

# Langenscheidt's German-English English-German Dictionary and the Graphical Law

Anindya Kumar Biswas\*

*Department of Physics;*

*North-Eastern Hill University,*

*Mawkynroh-Umshing, Shillong-793022.*

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## Abstract

We study Langenscheidt's German-English English-German Dictionary. We draw the natural logarithm of the number of the German language entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised( unnormalised). We find that the words underlie a magnetisation curve of a Spin-Glass in the presence of little external magnetic field. Moreover, we compare the German language with two Romance languages, the Basque and the Romanian, respectively, with respect to Spin-Glass magnetisation.

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\* anindya@nehu.ac.in

## I. INTRODUCTION

Germany is a country in the center of Europe. The Germans are the people. The German is the language. To study the language we consult a dictionary of the German. This is Langenscheidt's German-English English-German Dictionary, [1]. Here, we introduce the German language by reproducing few entries from the dictionary,[1], in the following.

Adel in the German language means aristocracy, All means the Universe, 'Ansatz means start, Appell means roll-call, 'Arbeiter means worker, auf means on, Bau means building, 'Aufbau means building up, Auge means eye, Aula means hall, Bach means brook, Backe means cheek, Bagger means excavator, Bahn means course, Baum means tree, Becken means basin, be'denken means to consider, beharren means to persist, Berg means mountain, Bernstein means amber, be'vor means before, Blase means bubble, Blatt means leaf (of a book), Bohle means thick plank, Bonbón means sweet, braten means in oven, Braut means bride, breit means broad, Buch means book, Bündel means bundle, der or, die or, das means the, dir means (to) you, Dorf means village, Druck means pressure, egal means equal, eigen means own, ein means one, Eisen means iron, Endung means ending, Enge means narrowness, Enkel means grandchild, Ensemble means company, Ernst means seriousness, Er'satz means replacement, Falke means falcon, faul means fruit, Feder means feather, fidel means jolly, flügge means fledged, Freude means joy, Freund means friend, Friede(n) means peace, Fuchs means fox, 'Führer means leader, Funk means radio, Gärung means fermentation, ganz means all, gleich means equal, Göttin means goddess, Graf means earl, Gros means main body, Haar means hair, Halde means slope, 'hassen means hate, Hecht means pike, Heck means stern, Heirat means marriage, Helmat means home, Haus means house, Herr means lord, Henkel means handle, Herzog means duke, herz means heart, Huhn means fowl, Hund means dog, Januar means January, Juli means July, Kahn means boat, Kaiser means emperor, kalb means calf, Kampf means combat, Kante means edge, Kanu means canoe, kauf means purchase, Kaufmann means businessman, keck means bold, Kittel means overall, klein means little, Knorren means knot, Köch means cook, Köcher means quiver, Kohl means cabbage, König means king, kraus means curly, Kreuz means cross, Kübel means bucket, krug means jug, Kummer means grief, Kürbis means pumpkin, Kutte means cowl, Kur means course of treatment, Kür means sports, lang means long, Längte means length, 'Lehrer means teacher, leib means body, Lenz means spring, lieb means nice, Luft

means air, Mache means make believe, Magie means magic, Mahl means meal, Mai means May, Mal means mark or, time, Mandel means almond, Mann means man, Matinee means morning performance, mehr means more, mir means (to) me, mit means with, Möhre means carrot, Mond means moon, Montag means Monday, Morgen means morning, Müller means miller, Mund means mouth, Nadel means needle, nahm or, nehmen means to take, paar means pair, Passah fest means Passover, Pfund means pound, Perle means pearl, Puder means powder, ragen means tower, Rahe means yard, Radau means row, Rarität means rarity, Rauch means smoke, Regen means rain, Reif means white, 'Richter means judge, ringen means to wrestle or, struggle, Ritz means crack, Rolle means roll or, coil, Rubin means ruby, Rudel means troop, rund means round, Saul means hall, Saat means sowing, sagen means to say, sägen means to saw, Saison means season, Saite means string, Sakko means lounge coat, Salve means volley, Sänger means singer, Satz means sentence, Schaum means foam, schäumen means foam, Schiff means ship, Schild means shield, Schnee means snow, 'Schneider means tailor, Schreck means fright, shroff means rugged, Schule means school, Schuster means shoemaker, Schutz means protection, Schwager means brother-in-law, schwarz means black, Schwinge wing, Sule means soul, Segen means blessing, Sitte means custom, Sommer frische means summer-holidays, Sommer zeit means summer time, Span means chip, Spiegel means mirror, stark means strong, stärke means strength, Stein means stone, Stelle means place, Steven means stem, Strom means stream, Tal means valley, taub means deaf, Taube means pigeon, Teller means plate, töten means to destroy, über means over, Unruh means balance(wheel), 'Urkunde means document, Vati means dad(dy), Verlag means publishing house, Vogel means bird, Volk means people, Wald means wood, Wahl means choice, 'Weber means weaver, Wein means wine, Weinberg means vineyard, Wiener means Viennese, wiegen means rock, wittern means to suspect, Zauber means spell, Zeit means time, Zimmer means room, Ziffer means figure or, digit, Zinn means tin, Zittern means tremble, Zucker means sugar, Zügel means bridle, Zweig means branch and so on.

In this article, we study magnetic field pattern behind this dictionary of the German,[1]. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as graphical law.

Then, we moved on to investigate into, [3], dictionaries of five disciplines of knowledge

and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law behind the bengali language,[4] and the basque language[5]. This was pursued by finding of the graphical law behind the Romanian language, [6], five more disciplines of knowledge, [7], Onsager core of Abor-Miri, Mising languages,[8], Onsager Core of Romanised Bengali language,[9], the graphical law behind the Little Oxford English Dictionary, [10], the Oxford Dictionary of Social Work and Social Care, [11], the Visayan-English Dictionary, [12], Garo to English School Dictionary, [13], Mursi-English-Amharic Dictionary, [14] and Names of Minor Planets, [15], A Dictionary of Tibetan and English, [16], Khasi English Dictionary, [17], Turkmen-English Dictionary, [18], Websters Universal Spanish-English Dictionary, [19], A Dictionary of Modern Italian, [20], respectively.

The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. In the section III, we describe analysis of the entries of the German language, [1]. The section IV is comparisons with other Romance languages. Sections V and VI are Acknowledgment and Bibliography respectively.

## II. MAGNETISATION

### A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up

or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by  $L = \frac{1}{N}\sum_i\sigma_i$ , where  $\sigma_i$  is i-th spin, N being total number of spins. L can vary from minus one to one.  $N = N_+ + N_-$ , where  $N_+$  is the number of up spins,  $N_-$  is the number of down spins.  $L = \frac{1}{N}(N_+ - N_-)$ . As a result,  $N_+ = \frac{N}{2}(1 + L)$  and  $N_- = \frac{N}{2}(1 - L)$ . Magnetisation or, net magnetic moment,  $M$  is  $\mu\sum_i\sigma_i$  or,  $\mu(N_+ - N_-)$  or,  $\mu NL$ ,  $M_{max} = \mu N$ .  $\frac{M}{M_{max}} = L$ .  $\frac{M}{M_{max}}$  is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[21], for the lattice of spins, setting  $\mu$  to one, is  $-\epsilon\sum_{n,n}\sigma_i\sigma_j - H\sum_i\sigma_i$ , where n.n refers to nearest neighbour pairs. The difference  $\Delta E$  of energy if we flip an up spin to down spin is, [22],  $2\epsilon\gamma\bar{\sigma} + 2H$ , where  $\gamma$  is the number of nearest neighbours of a spin. According to Boltzmann principle,  $\frac{N_-}{N_+}$  equals  $exp(-\frac{\Delta E}{k_B T})$ , [23]. In the Bragg-Williams approximation,[24],  $\bar{\sigma} = L$ , considered in the thermal average sense. Consequently,

$$\ln \frac{1+L}{1-L} = 2 \frac{\gamma\epsilon L + H}{k_B T} = 2 \frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2 \frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where,  $c = \frac{H}{\gamma\epsilon}$ ,  $T_c = \gamma\epsilon/k_B$ , [25].  $\frac{T}{T_c}$  is referred to as reduced temperature.

Plot of  $L$  vs  $\frac{T}{T_c}$  or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field,  $c \neq 0$ , the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [22]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf

Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

**B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field**

In the approximation scheme which is improvement over the Bragg-Williams, [21],[22],[23],[24],[25], due to Bethe-Peierls, [26], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe data s generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$				$\frac{M}{M_{max}}$ ,
BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4, $\beta H = 0$ )	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE I. Datas for Reduced temperature[ for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field,  $c = 0$ ,  $c = \frac{H}{\gamma\epsilon} = 0.005$ ,  $c = \frac{H}{\gamma\epsilon} = 0.01$  respectively and in the Bethe-Peierls approximation, BP(4, $\beta H=0$ ), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature data set( say, data set BW(c=0)) is drawn along the x-axis and the corresponding Reduced magnetisation data set is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.[For example, for drawing BW(c=0), ".dat" file, say denoted as "0.dat", contains BW(c=0) data set in first column and reduced magnetisation data set in second column. Moreover, after (0.944,0.400), next pair of points will be (0.969,0.300), then (0.987,0.200), ...and so on in the "0.dat" file.]

