

Formulas of Feigenbaum Constants and Their Physical Meanings

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper supposes that Feigenbaum constants should be rational numbers in the world of nuclides, gives their formulas in fractional number format and exhibits the physical meanings of the factors in the formulas, especially their relationships with nuclides, the fine-structure constant and 2π . This paper also supposes that there would be the third Feigenbaum constant and gives its two possible approximate values.

Keywords: Feigenbaum constants; the fine-structure constant; nuclides.

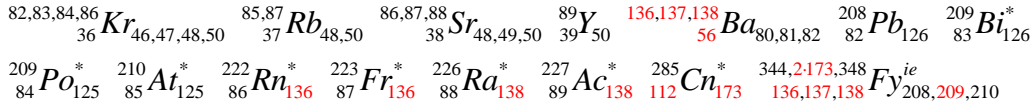
1. Introduction

Feigenbaum constants are characterizing constants in chaotic systems, it is assumed by scientific community that they would characterize chaos just as 2π stands for periodicity. The nuclei of nuclides except proton should be multiple-body systems and hence chaos should be the common state in the world of nuclei. So Feigenbaum constants should express some functions in nuclides.

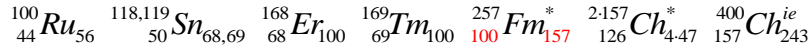
There are two Feigenbaum constants, i. e., $\delta=4,6692\dots$ and $\alpha=2.5029\dots$, and mathematically they should be irrational numbers with infinite digits.

In our previous papers^{1,2,3}, we had already given formulas of the fine-structure constant and given two values of it, i. e., $\alpha_1=1/137.035999037435$ and $\alpha_2=1/137.035999111818$ which are rational numbers with 15 digits. The values and formulas of the fine-structure constant show strong relationships with nuclides, and hereby some relevant examples of these relationships are listed as follows. And it

shows that the values of the fine-structure constant express as integer numbers of 136, 137 and 138 in the world of nuclides.



In our previous paper^{1,2,3}, we also exhibited the relationships of 2π with nuclides, for example, $2\pi \approx 6.28 = (4 \times 157)/100$ relates to nuclides as follows.



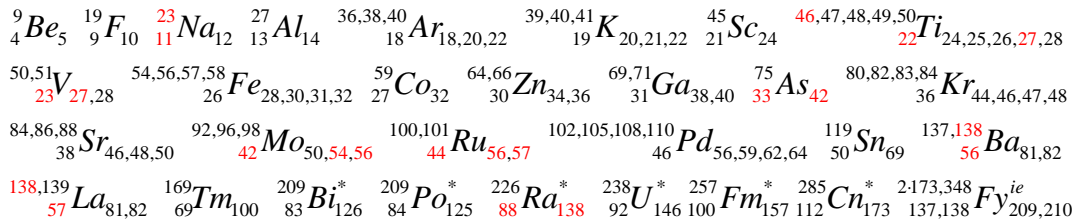
In this paper, we suppose Feigenbaum constants should also be rational numbers in the world of nuclides, give their formulas and exhibit their relationships with nuclides and hence with the fine-structure constant and 2π .

2. Formulas of Feigenbaum Constants in Fractional Numbers

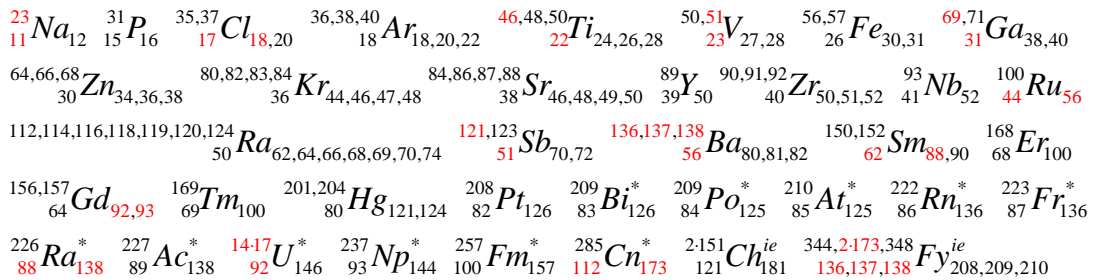
It is supposed that Feigenbaum constants should be rational numbers with 15 digits and have the following fractional formulas and relationships with nuclides.

