Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung and the Graphical Law

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(Dated: September 20, 2025)

Abstract

We study Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung, dictionary, 2014 edition. We draw the natural logarithm of the number of the German language words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised(unnormalised). We find that the words underlie a magnetisation curve of a Spin-Glass in the presence of little external magnetic field. We notice that there is no qualitative change compared to the Langenscheidt's German-English English-German Dictionary, 1970 edition.

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I. INTRODUCTION

"Size matters."

.... common wisdom.

To check that we embark on studying Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung, dictionary, 2014 edition, [1]. This is considerably expanded in terms of total number of words compared to the Langenscheidt's German-English English-German Dictionary, 1970 edition, [2]. We have found the graphical law to exist behind the german langauge entries of this dictionary, [2], in the paper, [3].

In this article, we study magnetic field pattern behind this dictionary of the German,[1]. We have started considering magnetic field pattern in [4], in the languages we converse with. We have studied there, a set of natural languages, [4] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as graphical law.

Then, we moved on to investigate into, [5], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law behind the bengali language,[6] and the basque language[7]. This was pursued by finding of the graphical law behind the Romanian language, [8], five more disciplines of knowledge, [9], Onsager core of Abor-Miri, Mising languages,[10], Onsager Core of Romanised Bengali language,[11], the graphical law behind the Little Oxford English Dictionary, [12], the Oxford Dictionary of Social Work and Social Care, [13], the Visayan-English Dictionary, [14], Garo to English School Dictionary, [15], Mursi-English-Amharic Dictionary, [16] and Names of Minor Planets, [17], A Dictionary of Tibetan and English, [18], Khasi English Dictionary, [19], Turkmen-English Dictionary, [20], Websters Universal Spanish-English Dictionary, [21], A Dictionary of Modern Italian, [22], Langenscheidt's German-English Dictionary, [3], Essential Dutch dictionary by G. Quist and D. Strik, [23], Swahili-English dictionary by C. W. Rechenbach, [24], Larousse Dictionnaire De Poche for the French, [25], the Onsager's solution behind the Arabic, [26], respectively.

The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. In the section III, we describe analysis of the words of the German language, [1]. The section IV is comparisons between two

Langenscheidt dictionaries, [1] and [2]. Section V is Acknowledgment. The last section is Bibliography.

II. MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[27], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [28], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [29]. In the Bragg-Williams approximation,[30], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L+c}{\frac{T}{T_c}} \quad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [31]. $\frac{T}{T_c}$ is referred to as reduced temperature.
Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [28]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [27],[28],[29],[30],[31], due to Bethe-Peierls, [32], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (2)$$

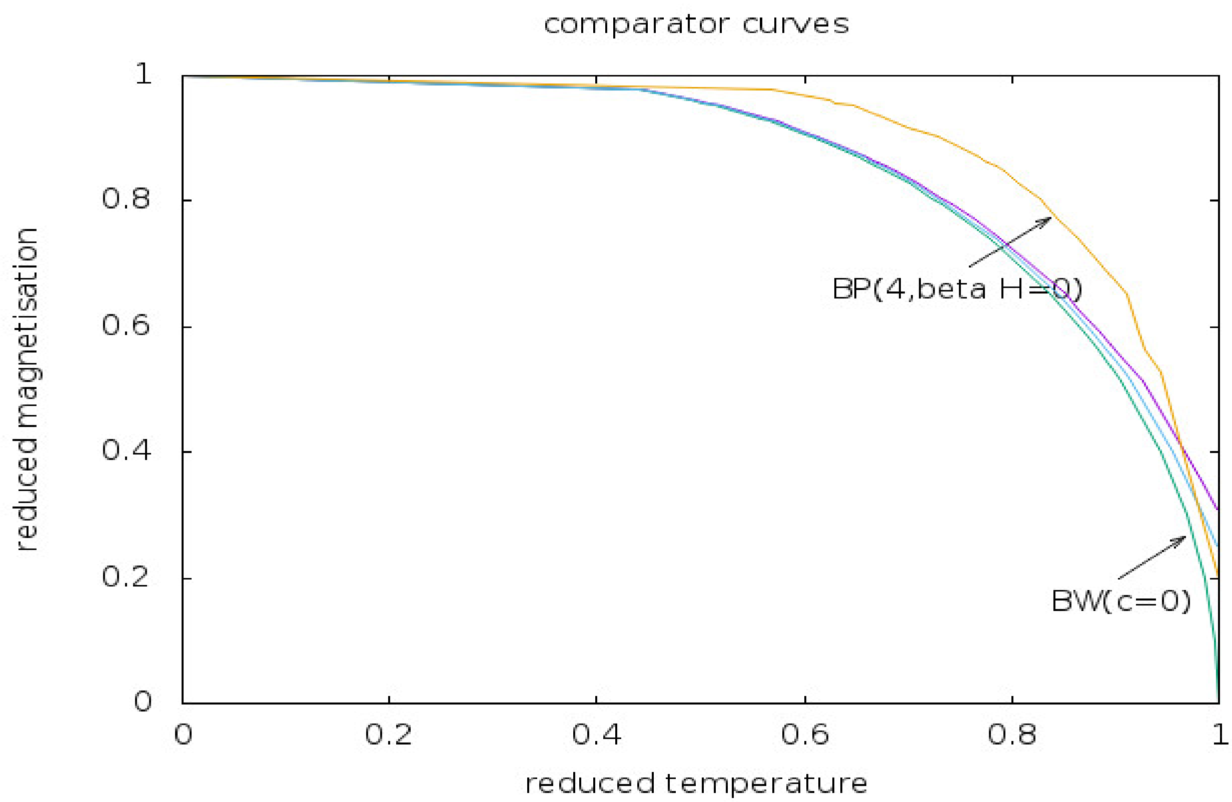

FIG. 1. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$, outwards; and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours (outer in the top).