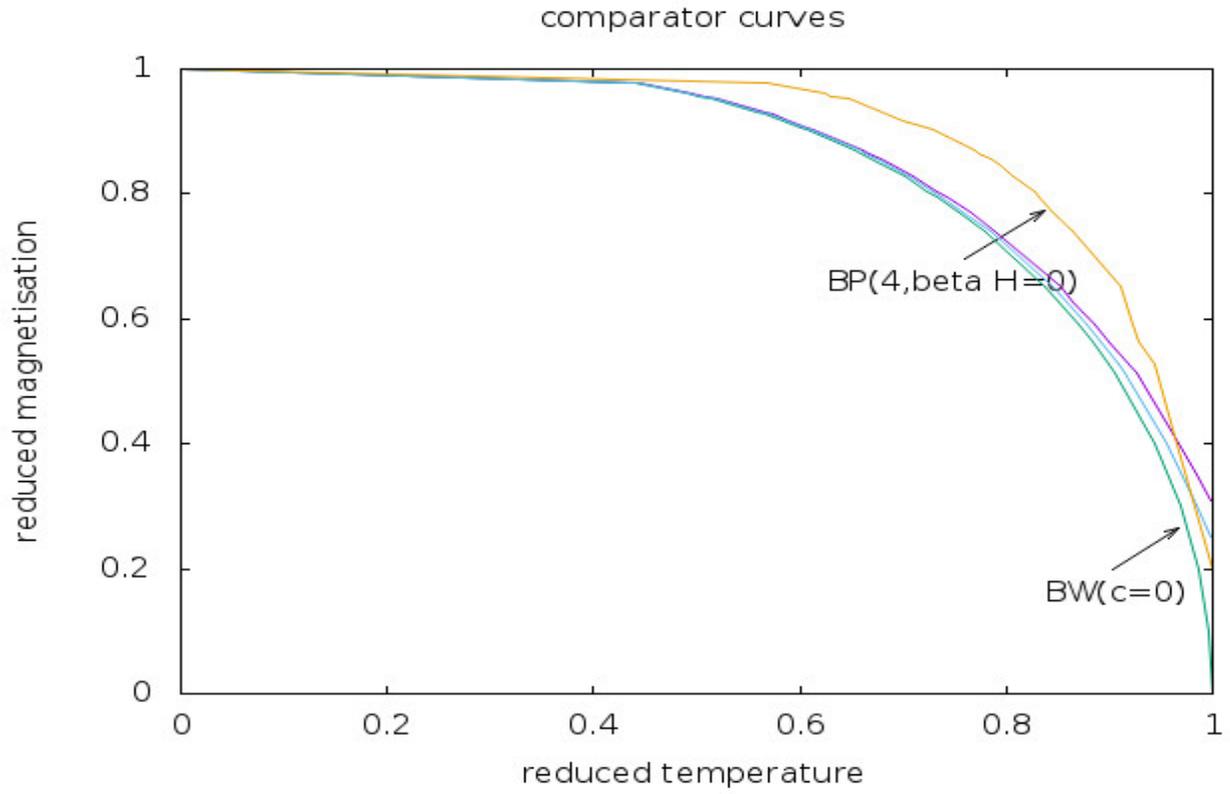


FIG. 1. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW( $c=0$ )) and in the presence (BW( $c=0.005$ ), BW( $c=0.01$ )) of magnetic field,  $c = 0$ ,  $c = \frac{H}{\gamma\epsilon} = 0.005$ ,  $c = \frac{H}{\gamma\epsilon} = 0.01$ , outwards; and in the Bethe-Peierls approximation, BP(4, $\beta H=0$ ), in the absence of magnetic field, for four nearest neighbours (outer in the top).

### C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [26], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor} - 1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula Ala [26] is given in the appendix of [7].

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. For four neighbours,

$$\frac{0.693}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor} - 1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe data s in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those data s. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.06$ . calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.05$ . calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.04$ . calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.02$ . calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.01$ . calculated from the equation(4). The data set is used to plot fig.2. Similarly, we plot fig.3. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

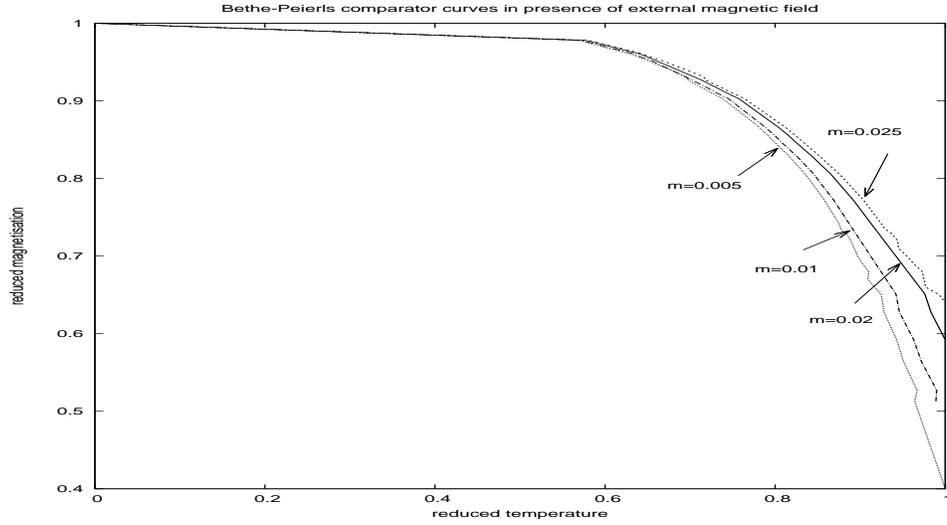


FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with  $\beta H = 2m$ .

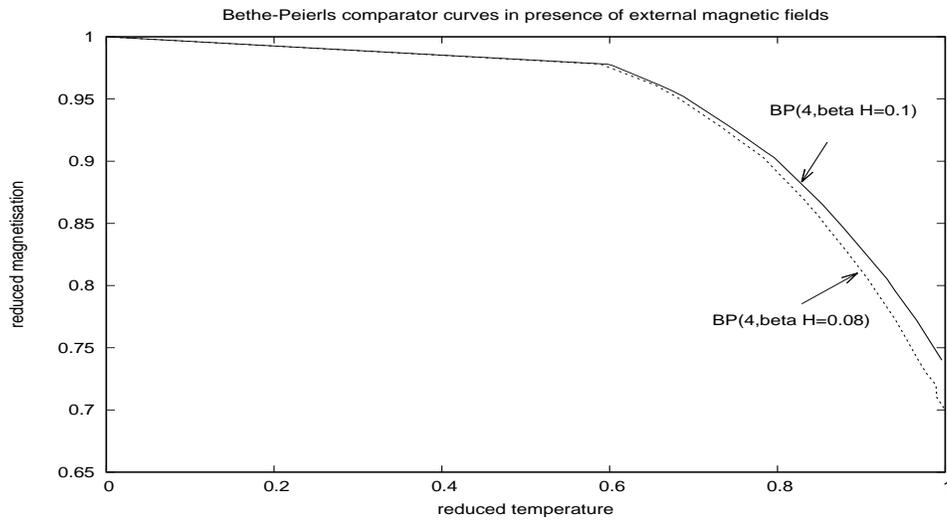


FIG. 3. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with  $\beta H = 2m$ .

reduced temperature, $\frac{T}{T_c}$				$\frac{M}{M_{max}}$ ,	
BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

#### D. Onsager solution

At a temperature  $T$ , below a certain temperature called phase transition temperature,  $T_c$ , for the two dimensional Ising model in absence of external magnetic field i.e. for  $H$  equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [27], [28], [29], [26],

$$\frac{M}{M_{max}} = [1 - (\sinh \frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

Graphically, the Onsager solution appears as in fig.4.

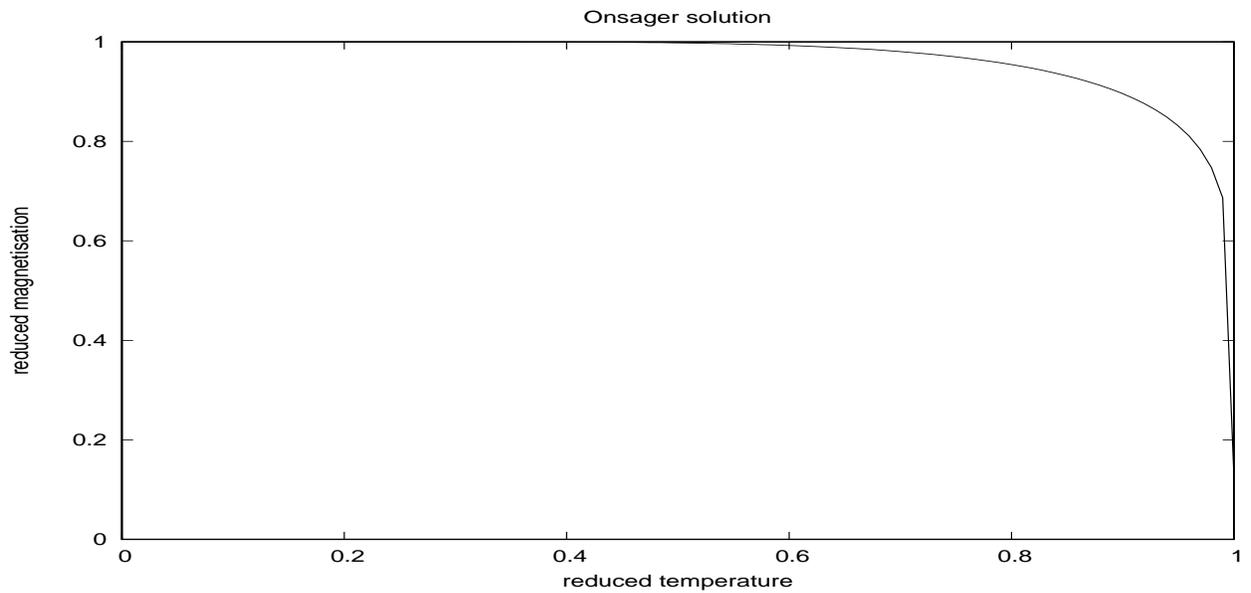


FIG. 4. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

## E. Spin-Glass

In the case coupling between (among) the spins, not necessarily n.n, for the Ising model is (are) random, we get Spin-Glass. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like  $\frac{1}{T-T_c}$  i.e. like the branch of rectangular hyperbola, up to the the phase transition temperature, followed by very little increase,[30–32], in magnetisation, as the ambient temperature continues to drop.