Note: $136=8 \cdot 17$, $138=6 \cdot 23$

$$\begin{aligned}
 \frac{1}{\delta} &= \frac{1}{4.66920160910299} = 0.214169377062326 \\
 &= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1)} + \frac{2 \cdot 23}{3 \cdot 19}
 \end{aligned}$$



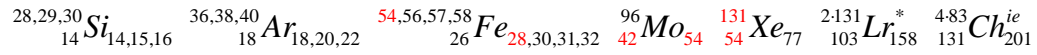
$$\begin{aligned}
 \frac{1}{\alpha} &= \frac{1}{2.50290787509589} = 0.399535280523135 \\
 &= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)}
 \end{aligned}$$



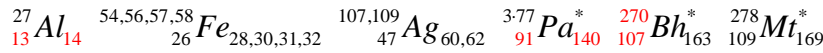
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3. Formulas of Feigenbaum Constants in Continued Fractional Numbers

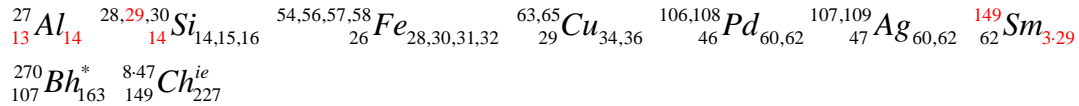
$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{1}{131 + \cfrac{1}{2 + \cfrac{1}{54 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{18}}}}}}}}$$



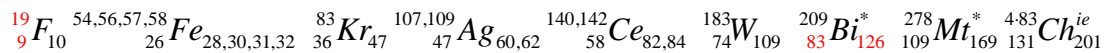
$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{1}{131 + \cfrac{1}{2 + \cfrac{1}{54 + \cfrac{1}{6 + \cfrac{7 \cdot 13}{109}}}}}}$$



$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{1}{131 + \cfrac{1}{2 + \cfrac{5 \cdot 149}{13 \cdot 29 \cdot 107}}}}$$



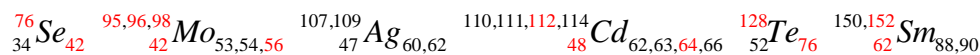
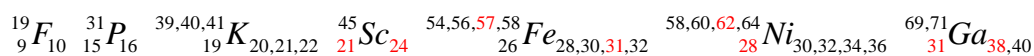
$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{9 \cdot 83 \cdot 109}{64 \cdot 7 \cdot (6 \cdot 7 \cdot (6 \cdot 5 \cdot 19 - 1) + 1)}}$$



$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326$$

$$= \frac{64 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 5 \cdot 19 - 1) + 1)}{(128 - 1) \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot 31 \cdot (4 \cdot 227 - 1) - 1))}$$

$$= \frac{64 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 5 \cdot 19 - 1) + 1)}{(128 - 1) \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot 31 \cdot (2 \cdot 3 \cdot (8 \cdot 19 - 1) + 1) - 1))} = \frac{32120256}{149975951}$$



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$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1 + \frac{1}{1 + \frac{1}{15}}}}}}}}}}}}}}$$

$$= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1 + \frac{1}{1 + \frac{1}{31}}}}}}}}}}}}}}$$

^{14,15}₇N_{7,8} ^{16,17,18}₈O_{8,9,10} ³¹₁₅P₁₆ ^{58,60,62}₂₈Ni_{30,32,34} ^{69,71}₃₁Ga_{38,40} ^{110,112,113,114}₄₈Cd_{62,64,65,68}
^{168,170}₆₈Er_{100,102} ^{170,171,172,173}₇₀Yb_{100,101,102,103} ²⁸¹₁₁₀Ds*₁₇₁ ²⁸⁵₁₁₂Cn*₁₇₃ ^{4-71,286}₁₁₃Nh^{ie}_{171,173}

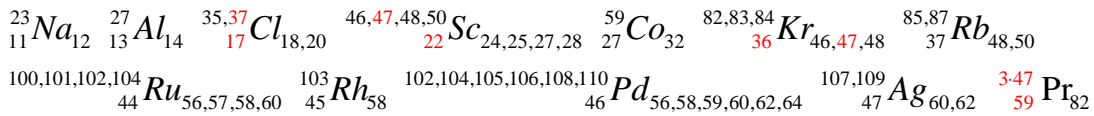
$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{8 \cdot 9 \cdot 5}}}}}}}}}}}}$$

