta w_2\right) = 0 \]

4. The occasion near the epicenter

When an occurring place of a cloud droplet is near the epicenter \((z^2 + \ell_o^2) \gg x^2, y^2\), \(\partial \sqrt{-z} / \partial x, \partial \sqrt{-z} / \partial y\) can resemble as follows and I fix on the plane of \(z = z_0\) and solve differential equation in a simple way.

\[ \frac{\sqrt{-z}}{dx} = \frac{2\ell_o}{x^2 + \ell_o^2} \left[3xz \sin \alpha - (z^2 + \ell_o^2) \cos \alpha\right] \]

\[ \frac{\sqrt{-z}}{dy} = \frac{6\ell_o y [z \sin \alpha + x \cos \alpha]}{\left[x^2 + \ell_o^2\right]^{5/2}} \]

\[ \int \frac{\sqrt{-z}}{dx} dx = \frac{\ell_o}{x^2 + \ell_o^2} \left[3xz \sin \alpha - 2(z^2 + \ell_o^2)x \cos \alpha\right] \]

\[ \int \frac{\sqrt{-z}}{dy} dy = \frac{3\ell_o y^2 z \sin \alpha}{\left[x^2 + \ell_o^2\right]^{5/2}} \]
When the special system is substituted for fundamental differential equation, an orbit of the belt of cloud near the epicenter is as follows.

\[
\frac{dy}{dx} = \frac{-Cy + \sqrt{\left(Cy + mV_y\right)^2 - 6mQ\ell_o y^2z_o\sin\alpha \left[z_o^2 + \ell_o^2\right]^{3/2}}}{-Cx + \sqrt{\left(Cx + mV_x\right)^2 - 2mQ\ell_o \left[3x^2z_o\sin\alpha - 2\left(z_o^2 + \ell_o^2\right)x\cos\alpha\right] \left[z_o^2 + \ell_o^2\right]^{3/2}}}
\]

4.1. The spiral of cloud near the epicenter (when it's windless)

In case of \( V_x = V_y = 0 \), differential equation is as follows.

\[
2mQ\ell_o \left[z_o^2 + \ell_o^2\right]^{3/2} dy = y\left[-C + \sqrt{\frac{C^2 - 6mQ\ell_o z_o \sin\alpha \left[z_o^2 + \ell_o^2\right]^{1/2}}{2 \left(z_o^2 + \ell_o^2\right) \cos\alpha - 3xz_o \sin\alpha}}\right] dx
\]

When it’s integrated about \( dx \) and \( dy \) using the integration formula(2), it is as follows.

\[
-\frac{2}{3z_o \sin\alpha} \log \left(\frac{\left(z_o^2 + \ell_o^2\right) \cos\alpha - 3xz_o \sin\alpha}{2}\right)
\]

\[
+ \frac{1}{\left(z_o^2 + \ell_o^2\right) \cos\alpha} \left(x \left(C^2x + \frac{4mQ\ell_o \cos\alpha}{\left[z_o^2 + \ell_o^2\right]^{3/2}}\right) + \frac{4mQ\ell_o \cos\alpha}{C^2} \log \left(\frac{\left(C^2x + \frac{4mQ\ell_o \cos\alpha}{\left[z_o^2 + \ell_o^2\right]^{3/2}}\right) + \sqrt{C^2x}}{\left(z_o^2 + \ell_o^2\right) \cos\alpha}\right)\right)
\]

When the boundary condition is \( y = y_o \) at \( x = 0 \), it is as follows.