Theoretical study of Spin Glass started with the paper by Edwards, Anderson,[33]. They were trying to explain two experimental results concerning continuous disordered freezing(phase transition) and sharp cusp in static magnetic susceptibility. This was followed by a paper by Sherrington, Kickpatrick, [34], who dealt with Ising model with interactions being present among all neighbours. The interaction is random, follows Gaussian distribution and does not distinguish one pair of neighbours from another pair of neighbours, irrespective of the distance between two neighbours. In presence of external magnetic field, they predicted in their next paper, [35], below spin-glass transition temperature a spin-glass phase with non-zero magnetisation. Almeida etal, [36], Gray and Moore, [37],finally Parisi, [38], [39] improved and gave final touch, [40], to their line of work. Parisi and collaborators, [41]-[45], wrote a series of papers in postscript, all revolving around a consistent assumption of constant magnetisation in the spin-glass phase in presence of little constant external magnetic field.

In another sequence of theoretical work, by Fisher etal,[46–48], concluded that for Ising model with nearest neighbour or, short range interaction of random type spin-glass phase does not exist in presence of external magnetic field.

For recent series of experiments on spin-glass, the references, [49, 50], are the places to look into.

For an in depth account, accessible to a commoner, the series of articles by late P. W. Anderson in Physics Today, [51]-[57], is probably the best place to look into. For a book to enter into the subject of spin-glass, one may start at [58].

Here, in our work to follow, spin-glass refers to spin-glass phase of a system with infinite range random interactions.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2066	1690	79	686	1332	1117	1547	1117	271	177	1286	811	991	597	288	845	71	894	2801	840	1199	1402	1004	7	1	842

TABLE III. German words

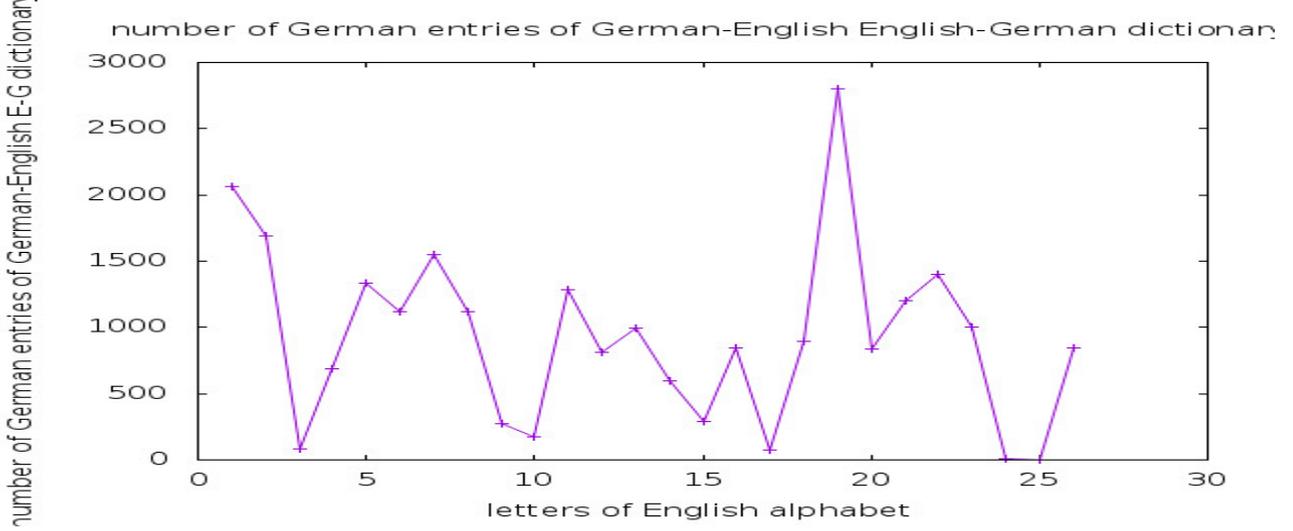


FIG. 5. Vertical axis is number of words of the German, [1], and horizontal axis is respective letters. Letters are represented by the sequence number in the alphabet or, dictionary sequence,[1].

### III. ANALYSIS OF WORDS OF THE GERMAN-ENGLISH DICTIONARY

The German language alphabet is composed of twenty six letters like English. We take a German-English dictionary,[1]. Then we count all the entries, [1], one by one from the beginning to the end, starting with different letters. The result is the table, III.

Highest number of entries, two thousand eight hundred one, starts with the letter S followed by entries numbering two thousand sixty six beginning with A, one thousand six hundred ninety with the letter B etc. To visualise we plot the number of words against respective letters in the dictionary sequence, [1], in the figure fig.5.

For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. The lowest value of  $f$  is one, corresponding to the letter Y. The respective rank,  $k$ , denoted as  $k_{lim}$  is twenty five. As a result both  $\frac{\ln f}{\ln f_{max}}$  and  $\frac{\ln k}{\ln k_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, IV and plot  $\frac{\ln f}{\ln f_{max}}$

against  $\frac{lnk}{lnk_{im}}$  in the figure fig.6. We then ignore the letter with the highest of words, tabulate in the adjoining table, IV and redo the plot, normalising the  $lnfs$  with next-to-maximum  $lnf_{nextmax}$ , and starting from  $k = 2$  in the figure fig.7. This program then we repeat up to  $k = 11$ , resulting in figures up to fig.16.

k	lnk	lnk/lnk <sub>im</sub>	f	lnf	lnf/lnf <sub>max</sub>	lnf/lnf <sub>n-max</sub>	lnf/lnf <sub>2nmax</sub>	lnf/lnf <sub>3nmax</sub>	lnf/lnf <sub>4nmax</sub>	lnf/lnf <sub>5nmax</sub>	lnf/lnf <sub>6nmax</sub>	lnf/lnf <sub>7nmax</sub>	lnf/lnf <sub>8nmax</sub>	lnf/lnf <sub>9nmax</sub>	lnf/lnf <sub>10nmax</sub>
1	0	0	2801	7.938	1	Blank									
2	0.69	0.214	2066	7.633	0.962	1	Blank								
3	1.10	0.342	1690	7.432	0.936	0.974	1	Blank							
4	1.39	0.432	1547	7.344	0.925	0.962	0.988	1	Blank						
5	1.61	0.500	1402	7.246	0.913	0.949	0.975	0.987	1	Blank	Blank	Blank	Blank	Blank	Blank
6	1.79	0.556	1332	7.194	0.906	0.942	0.968	0.980	0.993	1	Blank	Blank	Blank	Blank	Blank
7	1.95	0.606	1286	7.159	0.902	0.938	0.963	0.975	0.988	0.995	1	Blank	Blank	Blank	Blank
8	2.08	0.646	1199	7.089	0.893	0.929	0.954	0.965	0.978	0.985	0.990	1	Blank	Blank	Blank
9	2.20	0.683	1117	7.018	0.884	0.919	0.944	0.956	0.969	0.976	0.980	0.990	1	Blank	Blank
10	2.30	0.714	1004	6.912	0.871	0.906	0.930	0.941	0.954	0.961	0.965	0.975	0.985	1	Blank
11	2.40	0.745	991	6.899	0.869	0.904	0.928	0.939	0.952	0.959	0.964	0.973	0.983	0.998	1
12	2.48	0.770	894	6.796	0.856	0.890	0.914	0.925	0.938	0.945	0.949	0.959	0.968	0.983	0.985
13	2.56	0.795	845	6.739	0.849	0.883	0.907	0.918	0.930	0.937	0.941	0.951	0.960	0.975	0.977
14	2.64	0.820	842	6.736	0.849	0.882	0.906	0.917	0.930	0.936	0.941	0.950	0.960	0.975	0.976
15	2.71	0.842	840	6.733	0.848	0.882	0.906	0.917	0.929	0.936	0.940	0.950	0.959	0.974	0.976
16	2.77	0.860	811	6.698	0.844	0.878	0.901	0.912	0.924	0.931	0.936	0.945	0.954	0.969	0.971
17	2.83	0.879	686	6.531	0.823	0.856	0.879	0.889	0.901	0.908	0.912	0.921	0.931	0.945	0.947
18	2.89	0.898	597	6.392	0.805	0.837	0.860	0.870	0.882	0.889	0.893	0.902	0.911	0.925	0.927
19	2.94	0.913	288	5.663	0.713	0.742	0.762	0.771	0.782	0.787	0.791	0.799	0.807	0.819	0.821
20	3.00	0.932	271	5.602	0.706	0.734	0.754	0.763	0.773	0.779	0.783	0.790	0.798	0.810	0.812
21	3.04	0.944	177	5.176	0.652	0.678	0.696	0.705	0.714	0.719	0.723	0.730	0.738	0.749	0.750
22	3.09	0.960	79	4.369	0.550	0.572	0.588	0.595	0.603	0.607	0.610	0.616	0.623	0.632	0.633
23	3.14	0.975	71	4.263	0.537	0.558	0.574	0.580	0.588	0.593	0.595	0.601	0.607	0.617	0.618
24	3.18	0.988	7	1.946	0.245	0.255	0.262	0.265	0.269	0.271	0.272	0.275	0.277	0.282	0.282
25	3.22	1	1	0	0	0	0	0	0	0	0	0	0	0	0

