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Defining Arrow of Time at the start of Inflation by expansion of Entropy in a Taylor series and examining initial conditions

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Abstract:

First we do a Taylor series expansion of Entropy. Afterwards we define the arrow of time. After that we define what terms we will analyze in the Taylor series expansion of entropy to help in finding initial conditions which may allow for the earliest possible identification of the Arrow of Time in cosmology. Definition of the arrow of time will allow choosing different initial starting points. That is, that in the actual equations of classical GR, there is no reason to have time asymmetry after given initial conditions. Time asymmetry is built into initial conditions and we start to explore which initial conditions may assist in evaluating contributions to Entropy via an analysis of which terms in a Taylor series survive, and what their sign and contribution values are

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1. Introduction. Concerning the arrow of time and initial conditions in cosmology

In Cosmology, there is one outstanding datum, which is that in classical GR, outside of the initial conditions of the beginning of space-time, there is in reality no reason for times arrow. We will introduce times arrow, in the context of cosmology via initial conditions. We look at a Taylor series expansion of entropy and the relative import of terms in the series expansion in order to delineate if conditions for an arrow of time being defined as early as possible in cosmology are possible.. These evaluation of terms I the Taylor series expansion of entropy will be brought up in terms of the initial conditions of the arrow of time, which we maintain should be in fidelity to the t'Hooft article's caution as to initial conditions.

1a. Look first at a Taylor series expansion of Entropy.

Doing this in terms of energy leads to

$$S(E)=S(\Delta E)+(E-\Delta E)\left.\frac{dS(E)}{dE}\right|_{E=\Delta E}+\frac{(E-\Delta E)^2}{2}\left.\frac{d^2S(E)}{dE^2}\right|_{E=\Delta E}+H.O.T. \qquad (1)$$

Our analysis will be using the following, i.e. we declare an arrow of time, as we define in the next section will exist if, assuming the Higher order terms are neglectible for now

$$(E-\Delta E)\left.\frac{dS(E)}{dE}\right|_{E=\Delta E}+\frac{(E-\Delta E)^2}{2}\left.\frac{d^2S(E)}{dE^2}\right|_{E=\Delta E}\geq 0 \qquad (2)$$

We now supecify the early universe, which makes what we are doing a linkage to time,i.e.

We pick Entropy as represented by an energy term E, for the following reason[1][2][3]

Shalyt-Margolin and Tregubovich (2004, p.73)[1], Shalyt-Margolin (2005, p.62)[2][3]

$$\Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_P^2 \frac{\Delta E}{\hbar} \Rightarrow \left(\Delta E\right)^2 - \frac{\hbar \Delta t}{\gamma t_P^2}\left(\Delta E\right)^1 + \frac{\hbar^2}{\gamma t_P^2} = 0$$

$$\Rightarrow \Delta E = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 + \sqrt{1 - \frac{4\hbar^2}{\gamma t_P^2 \cdot \left(\frac{\hbar \Delta t}{2\gamma t_P^2}\right)^2}} \right) = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \sqrt{1 - \frac{16\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2}} \right) \quad (3)$$

For sufficiently small γ . The above could be represented by[3]

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \left(1 - \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2} \right) \right)$$

$$\Rightarrow \Delta E \approx either \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2}, or \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(2 - \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2} \right) \quad (4)$$

This would lead to a minimal relationship between change in E and change in time as

represented by Eq. (4), so that we could to first order, say be looking at something very close to the traditional Heisenberg uncertainty principle results of approximately

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{\left(\hbar \Delta t\right)^2} \equiv \frac{4\hbar}{\Delta t} \quad (5)$$