\[
-\frac{\left(z_o^2 + \ell_o^2\right)^{3/2}}{6mQ\ell_o \sin\alpha} \log \left(\frac{2\left(z_o^2 + \ell_o^2\right) \cos\alpha - 3xz_o \sin\alpha}{2 \left(z_o^2 + \ell_o^2\right) \cos\alpha}\right)
\]

\[
+ \frac{1}{C} \log \sqrt{\frac{C^2x + \frac{4mQ\ell_o \cos\alpha}{\left[z_o^2 + \ell_o^2\right]^{3/2}} + \sqrt{C^2x}}{\left(z_o^2 + \ell_o^2\right)} + \frac{4mQ\ell_o \cos\alpha}{\left[z_o^2 + \ell_o^2\right]^{3/2}}\right)
\]

\[
- \frac{1}{C} + \sqrt{\frac{\left(z_o^2 + \ell_o^2\right)^{3/2}}{\left(z_o^2 + \ell_o^2\right)^{3/2}} - \frac{6mQ\ell_o \sin\alpha}{\left[z_o^2 + \ell_o^2\right]^{3/2}}} \log \left(\frac{y}{y_o}\right)
\]

I can obtain the x coordinate at \( y = 0 \) when a numerator of the log function in the initial term is 0. \( x = 2\left(z_o^2 + \ell_o^2\right) \cos\alpha / 3z_o \sin\alpha \).

Therefore, the curved line is a semi-oval closed circle on \( (x, y) \) plane and is the spiral curved line which expands upward as a whole.
4.2. Multi-belts and two belts of cloud near the epicenter (when it's windy, $V_x \neq 0$ and $V_y = 0$)

I do series development of the coefficient of fundamental equation $dx$ and substitute $\int \frac{2n}{dx} dx$.

$$\frac{dx}{-C x + (C x + m V_x)^2} = \int \frac{m Q}{(C x + m V_x)^2} \frac{d\sqrt{\frac{3}{m Q}}} {dx}$$

$$\int \frac{dx}{C x + (C x + m V_x)^2} [1 - \frac{m Q}{(C x + m V_x)^2} \frac{d\sqrt{\frac{3}{m Q}}}{dx}] = \frac{m Q}{(C x + m V_x)^2} \frac{d\sqrt{\frac{3}{m Q}}}{dx}$$

Symbols A,B,C,D,E used in the integration formula are utilized to integrate this system and it's a simple way.

$$\int \frac{Dx + E}{Ax^2 + Bx + C} dx = \frac{D}{2A} \log(A x^2 + B x + C) + \left( E - \frac{DB}{2A} \right) \frac{2}{\sqrt{4AC - B^2}} \left( \frac{2Ax + B}{\sqrt{4AC - B^2}} + n\pi \right) \text{ in case of } 4AC - B^2 > 0$$

$$\int \frac{Dx + E}{Ax^2 + Bx + C} dx = \frac{D}{2A} \log(A x^2 + B x + C) + \left( E - \frac{DB}{2A} \right) \frac{1}{\sqrt{B^2 - 4AC}} \log\left( \frac{2Ax + B - \sqrt{B^2 - 4AC}}{2Ax + B + \sqrt{B^2 - 4AC}} \right) \text{ in case of } 4AC - B^2 < 0$$

On the other hand, it is as follow using above symbols A,D about dy of fundamental equation.

$$dy = \frac{2Cy}{-2mQ \int \frac{dy}{dy}}$$

Consequently, in case of $4AC - B^2 > 0$

$$y^2 = k (Ax^2 + Bx + C) \cdot \exp\left[ \frac{2(2AE - BD)}{D\sqrt{4AC - B^2}} \left( \arctan \frac{2Ax + B}{\sqrt{4AC - B^2}} + n\pi \right) \right]$$

At the same $x$, the value $y$ which changes to be conform to $n\pi$ and $n = \pm 0, 1, 2, 3$ generates a lot by geometric progression of

$$\exp\left[ \frac{2(2AE - BD)}{D\sqrt{4AC - B^2}} n\pi \right]$$. In other words, multi-belts of cloud occur.

In case of $4AC - B^2 < 0$

$$y^2 = k (Ax^2 + Bx + C) \left( \frac{2Ax + B + \sqrt{B^2 - 4AC}}{2Ax + B - \sqrt{B^2 - 4AC}} \right) \frac{2AE - BD}{D\sqrt{B^2 - 4AC}}$$

Two belts of cloud occur.