TABLE IV. German language words: ranking, natural logarithm, normalisations

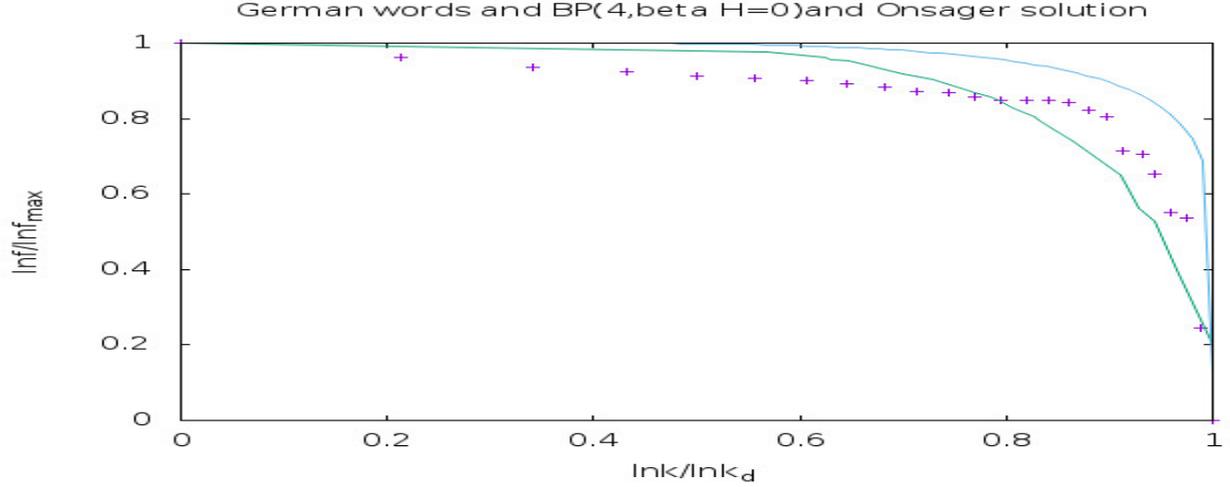


FIG. 6. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in the absence of external magnetic field. The uppermost curve is the Onsager solution.

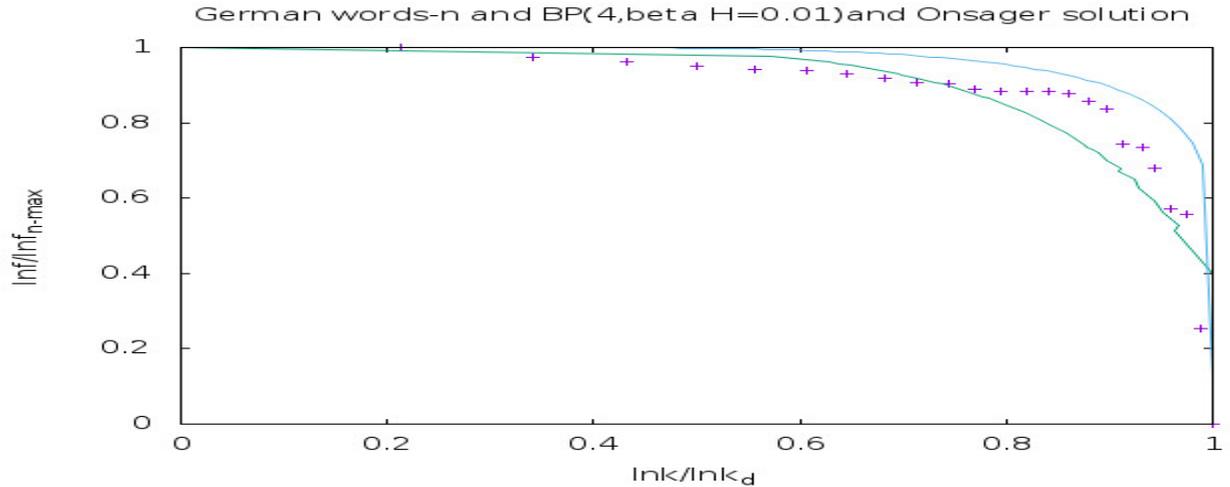


FIG. 7. The vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.005$  or,  $\beta H = 0.01$ . The uppermost curve is the Onsager solution.

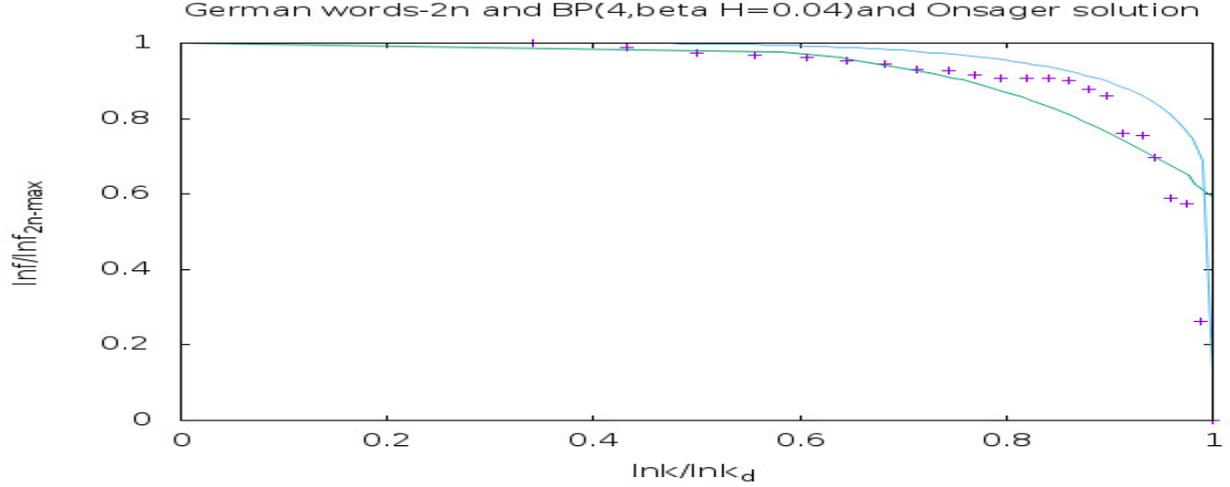


FIG. 8. The vertical axis is  $\frac{\ln f}{\ln f_{\text{nextnext-max}}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.02$  or,  $\beta H = 0.04$ . The uppermost curve is the Onsager solution.

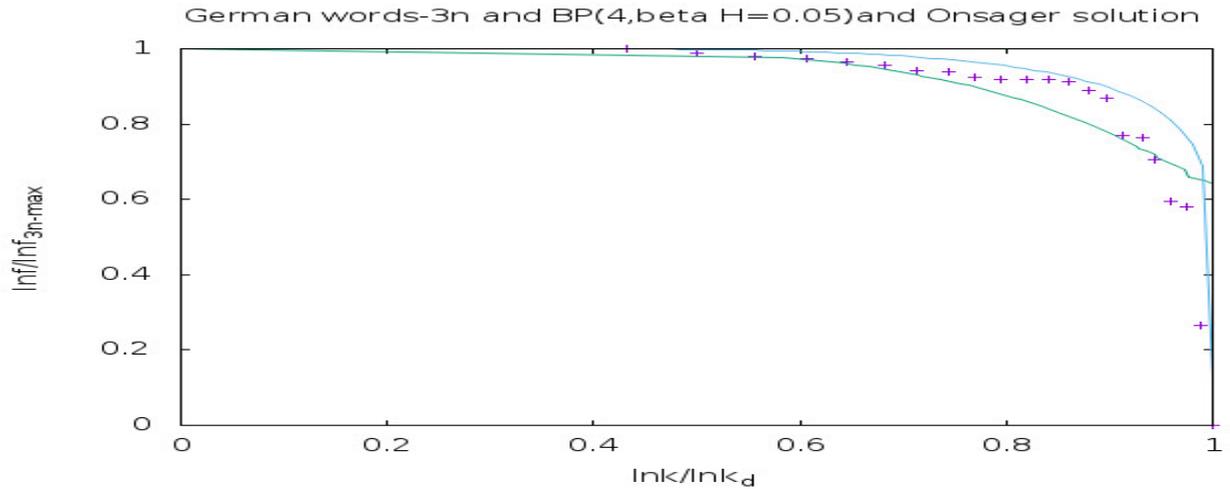


FIG. 9. The vertical axis is  $\frac{\ln f}{\ln f_{\text{nextnextnext-max}}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.025$  or,  $\beta H = 0.05$ . The uppermost curve is the Onsager solution.