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe data s generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$				$\frac{M}{M_{max}}$,
BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE I. Datas for Reduced temperature[for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$ respectively and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature data set(say, data set BW(c=0)) is drawn along the x-axis and the corresponding Reduced magnetisation data set is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.[For example, for drawing BW(c=0), ".dat" file, say denoted as "0.dat", contains BW(c=0) data set in first column and reduced magnetisation data set in second column. Moreover, after (0.944,0.400), next pair of points will be (0.969,0.300), then (0.987,0.200), ...and so on in the "0.dat" file.]

C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [32], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c};factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula Ala [32] is given in the appendix of [9].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma=4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c};factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe data s in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those data s. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.01$. calculated from the equation(4). The data set is used to plot fig.2. Similarly, we plot fig.3. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

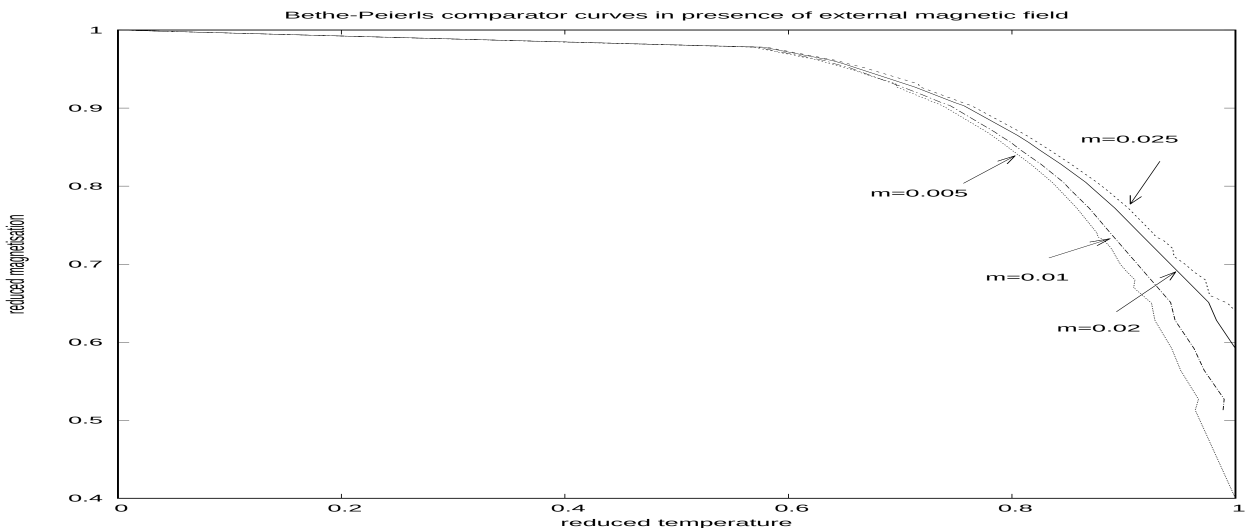

FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

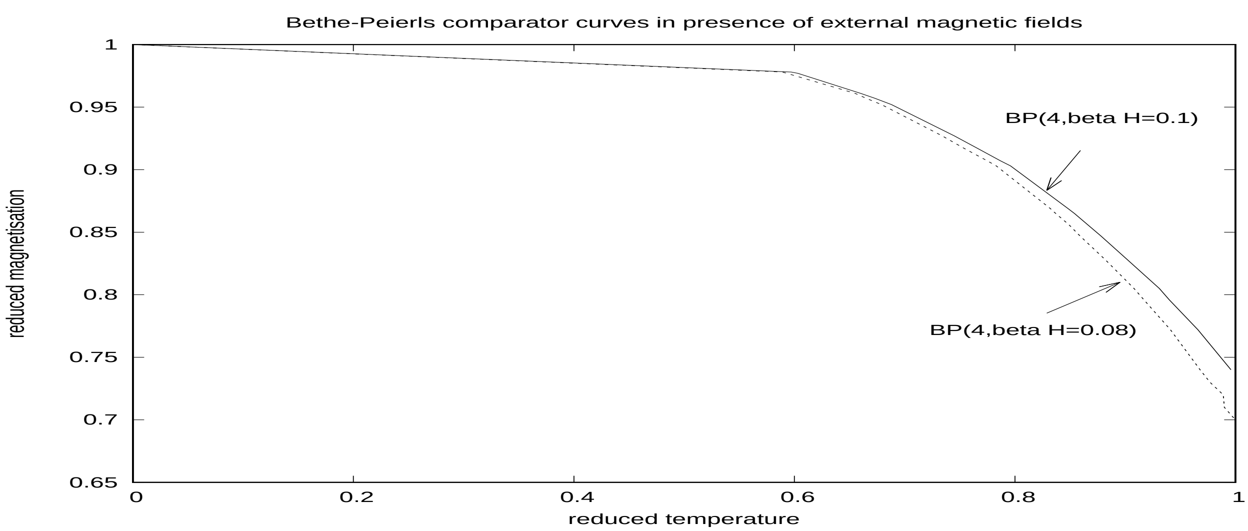

FIG. 3. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

reduced temperature, $\frac{T}{T_c}$					$\frac{M}{M_{max}}$,
BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

D. Onsager solution

At a temperature T, below a certain temperature called phase transition temperature, T_c, for the two dimensional Ising model in absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [33], [34], [35], [32],

$$\frac{M}{M_{max}} = [1 - (sinh\frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

Graphically, the Onsager solution appears as in fig.4.