Or

$$\Delta E \Delta t \approx 4\hbar \quad (6)$$

Assuming that we are using Eq. (2) to define the genesis of an arrow of time, we by Eq.(2) and Eq.(6) could be defining a necessary condition for the start of an arrow of time. So first we state some particular constraints on the arrow of time, and then go to our jcorresponding Entropy expressions in cosmology as defined by using the results of [4], page 47 for a Rindler space representation of entropy density of say massless bosons in "low dimensions " as

$$\frac{S}{L} = \frac{\pi}{3} T \quad (7)$$

Where S is entropy, L is a length, specified for a space-time lattics, and T is the temperature, wheras we use the following[5] for energy, E and Temperature

$$E = \frac{d(space-time)k_B T}{2} \qquad (8)$$

If, say we use Eq.(6), Eq. (7) and Eq. (8) , we could write, say the following for a hoson "gas"

$$S = \frac{2\pi LE}{3d(space-time)k_B} \qquad (9)$$

If so then, to first order, we have for an arrow of time, the situation where

$$\begin{aligned}
&(E-\Delta E)\left.\frac{dS(E)}{dE}\right|_{E=\Delta E} + \frac{(E-\Delta E)^2}{2}\left.\frac{d^2S(E)}{dE^2}\right|_{E=\Delta E} \\
&\approx (E-\Delta E)\frac{2\pi L}{3d(space-time)k_B} \qquad (10) \\
&\approx \frac{L(T-\Delta T)}{3} \geq 0, iff \quad T \geq \Delta T
\end{aligned}$$

This is for 2 dimensional space-time where we can presume L approximately a Planck Length, and time T proportinal to energy E, due to

Oops. I.e. this is saying that the initial temperature T, would have to be in an initial space-time lattice greater than the change in temperatures, afterwards. For forming an arrow of time. It gets worse, taking Eq. (10) and isolating the time step factor, according to [4] we are looking at for an arrow of time, the situation for which we have if we employ Eq.(8) for energy

$$\frac{\hbar}{t} \geq \frac{\hbar}{\Delta t} \Rightarrow \Delta t \geq t \qquad (11)$$

If t is initial time, then what this is saying is that the change in time from the initial time would have to be greater than the initial time. i.e. this seems to be specifying a one way increase in time. That may be sufficient for saying we have an arrow of entropy. But it means that we would likely have to think of t, in Eq.(11) as a minimum time step.

If we are higher than 2 spatial dimensions, it is still very likely we will be looking at the increase in time stepping to be given by a higher dimensional analogue to Eq. (11) above

How likely would this be in terms of early universe dynamics ? Before we go there we should review what is known about the arrow of time, and initial conditions

1b. generic arrow of time defined with heuristics

First of all consider the quote given by Eddington which states some of the problem

Let us draw an arrow arbitrarily. If as we follow the arrow we find more and more of the random element in the state of the world, then the arrow is pointing towards the future, if the random element decreases the arrow points towards the past. That is the only distinction known to physics. This follows at once if our fundamental contention is admitted that the introduction of randomness is the only thing which cannot be undone. I shall use the phrase 'time's arrow' to express this one-way property of time which has no analogue in space [5].

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In a word we have that the entire discussion of entropy, its production and all that start with the 2nd law of thermodynamics [5], which we can simply state as

$$\frac{dS(entropy)}{dt} \geq 0 \qquad (13)$$

Whereas the question raised, in [5] can be rendered in the following.

This law is certainly not symmetric in time; if we interchanged past and future the entropy would tend to decrease. How did we get, from basic reversible equations to a manifestly irreversible result?.

As a given, we may consider what it takes to form initial conditions. One thought to keep in mind is that we will be, when establishing an order of time be affected, as brought up by t'Hooft [6]:

If we adhere to the quantum mechanical description of all microscopical dynamical laws, we find the CPT theorem on our way, which implies that if we combine time reversal T with parity reversal P and particle-antiparticle interchange C, then this symmetry is perfect. We could well stick to our verdict that Nature's boundary conditions in the time direction suffice to explain the arrow of time.