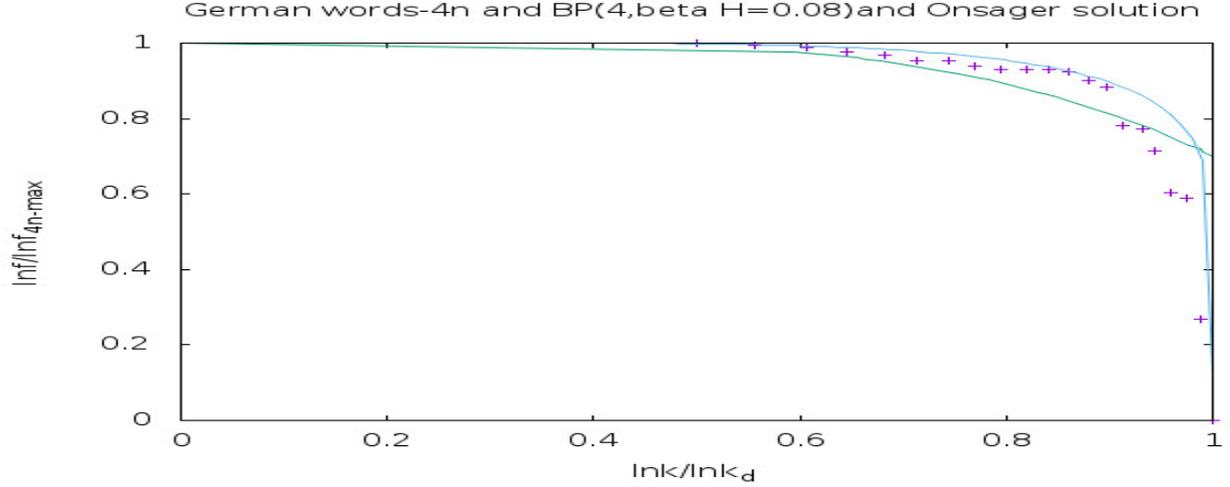


FIG. 10. The vertical axis is  $\frac{\ln f}{\ln f_{nextnextnextnext-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.04$  or,  $\beta H = 0.08$ . The uppermost curve is the Onsager solution.

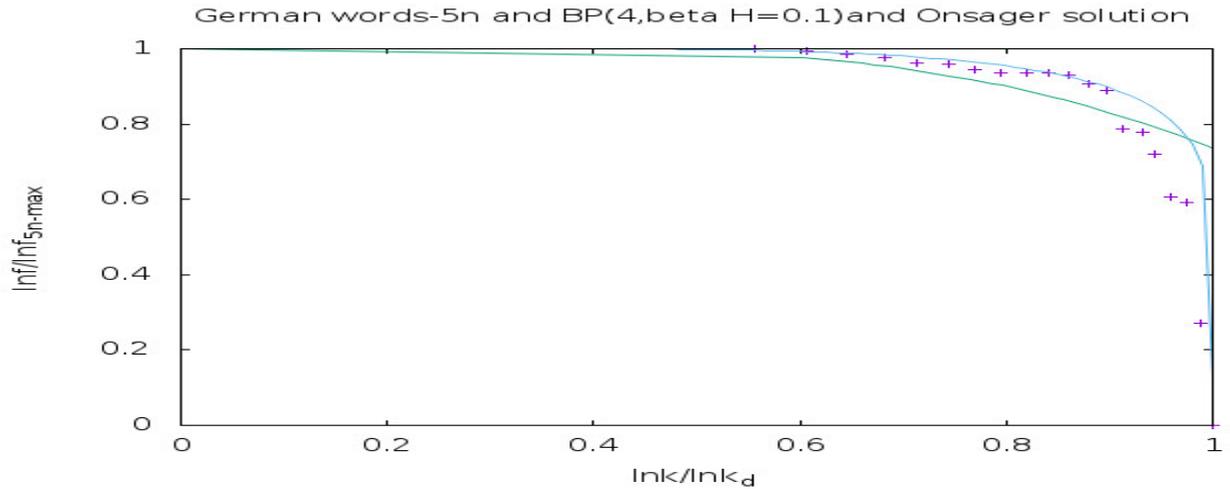


FIG. 11. The vertical axis is  $\frac{\ln f}{\ln f_{nnnnn-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.05$  or,  $\beta H = 0.1$ . The uppermost curve is the Onsager solution.

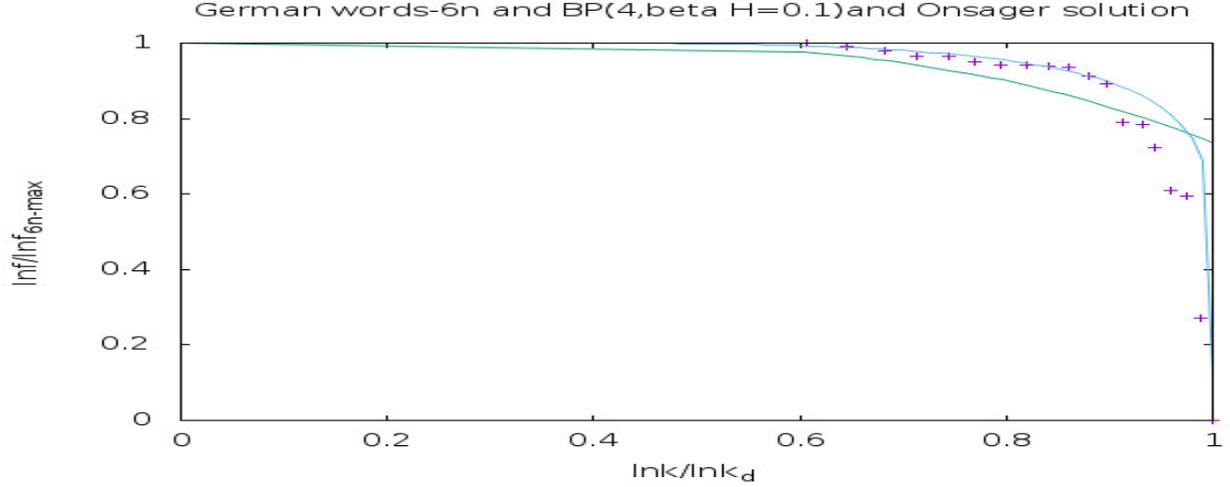


FIG. 12. The vertical axis is  $\frac{\ln f}{\ln f_{6n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.05$  or,  $\beta H = 0.1$ . The uppermost curve is the Onsager solution.

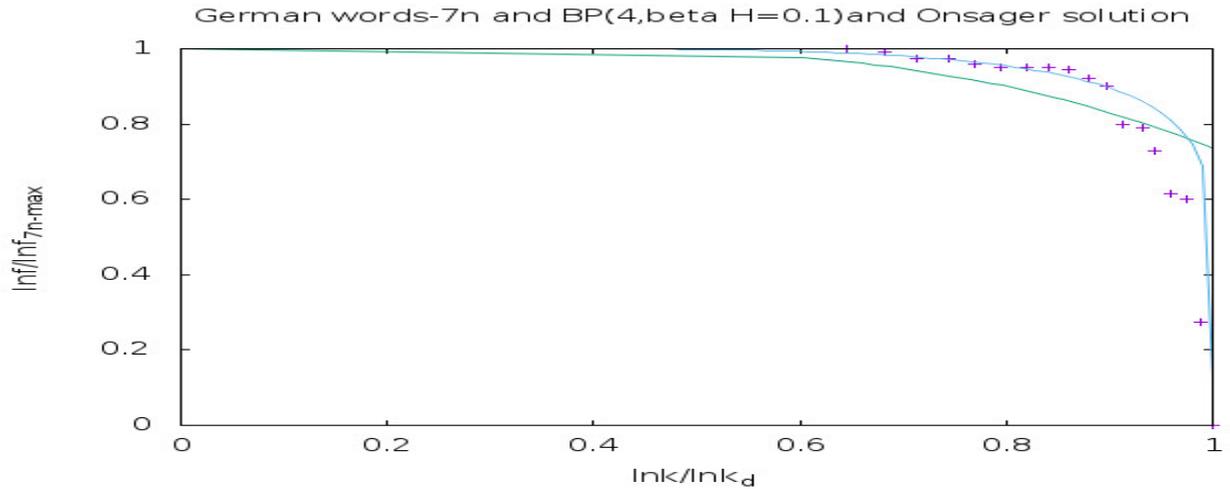


FIG. 13. The vertical axis is  $\frac{\ln f}{\ln f_{7n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.05$  or,  $\beta H = 0.1$ . The uppermost curve is the Onsager solution.

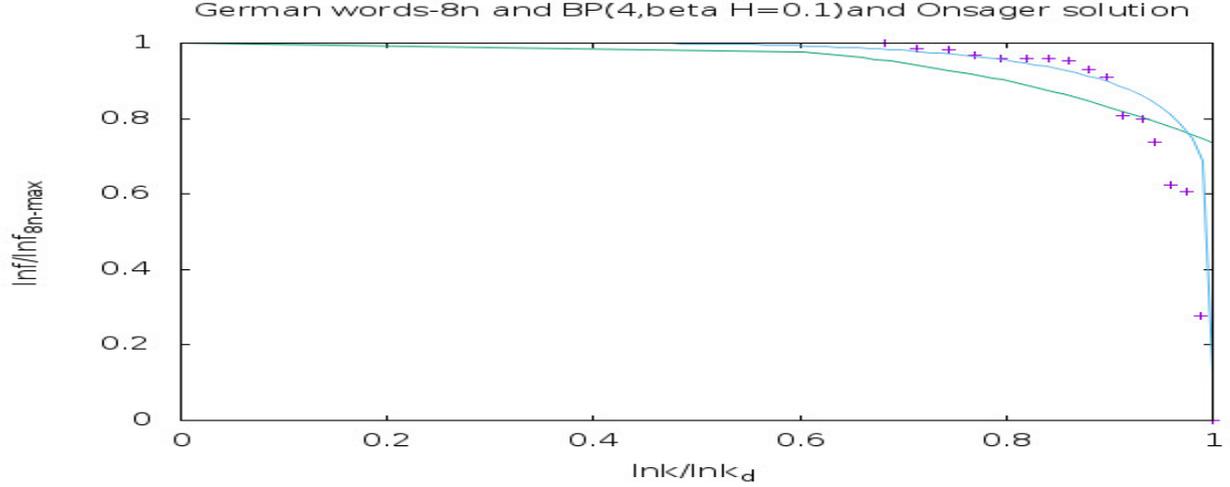


FIG. 14. The vertical axis is  $\frac{\ln f}{\ln f_{8n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.05$  or,  $\beta H = 0.1$ . The uppermost curve is the Onsager solution.