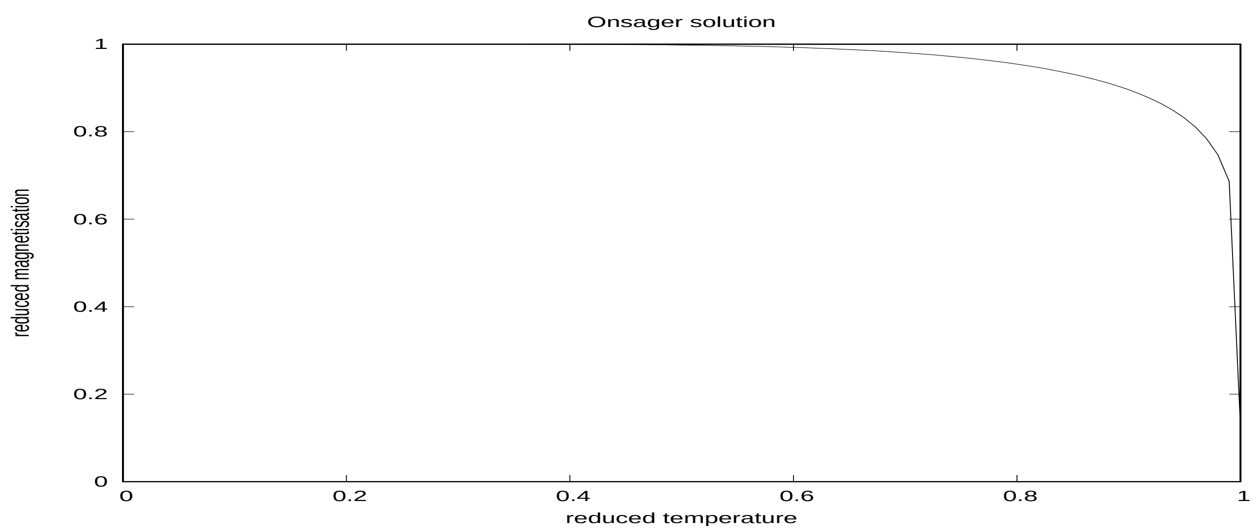

FIG. 4. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

E. Spin-Glass

In the case coupling between(among) the spins, not necessarily n.n, for the Ising model is(are) random, we get Spin-Glass. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like $\frac{1}{T-T_c}$ i.e. like the branch of rectangular hyperbola, up to the the phase transition temperature, followed by very little increase,[36–38], in magnetisation, as the ambient temperature continues to drop.

Theoretical study of Spin Glass started with the paper by Edwards, Anderson,[39]. They were trying to explain two experimental results concerning continuous disordered freezing(phase transition) and sharp cusp in static magnetic susceptibility. This was followed by a paper by Sherrington, Kickpatrick, [40], who dealt with Ising model with interactions being present among all neighbours. The interaction is random, follows Gaussian distribution and does not distinguish one pair of neighbours from another pair of neighbours, irrespective of the distance between two neighbours. In presence of external magnetic field, they predicted in their next paper, [41], below spin-glass transition temperature a spin-glass phase with non-zero magnetisation. Almeida etal, [42], Gray and Moore, [43],finally Parisi, [44], [45] improved and gave final touch, [46], to their line of work. Parisi and collaborators, [47]-[51], wrote a series of papers in postscript, all revolving around a consistent assumption of constant magnetisation in the spin-glass phase in presence of little constant external magnetic field.

In another sequence of theoretical work, by Fisher etal,[52–54], concluded that for Ising model with nearest neighbour or, short range interaction of random type spin-glass phase does not exist in presence of external magnetic field.

For recent series of experiments on spin-glass, the references, [55, 56], are the places to look into.

For an in depth account, accessible to a commoner, the series of articles by late P. W. Anderson in Physics Today, [57]-[63], is probably the best place to look into. For a book to enter into the subject of spin-glass, one may start at [64].

Here, in our work to follow, spin-glass refers to spin-glass phase of a system with infinite range random interactions.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
3290	2605	277	1489	2193	1958	2403	2145	660	308	2813	1381	1716	1043	603	1838	127	1649	4646	1320	1452	1946	1523	14	11	1260

TABLE III. German words

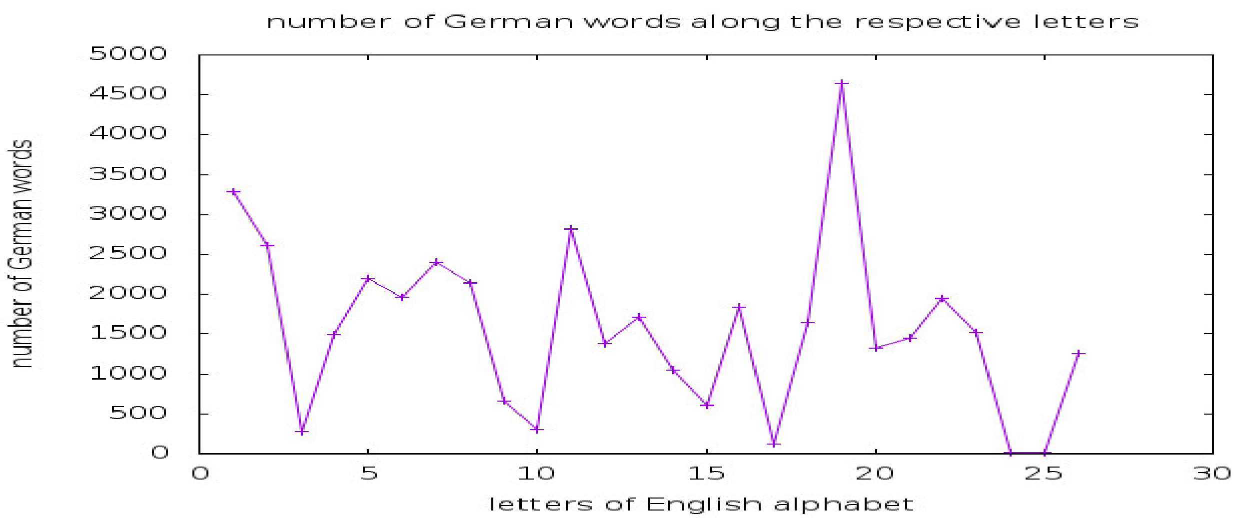

FIG. 5. Vertical axis is number of words of the German, [1], and horizontal axis is respective letters. Letters are represented by the sequence number in the alphabet or, dictionary sequence,[1].