In a word, we get times ARROW of time, going back to the ideas of Eddington [5], and [5] as a consequence of how we choose the initial conditions. To do so we first of all start with the initial

2. Methods, here we will be examining the different cosmological models and their relations to items given above

At the moment of the **Big Bang**, almost all of the **entropy** was due to radiation, and the total **entropy of the Universe** was about $S = 10^{88} k_B$. Or slightly higher

There was a sea of particles, including matter, antimatter, gluons, neutrinos and photons, all around at energies billions of times higher than what the LHC can obtain today. There were so many of them -- perhaps 10^90 in total. If there was a traditional model of the big bang and inflation [7]

$$S \sim 3 \frac{m_{Plank}^2 \left[H = 1.66 \cdot \sqrt{\tilde{g}_*} \cdot T^2 / m_{planck} \cdot \right]^2}{T} \sim 3 \cdot \left[1.66 \cdot \sqrt{\tilde{g}_*} \right]^2 T^3 \qquad (14)$$

If we have a beach ball sized "universe" at the end of the inflationary era, with say temperature of T proportional to Planck temperature, of T 1.416785(71) $\times 10^{32}$ kelvin we can approach $S = 10^{88} k_B$ On the other hand, we may have a value slightly larger. Is this due to thermal versus particle generation? If there was a traditional model of the big bang and inflation[7] We will then have the situation which has Eq. (14) holding due to superhot Planckian temperatures holding where we also would have g_* being the initial degrees of freedom which according to Kolb and Turner[8]would take the value of about 100 to 120,

To measure entropy in cosmology we can count photons. If the number of photons in a given Volume is N, then the entropy of that volume is S ~ kN where k is Boltzmann' s constant.

Is there a way before the generation of the CMBR to do the same thing in terms of a counting procedure, like S ~ kN , with N a number or count of "particles" in order to compliment Eq. (14) above ? Any such attempt would have to adhere to the following outline for an arrow of time

In order to have the value of the increasing onset of the entropy we would like to have the following, namely by using Eq. (1) we would assert a causal ordering following the given values of:

$$S + \Delta S \approx n + \Delta n \geq n \quad iff \quad t + \Delta t \geq t \qquad \textbf{(15)}$$

Note that Y. Jack Ng. has [9] , from a very different stand point derived $S \sim n$ based upon string theory derived ideas , with n a 'particle' count , which in Y. Jack Ng's procedure is based upon the number of dark matter candidates in a given region of phase space. Y. Jack Ng's idea was partly based upon the idea of quantum 'infinite ' statistics, and a partition function [9]

2a. What about breaking up of initial black holes, right after the birth of a new universe?

In [10], there is a reference to the destruction of primordial black holes which is given as when the density of universe climbs to a value given as $\omega_Q = p_Q / \rho_Q$ is defined, with the numerator being the pressure, and denominator density of phantom fields. which leads to by [10] a density for which there is breakup of primordial black holes

$$\rho_{BH} \approx M_p^4 \cdot \left(\frac{M_p^2}{M^2} \right) \cdot \left(\frac{3}{32\pi} \right) \cdot \frac{1}{\left| 1 + \omega_Q \right|} \qquad \textbf{(16)}$$

If the black holes being broken up lead to particle generation, which could then feed into writing say

$$S_{bounce} \approx n_Q = Gravitons - from - black - holes \qquad \textbf{(17)}$$

The problem would then be to delineate conditions for which the Eq.(16) would lead from a low to a high entropy build up, which would require a lot of computer simulation work to ascertain, but it may, if done carefully yield conditions as to the causal conditions for creation of an arrow of time;. The problem would be then to ascertain if and when the causal conditions lead to the density of the Universe yielding a value say of the order of magnitude of Eq.(16) above

Keep in mind that according to[11] Khlopov, has the following for black hole density, namely

$$\rho_{BH} \approx \frac{M}{\left(r_g = 2GM / c^2 \right)^3} \equiv \frac{c^6}{8G^3 M^2} \qquad \textbf{(18)}$$

Here, M is the presumed mass of a black hole, and the result is counter intuitive to say the least, as r_g is the mass of the configuration with mass M

We state that in this situation we have that there may be

$$S_{gravitons} \approx n_{gravitons} \propto S_{Thermal} \approx T^3_{thermal-temp} \qquad (19)$$

