On the geometrical optics and the Atiyah-Singer index theorem

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We assume that the curvature in the Atiyah-Singer index theorem is related with, a function of the Riemann-Christoffel curvature tensor, so I_M is also a function of the Riemann-Christoffel curvature tensor.

I. GEOMETRICAL OPTICS

In our previous work on the geometrical optics, we obtain the result as below[1]

$$\begin{aligned} & gN_\sigma\ \partial_\rho \ln \Big| \partial_\nu \Big\{ \frac{c}{\omega} \arccos \\ & \left(A_\mu^{U(1)} \hat{m}^{U(1)} - \frac{1}{g} \hat{m}^{U(1)} \times \partial_\mu \hat{m}^{U(1)} \right) a_\mu^{-1} + ct \Big\} \Big| \\ & = \ R_{\mu\nu\rho\sigma} \end{aligned} \tag{1}$$

where $R_{\mu\nu\rho\sigma}$ is the Riemann-Christoffel curvature tensor.

II. THE ATIYAH-SINGER INDEX THEOREM

Roughly, the Atiyah-Singer index theorem can be written as[2]

$$\text{Index} = \int_M I_M \ \cdot \ \text{ch}\ (\sigma) \tag{2}$$

where I_M is a differential form determined by the curvature of the manifold, M, on which the equation is defined and ch (σ) is a differential form obtained from the symbol of the equation.

We assume that the curvature in eq.(2) is related with, a function of the Riemann-Christoffel curvature tensor, so I_M is also a function of the Riemann-Christoffel curvature tensor

$$I_M = f(R_{\mu\nu\rho\sigma}) \tag{3}$$

where

$$\begin{aligned} & gN_\sigma\ \partial_\rho \ln \Big| \partial_\nu \Big\{ \frac{c}{\omega} \arccos \\ & \left(A_\mu^{U(1)} \hat{m}^{U(1)} - \frac{1}{g} \hat{m}^{U(1)} \times \partial_\mu \hat{m}^{U(1)} \right) a_\mu^{-1} + ct \Big\} \Big| \\ & = \ R_{\mu\nu\rho\sigma} \end{aligned} \tag{4}$$

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[1] Miftachul Hadi, *Magnetic symmetry of geometrical optics*, https://vixra.org/abs/2104.0188, 2021.

[2] Nigel Higson, John Roe, *The Atiyah-Singer Index Theorem.*