On the Inverse Fourier Transform of the Planck-Einstein-de Broglie law

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Abstract

Using results derived earlier by the author, in this letter after generalising $E = hf$, its inverse Fourier transform is calculated.

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In [1] I proposed the notion of *angles-time space* and the *principle of maximum frequency* based on the fundamental guiding principle that the so-called wave-particle duality is a purely classical result of spin motion of particles and argued that the Planck–Einstein relation $E = \hbar\omega$ is nothing but $E = I\omega^2$ of classical mechanics with a change of dress. There I argued that the natural implementation of the principle of maximum frequency results in the following metric for angles-time space in absence of all interactions

$$d\Theta^2 = \omega_P^2 dt^2 - d\boldsymbol{\theta}^2,$$

where ω_P is the Planck frequency. In this letter I prove that the principle of maximum frequency solves one of the fundational problems of quantum mechanics: Taking Planck-Einstein relation $E = \hbar\omega$ as a fundamental law of nature and abiding by the principle that **a fundamental law of nature must not depend on our choice of basis for the function space (Fourier basis)**, we expect the Inverse Fourier Transform of $E = \hbar\omega$ to yield the energy of electromagnetic field as a function of time, viz.

$$E(t) = \frac{\hbar}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t}\omega \; d\omega,$$

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but this integral is wildly divergent, and I maintain that *this is the root of the problem of renormalisation in QFT*. I now prove that this problem is solved by implementing the principle of maximum frequency. As a result of the principle of maximum frequency the complete exact form of Planck–Einstein relation is given by[1]

$$\boxed{E(\omega) = \frac{2\hbar\omega_P^2}{\omega}\left(\frac{1}{\sqrt{1-(\omega/\omega_P)^2}} - 1\right)} \tag{1}$$

Note that $E = \hbar\omega$ is an approximation to the above equation, by being the first term in the Taylor expansion of the angles-time space factor. Unlike $E = \hbar\omega$ however, my proposed generalisation of Planck–Einstein relation is not plagued by infinities and it is perfectly possible to take its inverse Fourier transform. To this purpose, recall that

$$\mathcal{F}[J_0(\omega_P t)] = \mathbf{1}_{-1\leq\frac{\omega}{\omega_P}\leq 1}\frac{2}{\sqrt{1-(\omega/\omega_P)^2}}, \tag{2}$$

where $J_0(t)$ is the zero-order Bessel function of the first kind. Note that according to the principle of maximum frequency

$$0 < \frac{\omega}{\omega_P} \leq 1,$$

therefore

$$\mathcal{F}[J_0(\omega_P t)] = \frac{2}{\sqrt{1-(\omega/\omega_P)^2}}.$$

Now observe that for $\omega \neq 0$

$$i\omega E(\omega) = 2i\hbar\omega_P^2\left(\frac{1}{\sqrt{1-(\omega/\omega_P)^2}} - 1\right),$$

which is

$$\mathcal{F}[\frac{d}{dt}E(t)] = i\hbar\omega_P^2\mathcal{F}[J_0(\omega_P t) - 2\delta(t)],$$

therefore

$$\boxed{E(t) = i\hbar\omega_P^2\left(-2H(t) + \int_{-\infty}^{t} J_0(\omega_P\tau)d\tau\right)} \tag{3}$$

where $H(t)$ is the Heaviside step function.

References

[1] Alireza Jamali. Towards an einsteinian quantum mechanics (preprint). *viXra:2103.0006*, 2021.