III. ANALYSIS OF WORDS OF LANGENSCHEIDT TASCHENWöRTERBUCH DEUTSCH-ENGLISCH / ENGLISCH-DEUTSCH, VöLLIGE NEUBEARBEITUNG

The German language alphabet is composed of twenty six letters like English. We take a German-English dictionary,Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung[1]. Then we count all the "simple" words, [1], one by one from the beginning to the end, starting with different letters. The result is the table, III. Highest number of words, four thousand six hundred forty six, starts with the letter S followed by words numbering three thousand two hundred ninety beginning with A, two thousand eight hundred thirteen with the letter K etc. To visualise we plot the number of words against respective letters in the dictionary sequence, [1], in the figure fig.5.

For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by f and the respective rank, denoted by k. k is a positive integer starting from one. The lowest value of f is eleven, corresponding to the letter Y. Hence we attach a limiting f equal to one. The corresponding rank, k, denoted as k_{lim} is twenty seven. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, IV and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.6. We then ignore the letter with the highest of words, tabulate in the adjoining table, IV and redo the plot, normalising the lnfs with next-to-maximum $lnf_{nextmax}$, and starting from $k = 2$ in the figure fig.7. This program then we repeat up to $k = 5$, resulting in figures up to fig.10.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{nmax}	lnf/lnf_{nnmax}	lnf/lnf_{nnnmax}	lnf/$lnf_{nnnnmax}$
1	0	0	4646	8.444	1	Blank	Blank	Blank	Blank
2	0.69	0.209	3290	8.099	0.959	1	Blank	Blank	Blank
3	1.10	0.333	2813	7.942	0.941	0.981	1	Blank	Blank
4	1.39	0.421	2605	7.865	0.931	0.971	0.990	1	Blank
5	1.61	0.488	2403	7.784	0.922	0.961	0.980	0.990	1
6	1.79	0.542	2193	7.693	0.911	0.950	0.969	0.978	0.990
7	1.95	0.591	2145	7.671	0.908	0.947	0.966	0.975	0.988
8	2.08	0.630	1958	7.580	0.898	0.936	0.954	0.964	0.982
9	2.20	0.667	1946	7.574	0.897	0.935	0.954	0.963	0.977
10	2.30	0.697	1838	7.516	0.890	0.928	0.946	0.956	0.965
11	2.40	0.727	1716	7.448	0.882	0.920	0.938	0.947	0.962
12	2.48	0.752	1649	7.408	0.877	0.915	0.933	0.942	0.952
13	2.56	0.776	1523	7.328	0.868	0.905	0.923	0.932	0.936
14	2.64	0.800	1489	7.306	0.865	0.902	0.920	0.929	0.922
15	2.71	0.821	1452	7.281	0.862	0.899	0.917	0.926	0.918
16	2.77	0.839	1381	7.231	0.856	0.893	0.910	0.919	0.897
17	2.83	0.858	1320	7.185	0.851	0.887	0.905	0.914	0.893
18	2.89	0.876	1260	7.139	0.845	0.881	0.899	0.908	0.868
19	2.94	0.891	1043	6.950	0.823	0.858	0.875	0.884	0.845
20	3.00	0.909	660	6.492	0.769	0.802	0.817	0.825	0.830
21	3.04	0.921	603	6.402	0.758	0.790	0.806	0.814	0.763
22	3.09	0.936	308	5.730	0.679	0.707	0.721	0.729	0.737
23	3.14	0.952	277	5.624	0.666	0.694	0.708	0.715	0.686
24	3.18	0.964	127	4.844	0.574	0.598	0.610	0.616	0.642
25	3.22	0.976	14	2.639	0.313	0.326	0.332	0.336	0.545
26	3.26	0.988	11	2.398	0.284	0.296	0.302	0.305	0.359
27	3.30	1	1	0	0	0	0	0	0

TABLE IV. German language words: ranking,natural logarithm, normalisations

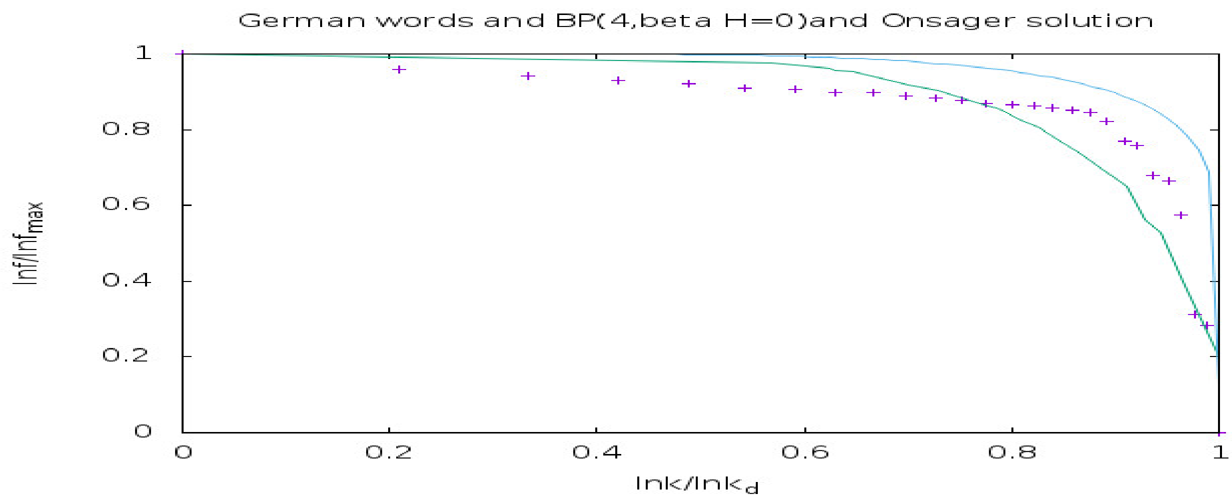

FIG. 6. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in the absence of external magnetic field. The uppermost curve is the Onsager solution.