4

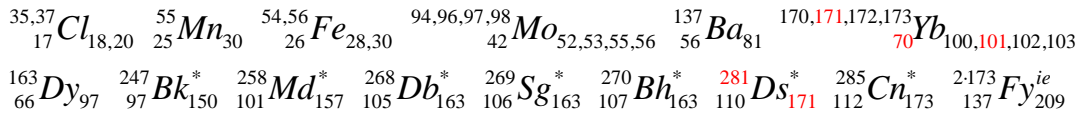
$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{1}{1 + \frac{1}{4 + \frac{11 \cdot 37}{13 \cdot 59}}}}}}}}$$



$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

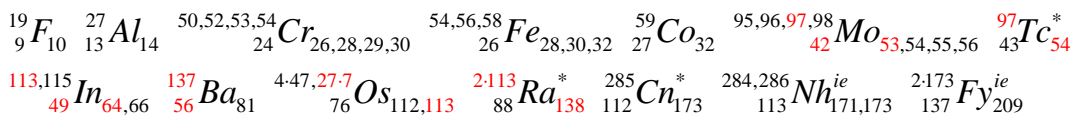
$$= \frac{2}{5 + \frac{1}{171 + \frac{1}{1 + \frac{1}{17 + \frac{25 \cdot (4 \cdot 5 \cdot 7 - 1)}{2 \cdot 3 \cdot 7 \cdot 101}}}}} = \frac{2}{5 + \frac{1}{171 + \frac{269 \cdot 281}{97 \cdot (6 \cdot 137 + 1)}}$$



$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

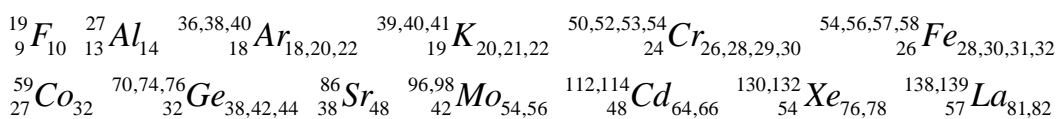
$$= \frac{2}{5 + \frac{97 \cdot (6 \cdot 137 + 1)}{2 \cdot 5 \cdot (2 \cdot 113 + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}} = \frac{2}{5 + \frac{(2 \cdot 49 - 1) \cdot (6 \cdot 137 + 1)}{2 \cdot 5 \cdot (2 \cdot 113 + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}}$$

$$= \frac{4 \cdot 5 \cdot (2 \cdot 113 + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}{9 \cdot 7 \cdot 13 \cdot 53 \cdot (2 \cdot 7 \cdot 113 + 1)} = \frac{27453380}{68713281}$$



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$$\frac{1}{\alpha} = \frac{4 \cdot 5 \cdot (2 \cdot (2 \cdot 3 \cdot 19 - 1) + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}{9 \cdot 7 \cdot 13 \cdot (2 \cdot 27 - 1) \cdot (2 \cdot 7 \cdot (2 \cdot 3 \cdot 19 - 1) + 1)} = \frac{27453380}{68713281}$$



2021/1/28

4. The Third Feigenbaum Constant

There would be the third Feigenbaum constant γ , and we could only give some guesses at present stage as follows.

Note: $136=8 \cdot 17$, $138=6 \cdot 23$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135 = f(2, 3, 17, 23, 11, \dots)$$

$$= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)}$$

$$= \frac{2}{5 + \frac{1}{171 + \frac{269 \cdot 281}{97 \cdot (6 \cdot 137 + 1)}}}$$

$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = f(2, 3, 23, 11, \dots)$$

$$= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1)} + \frac{2 \cdot 23}{3 \cdot 19}$$

$$= \frac{3}{14 + \frac{1}{131 + \frac{13 \cdot 29 \cdot 107}{9 \cdot 83 \cdot 109}}}$$

$$\frac{1}{\gamma} = f(2, 3, 17, \dots) \approx 1 - \frac{1}{3} + \frac{1}{8 \cdot 17} = \frac{25 \cdot 11}{8 \cdot 3 \cdot 17} \approx \frac{2}{3 - \frac{1}{31 + \frac{1}{2}}} = \frac{2 \cdot 9 \cdot 7}{11 \cdot 17} \approx \frac{1}{1.484}$$

or

$$\frac{1}{\gamma} = f(2, 3, 17, \dots) \approx \frac{1}{8} - \frac{1}{81} + \frac{1}{8 \cdot 3 \cdot 17} = \frac{317}{2 \cdot 81 \cdot 17} \approx \frac{3}{26 + \frac{1}{15}} = \frac{9 \cdot 5}{17 \cdot 23} \approx \frac{1}{8.689}$$