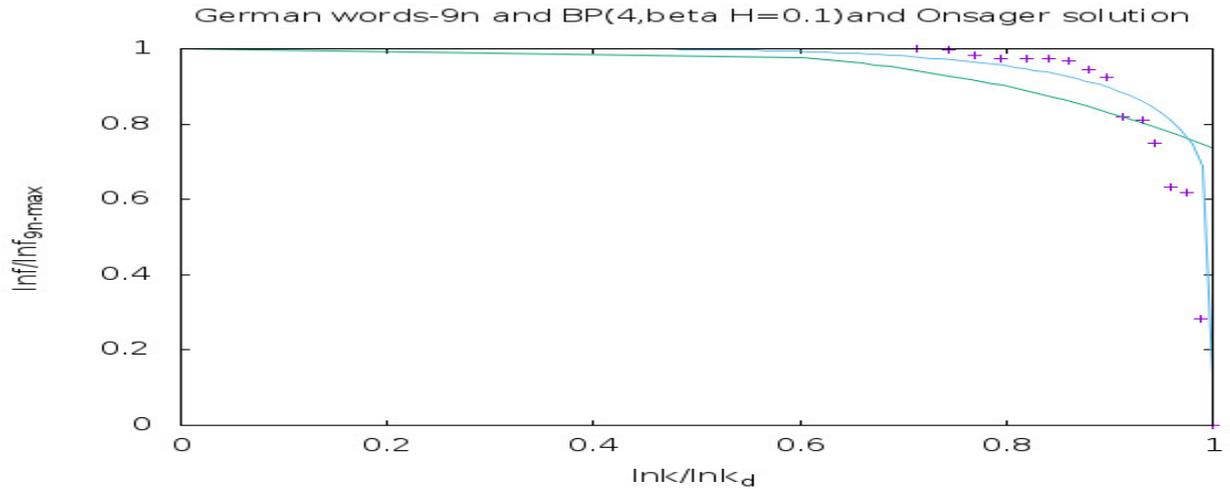


FIG. 15. The vertical axis is  $\frac{\ln f}{\ln f_{9n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language with fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.05$  or,  $\beta H = 0.1$ . The uppermost curve is the Onsager solution.

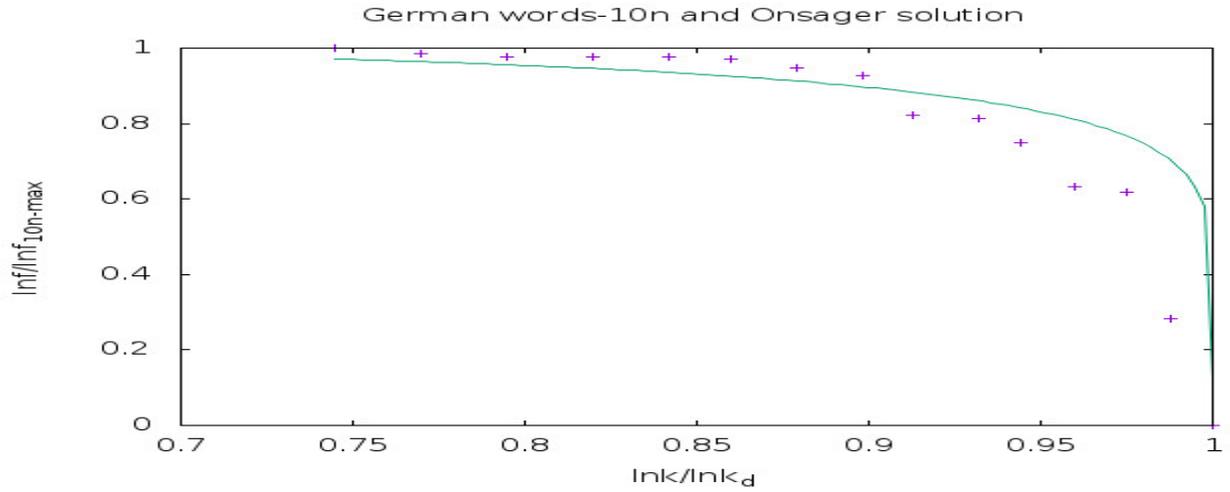


FIG. 16. The vertical axis is  $\frac{\ln f}{\ln f_{10n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the German language. The reference curve is the Onsager solution.

Matching of the plots in the figures fig.(6-16), with comparator curves i.e. the magnetisation curves of Bethe-Peierls approximations, is with dispersion and dispersion does not reduce to zero over higher orders of normalisations. On the top of it, on successive higher normalisations, entries of the German language,[1], do not go over to Onsager solution in the normalised  $\ln f$  vs  $\frac{\ln k}{\ln k_{lim}}$  graphs.

To explore for possible existence of spin-glass transition, in presence of little external magnetic field,  $\frac{\ln f}{\ln f_{max}}$ ,  $\frac{\ln f}{\ln f_{next-max}}$  and  $\frac{\ln f}{\ln f_{nn-max}}$  are drawn against  $\ln k$  in the figures fig.17-fig.19.

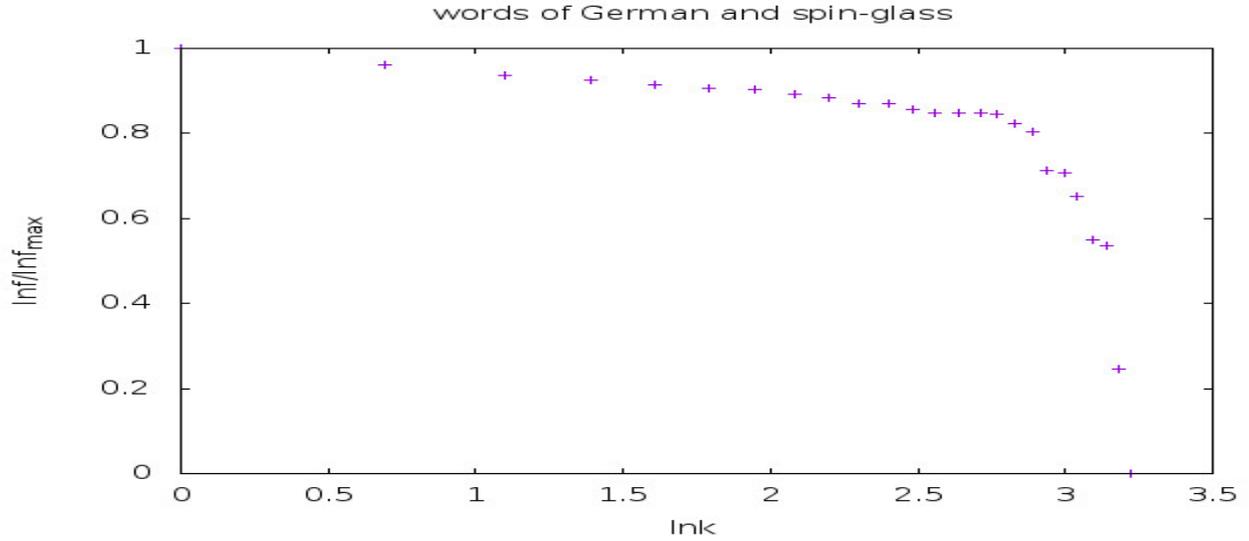


FIG. 17. The vertical axis is  $\frac{\ln f}{\ln f_{\max}}$  and the horizontal axis is  $\ln k$ . The + points represent the entries of the German language.

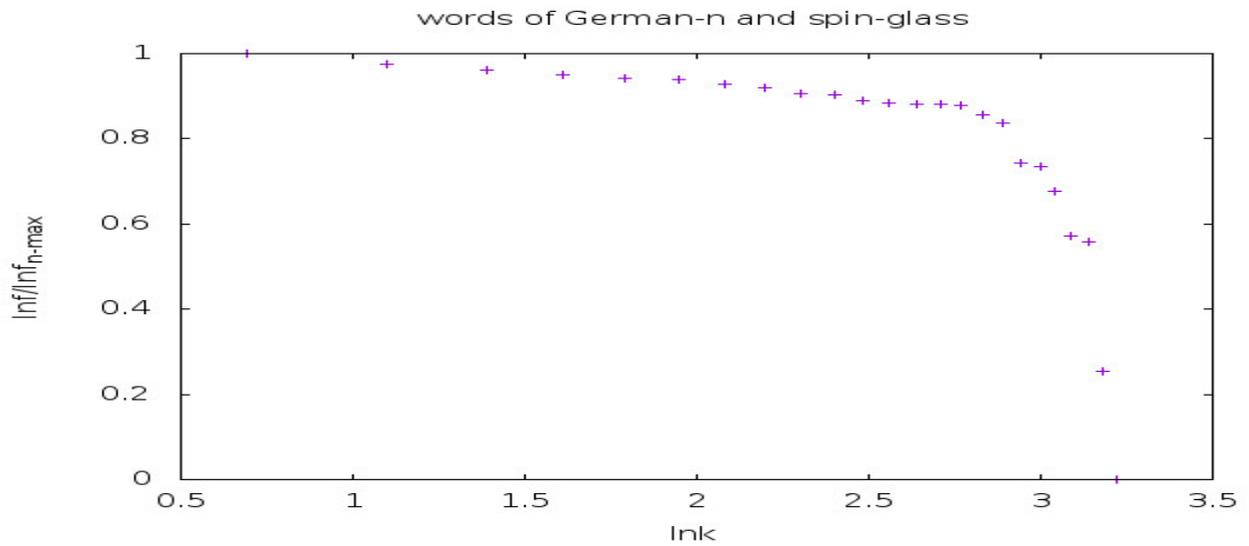


FIG. 18. The vertical axis is  $\frac{\ln f}{\ln f_{\text{next-max}}}$  and the horizontal axis is  $\ln k$ . The + points represent the entries of the German language.