By means of discriminant $4AC - B^2$, it seems not exactly but roughly that multi-belts of cloud occur in condition of angle $\alpha > 0$ and $Q < 0$, and that two belts of cloud occur in condition of angle $\alpha > 0$ and $Q > 0$.

4.3. Multi-belts and two belts of cloud near the epicenter (when it's windy, $V_x = 0$ and $V_y \neq 0$)

In the same way with the preceding section, fundamental differential equation is transformed and solved as follows in case of $V_x = 0$, $V_y \neq 0$.

$$\frac{C dx}{-3m Q \ell \bar{z}_o \bar{sin}^2 \left[ \bar{z}_o^2 + \ell_o^2 \right]^{3/2}} = \frac{(Cy + m V_y) dy}{-3m Q \ell \bar{z}_o \bar{sin}^2 \left[ \bar{z}_o^2 + \ell_o^2 \right]^{3/2}}$$
In case of \(4AC - B^2 > 0\)
\[
\left( x - 2(z_o^2 + \ell_o^2) / 3 \tan \alpha \right)^2 = K (A y^2 + B y + C) \exp \left( \frac{2(2AE - BD)}{D \sqrt{4AC - B^2}} \left( \arctan \frac{2Ay + B}{\sqrt{4AC - B^2}} + n \pi \right) \right)
\]

At the same \(y\), the value \(x\) which changes to be conform to \(n \pi\) and \(n = \pm 0, 1, 2, 3\) generates a lot by geometric progression of \(\exp \left( \frac{2(2AE - BD)}{D \sqrt{4AC - B^2}} n \pi \right)\). In other words, multi-belts of cloud occur.

In case of \(4AC - B^2 < 0\)
\[
\left( x - 2(z_o^2 + \ell_o^2) / 3 \tan \alpha \right)^2 = K (A y^2 + B y + C) \cdot \left( \frac{2Ay + B - \sqrt{B^2 - 4AC}}{2Ay + B + \sqrt{B^2 - 4AC}} \right) \exp \left( \frac{2AE - BD}{D \sqrt{B^2 - 4AC}} \right)
\]

Two belts of cloud occur.

By means of discriminant \(4AC - B^2\), it seems not exactly but roughly that multi-belts of cloud occur in condition of angle \(\alpha > 0\) and \(Q < 0\) and that two belts of cloud occur in condition of angle \(\alpha > 0\) and \(Q > 0\).

4.4. Multi-belts and two belts of cloud near the epicenter (when it's windy, \(V_x \neq 0\) and \(V_y \neq 0\))

In the same way with the preceding section, fundamental differential equation is transformed and solved as follows in case of \(V_x \neq 0\), \(V_y \neq 0\).

In case of \(4AC - B^2 > 0\)
\[
\frac{D}{2A} \log (Ax^2 + Bx + C) \left( E_x - \frac{DB_x}{2A} \right) \left( \frac{2Ax + Bx - \sqrt{B_x^2 - 4AC_x}}{2Ax + Bx + \sqrt{B_x^2 - 4AC_x}} \right) \left( \frac{2Ay + By + \sqrt{B_y^2 - 4AC_y}}{2Ay + By - \sqrt{B_y^2 - 4AC_y}} \right) + K
\]

A and D are common to \(x, y\) differential equation. \(n_x\) and \(n_y\) appear because multi-belts of cloud which incline toward \(x\) axis and \(y\) axis is being judged vertically from \(x\) axis and \(y\) axis.

In case of \(4AC - B^2 < 0\)
\[
\frac{D}{2A} \log (Ax^2 + Bx + C) \left( E_x - \frac{DB_x}{2A} \right) \left( \frac{2Ax + Bx - \sqrt{B_x^2 - 4AC_x}}{2Ax + Bx + \sqrt{B_x^2 - 4AC_x}} \right) \left( \frac{2Ay + By + \sqrt{B_y^2 - 4AC_y}}{2Ay + By - \sqrt{B_y^2 - 4AC_y}} \right) + K
\]

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