The energy of the static gravitational field

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Abstract: By adopting the assumption of elastic space-time, we can derive the wave equation of gravitational waves. Inspired by this, this article attempts to discuss the bending of spacetime by static mass. Because mass is also a form of energy, mass can also have a bending effect on spacetime. This article assumes that space-time is an elastic substance, so the existence of mass will squeeze space-time and cause the phenomenon of space-time bending. This article believes that the existence of mass will result in a Schwarzschild radius hole in spacetime. The void will squeeze the surrounding spacetime. Considering that mass squeezing spacetime requires "force", therefore, to squeeze out a Schwarzschild radius cavity in spacetime, work is required. This article believes that the energy required to squeeze out this cavity is equal to the energy corresponding to the mass. This is the energy possessed by the static gravitational field. After analyzing this squeezing effect with the knowledge of elasticity, it can be found that the existence of space-time is related to mass. Only the location where mass or energy exists can form space-time, and the radius of this space-time is proportional to the third power of mass. However, the analysis in this article also has shortcomings, including whether the space-time radius exists or not. It is just a conjecture at present, which directly affects the calculation of the space-time elastic modulus and other constants involved.

1 Energy's bending of spacetime

It can be seen from Einstein's field equation that the existence of energy momentum tensor causes space-time bending, which is recognized by everyone. Considering that in Einstein's field equation, the energy is gradually reduced, and the curvature of space-time will gradually decrease. From this point, it can be judged that space-time should be elastic, that is, it can become curved due to the squeezing of energy. With the disappearance of energy, spacetime will become flat again.

Starting from the ability of energy to bend space-time, we can use a very simple method to obtain the wave equation of gravitational waves and prove the existence of gravitational waves[1].

Mass is also a form of energy. But the mass is static, so it will only bend space-time without generating gravitational waves. So does the static mass also have such a law for the bending of spacetime?
2 Elastic mechanics analysis of spherically symmetric space-time

Assuming that space-time is uniformly distributed and isotropic, space-time has the characteristics of spherical symmetry. In addition, it is assumed that space-time is finite, so a space-time has a radius R.

When there is a mass $M$ in this space-time, the Schwarzschild radius corresponding to this mass is

$$r_s = \frac{2GM}{c^2}$$

Since the event horizon has been exceeded within the Schwarzschild radius, it can be considered that this is no longer the space-time we can observe, and we can regard the sphere with the size of the Schwarzschild radius as a hole in the entire space-time. The existence of this void will compress the surrounding space-time, causing the surrounding space-time to bend. As shown in Figure 1.

In this way, for a space-time sphere with a radius of R, two strains will be generated inside:

Radial strain

$$\varepsilon_r = \frac{r}{R}$$

And tangential strain

$$\varepsilon_t = \frac{2\pi r}{R}$$

Then, based on the knowledge of elastic mechanics, we can calculate that to squeeze out a cavity
with a radius of \( r \), the required stresses are respectively:\(^2\)

\[
\sigma_r = \frac{E}{(1 + \mu)(1 - 2\mu)}[(1 - \mu)\varepsilon_r + 2\mu\varepsilon_t]
\]

\[
\sigma_t = \frac{E}{(1 + \mu)(1 - 2\mu)}[\mu\varepsilon_r + \varepsilon_t]
\]

Where \( E \) is the modulus of elasticity and \( \mu \) is the Poisson's ratio. The above formula can be transformed into:

such

\[
\sigma_r = \frac{E}{(1 + \mu)(1 - 2\mu)R}[(1 - \mu)r + 4\pi\mu r] = \frac{E(1 - \mu + 4\pi\mu)}{(1 + \mu)(1 - 2\mu)R}r
\]

\[
\sigma_t = \frac{E}{(1 + \mu)(1 - 2\mu)R}[\mu r + 2\pi r] = \frac{E(\mu + 2\pi)}{(1 + \mu)(1 - 2\mu)R}r
\]

3 Energy required for mass squeeze spacetime

Now let's look at the amount of energy required for the spacetime with mass \( M \) extrusion radius \( R \). According to the calculation formula of force \( F \) to do work, we can get

\[
U = \int_0^{r_s} F dr
\]

That is

\[
U = \int_0^{r_s} (\sigma_r S + 2\pi\sigma_t S) dr = 4\pi \int_0^{r_s} (\sigma_r + 2\pi\sigma_t)r^2 dr
\]

\[
= \frac{4\pi A}{R} \int_0^{r_s} r^3 dr = \frac{\pi A}{R} r_s^4
\]

Among them

\[
A = \frac{E(1 - \mu + 4\pi\mu)}{(1 + \mu)(1 - 2\mu)} + \frac{E(2\pi\mu + 4\pi^2)}{(1 + \mu)(1 - 2\mu)} = \frac{E(1 - \mu + 6\pi\mu + 4\pi^2)}{(1 + \mu)(1 - 2\mu)}
\]
If the work done by this extrusion process is exactly equal to the mass $M$, then

$$\frac{\pi A}{R} r_s^4 = M c^2$$

We can get

$$r_s = \left(\frac{RM c^2}{\pi A}\right)^{1/4}$$

Considering

$$r_s = \frac{2GM}{c^2}$$

Then

$$\frac{A}{R} = \frac{M c^2}{\pi} \left(\frac{c^2}{2GM}\right)^4 = \frac{c^{10}}{4G^4 \pi M^3}$$

$$\frac{E (1 - \mu + 6\pi\mu + 4\pi^2)}{(1 + \mu)(1 - 2\mu) R} = \frac{c^{10}}{4G^4 \pi M^3}$$

If the elastic modulus of spacetime and Poisson’s ratio are constant, then

$$R = \frac{4\pi E (1 - \mu + 6\pi\mu + 4\pi^2) G^4}{(1 + \mu)(1 - 2\mu) c^{10} M^3} = k M^3$$

In this case, the mass $M$ extrudes a spacetime cavity with a Schwarzschild radius, and the energy required for the extrusion process is equal to the mass. And this energy can also be regarded as the energy of the gravitational field.

One conclusion obtained in this way is that spacetime is limited, and it is formed due to the existence of mass. The radius of the spacetime is proportional to the cube of the mass. The larger the mass, the larger the spacetime radius formed.

### 4 Conclusions

From the analysis results of this article, the static mass can also produce a squeezing effect on spacetime. The energy required for the extrusion process should be equal to the energy of the gravitational field generated. The energy of the generated gravitational field is equal to the mass that forms the gravitational field.
The advantage of this analysis is that it can make the origin of gravity more concrete and easier to understand.

The analysis of this article can also draw a new conclusion. If spacetime is an elastic substance with constant elastic modulus and Poisson's ratio, then spacetime must be finite, and the existence of spacetime is directly related to the mass of the gravitational field. The radius of the spacetime is proportional to the cube of the mass.

Then this brings a new problem: if there is no mass or energy, according to the analysis in this article, there should be no spacetime. How to use the laws of physics to describe the nonexistential of spacetime?

There are still some shortcomings in the analysis of this article. Although from an analysis point of view, it is concluded that spacetime is limited. But compared with my last model, the relationship between spacetime radius and mass is very different [3].

Since the spacetime radius is just a conjecture at present, there is no way to calculate the elasticity modulus and Poisson's ratio of spacetime with the known laws of physics. I believe these constants should be closely related to the known physical constants.

**References**