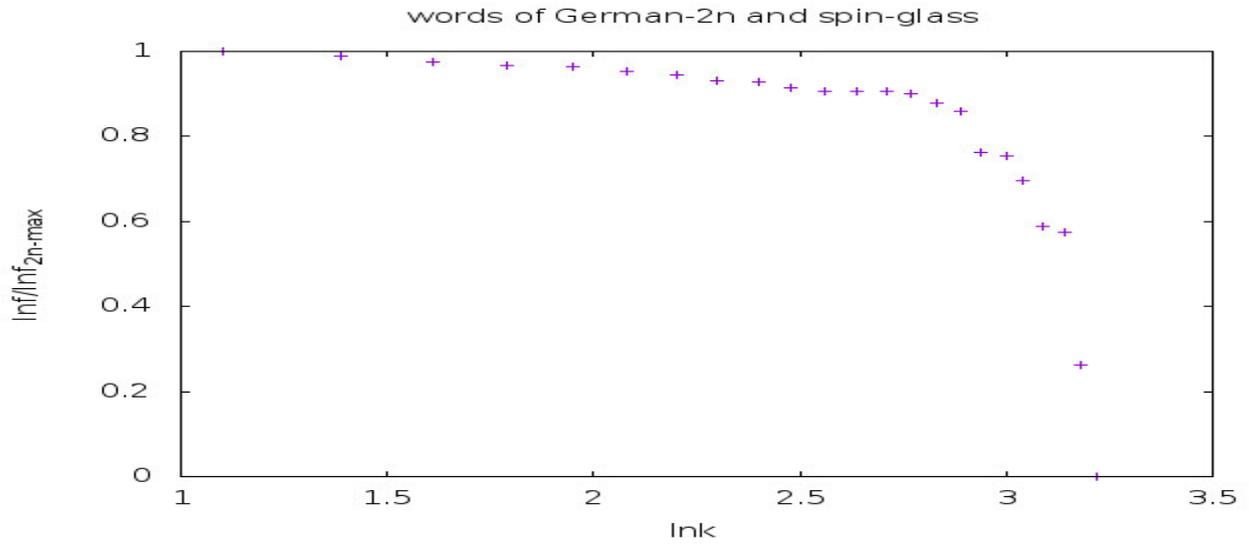


FIG. 19. The vertical axis is  $\frac{\ln f}{\ln f_{nn-\max}}$  and the horizontal axis is  $\ln k$ . The + points represent the entries of the German language.

## A. conclusion

In the figures Fig.17-Fig.19, the points has a clear-cut transition. Above the transition point(s), the lines are almost horizontal and below the transition point(s), points-line rises like the branch of a rectangular hyperbola. Hence, the words of the German, [1], is well-suited to be described by a Spin-Glass magnetisation curve, [30], in the presence of little magnetic field.

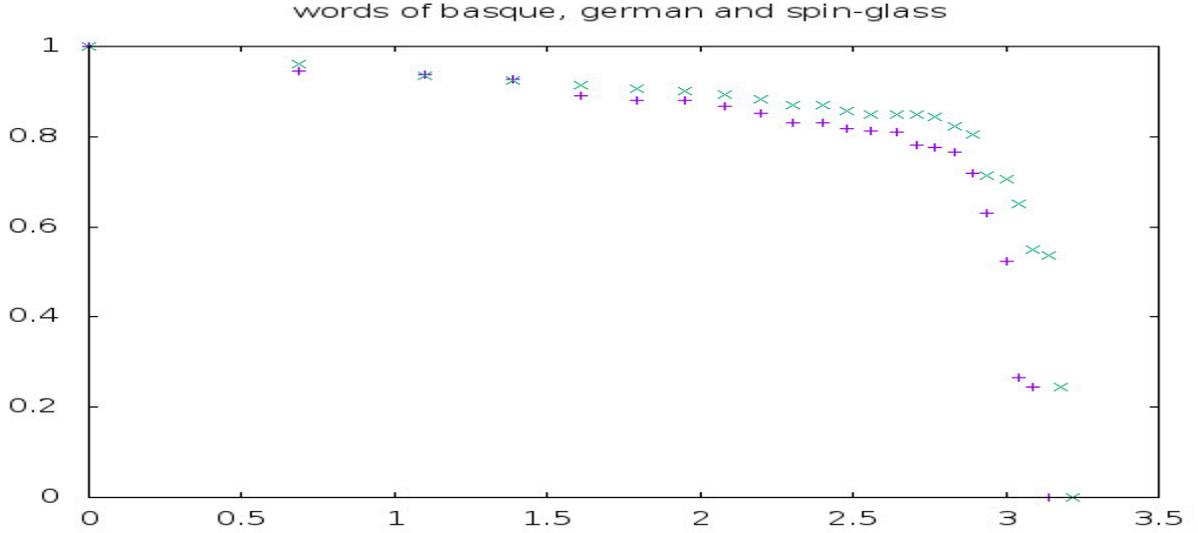


FIG. 20. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\ln k$ . The + points represent the entries of the Basque language, [5]. The × points represent the entries of the German language, [1].

#### IV. GERMAN AND ROMANCE LANGUAGES

We have studied two Romance languages Romanian, [6] and Basque, [5], in detail before. Those were better fit by spin-glass magnetisation curves in presence of little external magnetic field. We compare the spin-glass magnetisation curve nature of the Romanian and the Basque languages with the German in the figures 20 to 21. We conclude from the figures fig.20 and fig.21, that the German language is closest to the spin-glass behaviour whereas the Romanian language and the Basque language are closer.

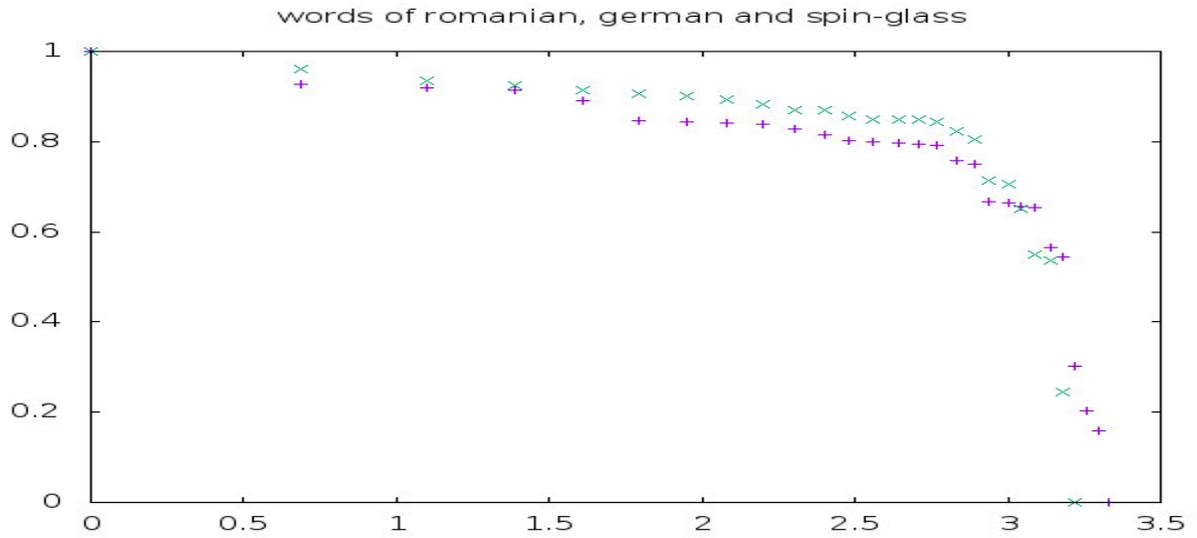


FIG. 21. The vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and the horizontal axis is  $\ln k$ . The + points represent the entries of the Romanian language, [6]. The × points represent the entries of the German language, [1].