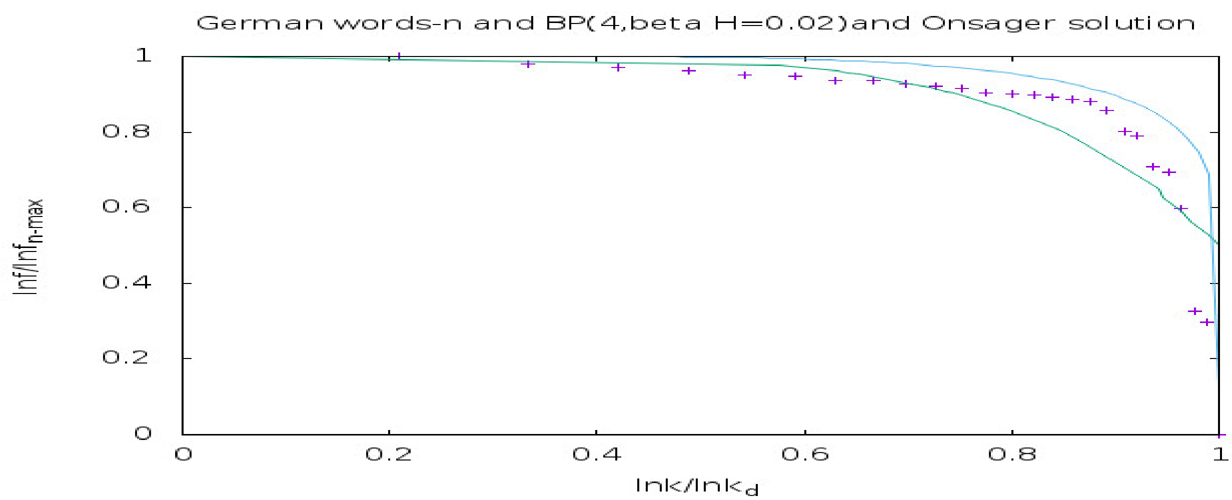

FIG. 7. The vertical axis is $\frac{lnf}{lnf_{next-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.01 or, $\beta H = 0.02$. The uppermost curve is the Onsager solution.

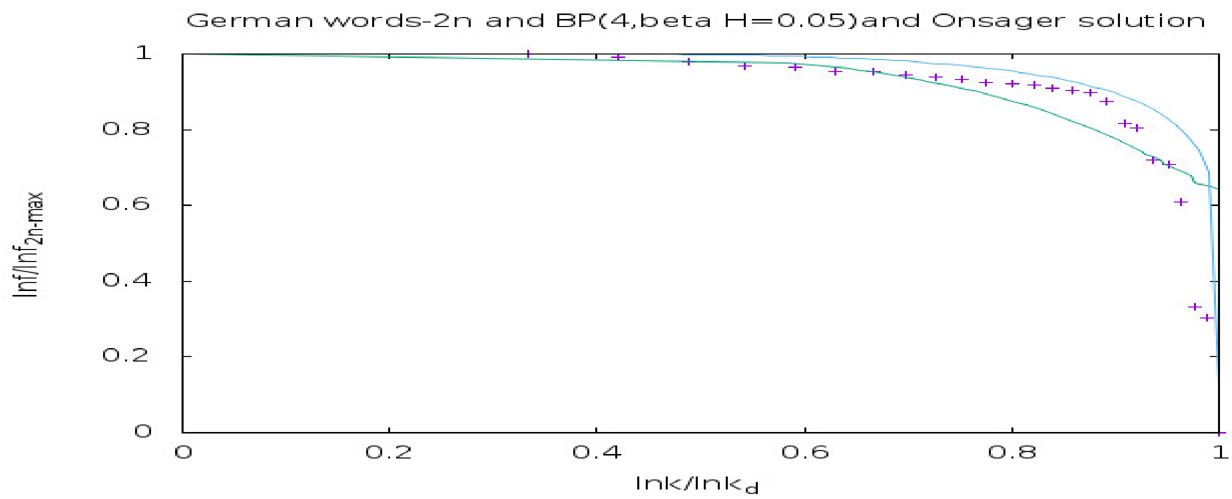

FIG. 8. The vertical axis is $\frac{lnf}{lnf_{nextnext-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.025 or, $\beta H = 0.05$. The uppermost curve is the Onsager solution.

