Bifurcations of the Higgs Potential and the Top Quark Mass

Ervin Goldfain

Ronin Institute, Montclair, New Jersey 07043, USA

E-mail ervin.goldfain@ronininstitute.org

Abstract

We have recently conjectured that the Standard Model gauge group unfolds under successive bifurcations of the Higgs potential. This brief report points out that the maximal fixed-point solution of the bifurcation process corresponds to a top-antitop quark condensate.

Key words: Bifurcations, Feigenbaum route to chaos, gauge symmetries, Higgs potential, top quark.

It can be shown that the flow of the classical Higgs potential with the Renormalization scale takes the form [1]

\[ \dot{y} = my(1 - y^2) \]  

(1)

in which \( y \) is given by
Here, $\phi$ denotes the amplitude of the complex-scalar field whose vacuum expectation value is $\nu = 246$ GeV. Eq. (1) follows from the theory of bi-stable systems embedded in a *double-well potential* [8]. The control parameter of (1) contains the self-interaction coupling $\lambda$ and a reference scale $m_0$ as in

$$m = \frac{2\lambda \nu^2}{m_0^2}$$

The differential equation (1) may be cast as the iterated map shown below

$$y_{n+1} = f(m, y_n) = my_n(1 - y_n^2)$$

There are two trivial fixed points of (1) and (4), given by: a) $y^* = 0, m = 0, \lambda = 0$ - which resembles massless photons in an “effective” approximation, and b) a pair of maximal solutions arisen in the limit of large number of map iterations ($n \to \infty$), namely,

$$y^*_\infty = \pm 1$$

whose separation along the $y$-axis is
\[
\Delta y^*_\infty = +1 - (-1) = 2
\]

As suggested in [2-5], the fermionic sector of the Standard Model unfolds as the last segment of the bifurcation diagram. By (6) and (2), this conjecture leads to a separation in field space closely approximating a \textit{top-antitop condensate}, that is,

\[
\Delta \phi^*_\infty = \sqrt{2} v = 347.9 \text{ GeV}
\]  

(7a)

\[
\Delta \phi^*_\infty \approx 2m_t
\]  

(7b)

where \( m_t \approx 173 \text{ GeV} \) is the experimental value of the top quark mass [6]. As the top quark is the heaviest known fermion, relation (7) brings additional support for the self-contained flavor composition of the Standard Model near the electroweak scale [7].

\textbf{References}

1. Available at the following site

https://www.researchgate.net/publication/357093456_Bifurcations_and_the\_Gauge_Structure_of_the_Standard_Model