## V. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper.

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- [1] Langenscheidt's German-English English-German Dictionary, Two Volumes in One, Edited by The Langenscheidt Editorial Staff, published by Pocket Books New York, 1970. ISBN: 0-671-54107-2.
- [2] Anindya Kumar Biswas, "Graphical Law beneath each written natural language", arXiv:1307.6235v3[physics.gen-ph]. A preliminary study of words of dictionaries of twenty six languages, more accurate study of words of dictionary of Chinese usage and all parts of speech of dictionary of Lakher(Mara) language and of verbs, adverbs and adjectives of dictionaries of six languages are included.
- [3] Anindya Kumar Biswas, "A discipline of knowledge and the graphical law", IJARPS Volume 1(4), p 21, 2014; viXra: 1908:0090[Linguistics].
- [4] Anindya Kumar Biswas, "Bengali language and Graphical law", viXra: 1908:0090[Linguistics].
- [5] Anindya Kumar Biswas, "Basque language and the Graphical Law", viXra: 1908:0414[Linguistics].
- [6] Anindya Kumar Biswas, "Romanian language, the Graphical Law and More", viXra: 1909:0071[Linguistics].
- [7] Anindya Kumar Biswas, "Discipline of knowledge and the graphical law, part II", viXra:1912.0243 [Condensed Matter],International Journal of Arts Humanities and Social Sciences Studies Volume 5 Issue 2 February 2020.
- [8] Anindya Kumar Biswas, "Onsager Core of Abor-Miri and Mising Languages", viXra: 2003.0343[Condensed Matter].
- [9] Anindya Kumar Biswas, "Bengali language, Romanisation and Onsager Core", viXra: 2003.0563[Linguistics].
- [10] Anindya Kumar Biswas, "Little Oxford English Dictionary and the Graphical Law", viXra: 2008.0041[Linguistics].
- [11] Anindya Kumar Biswas, "Oxford Dictionary Of Social Work and Social Care and the Graphical law", viXra: 2008.0077[Condensed Matter].

- [12] Anindya Kumar Biswas, "Visayan-English Dictionary and the Graphical law", viXra: 2009.0014[Linguistics].
- [13] Anindya Kumar Biswas, "Garo to English School Dictionary and the Graphical law", viXra: 2009.0056[Condensed Matter].
- [14] Anindya Kumar Biswas, "Mursi-English-Amharic Dictionary and the Graphical law", viXra: 2009.0100[Linguistics].
- [15] Anindya Kumar Biswas, "Names of Minor Planets and the Graphical law", viXra: 2009.0158[History and Philosophy of Physics].
- [16] Anindya Kumar Biswas, "A Dictionary of Tibetan and English and the Graphical law", viXra: 2010.0237[Condensed Matter].
- [17] Anindya Kumar Biswas, "Khasi English Dictionary and the Graphical law", viXra: 2011.0011[Linguistics].
- [18] Anindya Kumar Biswas, "Turkmen-English Dictionary and the Graphical law", viXra: 2011.0069[Linguistics].
- [19] Anindya Kumar Biswas, " Webster's Universal German-English Dictionary, the Graphical law and A Dictionary of Geography of Oxford University Press", viXra: 2103.0175[Condensed Matter].
- [20] Anindya Kumar Biswas, "A Dictionary of Modern Italian, the Graphical law and Dictionary of Law and Administration, 2000, National Law Development Foundation", viXra: 2107.0171[Condensed Matter].
- [21] E. Ising, Z.Physik 31,253(1925).
- [22] R. K. Pathria, Statistical Mechanics, p400-403, 1993 reprint, Pergamon Press,© 1972 R. K. Pathria.
- [23] C. Kittel, Introduction to Solid State Physics, p. 438, Fifth edition, thirteenth Wiley Eastern Reprint, May 1994, Wiley Eastern Limited, New Delhi, India.
- [24] W. L. Bragg and E. J. Williams, Proc. Roy. Soc. A, vol.145, p. 699(1934);
- [25] P. M. Chaikin and T. C. Lubensky, Principles of Condensed Matter Physics, p. 148, first edition, Cambridge University Press India Pvt. Ltd, New Delhi.
- [26] Kerson Huang, Statistical Mechanics, second edition, John Wiley and Sons(Asia) Pte Ltd.
- [27] S. M. Bhattacharjee and A. Khare, "Fifty Years of the Exact solution of the Two-dimensional Ising Model by Onsager", arXiv:cond-mat/9511003v2.

- [28] L. Onsager, *Nuovo Cim. Supp.*6(1949)261.
- [29] C. N. Yang, *Phys. Rev.* 85, 809(1952).
- [30] R. V. Chamberlin, M. Hardiman, L. A. Turkevich and R. Orbach, "H-T phase diagram for spin-glasses: An experimental study of Ag:Mn", *PRB* 25(11), 6720-6729, 1982.
- [31] R. V. Chamberlin, George Mozurkewich and R. Orbach, "Time Decay of the Remanent Magnetization in Spin-Glasses", *PRL* 52(10), 867-870, 1984.
- [32] [http://en.wikipedia.org/wiki/Spin\\_glass](http://en.wikipedia.org/wiki/Spin_glass)
- [33] S. F Edwards and P. W. Anderson, "Theory of spin glasses", *J. Phys.F: Metal Phys.* 5, 965-74, 1975.
- [34] D. Sherrington and S. Kirkpatrick, "Solvable model of a Spin-Glass", *PRL* 35, 1792-6, 1975.
- [35] D. Sherrington and S. Kirkpatrick, "Infinite-ranged models of spin-glasses", *PRB* 17(11), 4384-4403, 1978.
- [36] J. R. L. de Almeida and D. J. Thouless, "Stability of the Sherrington-Kirkpatrick solution of a spin glass model", *J. Phys. A: Math.Gen.*, Vol. 11, No. 5, 1978.
- [37] A. J. Bray and M. A. Moore, "Replica-Symmetry Breaking in Spin-Glass Theories", *PRL* 41, 1068-1072, 1978.
- [38] G Parisi, "A sequence of approximated solutions to the S-K model for spin glasses", *J. Phys. A: Math.Gen.*13 L115, 1980.
- [39] G Parisi, "Infinite Number of Order Parameters for Spin-Glasses", *PRL* 43, 1754-1756, 1979.
- [40] D. J. Thouless, J. R. L. de Almeida and J. M. Kosterlitz, "Stability and susceptibility in Parisi's solution of a spin glass model", *J. Phys. C: Solid State Phys.* 13, 3271-80, 1980.
- [41] G. Parisi, G. Toulouse, "A simple hypothesis for the spin glass phase of the infinite-ranged SK model", *Journal de Physique Lettres, Edp sciences*, 41(15), pp.361-364, 1980; <http://hal.archives-ouvertes.fr/jpa-00231798>.
- [42] G. Toulouse, "On the mean field theory of mixed spin glass-ferromagnetic phases", *Journal de Physique Lettres, Edp sciences*, 41(18), pp.447-449, 1980; <http://hal.archives-ouvertes.fr/jpa-00231818>.
- [43] G. Toulouse, M. Gabay, "Mean field theory for Heisenberg spin glasses", *Journal de Physique Lettres, Edp sciences*, 42(5), pp.103-106, 1981; <http://hal.archives-ouvertes.fr/jpa-00231882>.

- [44] Marc Gabay and Gérard Toulouse, "Coexistence of Spin-Glass and Ferromagnetic Orderings", PRL 47, 201-204, 1981.
- [45] J. Vannimenus, G. Toulouse, G. Parisi, "Study of a simple hypothesis for the mean-field theory of spin glasses", Journal de Physique, 42(4), pp.565-571, 1981; <http://hal.archives-ouvertes.fr/jpa-00209043>.
- [46] W L McMillan, "Scaling theory of Ising spin glasses", J. Phys. C: Solid State Phys., 17(1984) 3179-3187.
- [47] Daniel S. Fisher and David A. Huse, "Ordered Phase of Short-Range Ising Spin-Glasses", PRL 56(15), 1601-1604, 1986.
- [48] Daniel S. Fisher and David A. Huse, "Equilibrium behavior of the spin-glass ordered phase", PRB 38(1), 386-411, 1988.
- [49] S. Guchhait and R. L. Orbach, "Magnetic Field Dependence of Spin Glass Free Energy Barriers", PRL 118, 157203 (2017).
- [50] M. E. Baity-Jesi, A. Calore, A. Cruz, L. A. Fernandez, J. M. Gil-Narvion,..., D. Yllanes, "Matching Microscopic and macroscopic responses in Glasses", PRL 118, 157202(2017).
- [51] P. W. Anderson, "Spin-Glass I: A SCALING LAW RESCUED", Physics Today, pp.9-11, January(1988).
- [52] P. W. Anderson, "Spin-Glass II: IS THERE A PHASE TRANSITION?", Physics Today, pp.9, March(1988).
- [53] P. W. Anderson, "Spin-Glass III: THEORY RAISES ITS HEAD", Physics Today, pp.9-11, June(1988).
- [54] P. W. Anderson, "Spin-Glass IV: GLIMMERINGS OF TROUBLE", Physics Today, pp.9-11, September(1988).
- [55] P. W. Anderson, "Spin-Glass V: REAL POWER BROUGHT TO BEAR", Physics Today, pp.9-11, July(1989).
- [56] P. W. Anderson, "Spin-Glass VI: SPIN GLASS AS CORNUCOPIA", Physics Today, pp.9-11, September(1989).
- [57] P. W. Anderson, "Spin-Glass VII: SPIN GLASS AS PARADIGM", Physics Today, pp.9-11, March(1990).
- [58] J. K. Bhattacharjee, "Statistical Physics: Equilibrium and Non-Equilibrium Aspects", Ch. 26, Allied Publishers Limited, New Delhi, 1997.

- [59] A. M. Gun, M. K. Gupta and B. Dasgupta, Fundamentals of Statistics Vol 1, Chapter 12, eighth edition, 2012, The World Press Private Limited, Kolkata.
- [60] Sonntag, Borgnakke and Van Wylen, Fundamentals of Thermodynamics, p206-207, fifth edition, John Wiley and Sons Inc.
- [61] Oxford Dictionary of Science, edited by J. Daintith and E. Martin, sixth edition, 2010; Oxford University Press, Great Clarendon Street, Oxford OX26DP.