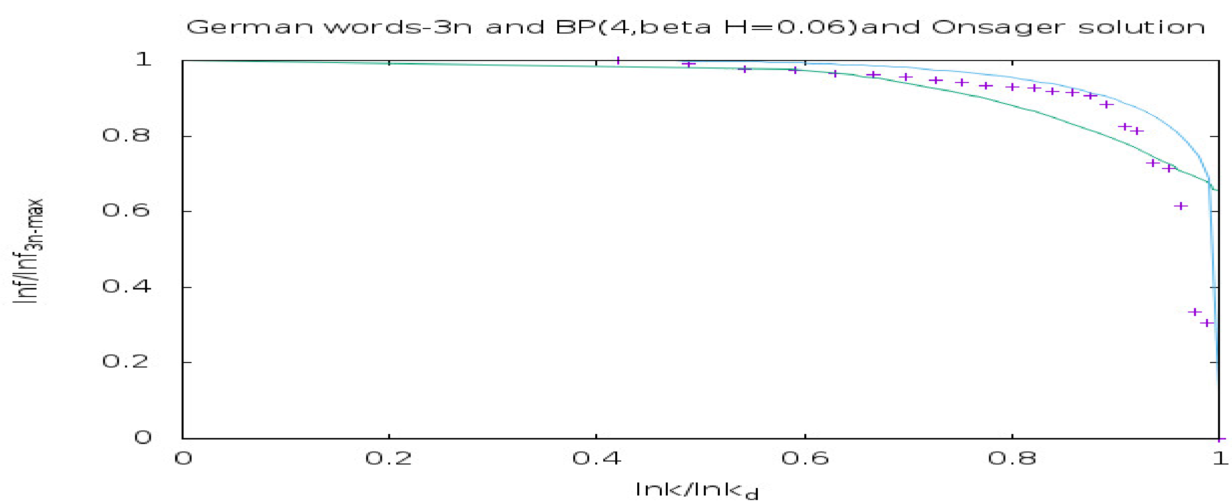

FIG. 9. The vertical axis is $\frac{lnf}{lnf_{nextnextnext-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.03 or, $\beta H = 0.06$. The uppermost curve is the Onsager solution.

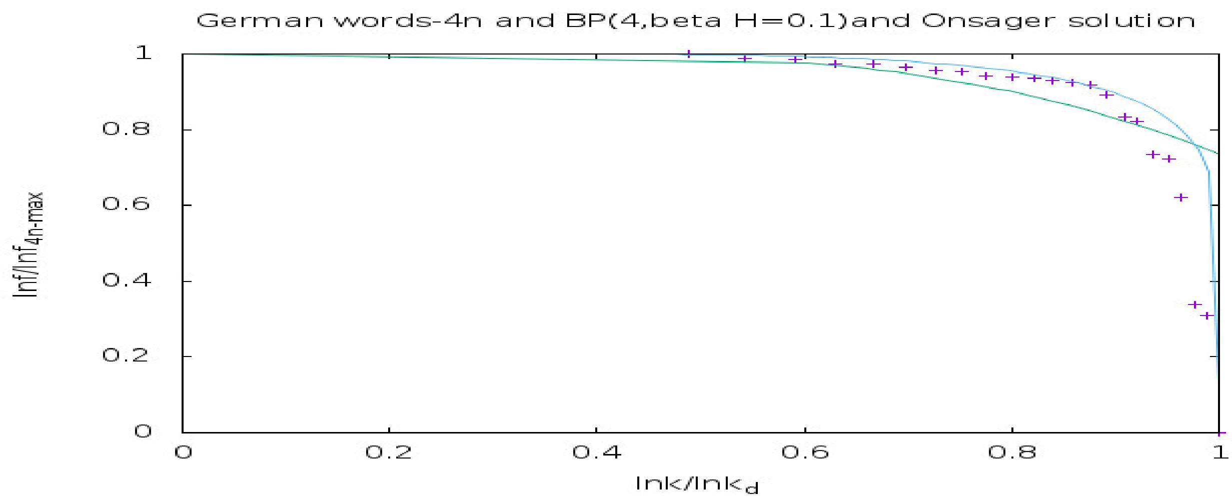

FIG. 10. The vertical axis is $\frac{lnf}{lnf_{nextnextnextnext-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the German language with the fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.05 or, $\beta H = 0.1$. The uppermost curve is the Onsager solution.

Matching of the plots in the figures fig.(6-10), with comparator curves i.e. the magnetisation curves of Bethe-Peierls approximations, is with dispersion and dispersion does not reduce over higher orders of normalisations. On the top of it, on successive higher normalisations, words of the German language,[1], do not go over to Onsager solution in the normalised lnf vs $\frac{lnk}{lnk_{lim}}$ graphs.

To explore for possible existence of spin-glass transition, in presence of little external magnetic field, $\frac{lnf}{lnf_{max}}$, $\frac{lnf}{lnf_{next-max}}$ and $\frac{lnf}{lnf_{nn-max}}$ are drawn against lnk in the figures fig.11-fig.13.

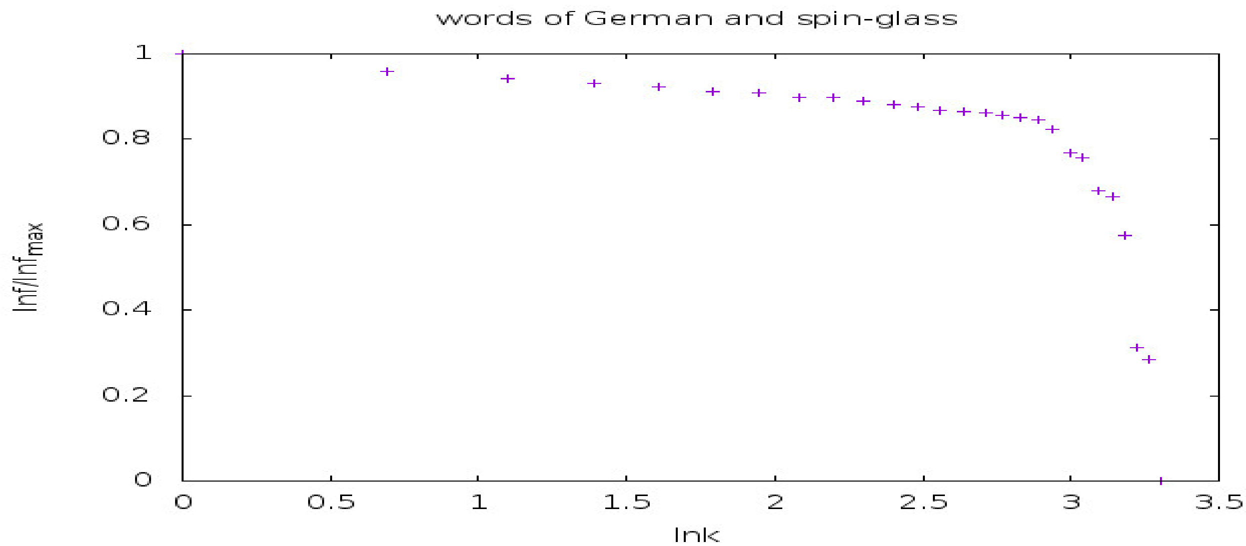

FIG. 11. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is lnk. The + points represent the words of the German language.