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$$\frac{1}{\gamma} = f(2, 3, 17, \dots) \approx \frac{1}{8} - \frac{1}{101} = \frac{3 \cdot 31}{8 \cdot 101} \approx \frac{3}{26 + \frac{1}{151}} = \frac{151}{7 \cdot 11 \cdot 17} \approx \frac{1}{8.6689}$$

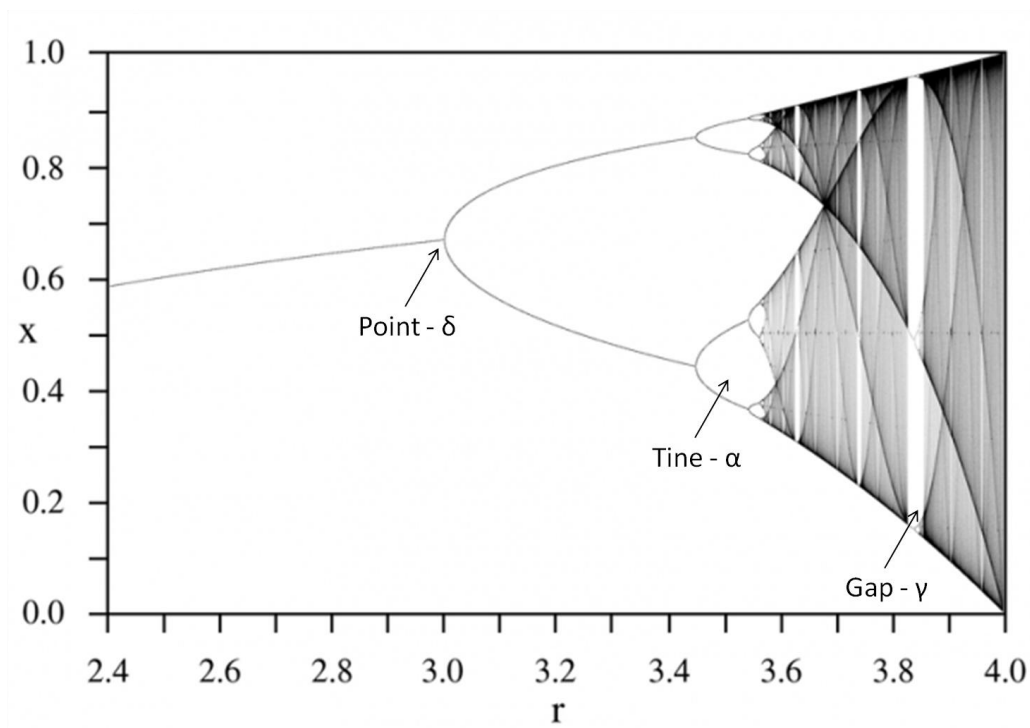
$$\frac{1}{\gamma} = f(2, 3, 17, \dots) \approx \frac{1}{8} - \frac{1}{103} = \frac{5 \cdot 19}{8 \cdot 103} \approx \frac{3}{26 + \frac{1}{157}} = \frac{157}{6 \cdot 227 - 1} \approx \frac{1}{8.6688}$$

2021/1/30-31

Formula of α in fractional number format contains the factors of 2, 3, 17, 23 and 11, Formula of δ in fractional number format contains the factors of 2, 3, 23 and 11, so it is supposed that there would be the third Feigenbaum constant γ which should relate to factors of 2, 3, 17 and 11.

5. Bifurcation Diagram and Feigenbaum Constants

There are three features in a typical bifurcation diagram, which are “point”, “tine” and “gap”. These three features should correspond to three Feigenbaum constants δ , α and γ (**Fig. 1**), so there should be three Feigenbaum constants.



Bifurcation Diagram and Three Feigenbaum Constants

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2021/1/28-30

Fig. 1

References:

1. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2002.0203.
2. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2008.0020.
3. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2012.0107.

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Appendix I: Research History

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