But this depends upon having

$$\rho_{BH} \approx \frac{c^6}{8G^3 M^2} \approx \frac{3M_P^6}{32\pi M^2 \left| 1 + \omega_Q \right|} \qquad \textbf{(20)}$$

If we use $\left| 1 + \omega_Q \right| \approx \frac{3}{4\pi}$ and $M_P = G = c = 1$, we have a $\omega_Q \approx -\left(\frac{4\pi - 3}{4\pi} \right)$ so that then pressure and density are approximate negative values of each other, which is implying the following. i.e., The cosmological constant has negative pressure, but positive energy. The negative pressure ensures that as the volume expands then matter loses energy (photons get red shifted, particles slow down); this loss of energy by matter causes the expansion to slow down - but the increase in energy of the increased volume is more important. The increase of energy associated with the extra space the cosmological constant fills has to be balanced by a decrease in the gravitational energy of the expansion - and this expansion energy is negative, allowing the universe to carry on expanding.

3. COMPARING TIMES ARROW as being created by a threshold information release criterion as compared to Seth Lloyd's linkage of entropy and bits of information

Seth Lloyd in 1999 [12] obtained the following and this is to a certain degree duplicated in our work but it has limitations.

A way to obtain traces of information exchange, from prior to present universe cycles is finding linkage between information and entropy. If such a parameterization can be found and analyzed, then Seth Lloyd's [12] shorthand for entropy,

$$I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} = [\rho \cdot c^5 \cdot t^4 / \hbar]^{3/4} \quad (21)$$

could be utilized as a way to represent information which can be transferred from a prior to the present universe. The question to ask, if does Eq. (21) permit a linkage of gravitons as information carriers, and can there be a linkage of information, in terms of the appearance of gravitons in the time interval of, say $0 < t < t_{Planck}$ either by vacuum nucleation of gravitons / information packets Oops. What is the problem? No special initial conditions as specified by 'tHooft in [6] in the setup of an initial arrow of time configuration. Eq. (21) is completely general, and does not tie in with also how we can have a satisfaction as to Eq. (16) given above

4. Conclusion.

It is a much harder problem than what most physics people think that of satisfying all of **the arrow of times** constituent parts. In the 1980s, Hawking [13] in his 1985 in his paper specifically also added a continually expanding volume of space-time as a reset of initial conditions for an arrow of time . However,in the Hawking problem, we do not have the special initial conditions for the arrow of time, and in addition if there is a singularity

it may be difficult to have anything like Eq.(15) with the confluence of Eq. (19) in our

present cosmological models. In which then new thinking will be required, which will be difficult for a lot of cosmologists to accept . And even good cosmologists as in [14], Linde come up with what I regard as fanciful suggestions in a field which has still not enough data and work behind it, to falsify our ideas with concrete data In [14] its author comes up with a suggested likelihood of the Cosmological constant having its present value based upon the Hartle-Hawking wavefunction of the universe, involving taking the actual exponential of a negative of the Hartle Hawking wavefunction of the universe. In doing so he obtained

having a given value of Λ via Hartle-Hawking theory having a given probability of the square of the Hartle-Hawking wavefunction, i.e.,

$$P(probability) \sim \exp(-24\pi^2 / \Lambda) = \exp(-S_A) \quad \textbf{(22)}$$

This probability would lead to a ridiculously large time value one would have to wait for any such occurrence happening with a time of a value infinitely larger than the age of the expected universe.

$$t \sim \exp(S_A) \sim 10^{10^{12}} \quad (23)$$

In short we can and must do better than this. And this requires new models and geometric paradigms to access what we may eventually be able to vet via experimental data sets

For the record, I have read in detail [15] and used a part of his ideas in the discussion of deformed special relativity and quantum uncertainty . I also was cognizant of [16] and nearly used it, but stopped when the author was intent upon using a version of entropy which automatically mandates, a nonexistent entropy at the very start to the expansion universe. In so many words, the jury is out on that one and there may be a different venue which shows up later.

Conflicts of Interest: "The authors declare no conflict of interest."

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