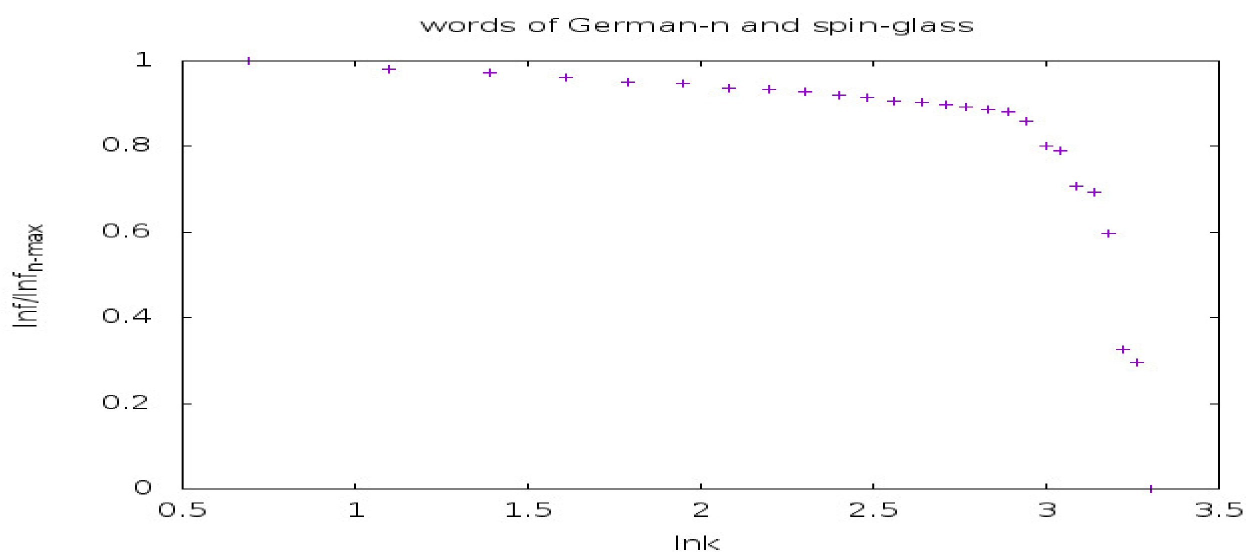

FIG. 12. The vertical axis is $\frac{lnf}{lnf_{next-max}}$ and the horizontal axis is lnk. The + points represent the words of the German language.

A. conclusion

In the figures Fig.11-Fig.13, the points has a clear-cut transition. Above the transition point(s), the lines are almost horizontal and below the transition point(s), points-line rises like the branch of a rectangular hyperbola. Hence, the words of the German, [1], is well-

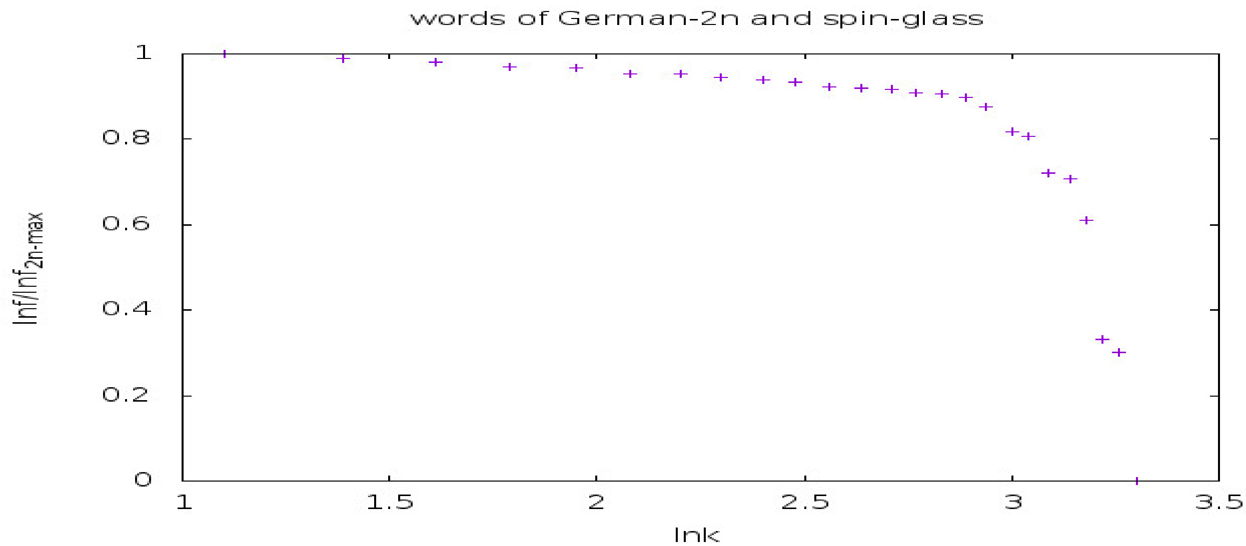

FIG. 13. The vertical axis is $\frac{lnf}{lnf_{nn-max}}$ and the horizontal axis is lnk. The + points represent the words of the German language.

suited to be described by a Spin-Glass magnetisation curve, [36], in the presence of little magnetic field.

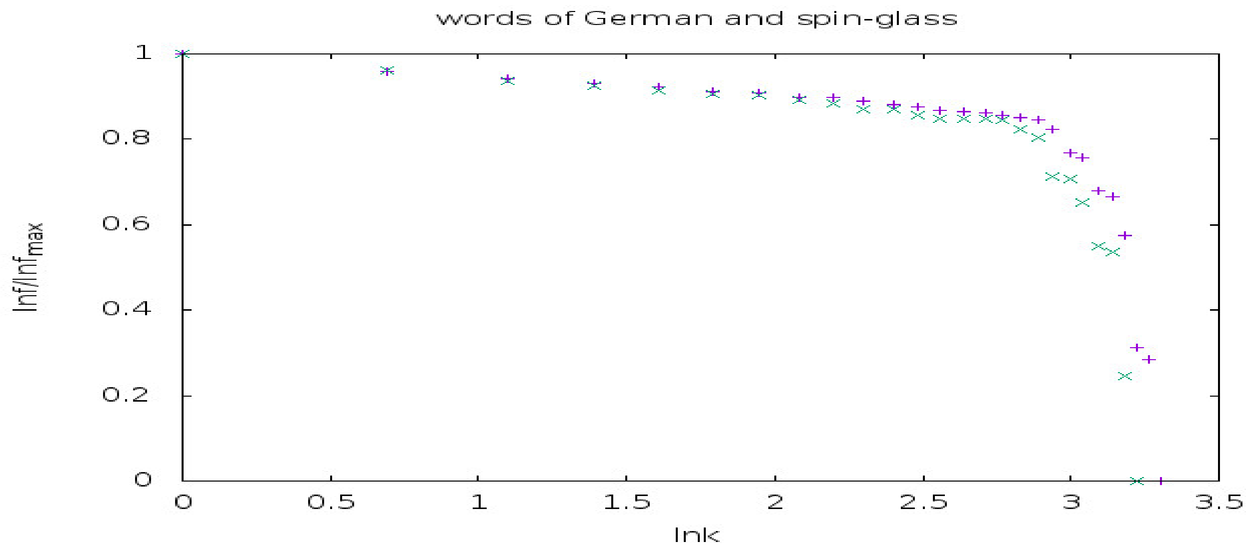

FIG. 14. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is lnk. The + points represent the words of the German language, [1]. The × points represent the words of the German language, [2]. Spin-glass behaviour is remaining the same.

IV. GERMAN WORDS: LANGENSCHEIDT, LANGENSCHEIDT

We compare the two Langenscheidt dictionaries, [1] and [2]. We compare spin-glass behaviour of these two in the figure14. In the figure15, we compare the frequencies. Next in the subsection, we go on comparing the naturalness number.

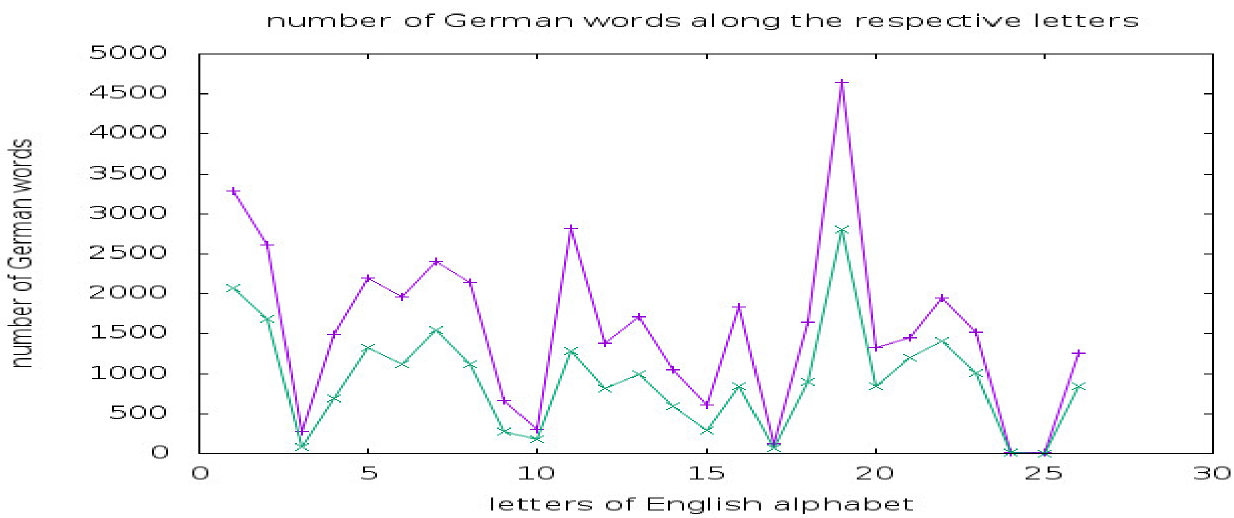

FIG. 15. The vertical axis is number of words of the German and the horizontal axis is the respective letters. Letters are represented by the sequence number in the English alphabet. The + points represent the words of the German language, [1]. The × points represent the words of the German language, [2].

A. Naturalness number

It is noticable of the lessness of the number of major peaks compared to the number of minor peaks. It was proposed in [24], that it may be reasonable to define naturalness of a language by the ratio of number of major peaks to the number of minor peaks. One may take major peaks as those with height up to the half of the height of the highest peak. The naturalness number of the French, [25], tuned out to be 11/5.

In this case,[1], the highest peak has the frequency 4646. Half of it is 2323. Number of peaks greater than 2323 is $3\frac{1}{2}$. Number of peaks less than 2323 is $4\frac{1}{2}$. Hence the naturalness number is 7/9. In the earlier case, [2], the highest peak has the frequency 2801. Half of it is 1400. Number of peaks greater than 1400 is $3\frac{1}{2}$. Number of peaks less than 1400 is $4\frac{1}{2}$. Hence the naturalness number is 7/9. Hence, the naturalness number is not changing as we are moving from [2] to [1].

V. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper.

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